Computer algebra independent integration tests

7-Inverse-hyperbolic-functions/7.2-Inverse-hyperbolic-cosine/7.2.4-f-x-^m-d+e-x^2-^p-a+b-arccosh-c-x-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [569]. This is test number [190].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (569)	% 0.00 (0)
Mathematica	% 98.07 (558)	% 1.93 (11)
Maple	% 82.78 (471)	% 17.22 (98)
Maxima	% 43.06 (245)	% 56.94 (324)
Fricas	% 41.48 (236)	% 58.52 (333)
Sympy	% 27.07 (154)	% 72.93 (415)
Giac	% 18.63 (106)	% 81.37 (463)
Mupad	% 25.31 (144)	% 74.69 (425)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

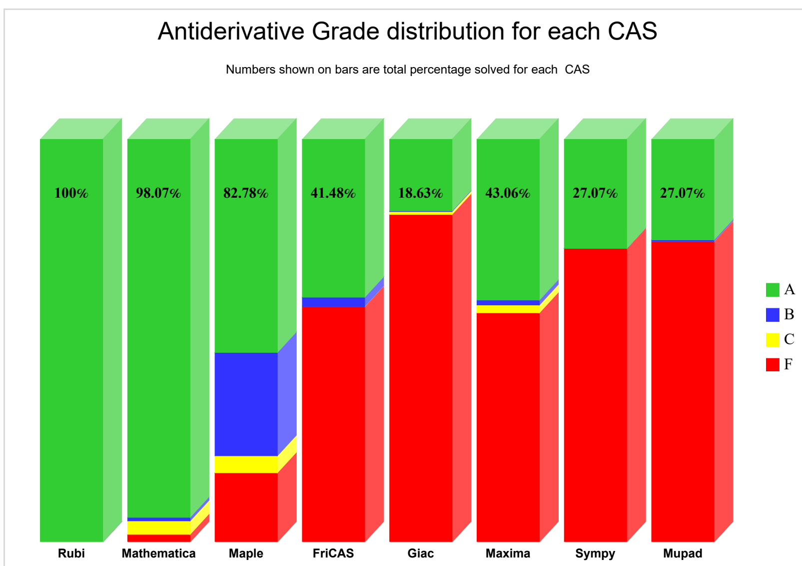
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

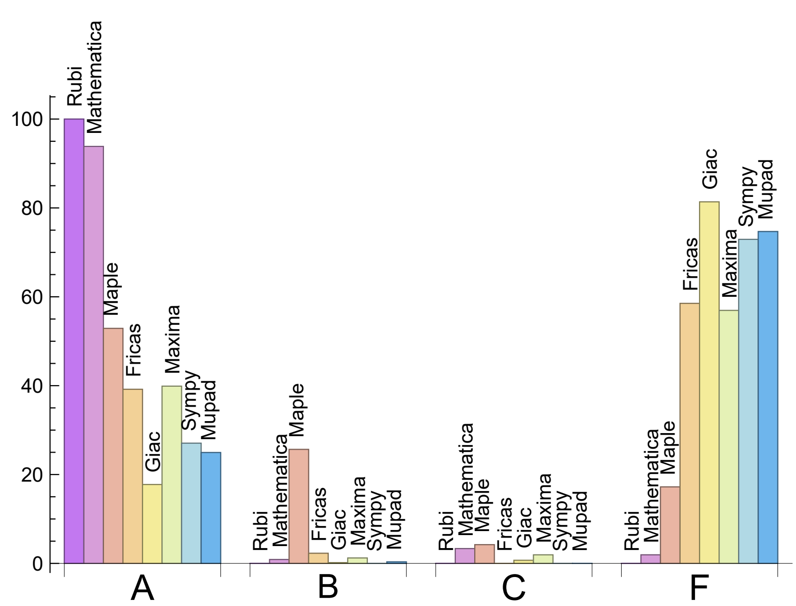
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	93.85	0.88	3.34	1.93
Maple	52.90	25.66	4.22	17.22
Maxima	39.89	1.23	1.93	56.94
Fricas	39.19	2.28	0.00	58.52
Sympy	27.07	0.00	0.00	72.93
Giac	17.75	0.18	0.70	81.37
Mupad	24.96	0.35	0.00	74.69

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	11	54.55 %	45.45 %	0.00 %
Maple	98	65.31 %	0.00 %	34.69 %
Maxima	324	96.30 %	0.00 %	3.70 %
Fricas	333	79.88 %	0.00 %	20.12 %
Sympy	415	78.31 %	21.69 %	0.00 %
Giac	463	53.13 %	1.51 %	45.36 %
Mupad	425	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

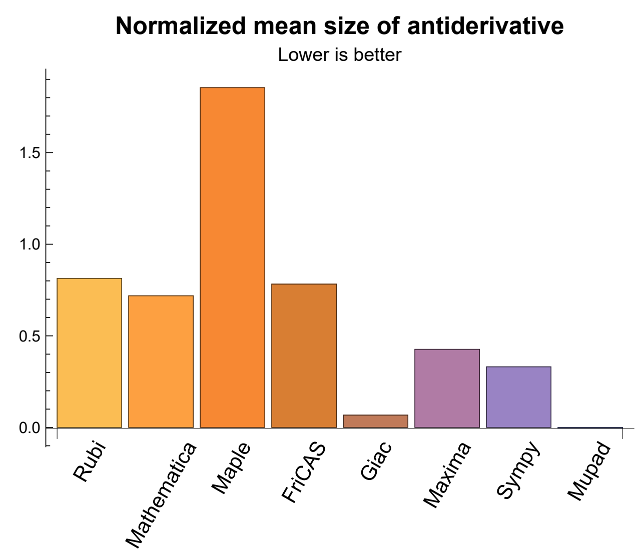
1.3 Performance

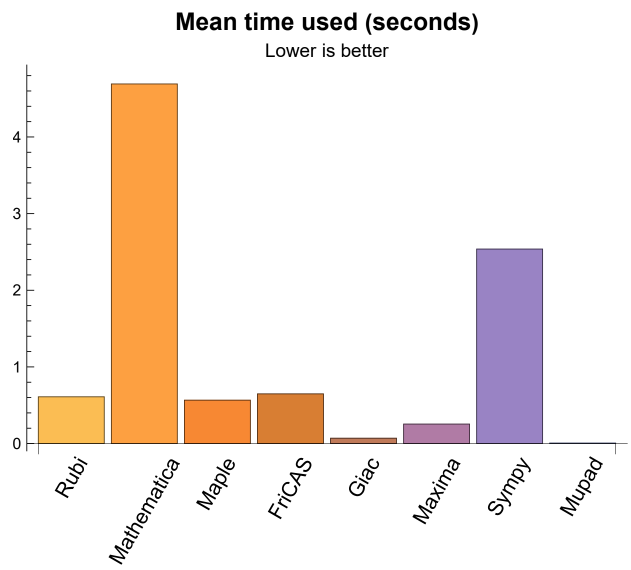
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.61	231.91	0.81	186.00	1.00
Mathematica	4.69	195.56	0.72	141.50	0.74
Maple	0.57	528.18	1.85	227.00	1.49
Maxima	0.25	91.65	0.43	0.00	0.00
Fricas	0.65	145.21	0.78	0.00	0.00
Sympy	2.54	79.21	0.33	0.00	0.00
Giac	0.07	5.21	0.07	0.00	0.00
Mupad	0.01	-0.24	-0.00	-1.00	-0.03

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{148, 149, 150, 233, 234, 235, 236, 237, 238, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {6, 8, 15, 17, 24, 26, 175, 177, 183, 185, 191, 193, 201, 203, 494, 496, 500, 501, 509, 510}

Mathematica {4, 6, 8, 11, 13, 15, 17, 20, 22, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 110, 112, 115, 117, 120, 122, 125, 130, 132, 140, 142, 151, 152, 160, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 230, 231, 232, 243, 244, 245, 246, 247, 250, 251, 252, 258, 259, 260, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 293, 315, 316, 317, 328, 329, 330, 335, 336, 337, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 384, 385, 388, 389, 392, 393, 397, 398, 401, 402, 403, 404, 408, 409, 410, 414, 415, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 462, 464, 466, 468, 471, 473, 475, 477, 480, 482, 484, 486, 489, 490, 491, 492, 494, 495, 496, 497, 498, 502, 503, 504, 505, 506, 511, 512, 513, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

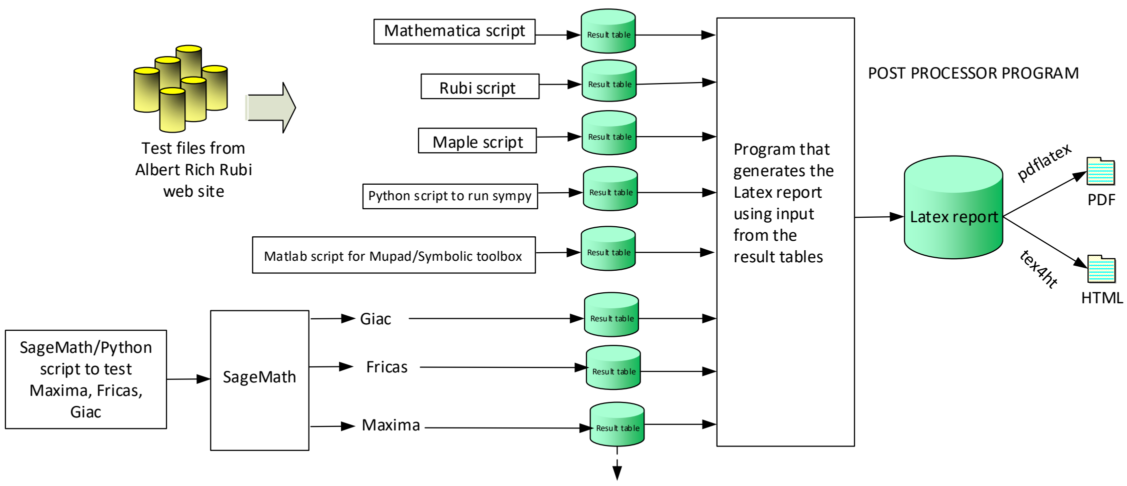
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

    The following field present only in Rubi and Mathematica Tables

13. integer. 1 if result was verified or 0 if not verified.

    The following fields present only in Rubi Tables

14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,..}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
May 11, 2021

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 253, 254, 255, 256, 257, 258, 259, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 320, 321, 322, 323, 324, 325, 326, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 490, 492, 493, 495, 497, 499, 507, 508, 514, 515, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 548, 549, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

B grade: { 44, 260, 315, 316, 317 }

C grade: { 160, 251, 252, 489, 491, 494, 496, 498, 502, 503, 504, 505, 506, 511, 512, 513, 516, 517, 518 }

F grade: { 161, 162, 327, 334, 500, 501, 509, 510, 547, 550, 551 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 33, 34, 36, 37, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49, 50, 52, 54, 56, 57, 58, 68, 69, 70, 71, 72, 84, 85, 86, 87, 88, 89, 100, 101, 102, 109, 110, 121, 130, 132, 134, 139, 142, 148, 149, 150, 164, 165, 166, 167, 168, 169, 187, 227, 228, 229, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 254, 255, 256, 257, 259, 261, 262, 263, 264, 265, 266, 267, 269, 271, 272, 273, 274, 275, 277, 279, 280, 281, 282, 283, 287, 288, 289, 290, 291, 296, 297, 298, 300, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 381, 382, 383, 386, 387, 390, 391, 394, 395, 396, 399, 400, 405, 406, 407, 411, 412, 413, 416, 417, 418, 422, 423, 427, 428, 432, 433, 437, 438, 439, 443, 444, 445, 447, 448, 449, 450, 451, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 514, 515, 522, 523, 524, 525, 526, 527, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 35, 42, 51, 53, 59, 60, 61, 62, 63, 64, 65, 66, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 123, 124, 125, 126, 127, 128, 129, 131, 133, 135, 136, 137, 138, 140, 141, 170, 171, 172, 173, 175, 177, 178, 179, 180, 181, 183, 185, 186, 188, 189, 191, 193, 194, 195, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 211, 213, 214, 215, 216, 217, 218, 219, 221, 223, 224, 225, 226, 250, 251, 252, 253, 268, 270, 276, 278, 284, 285, 286, 292, 293, 294, 295, 299, 301, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 499, 507, 508, 543 }

C grade: { 32, 55, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513 }

F grade: { 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 174, 176, 182, 184, 190, 192, 200, 202, 210, 212, 220, 222, 230, 232, 258, 260, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 384, 385, 388, 389, 392, 393, 397, 398, 401, 402, 403, 404, 408, 409, 410, 414, 415, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 446, 452, 453, 516, 517, 518, 519, 520, 521, 529, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 12, 14, 16, 18, 23, 25, 27, 63, 64, 65, 66, 67, 77, 78, 79, 80, 81, 82, 83, 94, 95, 96, 97, 98, 99, 104, 106, 108, 113, 116, 119, 121, 126, 127, 129, 133, 134, 148, 149, 150, 164, 165, 166, 170, 172, 178, 180, 186, 188, 194, 196, 198, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 485, 487, 488, 514, 515, 522, 523, 524, 525, 526, 527, 528, 530, 531, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 11, 13, 19, 20, 21, 22, 40 }

C grade: { 62, 76, 93, 111, 136, 138, 141, 226, 228, 254, 256 }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 105, 107, 109, 110, 112, 114, 115, 117, 118, 120, 122, 123, 124, 125, 128, 130, 131, 132, 135, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 347, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 529, 532, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 16, 18, 19, 20, 21, 22, 23, 25, 27, 40, 47, 49, 63, 64, 65, 66, 67, 76, 77, 78, 79, 80, 81, 82, 83, 93, 94, 95, 96, 97, 98, 99, 104, 106, 108, 111, 113, 114, 116, 118, 124, 126, 128, 136, 138, 148, 149, 150, 164, 165, 166, 170, 172, 178, 180, 186, 188, 194, 196, 198, 226, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 254, 256, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 411, 416, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 469, 470, 471, 472, 473, 474, 476, 478, 479, 480, 481, 482, 483, 485, 487, 488, 514, 515, 516, 522, 523, 524, 525, 526, 527, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551 }

B grade: { 62, 296, 302, 347, 368, 437, 443, 499, 507, 508, 517, 518, 528 }

C grade: { }

F grade: { 6, 8, 15, 17, 24, 26, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 48, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 68, 69, 70, 71, 72, 73, 74, 75, 84, 85, 86, 87, 88, 89, 90, 91, 92, 100, 101, 102, 103, 105, 107, 109, 110, 112, 115, 117, 119, 120, 121, 122, 123, 125, 127, 129, 130, 131, 132, 133, 134, 135, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 171, 173, 174, 175, 176, 177, 179, 181, 182, 183, 184, 185, 187, 189, 190, 191, 192, 193, 195, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 299, 300, 301, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 412, 413, 414, 415, 417, 418, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 440, 441, 442, 466, 468, 475, 477, 484, 486, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 500, 501, 502, 503, 504, 505, 506, 509, 510, 511, 512, 513, 519, 520, 521, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 148, 149, 164, 165, 166, 235, 236, 237, 239, 240, 241, 242, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 364, 365, 373, 378, 382, 383, 387, 395, 396, 400, 406, 407, 412, 422, 423, 427, 438, 439, 444, 445, 446, 447, 448, 449, 451, 454, 455, 459, 461, 462, 463, 464, 465, 470, 471, 472, 473, 474, 479, 480, 481, 482, 483, 488, 514, 515, 522, 525, 526, 527, 528, 530, 531, 532, 537, 539, 540, 541, 542, 546, 548, 549, 550, 551, 555, 556, 559, 564, 565, 568 }

B grade: { }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 341, 342, 343, 344, 345, 346, 347, 363, 366, 367, 368, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 391, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 413, 414, 415, 416, 417, 418, 419, 420, 421, 424, 425, 426, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 450, 452, 453, 456, 457, 458, 460, 466, 467, 468, 469, 475, 476, 477, 478, 484, 485, 486, 487, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 523, 524, 529, 533, 534, 535, 536, 538, 543, 544, 545, 547, 552, 553, 554, 557, 558, 560, 561, 562, 563, 566, 567, 569 }

2.1.7 Giac

A grade: { 134, 148, 149, 150, 236, 237, 238, 239, 261, 265, 266, 273, 275, 281, 283, 289, 291, 297, 298, 304, 305, 307, 309, 312, 313, 314, 318, 319, 325, 327, 332, 334, 339, 341, 349, 351, 353, 355, 356, 358, 360, 362, 365, 366, 367, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 455, 456, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 528 }

C grade: { 138, 141, 228, 256 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 272, 274, 276, 277, 278, 279, 280, 282, 284, 285, 286, 287, 288, 290, 292, 293, 294, 295, 296, 299, 300, 301, 302, 303, 306, 308, 310, 311, 315, 316, 317, 320, 321, 322, 323, 324, 326, 328, 329, 330, 331, 333, 335, 336, 337, 338, 340, 342, 343, 344, 345, 346, 347, 348, 350, 352, 354, 357, 359, 361, 363, 364, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 440, 441, 442, 443, 452, 453, 454, 457, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 525, 526, 527, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.1.8 Mupad

A grade: { 148, 149, 150, 233, 234, 235, 236, 237, 238, 239, 261, 265, 266, 272, 273, 274, 275, 280, 281, 282, 283, 288, 289, 290, 291, 297, 298, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 318, 319, 324, 325, 326, 327, 331, 332, 333, 334, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 373, 378, 382, 383, 387, 391, 395, 396, 400, 406, 407, 412, 413, 417, 418, 422, 423, 427, 428, 432, 433, 438, 439, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 514, 515, 522, 523, 524, 530, 531, 532, 533, 537, 538, 539, 540, 541, 542, 546, 547, 548, 549, 550, 551, 555, 556, 559, 560, 564, 565, 568, 569 }

B grade: { 347, 368 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 267, 268, 269, 270, 271, 276, 277, 278, 279, 284, 285, 286, 287, 292, 293, 294, 295, 296, 299, 300, 301, 302, 315, 316, 317, 320, 321, 322, 323, 328, 329, 330, 335, 336, 337, 342, 343, 344, 345, 346, 369, 370, 371, 372, 374, 375, 376, 377, 379, 380, 381, 384, 385, 386, 388, 389, 390, 392, 393, 394, 397, 398, 399, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 419, 420, 421, 424, 425, 426, 429, 430, 431, 434, 435, 436, 437, 440, 441, 442, 443, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 519, 520, 521, 525, 526, 527, 528, 529, 534, 535, 536, 543, 544, 545, 552, 553, 554, 557, 558, 561, 562, 563, 566, 567 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	91	98	184	113	158	0	-1
normalized size	1	1.00	0.60	0.65	1.22	0.75	1.05	0.00	-0.01
time (sec)	N/A	0.150	0.168	0.029	0.342	0.477	6.018	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	166	160	202	108	144	0	-1
normalized size	1	1.00	1.23	1.19	1.50	0.80	1.07	0.00	-0.01
time (sec)	N/A	0.139	0.100	0.018	0.335	0.418	3.980	0.000	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	90	145	103	133	0	-1
normalized size	1	1.00	0.74	0.74	1.20	0.85	1.10	0.00	-0.01
time (sec)	N/A	0.133	0.111	0.011	0.325	0.490	2.283	0.000	0.000

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	100	136	162	98	124	0	-1
normalized size	1	1.00	1.02	1.39	1.65	1.00	1.27	0.00	-0.01
time (sec)	N/A	0.042	0.158	0.014	0.317	0.761	1.274	0.000	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	71	73	97	83	97	0	-1
normalized size	1	1.00	0.83	0.85	1.13	0.97	1.13	0.00	-0.01
time (sec)	N/A	0.074	0.091	0.012	0.334	0.450	0.585	0.000	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	130	131	0	0	0	0	-1
normalized size	1	1.00	1.11	1.12	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.104	0.292	0.000	0.485	0.000	0.000	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	110	100	66	127	0	0	-1
normalized size	1	1.00	1.45	1.32	0.87	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.192	0.017	0.445	0.514	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	107	140	0	0	0	0	-1
normalized size	1	1.00	0.79	1.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.088	0.567	0.000	0.765	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	127	108	89	146	0	0	-1
normalized size	1	1.00	1.41	1.20	0.99	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.125	0.259	0.022	0.487	0.583	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	264	124	128	319	165	236	0	-1
normalized size	1	1.28	0.60	0.62	1.55	0.80	1.15	0.00	-0.00
time (sec)	N/A	0.293	0.214	0.015	0.354	0.634	16.463	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	284	194	230	346	161	224	0	-1
normalized size	1	1.42	0.97	1.15	1.73	0.80	1.12	0.00	-0.00
time (sec)	N/A	0.277	0.236	0.017	0.330	0.452	11.276	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	223	116	120	261	153	209	0	-1
normalized size	1	1.26	0.66	0.68	1.47	0.86	1.18	0.00	-0.01
time (sec)	N/A	0.248	0.178	0.012	0.344	0.498	6.134	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	126	204	287	149	197	0	-1
normalized size	1	1.00	0.93	1.50	2.11	1.10	1.45	0.00	-0.01
time (sec)	N/A	0.067	0.235	0.015	0.456	0.645	4.217	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	177	99	102	194	133	172	0	-1
normalized size	1	1.24	0.69	0.71	1.36	0.93	1.20	0.00	-0.01
time (sec)	N/A	0.152	0.166	0.012	0.374	0.644	2.231	0.000	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	162	201	0	0	0	0	-1
normalized size	1	1.00	0.88	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.280	0.337	0.000	0.620	0.000	0.000	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	182	131	167	143	201	0	0	-1
normalized size	1	1.35	0.97	1.24	1.06	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.230	0.171	0.017	0.462	0.637	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	182	220	0	0	0	0	-1
normalized size	1	1.00	0.91	1.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.266	0.565	0.000	0.707	0.000	0.000	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	186	135	167	137	213	0	0	-1
normalized size	1	1.31	0.95	1.18	0.96	1.50	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.174	0.019	0.459	0.575	0.000	0.000	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	326	147	158	465	201	296	0	-1
normalized size	1	1.27	0.57	0.62	1.82	0.79	1.16	0.00	-0.00
time (sec)	N/A	0.439	0.280	0.021	0.363	0.764	39.284	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	328	162	284	501	197	287	0	-1
normalized size	1	1.43	0.70	1.23	2.18	0.86	1.25	0.00	-0.00
time (sec)	N/A	0.281	0.368	0.023	0.369	0.586	28.394	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	285	139	150	388	189	272	0	-1
normalized size	1	1.26	0.61	0.66	1.71	0.83	1.20	0.00	-0.00
time (sec)	N/A	0.396	0.290	0.010	0.370	0.551	16.710	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	150	258	423	185	260	0	-1
normalized size	1	1.00	0.90	1.55	2.55	1.11	1.57	0.00	-0.01
time (sec)	N/A	0.079	0.390	0.015	0.366	0.474	11.615	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	237	123	132	302	169	228	0	-1
normalized size	1	1.24	0.64	0.69	1.58	0.88	1.19	0.00	-0.01
time (sec)	N/A	0.262	0.271	0.011	0.350	0.528	6.322	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	207	255	0	0	0	0	-1
normalized size	1	1.00	0.87	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	0.393	0.466	0.000	0.601	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	239	136	219	231	249	0	0	-1
normalized size	1	1.33	0.76	1.22	1.28	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.361	0.306	0.020	0.466	0.554	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	226	275	0	0	0	0	-1
normalized size	1	1.00	0.85	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.372	0.783	0.000	0.754	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	252	142	223	208	253	0	0	-1
normalized size	1	1.29	0.73	1.14	1.07	1.30	0.00	0.00	-0.01
time (sec)	N/A	0.390	0.309	0.022	0.562	0.660	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	227	263	0	0	0	0	-1
normalized size	1	1.00	1.44	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	0.343	0.416	0.000	0.477	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	151	244	0	0	0	0	-1
normalized size	1	1.00	1.08	1.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.198	0.309	0.293	0.000	0.657	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	155	208	0	0	0	0	-1
normalized size	1	1.00	1.52	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.161	0.268	0.000	0.557	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	85	179	0	0	0	0	-1
normalized size	1	1.00	1.15	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.089	0.092	0.000	0.539	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	64	326	0	0	0	0	-1
normalized size	1	1.00	1.08	5.53	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.071	0.606	0.000	0.457	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	93	91	0	0	0	0	-1
normalized size	1	1.00	1.52	1.49	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.127	0.159	0.136	0.000	0.625	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	132	161	0	0	0	0	-1
normalized size	1	1.00	1.39	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.298	0.273	0.000	0.406	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	144	301	0	0	0	0	-1
normalized size	1	1.00	1.22	2.55	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.197	0.567	0.284	0.000	0.518	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	223	225	0	0	0	0	-1
normalized size	1	1.00	1.42	1.43	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.364	0.350	0.000	0.716	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	244	300	0	0	0	0	-1
normalized size	1	1.00	1.38	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.225	1.093	0.608	0.000	0.592	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	209	309	0	0	0	0	-1
normalized size	1	1.00	1.17	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.680	0.606	0.000	0.456	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	206	255	0	0	0	0	-1
normalized size	1	1.00	1.66	2.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.759	0.314	0.000	0.554	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	53	64	134	65	0	0	-1
normalized size	1	1.00	0.87	1.05	2.20	1.07	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.143	0.016	0.372	0.739	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	189	252	0	0	0	0	-1
normalized size	1	1.00	1.58	2.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	1.408	0.099	0.000	0.587	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	149	339	0	0	0	0	-1
normalized size	1	1.00	1.28	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.808	0.285	0.000	0.431	0.000	0.000	0.000

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	283	259	0	0	0	0	-1
normalized size	1	1.00	1.66	1.52	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.717	0.364	0.000	0.538	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	319	371	0	0	0	0	-1
normalized size	1	1.00	2.10	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.598	0.338	0.000	0.556	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	377	352	0	0	0	0	-1
normalized size	1	1.00	1.52	1.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.291	1.703	0.473	0.000	0.501	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	287	383	0	0	0	0	-1
normalized size	1	1.00	1.15	1.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	1.788	0.845	0.000	0.459	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	83	136	0	101	0	0	-1
normalized size	1	1.00	0.61	1.00	0.00	0.74	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.252	0.023	0.000	0.719	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	287	380	0	0	0	0	-1
normalized size	1	1.00	1.54	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	1.668	0.581	0.000	0.536	0.000	0.000	0.000

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	64	86	0	98	0	0	-1
normalized size	1	1.00	0.70	0.95	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.219	0.013	0.000	0.620	0.000	0.000	0.000

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	316	378	0	0	0	0	-1
normalized size	1	1.00	1.76	2.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.134	1.175	0.156	0.000	0.562	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	210	508	0	0	0	0	-1
normalized size	1	1.00	1.23	2.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.261	1.538	0.520	0.000	0.537	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	362	392	0	0	0	0	-1
normalized size	1	1.00	1.57	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	1.781	0.453	0.000	0.607	0.000	0.000	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	273	641	0	0	0	0	-1
normalized size	1	1.00	1.09	2.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.367	3.033	0.647	0.000	0.851	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	471	504	0	0	0	0	-1
normalized size	1	1.00	1.52	1.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	1.933	0.577	0.000	0.569	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	77	309	0	0	0	0	-1
normalized size	1	1.00	1.45	5.83	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.049	0.016	0.000	0.466	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	184	0	0	0	0	-1
normalized size	1	1.00	1.10	1.69	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.886	0.104	0.000	0.624	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	223	276	0	0	0	0	-1
normalized size	1	1.00	1.36	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.113	2.486	0.298	0.000	0.468	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	198	449	0	0	0	0	-1
normalized size	1	1.00	0.71	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.782	1.218	0.813	0.000	0.624	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	151	346	0	0	0	0	-1
normalized size	1	1.00	0.75	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.579	1.097	0.381	0.000	0.581	0.000	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	144	239	0	0	0	0	-1
normalized size	1	1.00	1.16	1.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.204	0.608	0.237	0.000	0.451	0.000	0.000	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	137	286	0	0	0	0	-1
normalized size	1	1.00	1.16	2.42	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.368	0.467	0.493	0.000	0.577	0.000	0.000	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	127	88	1017	150	462	0	0	-1
normalized size	1	1.07	0.74	8.55	1.26	3.88	0.00	0.00	-0.01
time (sec)	N/A	0.285	0.127	0.729	0.489	0.572	0.000	0.000	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	226	128	1741	146	549	0	0	-1
normalized size	1	1.14	0.64	8.75	0.73	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.346	0.218	0.905	0.454	0.578	0.000	0.000	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	303	146	2534	207	615	0	0	-1
normalized size	1	1.09	0.52	9.08	0.74	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.378	0.269	0.931	0.440	0.754	0.000	0.000	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	302	152	988	205	203	0	0	-1
normalized size	1	1.11	0.56	3.63	0.75	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.352	0.368	0.678	0.452	0.519	0.000	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	214	128	640	144	176	0	0	-1
normalized size	1	1.10	0.66	3.28	0.74	0.90	0.00	0.00	-0.01
time (sec)	N/A	0.323	0.187	0.537	0.512	0.507	0.000	0.000	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	98	356	81	142	0	0	-1
normalized size	1	1.07	0.83	3.02	0.69	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.135	0.293	0.342	0.833	0.000	0.000	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	233	394	0	0	0	0	-1
normalized size	1	1.00	1.09	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	0.907	0.430	0.000	0.602	0.000	0.000	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	307	438	0	0	0	0	-1
normalized size	1	1.00	1.31	1.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	1.097	0.711	0.000	0.561	0.000	0.000	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	290	541	0	0	0	0	-1
normalized size	1	1.00	0.92	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.743	1.052	0.749	0.000	0.487	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	372	337	561	0	0	0	0	-1
normalized size	1	1.03	0.94	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.064	4.817	0.619	0.000	1.065	0.000	0.000	0.000

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	293	270	456	0	0	0	0	-1
normalized size	1	1.04	0.96	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	1.985	0.460	0.000	0.519	0.000	0.000	0.000

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	212	235	344	0	0	0	0	-1
normalized size	1	1.06	1.18	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.321	1.266	0.246	0.000	0.632	0.000	0.000	0.000

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	209	223	427	0	0	0	0	-1
normalized size	1	1.06	1.13	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.536	1.080	0.516	0.000	0.660	0.000	0.000	0.000

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	203	215	259	1181	0	0	0	0	-1
normalized size	1	1.06	1.28	5.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.634	0.838	0.745	0.000	0.707	0.000	0.000	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	179	94	2171	189	572	0	0	-1
normalized size	1	1.08	0.57	13.08	1.14	3.45	0.00	0.00	-0.01
time (sec)	N/A	0.354	0.081	0.875	0.465	0.623	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	322	136	3144	163	648	0	0	-1
normalized size	1	1.30	0.55	12.73	0.66	2.62	0.00	0.00	-0.00
time (sec)	N/A	0.445	0.148	0.900	0.472	1.456	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	328	401	154	4259	225	720	0	0	-1
normalized size	1	1.22	0.47	12.98	0.69	2.20	0.00	0.00	-0.00
time (sec)	N/A	0.513	0.334	1.035	0.730	0.666	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	480	170	5518	287	792	0	0	-1
normalized size	1	1.17	0.42	13.49	0.70	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.606	0.453	1.239	0.715	0.652	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	460	182	1846	285	275	0	0	-1
normalized size	1	1.15	0.46	4.63	0.71	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.496	0.234	1.004	0.925	0.687	0.000	0.000	0.000

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	366	164	1376	223	245	0	0	-1
normalized size	1	1.14	0.51	4.29	0.69	0.76	0.00	0.00	-0.00
time (sec)	N/A	0.437	0.183	0.649	1.360	0.648	0.000	0.000	0.000

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	272	150	966	161	215	0	0	-1
normalized size	1	1.12	0.62	3.98	0.66	0.88	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.238	0.582	1.091	0.634	0.000	0.000	0.000

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	178	107	620	102	185	0	0	-1
normalized size	1	1.08	0.65	3.76	0.62	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.265	0.217	0.307	0.769	0.571	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	292	304	336	499	0	0	0	0	-1
normalized size	1	1.04	1.15	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.789	1.250	0.475	0.000	0.529	0.000	0.000	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	311	323	500	542	0	0	0	0	-1
normalized size	1	1.04	1.61	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	1.578	0.644	0.000	0.807	0.000	0.000	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	333	574	570	0	0	0	0	-1
normalized size	1	1.04	1.79	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.837	1.282	0.727	0.000	0.610	0.000	0.000	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	454	485	581	690	0	0	0	0	-1
normalized size	1	1.07	1.28	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.385	6.590	0.809	0.000	0.448	0.000	0.000	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	402	415	581	0	0	0	0	-1
normalized size	1	1.08	1.12	1.57	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.171	4.547	0.493	0.000	0.462	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	324	347	462	0	0	0	0	-1
normalized size	1	1.11	1.18	1.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.543	2.474	0.307	0.000	0.605	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	315	305	550	0	0	0	0	-1
normalized size	1	1.11	1.07	1.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	1.830	0.566	0.000	0.513	0.000	0.000	0.000

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	324	319	1407	0	0	0	0	-1
normalized size	1	1.11	1.09	4.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	1.426	0.787	0.000	0.796	0.000	0.000	0.000

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	293	324	400	2429	0	0	0	0	-1
normalized size	1	1.11	1.37	8.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.947	3.593	0.856	0.000	0.544	0.000	0.000	0.000

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	234	105	3775	224	703	0	0	-1
normalized size	1	1.07	0.48	17.24	1.02	3.21	0.00	0.00	-0.00
time (sec)	N/A	0.379	0.100	0.942	0.959	0.589	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	448	147	5006	187	795	0	0	-1
normalized size	1	1.43	0.47	15.94	0.60	2.53	0.00	0.00	-0.00
time (sec)	N/A	0.526	0.173	1.091	0.518	0.707	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	519	165	6379	251	879	0	0	-1
normalized size	1	1.35	0.43	16.57	0.65	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.579	0.214	1.313	0.801	0.623	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	458	527	193	2374	313	353	0	0	-1
normalized size	1	1.15	0.42	5.18	0.68	0.77	0.00	0.00	-0.00
time (sec)	N/A	0.516	0.264	0.878	0.952	0.628	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	429	175	1840	249	317	0	0	-1
normalized size	1	1.13	0.46	4.87	0.66	0.84	0.00	0.00	-0.00
time (sec)	N/A	0.468	0.200	0.736	0.883	0.502	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	331	160	1102	185	281	0	0	-1
normalized size	1	1.11	0.54	3.70	0.62	0.94	0.00	0.00	-0.00
time (sec)	N/A	0.424	0.154	0.542	0.950	0.446	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	233	117	956	118	241	0	0	-1
normalized size	1	1.07	0.54	4.39	0.54	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.282	0.254	0.351	0.684	0.568	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	410	471	620	0	0	0	0	-1
normalized size	1	1.08	1.24	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.059	3.854	0.529	0.000	0.630	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	404	435	596	667	0	0	0	0	-1
normalized size	1	1.08	1.48	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.064	4.288	0.740	0.000	0.525	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	407	438	660	691	0	0	0	0	-1
normalized size	1	1.08	1.62	1.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.083	1.501	0.835	0.000	0.502	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	84	54	152	0	0	0	0	-1
normalized size	1	1.27	0.82	2.30	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.104	0.115	0.388	0.000	0.462	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	260	140	670	195	176	0	0	-1
normalized size	1	1.10	0.59	2.84	0.83	0.75	0.00	0.00	-0.00
time (sec)	N/A	0.709	0.298	0.741	0.927	0.742	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	228	171	408	0	0	0	0	-1
normalized size	1	1.08	0.81	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.648	0.933	0.825	0.000	0.570	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	172	113	382	131	146	0	0	-1
normalized size	1	1.10	0.72	2.45	0.84	0.94	0.00	0.00	-0.01
time (sec)	N/A	0.496	0.239	0.559	0.843	0.517	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	140	141	291	0	0	0	0	-1
normalized size	1	1.06	1.07	2.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.398	0.665	0.534	0.000	0.570	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	80	85	158	63	117	0	0	-1
normalized size	1	1.11	1.18	2.19	0.88	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.163	0.248	1.251	0.523	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	89	0	0	0	0	-1
normalized size	1	1.00	1.00	1.68	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.121	0.037	0.061	0.000	0.568	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	153	327	0	0	0	0	-1
normalized size	1	1.00	1.01	2.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.333	0.289	0.374	0.000	0.482	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	79	71	219	116	265	0	0	-1
normalized size	1	1.11	1.00	3.08	1.63	3.73	0.00	0.00	-0.01
time (sec)	N/A	0.303	0.066	0.381	0.541	0.584	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	238	246	309	489	0	0	0	0	-1
normalized size	1	1.03	1.30	2.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	1.088	0.648	0.000	0.590	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	171	174	854	134	479	0	0	-1
normalized size	1	1.10	1.12	5.51	0.86	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.506	0.334	0.664	0.972	0.862	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	262	145	431	0	489	0	0	-1
normalized size	1	1.12	0.62	1.85	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.434	0.108	0.681	0.000	0.612	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	237	192	445	0	0	0	0	-1
normalized size	1	1.05	0.85	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.681	1.478	0.757	0.000	0.402	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	163	97	313	157	429	0	0	-1
normalized size	1	1.09	0.65	2.09	1.05	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.385	0.074	0.535	0.929	0.461	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	159	279	0	0	0	0	-1
normalized size	1	1.00	1.11	1.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.454	0.676	0.472	0.000	0.466	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	90	198	0	327	0	0	-1
normalized size	1	1.00	1.18	2.61	0.00	4.30	0.00	0.00	-0.01
time (sec)	N/A	0.248	0.238	0.241	0.000	0.783	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	72	180	70	0	0	0	-1
normalized size	1	1.00	0.86	2.14	0.83	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.032	0.203	0.704	0.719	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	301	511	0	0	0	0	-1
normalized size	1	1.00	1.31	2.23	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.591	2.573	0.423	0.000	0.503	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	159	114	242	144	0	0	0	-1
normalized size	1	1.01	0.72	1.53	0.91	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.397	0.089	0.357	0.742	1.766	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	405	648	0	0	0	0	-1
normalized size	1	1.00	1.23	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.862	4.450	0.639	0.000	0.478	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	161	1050	0	0	0	0	-1
normalized size	1	1.00	0.64	4.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.458	0.125	0.657	0.000	0.952	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	280	167	466	0	529	0	0	-1
normalized size	1	1.15	0.69	1.92	0.00	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.443	0.169	0.650	0.000	0.831	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	251	225	1519	0	0	0	0	-1
normalized size	1	1.12	1.00	6.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.757	0.760	0.841	0.000	0.662	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	243	122	313	175	469	0	0	-1
normalized size	1	1.54	0.77	1.98	1.11	2.97	0.00	0.00	-0.01
time (sec)	N/A	0.448	0.131	0.539	0.695	0.694	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	160	101	1228	169	0	0	0	-1
normalized size	1	1.20	0.76	9.23	1.27	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.405	0.209	0.540	0.679	0.601	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	154	119	249	0	421	0	0	-1
normalized size	1	1.21	0.94	1.96	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.259	0.292	0.000	0.603	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	189	132	1073	157	0	0	0	-1
normalized size	1	1.17	0.81	6.62	0.97	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.268	0.088	0.296	0.705	1.266	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	332	377	619	0	0	0	0	-1
normalized size	1	1.05	1.19	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.832	7.085	0.476	0.000	0.650	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	279	147	1350	0	0	0	0	-1
normalized size	1	1.12	0.59	5.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	0.378	0.458	0.000	0.877	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	509	500	801	0	0	0	0	-1
normalized size	1	1.06	1.04	1.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.143	7.286	0.737	0.000	0.631	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	383	169	1878	276	0	0	0	-1
normalized size	1	1.13	0.50	5.56	0.82	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	0.439	0.671	0.708	1.079	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	276	116	419	191	0	0	142	-1
normalized size	1	1.12	0.47	1.70	0.78	0.00	0.00	0.58	-0.00
time (sec)	N/A	0.339	0.090	0.465	0.444	0.663	0.000	4.410	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	206	93	456	0	0	0	0	-1
normalized size	1	1.42	0.64	3.14	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.501	0.259	0.796	0.000	0.667	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	158	74	311	62	101	0	0	-1
normalized size	1	1.44	0.67	2.83	0.56	0.92	0.00	0.00	-0.01
time (sec)	N/A	0.392	0.128	0.533	0.899	1.181	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	125	75	223	0	0	0	0	-1
normalized size	1	1.42	0.85	2.53	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.324	0.155	0.482	0.000	0.538	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	73	55	123	28	72	0	40	-1
normalized size	1	1.49	1.12	2.51	0.57	1.47	0.00	0.82	-0.02
time (sec)	N/A	0.177	0.086	0.211	0.664	0.607	0.000	0.730	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0	-1
normalized size	1	1.41	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.098	0.021	0.060	0.000	0.864	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	142	113	270	0	0	0	0	-1
normalized size	1	1.38	1.10	2.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.276	0.145	0.355	0.000	0.583	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	F(-2)	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	72	57	168	73	0	0	80	-1
normalized size	1	1.50	1.19	3.50	1.52	0.00	0.00	1.67	-0.02
time (sec)	N/A	0.254	0.030	0.355	0.856	0.000	0.000	0.523	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	167	230	234	349	0	0	0	0	-1
normalized size	1	1.38	1.40	2.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.471	0.288	0.612	0.000	2.266	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	111	100	0	0	0	0	0	-1
normalized size	1	1.13	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.358	0.112	180.000	0.000	1.703	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	115	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.383	0.035	180.000	0.000	0.678	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	387	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.755	1.178	180.000	0.000	1.612	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	290	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	0.451	180.000	0.000	0.996	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	191	0	0	0	0	0	-1
normalized size	1	1.00	1.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.238	180.000	0.000	1.528	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.072	4.101	0.785	0.000	1.114	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	161	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.207	11.210	1.249	0.000	1.236	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	294	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.335	15.021	1.367	0.000	0.691	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	723	764	350	0	0	0	0	0	-1
normalized size	1	1.06	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.388	1.310	2.216	0.000	0.894	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	455	477	274	0	0	0	0	0	-1
normalized size	1	1.05	0.60	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.904	0.788	1.954	0.000	0.987	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	288	223	0	0	0	0	0	-1
normalized size	1	1.04	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.577	0.294	1.878	0.000	0.490	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	147	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.352	0.077	0.652	0.000	0.742	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	216	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.660	0.250	0.993	0.000	0.605	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	450	465	319	0	0	0	0	0	-1
normalized size	1	1.03	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.993	0.709	0.911	0.000	0.633	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	827	387	0	0	0	0	0	-1
normalized size	1	1.01	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.580	2.349	2.966	0.000	0.656	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	503	513	288	0	0	0	0	0	-1
normalized size	1	1.02	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.991	1.047	2.599	0.000	0.660	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	312	229	0	0	0	0	0	-1
normalized size	1	1.03	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.557	0.221	2.004	0.000	0.527	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	264	0	0	0	0	0	-1
normalized size	1	1.00	1.40	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.559	6.266	0.701	0.000	0.512	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.943	2.561	0.957	0.000	0.658	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	504	504	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.452	2.699	0.971	0.000	0.701	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	141	124	0	0	0	0	0	-1
normalized size	1	1.10	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.083	0.573	0.000	0.546	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	125	188	178	175	243	0	-1
normalized size	1	1.00	0.47	0.71	0.67	0.66	0.91	0.00	-0.00
time (sec)	N/A	0.676	0.278	0.207	0.852	0.639	11.249	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	101	140	134	142	182	0	-1
normalized size	1	1.00	0.52	0.72	0.69	0.73	0.93	0.00	-0.01
time (sec)	N/A	0.462	0.225	0.090	0.822	0.550	3.902	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	90	76	95	105	0	-1
normalized size	1	1.00	0.65	0.80	0.68	0.85	0.94	0.00	-0.01
time (sec)	N/A	0.263	0.117	0.072	1.076	0.623	1.062	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	95	201	0	0	0	0	-1
normalized size	1	1.00	0.97	2.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.078	0.084	0.000	0.555	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	191	288	0	0	0	0	-1
normalized size	1	1.00	1.17	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.287	0.940	0.254	0.000	0.475	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	319	443	0	0	0	0	-1
normalized size	1	1.00	1.24	1.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	4.810	0.381	0.000	0.541	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	237	1284	326	349	0	0	-1
normalized size	1	1.00	0.64	3.46	0.88	0.94	0.00	0.00	-0.00
time (sec)	N/A	1.056	0.439	0.727	0.999	0.516	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	241	767	0	0	0	0	-1
normalized size	1	1.00	0.76	2.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.930	2.106	0.641	0.000	0.569	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	194	181	726	204	280	0	0	-1
normalized size	1	1.04	0.97	3.90	1.10	1.51	0.00	0.00	-0.01
time (sec)	N/A	0.370	0.367	0.357	1.236	0.568	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	235	528	0	0	0	0	-1
normalized size	1	1.00	1.15	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	1.085	0.296	0.000	0.648	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	449	0	0	0	0	0	-1
normalized size	1	1.00	1.12	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	1.342	0.655	0.000	0.742	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	270	582	0	0	0	0	-1
normalized size	1	1.00	1.15	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.631	1.707	0.619	0.000	0.600	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	547	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.877	85.467	0.870	0.000	0.639	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	344	304	2633	0	0	0	0	-1
normalized size	1	1.02	0.90	7.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.583	1.008	0.902	0.000	0.480	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	507	262	1952	388	432	0	0	-1
normalized size	1	1.02	0.53	3.94	0.78	0.87	0.00	0.00	-0.00
time (sec)	N/A	1.674	0.585	0.771	1.197	0.681	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	441	453	485	1021	0	0	0	0	-1
normalized size	1	1.03	1.10	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.477	4.371	0.957	0.000	0.638	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	361	208	1270	278	367	0	0	-1
normalized size	1	1.04	0.60	3.65	0.80	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.554	0.528	0.450	0.747	0.568	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	348	374	775	0	0	0	0	-1
normalized size	1	1.04	1.11	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.609	2.954	0.385	0.000	0.625	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	573	585	650	0	0	0	0	0	-1
normalized size	1	1.02	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.255	2.913	0.652	0.000	0.489	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	453	465	433	942	0	0	0	0	-1
normalized size	1	1.03	0.96	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.966	4.107	0.677	0.000	1.131	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	642	1129	0	0	0	0	0	-1
normalized size	1	1.02	1.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.374	176.117	0.892	0.000	0.558	0.000	0.000	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	438	583	2879	0	0	0	0	-1
normalized size	1	1.03	1.37	6.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.167	2.120	0.935	0.000	0.546	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	880	911	288	2224	471	558	0	0	-1
normalized size	1	1.04	0.33	2.53	0.54	0.63	0.00	0.00	-0.00
time (sec)	N/A	2.345	0.754	0.858	0.806	0.933	0.000	0.000	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	841	872	910	1312	0	0	0	0	-1
normalized size	1	1.04	1.08	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.124	5.910	0.911	0.000	0.418	0.000	0.000	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	485	234	1958	337	477	0	0	-1
normalized size	1	1.03	0.50	4.17	0.72	1.01	0.00	0.00	-0.00
time (sec)	N/A	0.680	0.615	0.569	0.674	0.475	0.000	0.000	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	486	517	740	1053	0	0	0	0	-1
normalized size	1	1.06	1.52	2.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.868	3.488	0.465	0.000	0.878	0.000	0.000	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	836	867	1031	0	0	0	0	0	-1
normalized size	1	1.04	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.776	7.245	0.770	0.000	0.536	0.000	0.000	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	638	554	1227	0	0	0	0	-1
normalized size	1	1.05	0.91	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.290	5.937	0.755	0.000	0.511	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	890	921	1384	0	0	0	0	0	-1
normalized size	1	1.03	1.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.985	90.306	1.010	0.000	1.056	0.000	0.000	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	638	669	803	3431	0	0	0	0	-1
normalized size	1	1.05	1.26	5.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.615	3.166	1.027	0.000	0.775	0.000	0.000	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	421	445	255	1314	407	348	0	0	-1
normalized size	1	1.06	0.61	3.12	0.97	0.83	0.00	0.00	-0.00
time (sec)	N/A	1.131	0.554	0.796	1.033	0.480	0.000	0.000	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	355	371	295	887	0	0	0	0	-1
normalized size	1	1.05	0.83	2.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	1.540	0.954	0.000	0.792	0.000	0.000	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	308	201	752	279	282	0	0	-1
normalized size	1	1.05	0.69	2.58	0.96	0.97	0.00	0.00	-0.00
time (sec)	N/A	0.769	0.454	0.698	1.021	0.621	0.000	0.000	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	234	228	624	0	0	0	0	-1
normalized size	1	1.04	1.01	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.626	0.883	0.718	0.000	0.617	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	163	149	314	145	218	0	0	-1
normalized size	1	1.05	0.96	2.03	0.94	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.342	0.393	0.307	0.391	1.107	0.000	0.000	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	149	0	0	0	0	-1
normalized size	1	1.00	1.00	2.81	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.195	0.048	0.086	0.000	0.526	0.000	0.000	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	315	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.521	0.626	0.508	0.000	0.650	0.000	0.000	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	194	179	513	0	0	0	0	-1
normalized size	1	1.04	0.96	2.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.519	0.864	0.485	0.000	0.604	0.000	0.000	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	438	551	0	0	0	0	0	-1
normalized size	1	1.02	1.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	84.830	0.806	0.000	0.654	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	328	344	346	2198	0	0	0	0	-1
normalized size	1	1.05	1.05	6.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.869	1.641	0.898	0.000	0.503	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	556	578	358	1099	0	0	0	0	-1
normalized size	1	1.04	0.64	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.367	3.839	0.862	0.000	0.493	0.000	0.000	0.000

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	440	451	343	1141	0	0	0	0	-1
normalized size	1	1.02	0.78	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.205	2.004	1.011	0.000	0.571	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	424	302	836	0	0	0	0	-1
normalized size	1	1.03	0.73	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.921	1.644	0.693	0.000	0.658	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	270	738	0	0	0	0	-1
normalized size	1	1.00	1.05	2.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.795	2.047	0.658	0.000	0.495	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	210	542	0	0	0	0	-1
normalized size	1	1.00	1.07	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.445	1.005	0.345	0.000	0.662	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	126	578	0	0	0	0	-1
normalized size	1	1.00	0.64	2.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.351	0.459	0.308	0.000	0.587	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	471	471	577	0	0	0	0	0	-1
normalized size	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.964	3.622	0.597	0.000	0.589	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	315	826	0	0	0	0	-1
normalized size	1	1.00	0.92	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.936	1.691	0.487	0.000	0.452	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	979	0	0	0	0	0	-1
normalized size	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.486	92.283	0.924	0.000	0.674	0.000	0.000	0.000

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	529	2868	0	0	0	0	-1
normalized size	1	1.00	1.07	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.461	2.425	0.946	0.000	0.563	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	568	594	437	1211	0	0	0	0	-1
normalized size	1	1.05	0.77	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.476	5.548	0.890	0.000	0.897	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	482	497	382	4074	0	0	0	0	-1
normalized size	1	1.03	0.79	8.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.333	2.561	1.007	0.000	0.560	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	351	341	835	0	0	0	0	-1
normalized size	1	1.04	1.01	2.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.969	4.535	0.744	0.000	0.582	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	389	404	264	3445	0	0	0	0	-1
normalized size	1	1.04	0.68	8.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.731	1.783	0.756	0.000	0.583	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	313	332	720	0	0	0	0	-1
normalized size	1	1.05	1.11	2.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.473	2.541	0.434	0.000	0.672	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	346	289	3050	0	0	0	0	-1
normalized size	1	1.05	0.87	9.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	1.522	0.439	0.000	0.729	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	597	612	806	0	0	0	0	0	-1
normalized size	1	1.03	1.35	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.330	10.450	0.634	0.000	0.664	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	506	457	3798	0	0	0	0	-1
normalized size	1	1.06	0.96	7.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.191	3.081	0.597	0.000	0.608	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	796	826	1181	0	0	0	0	0	-1
normalized size	1	1.04	1.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.938	99.156	0.973	0.000	0.619	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	607	534	5251	0	0	0	0	-1
normalized size	1	1.08	0.95	9.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.821	3.677	0.885	0.000	0.537	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	459	220	794	0	0	0	0	-1
normalized size	1	1.07	0.51	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.672	1.382	0.503	0.000	0.828	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	329	116	488	0	0	0	0	-1
normalized size	1	1.35	0.48	2.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.265	0.769	0.000	0.545	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	237	123	343	105	150	0	0	-1
normalized size	1	1.34	0.69	1.94	0.59	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.592	0.148	0.501	0.469	0.623	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	207	87	239	0	0	0	0	-1
normalized size	1	1.37	0.58	1.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.510	0.176	0.462	0.000	0.555	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	109	54	139	50	114	0	76	-1
normalized size	1	1.38	0.68	1.76	0.63	1.44	0.00	0.96	-0.01
time (sec)	N/A	0.272	0.098	0.200	0.376	0.517	0.000	0.562	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0	-1
normalized size	1	1.41	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.150	0.023	0.058	0.000	0.531	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	183	248	151	0	0	0	0	0	-1
normalized size	1	1.36	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.420	0.189	0.343	0.000	0.475	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	174	111	241	0	0	0	0	-1
normalized size	1	1.40	0.90	1.94	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.444	0.477	0.358	0.000	0.538	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	296	398	233	0	0	0	0	0	-1
normalized size	1	1.34	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	1.009	0.667	0.000	0.484	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1154	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.545	1.591	2.232	0.000	0.433	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	584	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	0.527	1.753	0.000	0.497	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	240	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	0.342	1.542	0.000	0.610	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.474	3.537	0.594	0.000	0.582	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.561	4.588	0.834	0.000	0.783	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.557	4.777	0.862	0.000	0.731	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.370	0.755	0.524	0.000	0.551	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	505	505	179	294	276	248	367	0	-1
normalized size	1	1.00	0.35	0.58	0.55	0.49	0.73	0.00	-0.00
time (sec)	N/A	1.408	0.417	0.132	0.505	0.466	17.364	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	147	218	210	204	274	0	-1
normalized size	1	1.00	0.38	0.56	0.54	0.53	0.71	0.00	-0.00
time (sec)	N/A	0.844	0.219	0.102	0.418	0.623	6.337	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	109	140	124	140	160	0	-1
normalized size	1	1.00	0.62	0.80	0.71	0.80	0.91	0.00	-0.01
time (sec)	N/A	0.477	0.119	0.092	0.459	0.634	1.998	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	273	0	0	0	0	-1
normalized size	1	1.00	0.90	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.108	0.090	0.000	0.472	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	276	464	0	0	0	0	-1
normalized size	1	1.00	1.06	1.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	2.164	0.269	0.000	0.578	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	455	710	0	0	0	0	-1
normalized size	1	1.00	1.18	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	8.192	0.404	0.000	0.818	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	605	636	189	887	0	0	0	0	-1
normalized size	1	1.05	0.31	1.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.497	1.203	0.508	0.000	0.556	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	414	148	536	0	0	0	0	-1
normalized size	1	1.03	0.37	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.946	0.445	0.347	0.000	0.584	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	98	256	0	0	0	0	-1
normalized size	1	1.00	0.42	1.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.537	0.207	0.329	0.000	0.880	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	55	0	0	0	0	-1
normalized size	1	1.00	1.00	1.20	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.156	0.030	0.059	0.000	0.523	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	145	548	0	0	0	0	-1
normalized size	1	1.00	0.60	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.256	0.393	0.000	0.460	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	413	428	258	955	0	0	0	0	-1
normalized size	1	1.04	0.62	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.638	0.997	0.565	0.000	0.500	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	607	637	363	1319	0	0	0	0	-1
normalized size	1	1.05	0.60	2.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.077	1.853	0.586	0.000	1.300	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	427	136	520	0	0	0	0	-1
normalized size	1	1.36	0.43	1.65	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.442	0.451	0.711	0.000	0.647	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	329	140	375	131	205	0	0	-1
normalized size	1	1.35	0.58	1.54	0.54	0.84	0.00	0.00	-0.00
time (sec)	N/A	1.026	0.170	0.491	0.708	0.500	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	257	98	255	0	0	0	0	-1
normalized size	1	1.37	0.52	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.765	0.224	0.450	0.000	0.476	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	153	101	155	65	159	0	103	-1
normalized size	1	1.39	0.92	1.41	0.59	1.45	0.00	0.94	-0.01
time (sec)	N/A	0.393	0.097	0.197	0.327	0.549	0.000	0.678	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	0	0	0	-1
normalized size	1	1.41	1.41	1.59	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.161	0.022	0.055	0.000	0.630	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	356	488	0	0	0	0	0	-1
normalized size	1	1.34	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.656	0.329	0.000	0.505	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	229	137	313	0	0	0	0	-1
normalized size	1	1.38	0.83	1.89	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.492	0.541	0.375	0.000	0.670	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	460	614	1051	0	0	0	0	0	-1
normalized size	1	1.33	2.28	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.013	6.265	0.605	0.000	0.638	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.458	3.683	0.582	0.000	1.076	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	45	44	0	0	0	0	-1
normalized size	1	1.00	0.67	0.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.140	0.280	0.147	0.000	0.651	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	33	0	0	0	0	-1
normalized size	1	1.00	0.68	0.66	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.113	0.172	0.064	0.000	0.494	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	25	24	0	0	0	0	-1
normalized size	1	1.00	0.86	0.83	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	0.121	0.065	0.000	0.448	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	1.577	0.161	0.000	0.951	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.032	6.139	0.398	0.000	0.626	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	430	188	591	0	0	0	0	-1
normalized size	1	1.27	0.55	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.879	0.507	0.661	0.000	0.506	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	371	171	543	0	0	0	0	-1
normalized size	1	1.25	0.58	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.861	0.457	0.424	0.000	0.956	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	178	103	227	0	0	0	0	-1
normalized size	1	1.28	0.74	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.672	0.312	0.334	0.000	0.472	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	245	127	361	0	0	0	0	-1
normalized size	1	1.24	0.64	1.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.572	0.315	0.198	0.000	0.448	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	178	105	227	0	0	0	0	-1
normalized size	1	1.28	0.76	1.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.343	0.204	0.184	0.000	0.511	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.077	1.249	0.414	0.000	0.781	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.931	1.088	0.368	0.000	0.534	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.447	1.420	0.737	0.000	0.678	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.438	0.896	0.840	0.000	0.646	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	497	215	725	0	0	0	0	-1
normalized size	1	1.25	0.54	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.946	0.941	0.442	0.000	0.436	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	430	188	591	0	0	0	0	-1
normalized size	1	1.27	0.55	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.881	0.741	0.516	0.000	0.516	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	297	371	172	543	0	0	0	0	-1
normalized size	1	1.25	0.58	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.675	0.689	0.231	0.000	0.692	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	239	304	147	409	0	0	0	0	-1
normalized size	1	1.27	0.62	1.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.459	0.458	0.208	0.000	0.477	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	216	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.752	1.277	0.420	0.000	0.481	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	164	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.525	1.423	0.469	0.000	0.442	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.525	1.447	0.705	0.000	0.742	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.539	0.910	0.832	0.000	0.487	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	497	216	725	0	0	0	0	-1
normalized size	1	1.25	0.54	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.997	1.333	0.450	0.000	0.895	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	556	233	773	0	0	0	0	-1
normalized size	1	1.27	0.53	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.979	1.215	0.561	0.000	0.625	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	497	216	725	0	0	0	0	-1
normalized size	1	1.25	0.54	1.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.791	1.087	0.259	0.000	0.649	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	430	191	591	0	0	0	0	-1
normalized size	1	1.27	0.56	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.568	0.754	0.243	0.000	0.585	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	310	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.376	1.315	0.457	0.000	0.590	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.093	1.336	0.493	0.000	0.632	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.546	1.425	0.743	0.000	0.494	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.551	1.005	0.884	0.000	0.531	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	137	69	249	0	0	0	0	-1
normalized size	1	1.40	0.70	2.54	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.466	0.118	0.712	0.000	0.451	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	91	60	200	0	0	0	0	-1
normalized size	1	1.40	0.92	3.08	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.450	0.089	0.612	0.000	0.502	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	91	60	149	0	0	0	0	-1
normalized size	1	1.40	0.92	2.29	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.434	0.100	0.470	0.000	0.960	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	41	50	100	0	0	0	0	-1
normalized size	1	1.46	1.79	3.57	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.301	0.083	0.257	0.000	0.407	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	41	47	48	0	55	0	0	-1
normalized size	1	1.46	1.68	1.71	0.00	1.96	0.00	0.00	-0.04
time (sec)	N/A	0.162	0.062	0.172	0.000	0.561	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.378	0.587	0.330	0.000	0.699	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.377	0.721	0.372	0.000	0.495	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	245	130	349	0	0	0	0	-1
normalized size	1	1.24	0.66	1.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.690	0.293	0.451	0.000	0.664	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	178	99	232	0	0	0	0	-1
normalized size	1	1.28	0.71	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.632	0.282	0.570	0.000	1.255	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	114	81	173	0	0	0	0	-1
normalized size	1	1.24	0.88	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.426	0.217	0.194	0.000	0.522	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	48	54	55	0	65	0	0	-1
normalized size	1	1.37	1.54	1.57	0.00	1.86	0.00	0.00	-0.03
time (sec)	N/A	0.221	0.107	0.177	0.000	0.487	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.498	0.749	0.377	0.000	0.710	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.507	1.311	0.435	0.000	0.483	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.575	4.429	0.553	0.000	0.783	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.401	7.156	0.279	0.000	0.799	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.250	0.143	0.247	0.000	0.666	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.568	3.565	0.496	0.000	0.455	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.577	2.224	0.407	0.000	0.550	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.540	1.068	1.169	0.000	0.638	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.451	0.179	1.186	0.000	0.493	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.483	0.647	0.500	0.000	0.772	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.565	1.187	0.706	0.000	0.513	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.580	1.720	0.670	0.000	0.683	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	257	107	0	0	0	0	-1
normalized size	1	1.00	2.62	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.325	0.411	0.250	0.000	0.625	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	194	87	0	0	0	0	-1
normalized size	1	1.00	2.37	1.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	0.318	0.135	0.000	0.463	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	217	63	0	0	0	0	-1
normalized size	1	1.00	3.74	1.09	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.235	1.776	0.115	0.000	0.650	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.246	3.115	0.158	0.000	0.560	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	13.052	0.356	0.000	0.555	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	429	322	1029	0	0	0	0	-1
normalized size	1	1.23	0.92	2.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.085	0.769	0.735	0.000	0.566	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	185	130	422	0	0	0	0	-1
normalized size	1	1.20	0.84	2.74	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.879	0.489	0.567	0.000	0.557	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	418	217	622	0	0	0	0	-1
normalized size	1	1.69	0.88	2.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.689	0.430	0.319	0.000	0.664	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	177	121	361	0	0	0	0	-1
normalized size	1	1.21	0.83	2.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.335	0.224	0.296	0.000	0.582	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	182	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.570	33.137	0.563	0.000	0.520	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.493	10.091	0.356	0.000	0.619	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.461	163.257	0.693	0.000	0.603	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.456	180.009	0.859	0.000	0.512	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	439	338	1176	0	0	0	0	-1
normalized size	1	1.24	0.95	3.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.135	1.071	0.751	0.000	0.499	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	348	429	327	1029	0	0	0	0	-1
normalized size	1	1.23	0.94	2.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.056	0.946	0.408	0.000	0.609	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	305	232	737	0	0	0	0	-1
normalized size	1	1.24	0.94	3.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.524	0.622	0.336	0.000	0.623	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	291	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.872	34.297	0.647	0.000	0.578	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.662	64.941	0.696	0.000	0.654	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.542	160.491	1.043	0.000	0.620	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.612	180.002	0.826	0.000	1.015	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	454	565	446	1676	0	0	0	0	-1
normalized size	1	1.24	0.98	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.515	1.736	0.867	0.000	0.549	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	448	555	436	1499	0	0	0	0	-1
normalized size	1	1.24	0.97	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.331	1.434	0.562	0.000	0.485	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	436	343	1176	0	0	0	0	-1
normalized size	1	1.24	0.98	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.652	1.040	0.432	0.000	0.592	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	386	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.985	9.003	0.738	0.000	0.495	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.712	17.837	0.795	0.000	0.585	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.530	21.182	1.080	0.000	0.980	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.526	179.699	1.249	0.000	0.542	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	424	190	1046	0	0	0	0	-1
normalized size	1	1.26	0.56	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.860	0.703	0.838	0.000	0.676	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	301	149	758	0	0	0	0	-1
normalized size	1	1.28	0.63	3.21	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.787	0.511	0.874	0.000	0.562	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	298	144	634	0	0	0	0	-1
normalized size	1	1.26	0.61	2.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.768	0.484	0.651	0.000	0.586	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	175	117	377	0	0	0	0	-1
normalized size	1	1.29	0.86	2.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.626	0.294	0.538	0.000	0.681	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	169	107	283	0	0	0	0	-1
normalized size	1	1.30	0.82	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.431	0.226	0.280	0.000	0.467	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	50	50	57	0	75	0	0	59
normalized size	1	1.35	1.35	1.54	0.00	2.03	0.00	0.00	1.59
time (sec)	N/A	0.215	0.033	0.058	0.000	0.608	0.000	0.000	0.451

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.528	4.819	0.375	0.000	0.815	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.542	1.594	0.388	0.000	0.726	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.568	29.990	0.896	0.000	0.730	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.646	5.986	0.589	0.000	0.523	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.403	22.713	0.287	0.000	0.504	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.320	2.320	0.265	0.000	0.668	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.549	23.246	0.622	0.000	0.478	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.560	21.043	0.438	0.000	0.546	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.654	5.145	0.897	0.000	0.975	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.557	50.715	0.868	0.000	0.460	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.557	7.827	0.894	0.000	0.438	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.391	45.932	0.540	0.000	0.761	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.305	3.838	0.436	0.000	0.562	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.543	41.019	0.988	0.000	0.598	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.551	15.676	1.124	0.000	0.570	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.532	1.198	1.327	0.000	0.590	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.450	0.203	1.263	0.000	0.511	0.000	0.000	0.000

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.522	0.691	0.532	0.000	0.663	0.000	0.000	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.558	1.283	0.744	0.000	0.503	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.551	1.798	0.764	0.000	0.808	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	45	45	51	0	56	0	0	48
normalized size	1	1.41	1.41	1.59	0.00	1.75	0.00	0.00	1.50
time (sec)	N/A	0.148	0.026	0.070	0.000	0.602	0.000	0.000	0.412

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	269	300	0	0	0	0	0	-1
normalized size	1	1.04	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.747	2.646	180.000	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	350	384	0	0	0	0	0	-1
normalized size	1	1.03	1.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.772	1.560	180.000	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	241	251	331	0	0	0	0	0	-1
normalized size	1	1.04	1.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.154	4.273	180.000	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	246	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.722	1.698	180.000	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	187	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	1.662	4.461	180.000	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	479	491	527	0	0	0	0	0	-1
normalized size	1	1.03	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.133	3.777	180.000	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	462	474	498	0	0	0	0	0	-1
normalized size	1	1.03	1.08	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.267	2.989	180.000	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	363	375	508	0	0	0	0	0	-1
normalized size	1	1.03	1.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.775	7.292	180.000	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	387	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.924	1.900	180.000	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	F(-2)	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	289	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.496	3.424	180.000	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	363	154	0	0	0	0	0	-1
normalized size	1	1.03	0.44	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.723	0.255	0.799	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	117	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.375	0.141	0.713	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
normalized size	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.164	0.032	0.082	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.244	1.667	0.773	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	194	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.432	2.272	0.759	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	511	523	198	0	0	0	0	0	-1
normalized size	1	1.02	0.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.098	0.479	0.617	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	136	0	0	0	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.618	0.353	0.703	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
normalized size	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.161	0.032	0.080	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.232	1.752	0.641	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	592	213	0	0	0	0	0	-1
normalized size	1	1.02	0.37	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.526	0.514	0.627	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	148	0	0	0	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.707	0.518	0.688	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	0	0	0	-1
normalized size	1	1.00	1.00	0.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.153	0.032	0.084	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	95	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	1.641	0.632	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	376	165	0	0	0	0	0	-1
normalized size	1	1.02	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.779	0.287	0.704	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	121	0	0	0	0	0	-1
normalized size	1	1.00	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.392	0.151	0.687	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0	-1
normalized size	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.175	0.049	0.089	0.000	0.000	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.258	0.963	0.648	0.000	0.000	0.000	0.000	0.000

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.479	2.316	0.762	0.000	0.000	0.000	0.000	0.000

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	533	219	0	0	0	0	0	-1
normalized size	1	1.02	0.42	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.282	0.478	0.602	0.000	0.000	0.000	0.000	0.000

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	144	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.728	0.386	0.696	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	44	0	0	0	0	-1
normalized size	1	1.00	1.00	0.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.168	0.054	0.083	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	1.042	0.617	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	83	72	0	0	0	0	0	-1
normalized size	1	1.28	1.11	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.192	0.106	0.450	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	209	0	0	0	0	0	-1
normalized size	1	1.00	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.445	0.442	0.663	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	153	0	0	0	0	0	-1
normalized size	1	1.00	0.52	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.339	0.265	0.631	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	114	0	0	0	0	0	-1
normalized size	1	1.00	0.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.148	0.708	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	0	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.160	0.037	0.076	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.198	1.807	0.661	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.203	2.481	0.737	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	444	411	0	0	0	0	0	-1
normalized size	1	1.03	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.461	1.275	0.599	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	295	239	0	0	0	0	0	-1
normalized size	1	1.03	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.349	0.483	0.603	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	176	127	0	0	0	0	0	-1
normalized size	1	1.04	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	0.271	0.691	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	41	0	59	0	0	-1
normalized size	1	1.00	1.00	0.89	0.00	1.28	0.00	0.00	-0.02
time (sec)	N/A	0.154	0.036	0.082	0.000	0.496	0.000	0.000	0.000

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.234	1.792	0.640	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	2.343	0.726	0.000	0.000	0.000	0.000	0.000

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	337	317	0	0	0	0	0	-1
normalized size	1	1.02	0.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.745	0.595	0.611	0.000	0.000	0.000	0.000	0.000

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	207	141	0	0	0	0	0	-1
normalized size	1	1.03	0.70	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.223	0.321	0.688	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	41	0	59	0	0	-1
normalized size	1	1.00	1.00	0.85	0.00	1.23	0.00	0.00	-0.02
time (sec)	N/A	0.154	0.034	0.082	0.000	0.662	0.000	0.000	0.000

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.228	1.819	0.653	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	114	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.226	2.359	0.749	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	181	0	0	0	0	0	-1
normalized size	1	1.00	0.72	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.620	1.073	180.000	0.000	0.507	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	241	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.680	1.283	0.341	0.000	0.806	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	214	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.376	0.702	0.293	0.000	0.554	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	212	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.082	0.250	0.544	0.000	0.535	0.000	0.000	0.000

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.902	0.234	0.551	0.000	0.494	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	658	658	438	0	0	0	0	0	-1
normalized size	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.052	3.367	180.000	0.000	0.621	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	578	578	500	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	2.192	0.277	0.000	0.570	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	384	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.564	2.210	0.239	0.000	0.482	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	415	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.836	0.302	0.490	0.000	0.603	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	292	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.517	0.592	0.585	0.000	0.785	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	870	870	677	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.242	7.405	180.000	0.000	0.663	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	793	793	633	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.054	3.976	0.279	0.000	0.571	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	674	674	538	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.703	5.427	0.245	0.000	0.695	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	805	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.555	0.335	0.491	0.000	0.706	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	486	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.188	0.623	0.540	0.000	0.620	0.000	0.000	0.000

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	375	292	0	0	0	0	0	-1
normalized size	1	1.16	0.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	1.314	180.000	0.000	0.640	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	250	212	0	0	0	0	0	-1
normalized size	1	1.18	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.616	0.814	180.000	0.000	0.572	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	180	154	0	0	0	0	0	-1
normalized size	1	1.17	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.422	0.253	0.307	0.000	0.641	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	56	56	53	0	213	0	0	-1
normalized size	1	1.30	1.30	1.23	0.00	4.95	0.00	0.00	-0.02
time (sec)	N/A	0.208	0.041	0.055	0.000	0.548	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.435	2.664	0.404	0.000	0.574	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	31	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.446	1.396	0.417	0.000	0.707	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	291	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.809	1.103	180.000	0.000	0.698	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	213	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.640	0.774	180.000	0.000	1.231	0.000	0.000	0.000

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	153	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.419	0.231	0.319	0.000	0.564	0.000	0.000	0.000

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	57	54	0	221	0	0	-1
normalized size	1	1.00	1.00	0.95	0.00	3.88	0.00	0.00	-0.02
time (sec)	N/A	0.194	0.045	0.056	0.000	0.569	0.000	0.000	0.000

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.443	0.332	0.416	0.000	0.959	0.000	0.000	0.000

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.465	0.351	0.441	0.000	0.581	0.000	0.000	0.000

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.525	0.656	180.000	0.000	0.564	0.000	0.000	0.000

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.368	0.642	0.309	0.000	0.508	0.000	0.000	0.000

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	29	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.223	0.082	0.253	0.000	0.675	0.000	0.000	0.000

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.521	0.595	0.425	0.000	0.541	0.000	0.000	0.000

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.525	0.556	0.438	0.000	0.709	0.000	0.000	0.000

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.417	0.474	0.453	0.000	0.546	0.000	0.000	0.000

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.090	1.224	180.000	0.000	0.714	0.000	0.000	0.000

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-2)	A	A	F(-1)	F(-2)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.057	0.623	180.000	0.000	0.732	0.000	0.000	0.000

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.023	0.021	0.127	0.000	0.539	0.000	0.000	0.000

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.102	0.648	0.275	0.000	0.590	0.000	0.000	0.000

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.099	0.984	0.482	0.000	0.705	0.000	0.000	0.000

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.498	0.673	0.506	0.000	0.604	0.000	0.000	0.000

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.401	0.126	0.537	0.000	0.686	0.000	0.000	0.000

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.436	0.462	0.451	0.000	0.613	0.000	0.000	0.000

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.503	0.719	0.436	0.000	0.551	0.000	0.000	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	122	133	178	140	230	0	-1
normalized size	1	1.00	0.69	0.75	1.01	0.79	1.30	0.00	-0.01
time (sec)	N/A	0.142	0.112	0.029	0.451	0.693	5.642	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	140	250	196	136	212	0	-1
normalized size	1	1.00	0.87	1.55	1.22	0.84	1.32	0.00	-0.01
time (sec)	N/A	0.137	0.196	0.017	0.363	0.614	3.833	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	101	115	139	119	178	0	-1
normalized size	1	1.00	0.73	0.83	1.01	0.86	1.29	0.00	-0.01
time (sec)	N/A	0.121	0.105	0.012	0.380	0.604	2.150	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	120	202	156	114	160	0	-1
normalized size	1	1.00	0.98	1.66	1.28	0.93	1.31	0.00	-0.01
time (sec)	N/A	0.109	0.136	0.013	0.463	0.593	1.189	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	76	90	91	94	116	0	-1
normalized size	1	1.00	0.81	0.96	0.97	1.00	1.23	0.00	-0.01
time (sec)	N/A	0.080	0.093	0.010	0.388	0.470	0.562	0.000	0.000

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	119	130	0	0	0	0	-1
normalized size	1	1.00	0.45	0.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.676	0.266	0.316	0.000	0.519	0.000	0.000	0.000

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	105	95	63	132	0	0	-1
normalized size	1	1.00	1.40	1.27	0.84	1.76	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.134	0.018	0.493	0.688	0.000	0.000	0.000

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	101	126	0	0	0	0	-1
normalized size	1	1.00	0.40	0.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.611	0.142	0.607	0.000	0.538	0.000	0.000	0.000

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	128	146	85	139	0	0	-1
normalized size	1	1.00	1.36	1.55	0.90	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.266	0.019	0.473	0.880	0.000	0.000	0.000

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	192	227	305	231	422	0	-1
normalized size	1	1.00	0.60	0.71	0.96	0.72	1.32	0.00	-0.00
time (sec)	N/A	0.411	0.278	0.013	0.342	0.545	16.209	0.000	0.000

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	214	440	332	227	389	0	-1
normalized size	1	1.00	0.63	1.29	0.97	0.67	1.14	0.00	-0.00
time (sec)	N/A	0.360	0.354	0.017	0.332	0.702	11.230	0.000	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	163	195	247	198	340	0	-1
normalized size	1	1.00	0.63	0.75	0.95	0.76	1.31	0.00	-0.00
time (sec)	N/A	0.318	0.257	0.013	0.355	0.564	6.037	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	183	363	273	195	306	0	-1
normalized size	1	1.00	0.68	1.35	1.01	0.72	1.14	0.00	-0.00
time (sec)	N/A	0.249	0.344	0.016	0.983	0.667	4.199	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	130	157	180	163	246	0	-1
normalized size	1	1.00	0.66	0.80	0.92	0.83	1.26	0.00	-0.01
time (sec)	N/A	0.203	0.209	0.011	0.465	0.452	2.222	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	369	217	225	0	0	0	0	-1
normalized size	1	1.08	0.63	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	0.444	0.338	0.000	0.490	0.000	0.000	0.000

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	185	128	177	134	236	0	0	-1
normalized size	1	1.16	0.80	1.11	0.84	1.48	0.00	0.00	-0.01
time (sec)	N/A	0.302	0.263	0.017	0.770	0.858	0.000	0.000	0.000

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	173	198	0	0	0	0	-1
normalized size	1	1.00	0.54	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	0.455	0.537	0.000	0.631	0.000	0.000	0.000

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	133	196	126	216	0	0	-1
normalized size	1	1.00	0.72	1.07	0.68	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.279	0.265	0.019	0.547	0.774	0.000	0.000	0.000

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	276	335	451	334	638	0	-1
normalized size	1	1.00	0.63	0.77	1.04	0.77	1.47	0.00	-0.00
time (sec)	N/A	0.617	0.412	0.016	0.531	0.799	38.832	0.000	0.000

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	494	494	294	659	487	330	604	0	-1
normalized size	1	1.00	0.60	1.33	0.99	0.67	1.22	0.00	-0.00
time (sec)	N/A	0.648	0.550	0.023	0.402	0.808	28.219	0.000	0.000

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	236	289	374	289	532	0	-1
normalized size	1	1.00	0.65	0.79	1.02	0.79	1.46	0.00	-0.00
time (sec)	N/A	0.541	0.333	0.013	0.370	0.641	16.713	0.000	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	256	553	409	286	490	0	-1
normalized size	1	1.00	0.72	1.54	1.14	0.80	1.37	0.00	-0.00
time (sec)	N/A	0.364	0.409	0.016	0.461	0.543	11.724	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	193	235	287	241	396	0	-1
normalized size	1	1.00	0.67	0.82	1.00	0.84	1.38	0.00	-0.00
time (sec)	N/A	0.376	0.289	0.013	0.429	0.616	6.378	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	314	351	0	0	0	0	-1
normalized size	1	1.00	0.62	0.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.092	0.758	0.305	0.000	0.691	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	182	282	222	327	0	0	-1
normalized size	1	1.00	0.69	1.06	0.84	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.418	0.368	0.020	0.406	0.751	0.000	0.000	0.000

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	267	296	0	0	0	0	-1
normalized size	1	1.00	0.56	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.760	0.706	0.428	0.000	0.429	0.000	0.000	0.000

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	184	278	197	322	0	0	-1
normalized size	1	1.00	0.71	1.07	0.76	1.24	0.00	0.00	-0.00
time (sec)	N/A	0.462	0.394	0.022	0.574	0.808	0.000	0.000	0.000

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	265	331	415	333	600	0	-1
normalized size	1	1.00	0.67	0.84	1.05	0.84	1.52	0.00	-0.00
time (sec)	N/A	0.475	0.403	0.013	0.359	0.642	16.746	0.000	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	627	627	524	364	0	0	0	0	-1
normalized size	1	1.00	0.84	0.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.052	1.511	9.447	0.000	0.488	0.000	0.000	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	512	2912	0	0	0	0	-1
normalized size	1	1.00	0.98	5.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.911	0.563	0.682	0.000	0.584	0.000	0.000	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	457	284	0	0	0	0	-1
normalized size	1	1.00	0.84	0.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.902	0.748	1.360	0.000	0.751	0.000	0.000	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	447	2805	0	0	0	0	-1
normalized size	1	1.00	1.00	6.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.738	0.138	0.372	0.000	0.575	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	397	232	0	0	0	0	-1
normalized size	1	1.00	0.79	0.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.734	0.345	0.127	0.000	0.660	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	489	472	418	393	0	0	0	0	-1
normalized size	1	0.97	0.85	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.925	0.816	0.355	0.000	0.607	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	549	329	0	0	0	0	-1
normalized size	1	1.00	1.01	0.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.906	1.690	1.575	0.000	0.564	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	550	531	479	462	0	0	0	0	-1
normalized size	1	0.97	0.87	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.955	1.438	0.420	0.000	0.525	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	624	624	641	410	0	0	0	0	-1
normalized size	1	1.00	1.03	0.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.981	1.611	1.793	0.000	0.557	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	693	2964	0	0	0	0	-1
normalized size	1	1.00	1.23	5.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.991	2.146	0.929	0.000	0.663	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	123	638	0	537	0	0	-1
normalized size	1	1.00	1.09	5.65	0.00	4.75	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.328	0.039	0.000	0.542	0.000	0.000	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	598	581	0	529	0	0	0	0	-1
normalized size	1	0.97	0.00	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.062	5.074	0.444	0.000	0.624	0.000	0.000	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	634	616	0	723	0	0	0	0	-1
normalized size	1	0.97	0.00	1.14	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.087	6.027	0.639	0.000	0.586	0.000	0.000	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	839	839	777	1749	0	0	0	0	-1
normalized size	1	1.00	0.93	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.188	2.411	7.079	0.000	0.702	0.000	0.000	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	792	792	720	1689	0	0	0	0	-1
normalized size	1	1.00	0.91	2.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.969	1.916	2.916	0.000	0.645	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	804	804	734	1695	0	0	0	0	-1
normalized size	1	1.00	0.91	2.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.024	2.087	1.933	0.000	0.595	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	846	846	821	1821	0	0	0	0	-1
normalized size	1	1.00	0.97	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.022	2.852	6.415	0.000	0.672	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	737	737	1155	5196	0	0	0	0	-1
normalized size	1	1.00	1.57	7.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.182	7.260	1.564	0.000	0.620	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	241	192	2499	0	1217	0	0	-1
normalized size	1	1.04	0.83	10.82	0.00	5.27	0.00	0.00	-0.00
time (sec)	N/A	0.362	0.843	0.035	0.000	0.836	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	183	2443	0	1233	0	0	-1
normalized size	1	1.00	1.03	13.80	0.00	6.97	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.935	0.029	0.000	1.235	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	772	755	0	1478	0	0	0	0	-1
normalized size	1	0.98	0.00	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.246	8.337	0.503	0.000	0.677	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	C	F	F	F(-1)	F	F
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	834	815	0	1928	0	0	0	0	-1
normalized size	1	0.98	0.00	2.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.295	12.213	0.766	0.000	0.840	0.000	0.000	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1224	1224	1185	3125	0	0	0	0	-1
normalized size	1	1.00	0.97	2.55	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.962	6.903	4.243	0.000	0.545	0.000	0.000	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1193	2269	0	0	0	0	-1
normalized size	1	1.00	0.97	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.918	6.724	3.270	0.000	0.652	0.000	0.000	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1234	1234	1184	3128	0	0	0	0	-1
normalized size	1	1.00	0.96	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.460	6.395	3.127	0.000	0.565	0.000	0.000	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.033	5.780	0.558	0.000	0.539	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.030	3.809	0.602	0.000	0.542	0.000	0.000	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	556	0	0	332	0	0	-1
normalized size	1	1.00	5.50	0.00	0.00	3.29	0.00	0.00	-0.01
time (sec)	N/A	0.192	3.358	0.638	0.000	1.435	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	190	633	0	0	724	0	0	-1
normalized size	1	1.04	3.48	0.00	0.00	3.98	0.00	0.00	-0.01
time (sec)	N/A	0.183	2.407	0.712	0.000	0.870	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	685	0	0	1360	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	4.79	0.00	0.00	-0.00
time (sec)	N/A	0.801	4.238	0.832	0.000	0.827	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	558	529	397	0	0	0	0	0	-1
normalized size	1	0.95	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.809	1.417	180.000	0.000	0.715	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	332	293	0	0	0	0	0	-1
normalized size	1	0.94	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.560	0.485	180.000	0.000	0.657	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	187	186	0	0	0	0	0	-1
normalized size	1	0.94	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.607	180.000	0.000	0.637	0.000	0.000	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	9.994	1.200	0.000	0.478	0.000	0.000	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	9.376	1.233	0.000	0.685	0.000	0.000	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	20.398	1.084	0.000	1.220	0.000	0.000	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	609	609	453	632	684	586	996	0	-1
normalized size	1	1.00	0.74	1.04	1.12	0.96	1.64	0.00	-0.00
time (sec)	N/A	2.099	0.847	0.137	0.493	0.595	13.776	0.000	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	299	402	429	380	602	0	-1
normalized size	1	1.00	0.83	1.12	1.19	1.06	1.68	0.00	-0.00
time (sec)	N/A	1.199	0.541	0.117	0.425	0.717	4.768	0.000	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	174	217	218	209	286	0	-1
normalized size	1	1.00	1.04	1.29	1.30	1.24	1.70	0.00	-0.01
time (sec)	N/A	0.573	0.281	0.103	0.514	0.665	1.356	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	84	78	72	96	88	111	-1
normalized size	1	1.00	1.65	1.53	1.41	1.88	1.73	2.18	-0.02
time (sec)	N/A	0.157	0.084	0.068	0.309	0.698	0.274	0.480	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	763	763	623	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.308	0.575	0.629	0.000	0.658	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	15.858	0.378	0.000	0.563	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	11.417	0.466	0.000	0.724	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	3.766	0.444	0.000	0.571	0.000	0.000	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.047	7.529	0.444	0.000	0.612	0.000	0.000	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	388	380	254	380	0	0	0	0	-1
normalized size	1	0.98	0.65	0.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.793	0.535	0.443	0.000	1.148	0.000	0.000	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	176	125	178	0	0	0	0	-1
normalized size	1	1.27	0.90	1.28	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	0.226	0.416	0.000	0.815	0.000	0.000	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	56	0	0	0	0	-1
normalized size	1	1.00	0.85	1.04	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.070	0.064	0.035	0.000	0.672	0.000	0.000	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	0.603	0.456	0.000	0.667	0.000	0.000	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	3.302	0.666	0.000	0.672	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	1.179	0.340	0.000	0.579	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	1.106	0.389	0.000	0.594	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	1.562	0.375	0.000	1.203	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	3.922	0.374	0.000	0.455	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	498	663	1102	0	0	0	0	-1
normalized size	1	0.98	1.30	2.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.951	2.058	0.571	0.000	0.625	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	249	338	465	0	0	0	0	-1
normalized size	1	0.97	1.32	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.595	0.966	0.474	0.000	0.547	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	86	80	125	0	0	0	0	-1
normalized size	1	0.96	0.89	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.323	0.353	0.052	0.000	0.635	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	178.622	0.455	0.000	0.556	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.037	180.002	0.708	0.000	0.781	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.045	28.227	0.380	0.000	0.824	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.046	24.843	0.390	0.000	0.564	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	180.001	0.374	0.000	0.723	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	180.004	0.374	0.000	0.646	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	672	672	536	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.402	6.693	180.000	0.000	0.000	0.000	0.000	0.000

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	317	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.287	2.818	180.000	0.000	0.000	0.000	0.000	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	100	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.420	0.196	0.014	0.000	0.000	0.000	0.000	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	4.894	0.651	0.000	0.000	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.057	21.728	1.430	0.000	0.000	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	812	0	0	0	0	0	-1
normalized size	1	1.00	1.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.732	3.479	180.000	0.000	0.000	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	269	0	0	0	0	0	-1
normalized size	1	1.00	1.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.402	0.714	0.000	0.000	0.000	0.000	0.000	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.067	2.336	0.625	0.000	0.000	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	14.258	1.349	0.000	0.000	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	608	608	530	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.148	1.142	180.000	0.000	0.000	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	213	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.535	0.642	180.000	0.000	0.000	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	100	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.118	0.000	0.000	0.000	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	0.158	0.641	0.000	0.000	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	0.295	1.309	0.000	0.000	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-2)	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	268	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.831	1.969	180.000	0.000	0.000	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	132	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.416	0.138	0.002	0.000	0.000	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.069	0.177	0.634	0.000	0.000	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.064	0.298	1.299	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [486] had the largest ratio of [.9048]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	6	1.00	23	0.261
2	A	7	7	1.00	23	0.304
3	A	6	6	1.00	23	0.261
4	A	4	3	1.00	21	0.143
5	A	4	4	1.00	20	0.200
6	A	8	8	1.00	23	0.348
7	A	5	6	1.00	23	0.261
8	A	9	9	1.00	23	0.391
9	A	5	6	1.00	23	0.261
10	A	7	7	1.28	25	0.280
11	A	9	10	1.42	25	0.400
12	A	6	6	1.26	25	0.240
13	A	5	3	1.00	23	0.130
14	A	6	6	1.24	22	0.273
15	A	12	8	1.00	25	0.320
16	A	8	8	1.35	25	0.320
17	A	13	11	1.00	25	0.440
18	A	8	9	1.31	25	0.360
19	A	6	6	1.27	25	0.240
20	A	11	10	1.43	25	0.400
21	A	6	6	1.26	25	0.240
22	A	6	3	1.00	23	0.130
23	A	6	6	1.24	22	0.273
24	A	17	8	1.00	25	0.320
25	A	8	8	1.33	25	0.320
26	A	18	11	1.00	25	0.440
27	A	9	9	1.29	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
28	A	12	8	1.00	25	0.320
29	A	8	8	1.00	25	0.320
30	A	8	6	1.00	25	0.240
31	A	5	5	1.00	23	0.217
32	A	6	4	1.00	22	0.182
33	A	7	5	1.00	25	0.200
34	A	9	7	1.00	25	0.280
35	A	9	7	1.00	25	0.280
36	A	14	9	1.00	25	0.360
37	A	12	9	1.00	25	0.360
38	A	10	10	1.00	25	0.400
39	A	8	6	1.00	25	0.240
40	A	2	2	1.00	23	0.087
41	A	8	6	1.00	22	0.273
42	A	9	7	1.00	25	0.280
43	A	13	11	1.00	25	0.440
44	A	13	10	1.00	25	0.400
45	A	20	13	1.00	25	0.520
46	A	13	10	1.00	25	0.400
47	A	7	7	1.00	25	0.280
48	A	10	7	1.00	25	0.280
49	A	3	3	1.00	23	0.130
50	A	10	6	1.00	22	0.273
51	A	12	8	1.00	25	0.320
52	A	17	11	1.00	25	0.440
53	A	17	11	1.00	25	0.440
54	A	26	13	1.00	25	0.520
55	A	6	4	1.00	18	0.222
56	A	8	6	1.00	18	0.333
57	A	10	6	1.00	18	0.333
58	A	8	5	1.00	27	0.185
59	A	6	5	1.00	27	0.185
60	A	4	4	1.00	24	0.167
61	A	4	4	1.00	27	0.148
62	A	4	3	1.07	27	0.111
63	A	5	7	1.14	27	0.259

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	5	7	1.09	27	0.259
65	A	4	5	1.11	27	0.185
66	A	4	5	1.10	27	0.185
67	A	3	2	1.07	25	0.080
68	A	9	7	1.00	27	0.259
69	A	9	7	1.00	27	0.259
70	A	11	8	1.00	27	0.296
71	A	11	7	1.03	27	0.259
72	A	9	7	1.04	27	0.259
73	A	7	6	1.06	24	0.250
74	A	7	6	1.06	27	0.222
75	A	7	6	1.06	27	0.222
76	A	5	4	1.08	27	0.148
77	A	6	8	1.30	27	0.296
78	A	6	8	1.22	27	0.296
79	A	6	8	1.17	27	0.296
80	A	5	6	1.15	27	0.222
81	A	5	6	1.14	27	0.222
82	A	5	6	1.12	27	0.222
83	A	4	3	1.08	25	0.120
84	A	11	8	1.04	27	0.296
85	A	12	9	1.04	27	0.333
86	A	12	9	1.04	27	0.333
87	A	15	9	1.07	27	0.333
88	A	13	9	1.08	27	0.333
89	A	9	7	1.11	24	0.292
90	A	11	9	1.11	27	0.333
91	A	11	8	1.11	27	0.296
92	A	11	8	1.11	27	0.296
93	A	5	4	1.07	27	0.148
94	A	7	9	1.43	27	0.333
95	A	6	8	1.35	27	0.296
96	A	5	6	1.15	27	0.222
97	A	5	6	1.13	27	0.222
98	A	5	6	1.11	27	0.222
99	A	4	3	1.07	25	0.120

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
100	A	14	9	1.08	27	0.333
101	A	14	10	1.08	27	0.370
102	A	15	10	1.08	27	0.370
103	A	4	4	1.27	14	0.286
104	A	7	5	1.10	27	0.185
105	A	6	4	1.08	27	0.148
106	A	5	5	1.10	27	0.185
107	A	4	4	1.06	27	0.148
108	A	3	3	1.11	25	0.120
109	A	2	2	1.00	24	0.083
110	A	7	5	1.00	27	0.185
111	A	3	3	1.11	27	0.111
112	A	9	7	1.03	27	0.259
113	A	5	5	1.10	27	0.185
114	A	5	9	1.12	27	0.333
115	A	8	7	1.05	27	0.259
116	A	4	7	1.09	27	0.259
117	A	4	4	1.00	27	0.148
118	A	3	3	1.00	25	0.120
119	A	3	3	1.00	24	0.125
120	A	9	7	1.00	27	0.259
121	A	5	7	1.01	27	0.259
122	A	12	9	1.00	27	0.333
123	A	6	7	1.00	27	0.259
124	A	6	9	1.15	27	0.333
125	A	8	6	1.12	27	0.222
126	A	5	9	1.54	27	0.333
127	A	5	4	1.20	27	0.148
128	A	4	4	1.21	25	0.160
129	A	5	5	1.17	24	0.208
130	A	12	8	1.05	27	0.296
131	A	6	8	1.12	27	0.296
132	A	16	11	1.06	27	0.407
133	A	6	8	1.13	27	0.296
134	A	7	5	1.12	20	0.250
135	A	6	4	1.42	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
136	A	5	5	1.44	22	0.227
137	A	4	4	1.42	22	0.182
138	A	3	3	1.49	20	0.150
139	A	2	2	1.41	19	0.105
140	A	7	5	1.38	22	0.227
141	A	3	3	1.50	22	0.136
142	A	9	7	1.38	22	0.318
143	A	2	2	1.13	30	0.067
144	A	2	2	1.00	31	0.065
145	A	8	9	1.00	27	0.333
146	A	7	8	1.00	27	0.296
147	A	6	7	1.00	25	0.280
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	10	7	1.06	29	0.241
152	A	7	6	1.05	29	0.207
153	A	4	4	1.04	29	0.138
154	A	2	2	1.00	29	0.069
155	A	4	4	1.00	29	0.138
156	A	6	4	1.03	29	0.138
157	A	9	6	1.01	35	0.171
158	A	6	5	1.02	35	0.143
159	A	3	3	1.03	35	0.086
160	A	2	2	1.00	35	0.057
161	A	4	4	1.00	35	0.114
162	A	6	4	1.00	35	0.114
163	A	2	2	1.10	24	0.083
164	A	14	5	1.00	20	0.250
165	A	10	5	1.00	20	0.250
166	A	6	4	1.00	18	0.222
167	A	8	5	1.00	20	0.250
168	A	11	8	1.00	20	0.400
169	A	15	9	1.00	20	0.450
170	A	17	9	1.00	29	0.310
171	A	12	9	1.00	29	0.310

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
172	A	6	6	1.04	27	0.222
173	A	6	6	1.00	26	0.231
174	A	13	9	1.00	29	0.310
175	A	8	8	1.00	29	0.276
176	A	13	10	1.00	29	0.345
177	A	11	11	1.02	29	0.379
178	A	26	13	1.02	29	0.448
179	A	20	13	1.03	29	0.448
180	A	8	8	1.04	27	0.296
181	A	11	9	1.04	26	0.346
182	A	18	13	1.02	29	0.448
183	A	15	14	1.03	29	0.483
184	A	18	15	1.02	29	0.517
185	A	18	13	1.03	29	0.448
186	A	34	18	1.04	29	0.621
187	A	30	21	1.04	29	0.724
188	A	8	8	1.03	27	0.296
189	A	17	9	1.06	26	0.346
190	A	25	17	1.04	29	0.586
191	A	24	16	1.05	29	0.552
192	A	27	21	1.03	29	0.724
193	A	29	17	1.05	29	0.586
194	A	17	8	1.06	29	0.276
195	A	12	8	1.05	29	0.276
196	A	10	8	1.05	29	0.276
197	A	6	6	1.04	29	0.207
198	A	5	4	1.05	27	0.148
199	A	2	2	1.00	26	0.077
200	A	9	6	1.00	29	0.207
201	A	7	7	1.04	29	0.241
202	A	13	10	1.02	29	0.345
203	A	10	10	1.05	29	0.345
204	A	23	14	1.04	29	0.483
205	A	15	13	1.02	29	0.448
206	A	14	10	1.03	29	0.345
207	A	8	8	1.00	29	0.276

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
208	A	8	6	1.00	27	0.222
209	A	7	7	1.00	26	0.269
210	A	16	11	1.00	29	0.379
211	A	15	11	1.00	29	0.379
212	A	26	15	1.00	29	0.517
213	A	25	13	1.00	29	0.448
214	A	27	13	1.05	29	0.448
215	A	19	13	1.03	29	0.448
216	A	17	9	1.04	29	0.310
217	A	12	12	1.04	29	0.414
218	A	10	8	1.05	27	0.296
219	A	10	10	1.05	26	0.385
220	A	25	13	1.03	29	0.448
221	A	20	15	1.06	29	0.517
222	A	39	19	1.04	29	0.655
223	A	34	18	1.08	29	0.621
224	A	14	11	1.07	22	0.500
225	A	12	8	1.35	24	0.333
226	A	9	8	1.34	24	0.333
227	A	6	6	1.37	24	0.250
228	A	4	4	1.38	22	0.182
229	A	2	2	1.41	21	0.095
230	A	9	6	1.36	24	0.250
231	A	7	7	1.40	24	0.292
232	A	13	10	1.34	24	0.417
233	A	0	0	0.00	0	0.000
234	A	0	0	0.00	0	0.000
235	A	0	0	0.00	0	0.000
236	A	0	0	0.00	0	0.000
237	A	0	0	0.00	0	0.000
238	A	0	0	0.00	0	0.000
239	A	0	0	0.00	0	0.000
240	A	26	14	1.00	20	0.700
241	A	18	11	1.00	20	0.550
242	A	10	7	1.00	18	0.389
243	A	10	6	1.00	20	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
244	A	18	10	1.00	20	0.500
245	A	28	11	1.00	20	0.550
246	A	25	10	1.05	22	0.454
247	A	15	9	1.03	22	0.409
248	A	7	6	1.00	22	0.273
249	A	2	2	1.00	22	0.091
250	A	8	8	1.00	22	0.364
251	A	12	11	1.04	22	0.500
252	A	18	12	1.05	22	0.546
253	A	14	5	1.36	24	0.208
254	A	11	7	1.35	24	0.292
255	A	7	5	1.37	24	0.208
256	A	5	4	1.39	22	0.182
257	A	2	2	1.41	21	0.095
258	A	11	7	1.34	24	0.292
259	A	8	8	1.38	24	0.333
260	A	19	11	1.33	24	0.458
261	A	0	0	0.00	0	0.000
262	A	7	3	1.00	20	0.150
263	A	6	3	1.00	20	0.150
264	A	5	3	1.00	18	0.167
265	A	0	0	0.00	0	0.000
266	A	0	0	0.00	0	0.000
267	A	13	6	1.27	28	0.214
268	A	13	6	1.25	28	0.214
269	A	7	6	1.28	28	0.214
270	A	10	6	1.24	26	0.231
271	A	7	6	1.28	25	0.240
272	A	0	0	0.00	0	0.000
273	A	0	0	0.00	0	0.000
274	A	0	0	0.00	0	0.000
275	A	0	0	0.00	0	0.000
276	A	16	6	1.25	28	0.214
277	A	13	6	1.27	28	0.214
278	A	13	6	1.25	26	0.231
279	A	10	6	1.27	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	0	0	0.00	0	0.000
281	A	0	0	0.00	0	0.000
282	A	0	0	0.00	0	0.000
283	A	0	0	0.00	0	0.000
284	A	16	6	1.25	28	0.214
285	A	16	6	1.27	28	0.214
286	A	16	6	1.25	26	0.231
287	A	13	6	1.27	25	0.240
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	0	0	0.00	0	0.000
292	A	6	4	1.40	24	0.167
293	A	6	4	1.40	24	0.167
294	A	5	4	1.40	24	0.167
295	A	3	3	1.46	22	0.136
296	A	2	2	1.46	21	0.095
297	A	0	0	0.00	0	0.000
298	A	0	0	0.00	0	0.000
299	A	10	6	1.24	28	0.214
300	A	7	6	1.28	28	0.214
301	A	5	5	1.24	26	0.192
302	A	2	2	1.37	25	0.080
303	A	0	0	0.00	0	0.000
304	A	0	0	0.00	0	0.000
305	A	0	0	0.00	0	0.000
306	A	0	0	0.00	0	0.000
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000
309	A	0	0	0.00	0	0.000
310	A	0	0	0.00	0	0.000
311	A	0	0	0.00	0	0.000
312	A	0	0	0.00	0	0.000
313	A	0	0	0.00	0	0.000
314	A	0	0	0.00	0	0.000
315	A	8	4	1.00	20	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
316	A	7	4	1.00	20	0.200
317	A	6	4	1.00	18	0.222
318	A	0	0	0.00	0	0.000
319	A	0	0	0.00	0	0.000
320	A	23	7	1.23	28	0.250
321	A	17	8	1.20	28	0.286
322	A	15	8	1.69	26	0.308
323	A	8	8	1.21	25	0.320
324	A	0	0	0.00	0	0.000
325	A	0	0	0.00	0	0.000
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	20	7	1.24	28	0.250
329	A	23	9	1.23	26	0.346
330	A	11	7	1.24	25	0.280
331	A	0	0	0.00	0	0.000
332	A	0	0	0.00	0	0.000
333	A	0	0	0.00	0	0.000
334	A	0	0	0.00	0	0.000
335	A	29	7	1.24	28	0.250
336	A	29	9	1.24	26	0.346
337	A	14	7	1.24	25	0.280
338	A	0	0	0.00	0	0.000
339	A	0	0	0.00	0	0.000
340	A	0	0	0.00	0	0.000
341	A	0	0	0.00	0	0.000
342	A	14	7	1.26	28	0.250
343	A	11	7	1.28	28	0.250
344	A	11	7	1.26	28	0.250
345	A	8	8	1.29	28	0.286
346	A	6	6	1.30	26	0.231
347	A	2	2	1.35	25	0.080
348	A	0	0	0.00	0	0.000
349	A	0	0	0.00	0	0.000
350	A	0	0	0.00	0	0.000
351	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	0	0	0.00	0	0.000
353	A	0	0	0.00	0	0.000
354	A	0	0	0.00	0	0.000
355	A	0	0	0.00	0	0.000
356	A	0	0	0.00	0	0.000
357	A	0	0	0.00	0	0.000
358	A	0	0	0.00	0	0.000
359	A	0	0	0.00	0	0.000
360	A	0	0	0.00	0	0.000
361	A	0	0	0.00	0	0.000
362	A	0	0	0.00	0	0.000
363	A	0	0	0.00	0	0.000
364	A	0	0	0.00	0	0.000
365	A	0	0	0.00	0	0.000
366	A	0	0	0.00	0	0.000
367	A	0	0	0.00	0	0.000
368	A	2	2	1.41	21	0.095
369	A	27	7	1.04	27	0.259
370	A	32	7	1.03	27	0.259
371	A	17	9	1.04	25	0.360
372	A	14	7	1.00	24	0.292
373	A	0	0	0.00	0	0.000
374	A	32	7	1.03	29	0.241
375	A	42	7	1.03	29	0.241
376	A	32	9	1.03	27	0.333
377	A	19	7	1.00	26	0.269
378	A	0	0	0.00	0	0.000
379	A	25	12	1.03	24	0.500
380	A	11	10	1.00	24	0.417
381	A	2	2	1.00	24	0.083
382	A	0	0	0.00	0	0.000
383	A	0	0	0.00	0	0.000
384	A	27	13	1.02	24	0.542
385	A	12	10	1.00	24	0.417
386	A	2	2	1.00	24	0.083
387	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
388	A	40	15	1.02	24	0.625
389	A	14	12	1.00	24	0.500
390	A	2	2	1.00	24	0.083
391	A	0	0	0.00	0	0.000
392	A	25	12	1.02	24	0.500
393	A	11	10	1.00	24	0.417
394	A	2	2	1.00	24	0.083
395	A	0	0	0.00	0	0.000
396	A	0	0	0.00	0	0.000
397	A	27	13	1.02	24	0.542
398	A	12	10	1.00	24	0.417
399	A	2	2	1.00	24	0.083
400	A	0	0	0.00	0	0.000
401	A	7	6	1.28	19	0.316
402	A	19	7	1.00	24	0.292
403	A	14	7	1.00	24	0.292
404	A	9	7	1.00	24	0.292
405	A	2	2	1.00	24	0.083
406	A	0	0	0.00	0	0.000
407	A	0	0	0.00	0	0.000
408	A	20	8	1.03	24	0.333
409	A	15	8	1.03	24	0.333
410	A	10	9	1.04	24	0.375
411	A	2	2	1.00	24	0.083
412	A	0	0	0.00	0	0.000
413	A	0	0	0.00	0	0.000
414	A	19	11	1.02	24	0.458
415	A	8	7	1.03	24	0.292
416	A	2	2	1.00	24	0.083
417	A	0	0	0.00	0	0.000
418	A	0	0	0.00	0	0.000
419	A	7	5	1.00	29	0.172
420	A	10	5	1.00	27	0.185
421	A	7	5	1.00	26	0.192
422	A	0	0	0.00	0	0.000
423	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	13	5	1.00	29	0.172
425	A	13	5	1.00	27	0.185
426	A	10	5	1.00	26	0.192
427	A	0	0	0.00	0	0.000
428	A	0	0	0.00	0	0.000
429	A	16	5	1.00	29	0.172
430	A	16	5	1.00	27	0.185
431	A	13	5	1.00	26	0.192
432	A	0	0	0.00	0	0.000
433	A	0	0	0.00	0	0.000
434	A	10	5	1.16	28	0.179
435	A	7	5	1.18	28	0.179
436	A	5	4	1.17	26	0.154
437	A	2	2	1.30	25	0.080
438	A	0	0	0.00	0	0.000
439	A	0	0	0.00	0	0.000
440	A	10	5	1.00	29	0.172
441	A	7	5	1.00	29	0.172
442	A	5	4	1.00	27	0.148
443	A	2	2	1.00	26	0.077
444	A	0	0	0.00	0	0.000
445	A	0	0	0.00	0	0.000
446	A	0	0	0.00	0	0.000
447	A	0	0	0.00	0	0.000
448	A	0	0	0.00	0	0.000
449	A	0	0	0.00	0	0.000
450	A	0	0	0.00	0	0.000
451	A	0	0	0.00	0	0.000
452	A	0	0	0.00	0	0.000
453	A	0	0	0.00	0	0.000
454	A	0	0	0.00	0	0.000
455	A	0	0	0.00	0	0.000
456	A	0	0	0.00	0	0.000
457	A	0	0	0.00	0	0.000
458	A	0	0	0.00	0	0.000
459	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
460	A	0	0	0.00	0	0.000
461	A	7	5	1.00	19	0.263
462	A	6	6	1.00	19	0.316
463	A	5	5	1.00	19	0.263
464	A	4	4	1.00	17	0.235
465	A	3	3	1.00	16	0.188
466	A	13	13	1.00	19	0.684
467	A	4	4	1.00	19	0.210
468	A	11	11	1.00	19	0.579
469	A	4	4	1.00	19	0.210
470	A	7	7	1.00	21	0.333
471	A	9	10	1.00	21	0.476
472	A	6	6	1.00	21	0.286
473	A	7	7	1.00	19	0.368
474	A	6	6	1.00	18	0.333
475	A	16	15	1.08	21	0.714
476	A	7	7	1.16	21	0.333
477	A	14	15	1.00	21	0.714
478	A	7	8	1.00	21	0.381
479	A	6	6	1.00	21	0.286
480	A	10	9	1.00	21	0.429
481	A	6	6	1.00	21	0.286
482	A	8	7	1.00	19	0.368
483	A	6	6	1.00	18	0.333
484	A	23	15	1.00	21	0.714
485	A	7	7	1.00	21	0.333
486	A	18	19	1.00	21	0.905
487	A	9	9	1.00	21	0.429
488	A	6	6	1.00	18	0.333
489	A	27	12	1.00	21	0.571
490	A	23	9	1.00	21	0.429
491	A	23	9	1.00	21	0.429
492	A	18	6	1.00	19	0.316
493	A	18	6	1.00	18	0.333
494	A	25	8	0.97	21	0.381
495	A	23	10	1.00	21	0.476

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
496	A	27	10	0.97	21	0.476
497	A	28	12	1.00	21	0.571
498	A	24	10	1.00	21	0.476
499	A	4	4	1.00	19	0.210
500	A	29	12	0.97	21	0.571
501	A	31	14	0.97	21	0.667
502	A	49	12	1.00	21	0.571
503	A	46	10	1.00	21	0.476
504	A	26	9	1.00	18	0.500
505	A	49	13	1.00	21	0.619
506	A	29	11	1.00	21	0.524
507	A	9	10	1.04	21	0.476
508	A	5	5	1.00	19	0.263
509	A	34	13	0.98	21	0.619
510	A	36	15	0.98	21	0.714
511	A	80	11	1.00	21	0.524
512	A	62	11	1.00	21	0.524
513	A	34	10	1.00	18	0.556
514	A	0	0	0.00	0	0.000
515	A	0	0	0.00	0	0.000
516	A	7	8	1.00	20	0.400
517	A	8	10	1.04	20	0.500
518	A	9	11	1.00	20	0.550
519	A	8	9	0.95	23	0.391
520	A	7	8	0.94	23	0.348
521	A	5	5	0.94	21	0.238
522	A	0	0	0.00	0	0.000
523	A	0	0	0.00	0	0.000
524	A	0	0	0.00	0	0.000
525	A	26	7	1.00	20	0.350
526	A	17	7	1.00	20	0.350
527	A	10	7	1.00	18	0.389
528	A	3	3	1.00	10	0.300
529	A	22	7	1.00	20	0.350
530	A	0	0	0.00	0	0.000
531	A	0	0	0.00	0	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
532	A	0	0	0.00	0	0.000
533	A	0	0	0.00	0	0.000
534	A	27	7	0.98	20	0.350
535	A	15	7	1.27	18	0.389
536	A	4	4	1.00	10	0.400
537	A	0	0	0.00	0	0.000
538	A	0	0	0.00	0	0.000
539	A	0	0	0.00	0	0.000
540	A	0	0	0.00	0	0.000
541	A	0	0	0.00	0	0.000
542	A	0	0	0.00	0	0.000
543	A	26	7	0.98	20	0.350
544	A	15	7	0.97	18	0.389
545	A	5	5	0.96	10	0.500
546	A	0	0	0.00	0	0.000
547	A	0	0	0.00	0	0.000
548	A	0	0	0.00	0	0.000
549	A	0	0	0.00	0	0.000
550	A	0	0	0.00	0	0.000
551	A	0	0	0.00	0	0.000
552	A	42	9	1.00	22	0.409
553	A	23	9	1.00	20	0.450
554	A	7	6	1.00	12	0.500
555	A	0	0	0.00	0	0.000
556	A	0	0	0.00	0	0.000
557	A	32	12	1.00	20	0.600
558	A	8	7	1.00	12	0.583
559	A	0	0	0.00	0	0.000
560	A	0	0	0.00	0	0.000
561	A	39	8	1.00	22	0.364
562	A	21	8	1.00	20	0.400
563	A	6	5	1.00	12	0.417
564	A	0	0	0.00	0	0.000
565	A	0	0	0.00	0	0.000
566	A	21	8	1.00	20	0.400
567	A	7	6	1.00	12	0.500

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	0	0	0.00	0	0.000
569	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int x^4\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=151

$$-\frac{1}{7}c^2dx^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{152bd\sqrt{cx-1}\sqrt{cx+1}}{3675c^5}-\frac{76bdx^2\sqrt{cx-1}\sqrt{cx+1}}{3675c^3}+\frac{1}{49}b$$

```
[Out] 1/5*d*x^5*(a+b*arccosh(c*x))-1/7*c^2*d*x^7*(a+b*arccosh(c*x))-152/3675*b*d*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-76/3675*b*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3-19/1225*b*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/49*b*c*d*x^6*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 460, 100, 74}

$$-\frac{1}{7}c^2dx^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{76bdx^2\sqrt{cx-1}\sqrt{cx+1}}{3675c^3}-\frac{152bd\sqrt{cx-1}\sqrt{cx+1}}{3675c^5}+\frac{1}{49}b$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-152*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (76*b*d*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(3675*c^3) - (19*b*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/
(1225*c) + (b*c*d*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/49 + (d*x^5*(a + b*ArcC
osh[c*x]))/5 - (c^2*d*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{7}c^2dx^7\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{dx^5\,(}{35\sqrt{-1}}$$

$$= \frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{7}c^2dx^7\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{35}(bcd)\int\frac{x}{\sqrt{-}}$$

$$= \frac{1}{49}bcdx^6\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{7}c^2dx^7\left(a+b\right.$$

$$= -\frac{19bdx^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c}+\frac{1}{49}bcdx^6\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{5}dx^5\left(a+\right.$$

$$= -\frac{19bdx^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c}+\frac{1}{49}bcdx^6\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{5}dx^5\left(a+\right.$$

$$= -\frac{76bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^3}-\frac{19bdx^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c}+\frac{1}{49}bcdx^6\sqrt{-}$$

$$= -\frac{76bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^3}-\frac{19bdx^4\sqrt{-1+cx}\sqrt{1+cx}}{1225c}+\frac{1}{49}bcdx^6\sqrt{-}$$

$$= -\frac{152bd\sqrt{-1+cx}\sqrt{1+cx}}{3675c^5}-\frac{76bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{3675c^3}-\frac{19bdx^4\sqrt{-1}}{122}$$

Mathematica [A] time = 0.17, size = 91, normalized size = 0.60

$$\frac{d\left(-105ax^5\left(5c^2x^2-7\right)-105bx^5\left(5c^2x^2-7\right)\cosh^{-1}(cx)+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(75c^6x^6-57c^4x^4-76c^2x^2-152\right)}{c^5}\right)}{3675}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-105*a*x^5*(-7 + 5*c^2*x^2) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-152 - 7
6*c^2*x^2 - 57*c^4*x^4 + 75*c^6*x^6))/c^5 - 105*b*x^5*(-7 + 5*c^2*x^2)*ArcC
osh[c*x]))/3675
```

fricas [A] time = 0.48, size = 113, normalized size = 0.75

$$-\frac{525\,ac^7dx^7 - 735\,ac^5dx^5 + 105\left(5\,bc^7dx^7 - 7\,bc^5dx^5\right)\log\left(cx + \sqrt{c^2x^2 - 1}\right) - \left(75\,bc^6dx^6 - 57\,bc^4dx^4 - 76\,bc^2}{3675\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d*x^7 - 735*a*c^5*d*x^5 + 105*(5*b*c^7*d*x^7 - 7*b*c^5*d
*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d*x^6 - 57*b*c^4*d*x^4 - 76*
b*c^2*d*x^2 - 152*b*d)*sqrt(c^2*x^2 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 98, normalized size = 0.65

$$\frac{-da\left(\frac{1}{7}c^7x^7 - \frac{1}{5}c^5x^5\right) - db\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7} - \frac{\operatorname{arccosh}(cx)c^5x^5}{5} - \frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75c^6x^6-57c^4x^4-76c^2x^2-152\right)}{3675}\right)}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(-d*a*(1/7*c^7*x^7-1/5*c^5*x^5)-d*b*(1/7*arccosh(c*x)*c^7*x^7-1/5*arc
cosh(c*x)*c^5*x^5-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-57*c^4*x^4
-76*c^2*x^2-152)))
```

maxima [A] time = 0.34, size = 184, normalized size = 1.22

$$-\frac{1}{7}\,ac^2dx^7+\frac{1}{5}\,adx^5-\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx) - \left(\frac{5\,\sqrt{c^2x^2-1}\,x^6}{c^2} + \frac{6\,\sqrt{c^2x^2-1}\,x^4}{c^4} + \frac{8\,\sqrt{c^2x^2-1}\,x^2}{c^6} + \frac{16\,\sqrt{c^2x^2-}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^2*d*x^7 + 1/5*a*d*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x
^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6
 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
/c^6)*c)*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \, (a + b \operatorname{acosh}(c\,x)) \left(d - c^2\,d\,x^2\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

sympy [A] time = 6.02, size = 158, normalized size = 1.05

$$\begin{cases} -\frac{ac^2dx^7}{7} + \frac{adx^5}{5} - \frac{bc^2dx^7 \operatorname{acosh}(cx)}{7} + \frac{bcdx^6\sqrt{c^2x^2-1}}{49} + \frac{bdx^5 \operatorname{acosh}(cx)}{5} - \frac{19bdx^4\sqrt{c^2x^2-1}}{1225c} - \frac{76bdx^2\sqrt{c^2x^2-1}}{3675c^3} - \frac{152bd\sqrt{c^2x^2-1}}{3675c^5} & \text{for} \\ \frac{dx^5\left(a+\frac{i\pi b}{2}\right)}{5} & \text{oth} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**7/7 + a*d*x**5/5 - b*c**2*d*x**7*acosh(c*x)/7 + b*c
*d*x**6*sqrt(c**2*x**2 - 1)/49 + b*d*x**5*acosh(c*x)/5 - 19*b*d*x**4*sqrt(c
**2*x**2 - 1)/(1225*c) - 76*b*d*x**2*sqrt(c**2*x**2 - 1)/(3675*c**3) - 152*
b*d*sqrt(c**2*x**2 - 1)/(3675*c**5), Ne(c, 0)), (d*x**5*(a + I*pi*b/2)/5, T
rue))
```

3.2 $\int x^3 \left(d - c^2 d x^2\right) \left(a + b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=135

$$-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bd\cosh^{-1}(cx)}{24c^4}-\frac{bdx\sqrt{cx-1}\sqrt{cx+1}}{24c^3}+\frac{1}{36}bcdx^5\sqrt{cx-1}$$

```
[Out] -1/24*b*d*arccosh(c*x)/c^4+1/4*d*x^4*(a+b*arccosh(c*x))-1/6*c^2*d*x^6*(a+b*
arccosh(c*x))-1/24*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*d*x^3*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c+1/36*b*c*d*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {14, 5731, 12, 460, 100, 90, 52}

$$-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bdx\sqrt{cx-1}\sqrt{cx+1}}{24c^3}-\frac{bd\cosh^{-1}(cx)}{24c^4}+\frac{1}{36}bcdx^5\sqrt{cx-1}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^3) - (b*d*x^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/36 - (b*d*Ar
cCosh[c*x])/(24*c^4) + (d*x^4*(a + b*ArcCosh[c*x]))/4 - (c^2*d*x^6*(a + b*A
rcCosh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{dx^4\left(\right.}{12\sqrt{-1}}$$

$$= \frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{12}(bcd)\int\frac{x}{\sqrt{-}}$$

$$= \frac{1}{36}bcdx^5\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{6}c^2dx^6\left(a+b\right.$$

$$= -\frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{4}dx^4\left(a+b\right.$$

$$= -\frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{4}dx^4\left(a+b\right.$$

$$= -\frac{bdx\sqrt{-1+cx}\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+c}$$

$$= -\frac{bdx\sqrt{-1+cx}\sqrt{1+cx}}{24c^3}-\frac{bdx^3\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{36}bcdx^5\sqrt{-1+c}$$

Mathematica [A] time = 0.10, size = 166, normalized size = 1.23

$$-\frac{1}{6}ac^2dx^6+\frac{1}{4}adx^4-\frac{bd\tanh^{-1}\left(\frac{\sqrt{cx-1}}{\sqrt{cx+1}}\right)}{12c^4}-\frac{bdx\sqrt{cx-1}\sqrt{cx+1}}{24c^3}-\frac{1}{6}bc^2dx^6\cosh^{-1}(cx)+\frac{1}{36}bcdx^5\sqrt{cx-1}\sqrt{cx+1}+\frac{1}{4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*d*x^4)/4 - (a*c^2*d*x^6)/6 - (b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(24*c^
3) - (b*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(36*c) + (b*c*d*x^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/36 + (b*d*x^4*ArcCosh[c*x])/4 - (b*c^2*d*x^6*ArcCosh[c*x
])/6 - (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/(12*c^4)
```

fricas [A] time = 0.42, size = 108, normalized size = 0.80

$$-\frac{12\,ac^6dx^6 - 18\,ac^4dx^4 + 3\left(4\,bc^6dx^6 - 6\,bc^4dx^4 + bd\right)\log\left(cx + \sqrt{c^2x^2-1}\right) - \left(2\,bc^5dx^5 - 2\,bc^3dx^3 - 3\,bcdx\right)\sqrt{}}{72\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/72*(12*a*c^6*d*x^6 - 18*a*c^4*d*x^4 + 3*(4*b*c^6*d*x^6 - 6*b*c^4*d*x^4 +
 b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d*x^5 - 2*b*c^3*d*x^3 - 3*b*c
*d*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 160, normalized size = 1.19

$$-\frac{c^2da\,x^6}{6}+\frac{da\,x^4}{4}-\frac{c^2db\,\mathrm{arccosh}\,(cx)\,x^6}{6}+\frac{db\,\mathrm{arccosh}\,(cx)\,x^4}{4}+\frac{bcd\,x^5\sqrt{cx-1}\,\sqrt{cx+1}}{36}-\frac{bd\,x^3\sqrt{cx-1}\,\sqrt{cx+1}}{36c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/6*c^2*d*a*x^6+1/4*d*a*x^4-1/6*c^2*d*b*arccosh(c*x)*x^6+1/4*d*b*arccosh(c
*x)*x^4+1/36*b*c*d*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/36*b*d*x^3*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c-1/24*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/24/c^4*d*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.33, size = 202, normalized size = 1.50

$$-\frac{1}{6}\,ac^2dx^6+\frac{1}{4}\,adx^4-\frac{1}{288}\left(48\,x^6\,\mathrm{arcosh}\,(cx)-\left(\frac{8\,\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\,\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\,\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\,\log\left(2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/6*a*c^2*d*x^6 + 1/4*a*d*x^4 - 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x
^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6
 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d + 1/32*(8*x^4*ar
ccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

sympy [A] time = 3.98, size = 144, normalized size = 1.07

$$\begin{cases} -\frac{ac^2dx^6}{6} + \frac{adx^4}{4} - \frac{bc^2dx^6 \operatorname{acosh}(cx)}{6} + \frac{bcdx^5\sqrt{c^2x^2-1}}{36} + \frac{bdx^4 \operatorname{acosh}(cx)}{4} - \frac{bdx^3\sqrt{c^2x^2-1}}{36c} - \frac{bdx\sqrt{c^2x^2-1}}{24c^3} - \frac{bd \operatorname{acosh}(cx)}{24c^4} & \text{for } c \neq 0 \\ \frac{dx^4\left(a+\frac{i\pi b}{2}\right)}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**6/6 + a*d*x**4/4 - b*c**2*d*x**6*acosh(c*x)/6 + b*c
*d*x**5*sqrt(c**2*x**2 - 1)/36 + b*d*x**4*acosh(c*x)/4 - b*d*x**3*sqrt(c**2
*x**2 - 1)/(36*c) - b*d*x*sqrt(c**2*x**2 - 1)/(24*c**3) - b*d*acosh(c*x)/(2
4*c**4), Ne(c, 0)), (d*x**4*(a + I*pi*b/2)/4, True))
```

3.3 $\int x^2 \left(d - c^2 d x^2\right) \left(a + b \cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=121

$$-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{26bd\sqrt{cx-1}\sqrt{cx+1}}{225c^3}+\frac{1}{25}bcdx^4\sqrt{cx-1}\sqrt{cx+1}-\frac{13b}{}$$

```
[Out] 1/3*d*x^3*(a+b*arccosh(c*x))-1/5*c^2*d*x^5*(a+b*arccosh(c*x))-26/225*b*d*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-13/225*b*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+
1/25*b*c*d*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {14, 5731, 12, 460, 100, 74}

$$-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{26bd\sqrt{cx-1}\sqrt{cx+1}}{225c^3}+\frac{1}{25}bcdx^4\sqrt{cx-1}\sqrt{cx+1}-\frac{13b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-26*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (13*b*d*x^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])/(225*c) + (b*c*d*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/25 + (
d*x^3*(a + b*ArcCosh[c*x]))/3 - (c^2*d*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
```

```
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)-(bc)\int\frac{dx^3\left(\right.}{15\sqrt{-1}}$$

$$=\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}(bcd)\int\frac{x}{\sqrt{-}}$$

$$=\frac{1}{25}bcdx^4\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{5}c^2dx^5\left(a+b\right.$$

$$=-\frac{13bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{3}dx^3\left(a+\right.$$

$$=-\frac{13bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1+cx}\sqrt{1+cx}+\frac{1}{3}dx^3\left(a+\right.$$

$$=-\frac{26bd\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{13bdx^2\sqrt{-1+cx}\sqrt{1+cx}}{225c}+\frac{1}{25}bcdx^4\sqrt{-1}$$

Mathematica [A] time = 0.11, size = 89, normalized size = 0.74

$$-\frac{d\left(15ac^3x^3\left(3c^2x^2-5\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-9c^4x^4+13c^2x^2+26\right)+15bc^3x^3\left(3c^2x^2-5\right)\cosh^{-1}(cx)\right)}{225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/225*(d*(15*a*c^3*x^3*(-5 + 3*c^2*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
26 + 13*c^2*x^2 - 9*c^4*x^4) + 15*b*c^3*x^3*(-5 + 3*c^2*x^2)*ArcCosh[c*x]))
/c^3
```

fricas [A] time = 0.49, size = 103, normalized size = 0.85

$$-\frac{45\,ac^5dx^5-75\,ac^3dx^3+15\left(3\,bc^5dx^5-5\,bc^3dx^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4dx^4-13\,bc^2dx^2-26\,bd\right)\sqrt{c^2x^2-}}{225\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/225*(45*a*c^5*d*x^5 - 75*a*c^3*d*x^3 + 15*(3*b*c^5*d*x^5 - 5*b*c^3*d*x^3
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*d*x^4 - 13*b*c^2*d*x^2 - 26*b*d)*
sqrt(c^2*x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 90, normalized size = 0.74

$$\frac{-da\left(\frac{1}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-db\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-13c^2x^2-26)}{225}\right)}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(-d*a*(1/5*c^5*x^5-1/3*c^3*x^3)-d*b*(1/5*arccosh(c*x)*c^5*x^5-1/3*c^3
*x^3*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*x^4-13*c^2*x^2-2
6)))
```

maxima [A] time = 0.32, size = 145, normalized size = 1.20

$$-\frac{1}{5}\,ac^2dx^5-\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^2d+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*a*c^2*d*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2
 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^2*d + 1/3*
a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c
^2*x^2 - 1)/c^4))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

sympy [A] time = 2.28, size = 133, normalized size = 1.10

$$\begin{cases} -\frac{ac^2dx^5}{5}+\frac{adx^3}{3}-\frac{bc^2dx^5\operatorname{acosh}(cx)}{5}+\frac{bcdx^4\sqrt{c^2x^2-1}}{25}+\frac{bdx^3\operatorname{acosh}(cx)}{3}-\frac{13bdx^2\sqrt{c^2x^2-1}}{225c}-\frac{26bd\sqrt{c^2x^2-1}}{225c^3} & \text{for } c\neq 0\\ \frac{dx^3\left(a+\frac{i\pi b}{2}\right)}{3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**5/5 + a*d*x**3/3 - b*c**2*d*x**5*acosh(c*x)/5 + b*c
*d*x**4*sqrt(c**2*x**2 - 1)/25 + b*d*x**3*acosh(c*x)/3 - 13*b*d*x**2*sqrt(c
**2*x**2 - 1)/(225*c) - 26*b*d*sqrt(c**2*x**2 - 1)/(225*c**3), Ne(c, 0)), (
d*x**3*(a + I*pi*b/2)/3, True))
```

3.4 $\int x\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=98

$$-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{3bd\cosh^{-1}(cx)}{32c^2}+\frac{bdx(cx-1)^{3/2}(cx+1)^{3/2}}{16c}-\frac{3bdx\sqrt{cx-1}\sqrt{cx+1}}{32c}$$

```
[Out] 1/16*b*d*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c+3/32*b*d*arccosh(c*x)/c^2-1/4*d*(-
c^2*x^2+1)^2*(a+b*arccosh(c*x))/c^2-3/32*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c
```

Rubi [A] time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5716, 38, 52}

$$-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{3bd\cosh^{-1}(cx)}{32c^2}+\frac{bdx(cx-1)^{3/2}(cx+1)^{3/2}}{16c}-\frac{3bdx\sqrt{cx-1}\sqrt{cx+1}}{32c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c) + (b*d*x*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(16*c) + (3*b*d*ArcCosh[c*x])/(32*c^2) - (d*(1 - c^2*x^2)^2*
(a + b*ArcCosh[c*x]))/(4*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}+\frac{(bd)\int(-1+cx)^{3/2}(1+cx)^{3/2}\,dx}{4c}$$
$$=\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{4c^2}-\frac{(3bd}{}$$
$$=-\frac{3bdx\sqrt{-1+cx}\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}-\frac{d\left(1-c^2x^2\right)}{}$$
$$=-\frac{3bdx\sqrt{-1+cx}\sqrt{1+cx}}{32c}+\frac{bdx(-1+cx)^{3/2}(1+cx)^{3/2}}{16c}+\frac{3bd\cosh^{-1}}{32c^2}$$

Mathematica [A] time = 0.16, size = 100, normalized size = 1.02

$$-\frac{d\left(cx\left(8acx\left(c^2x^2-2\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(5-2c^2x^2\right)\right)+8bc^2x^2\left(c^2x^2-2\right)\cosh^{-1}(cx)+10b\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right.\right.}{32c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/32*(d*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5 - 2*c^2*x^2) + 8*a*c*x*(-2
 + c^2*x^2)) + 8*b*c^2*x^2*(-2 + c^2*x^2)*ArcCosh[c*x] + 10*b*ArcTanh[Sqrt[
(-1 + c*x)/(1 + c*x)]]))/c^2
```

fricas [A] time = 0.76, size = 98, normalized size = 1.00

$$-\frac{8\,ac^4dx^4-16\,ac^2dx^2+\left(8\,bc^4dx^4-16\,bc^2dx^2+5\,bd\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^3dx^3-5\,bcdx\right)\sqrt{c^2x^2-1}}{32\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/32*(8*a*c^4*d*x^4 - 16*a*c^2*d*x^2 + (8*b*c^4*d*x^4 - 16*b*c^2*d*x^2 + 5
*b*d)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*d*x^3 - 5*b*c*d*x)*sqrt(c^2*x
^2 - 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 136, normalized size = 1.39

$$-\frac{c^2da\,x^4}{4}+\frac{da\,x^2}{2}-\frac{c^2db\,\mathrm{arccosh}\,(cx)\,x^4}{4}+\frac{db\,\mathrm{arccosh}\,(cx)\,x^2}{2}+\frac{cdb\sqrt{cx-1}\,\sqrt{cx+1}\,x^3}{16}-\frac{5bdx\sqrt{cx-1}\,\sqrt{cx+1}}{32c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/4*c^2*d*a*x^4+1/2*d*a*x^2-1/4*c^2*d*b*arccosh(c*x)*x^4+1/2*d*b*arccosh(c
*x)*x^2+1/16*c*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-5/32*b*d*x*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c-5/32/c^2*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)
*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.32, size = 162, normalized size = 1.65

$$-\frac{1}{4}ac^2dx^4-\frac{1}{32}\left(8x^4\operatorname{arcosh}(cx)-\left(\frac{2\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^5}\right)c\right)bc^2d+\frac{1}{2}ad$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/4*a*c^2*d*x^4 - 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2
+ 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c
)*b*c^2*d + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/
c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\operatorname{acosh}(c\,x)\right)\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

sympy [A] time = 1.27, size = 124, normalized size = 1.27

$$\begin{cases} -\frac{ac^2dx^4}{4}+\frac{adx^2}{2}-\frac{bc^2dx^4\operatorname{acosh}(cx)}{4}+\frac{bcdx^3\sqrt{c^2x^2-1}}{16}+\frac{bdx^2\operatorname{acosh}(cx)}{2}-\frac{5bdx\sqrt{c^2x^2-1}}{32c}-\frac{5bd\operatorname{acosh}(cx)}{32c^2} & \text{for}\ c\neq 0\\ \frac{dx^2\left(a+\frac{i\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**4/4 + a*d*x**2/2 - b*c**2*d*x**4*acosh(c*x)/4 + b*c
*d*x**3*sqrt(c**2*x**2 - 1)/16 + b*d*x**2*acosh(c*x)/2 - 5*b*d*x*sqrt(c**2*
x**2 - 1)/(32*c) - 5*b*d*acosh(c*x)/(32*c**2), Ne(c, 0)), (d*x**2*(a + I*pi
*b/2)/2, True))
```

3.5 $\int \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=86

$$-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}bcdx^2\sqrt{cx-1}\,\sqrt{cx+1}-\frac{7bd\sqrt{cx-1}\,\sqrt{cx+1}}{9c}$$

```
[Out] d*x*(a+b*arccosh(c*x))-1/3*c^2*d*x^3*(a+b*arccosh(c*x))-7/9*b*d*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c+1/9*b*c*d*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5680, 12, 460, 74}

$$-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}bcdx^2\sqrt{cx-1}\,\sqrt{cx+1}-\frac{7bd\sqrt{cx-1}\,\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-7*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c) + (b*c*d*x^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])/9 + d*x*(a + b*ArcCosh[c*x]) - (c^2*d*x^3*(a + b*ArcCosh[c*x]))
/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{dx\left(1-\frac{c^2}{3}\right.}{\sqrt{-1+cx}\sqrt{1}}$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-1}(cx)\right)-(bcd)\int \frac{x\left(1-\frac{c^2}{}\right.}{\sqrt{-1+cx}\sqrt{}}$$

$$= \frac{1}{9}bcdx^2\sqrt{-1+cx}\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2dx^3\left(a+b\cosh^{-}\right.$$

$$= -\frac{7bd\sqrt{-1+cx}\sqrt{1+cx}}{9c}+\frac{1}{9}bcdx^2\sqrt{-1+cx}\sqrt{1+cx}+dx\left(a+b\cosh^{-1}(\right.$$

Mathematica [A] time = 0.09, size = 71, normalized size = 0.83

$$\frac{d\left(a\left(9cx-3c^3x^3\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-7\right)-3bcx\left(c^2x^2-3\right)\cosh^{-1}(cx)\right)}{9c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-7 + c^2*x^2) + a*(9*c*x - 3*c^3*x^3) -
 3*b*c*x*(-3 + c^2*x^2)*ArcCosh[c*x]))/(9*c)
```

fricas [A] time = 0.45, size = 83, normalized size = 0.97

$$-\frac{3\,ac^3dx^3-9\,acdx+3\left(bc^3dx^3-3\,bcdx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^2dx^2-7\,bd\right)\sqrt{c^2x^2-1}}{9\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/9*(3*a*c^3*d*x^3 - 9*a*c*d*x + 3*(b*c^3*d*x^3 - 3*b*c*d*x)*log(c*x + sqr
t(c^2*x^2 - 1)) - (b*c^2*d*x^2 - 7*b*d)*sqrt(c^2*x^2 - 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 73, normalized size = 0.85

$$\frac{-da\left(\frac{1}{3}c^3x^3-cx\right)-db\left(\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-7\right)}{9}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-d*a*(1/3*c^3*x^3-c*x)-d*b*(1/3*c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-
1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*x^2-7)))
```

maxima [A] time = 0.33, size = 97, normalized size = 1.13

$$-\frac{1}{3}ac^2dx^3-\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^2d+adx+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/3*a*c^2*d*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x
^2 - 1))*b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2), x)
```

sympy [A] time = 0.58, size = 97, normalized size = 1.13

$$\begin{cases} -\frac{ac^2dx^3}{3}+adx-\frac{bc^2dx^3\operatorname{acosh}(cx)}{3}+\frac{bcdx^2\sqrt{c^2x^2-1}}{9}+bdx\operatorname{acosh}(cx)-\frac{7bd\sqrt{c^2x^2-1}}{9c} & \text{for } c\neq 0 \\ dx\left(a+\frac{i\pi b}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**2*d*x**3/3 + a*d*x - b*c**2*d*x**3*acosh(c*x)/3 + b*c*d*x*
*2*sqrt(c**2*x**2 - 1)/9 + b*d*x*acosh(c*x) - 7*b*d*sqrt(c**2*x**2 - 1)/(9*
c), Ne(c, 0)), (d*x*(a + I*pi*b/2), True))
```

3.6 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{2}bd\text{Li}_2\left(-e\right.$$

```
[Out] -1/4*b*d*arccosh(c*x)+1/2*d*(-c^2*x^2+1)*(a+b*arccosh(c*x))+1/2*d*(a+b*arcc
osh(c*x))^2/b+d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2)-1/2*b*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/4*b*c*d*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d\log\left(e^{2\cosh^{-1}(cx)}+1\right)\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*d*ArcCosh[c*x])/4 + (d*(1 - c
^2*x^2)*(a + b*ArcCosh[c*x]))/2 - (d*(a + b*ArcCosh[c*x])^2)/(2*b) + d*(a +
 b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] + (b*d*PolyLog[2, -E^(2*ArcCos
h[c*x])])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx+\frac{1}{2}(bcd)$$

$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+d\,\mathrm{Subst}$$

$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh\right.$$

$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh\right.$$

$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh\right.$$

$$= \frac{1}{4}bcdx\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{4}bd\cosh^{-1}(cx)+\frac{1}{2}d\left(1-c^2x^2\right)\left(a+b\cosh\right.$$

Mathematica [A] time = 0.10, size = 130, normalized size = 1.11

$$-\frac{1}{2}ac^2dx^2+ad\log(x)-\frac{1}{2}bc^2dx^2\cosh^{-1}(cx)+\frac{1}{2}bd\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right)-\mathrm{Li}_2\left(-e\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/2*(a*c^2*d*x^2) + (b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c^2*d*x^
2*ArcCosh[c*x])/2 + (b*d*ArcTanh[Sqrt[-1 + c*x]/Sqrt[1 + c*x]])/2 + a*d*Log
[x] + (b*d*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) -
PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{ac^2dx^2-ad+\left(bc^2dx^2-bd\right)\mathrm{arcosh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.29, size = 131, normalized size = 1.12

$$-\frac{da\,c^2x^2}{2}+da\ln(cx)-\frac{db\operatorname{arccosh}(cx)^2}{2}+\frac{bcdx\sqrt{cx-1}\,\sqrt{cx+1}}{4}-\frac{db\operatorname{arccosh}(cx)\,c^2x^2}{2}+\frac{bd\operatorname{arccosh}(cx)}{4}+db\operatorname{arccos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] -1/2*d*a*c^2*x^2+d*a*ln(c*x)-1/2*d*b*arccosh(c*x)^2+1/4*b*c*d*x*(c*x-1)^(1/
2)*(c*x+1)^(1/2)-1/2*d*b*arccosh(c*x)*c^2*x^2+1/4*b*d*arccosh(c*x)+d*b*arcc
osh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*d*b*polylog(2,-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,ac^2dx^2+ad\log(x)-\int bc^2dx\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)-\frac{bd\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/2*a*c^2*d*x^2 + a*d*log(x) - integrate(b*c^2*d*x*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) - b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x}\right)dx+\int ac^2x\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x}\right)dx+\int bc^2x\operatorname{acosh}(cx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d*(Integral(-a/x, x) + Integral(a*c**2*x, x) + Integral(-b*acosh(c*x)/x, x
) + Integral(b*c**2*x*acosh(c*x), x))
```

3.7 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=76

$$c^2(-d)x\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+bcd\sqrt{cx-1}\,\sqrt{cx+1}+bcd\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)$$

```
[Out] -d*(a+b*arccosh(c*x))/x-c^2*d*x*(a+b*arccosh(c*x))+b*c*d*arctan((c*x-1)^(1/
2)*(c*x+1)^(1/2))+b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 460, 92, 205}

$$c^2(-d)x\left(a+b\cosh^{-1}(cx)\right)-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+bcd\sqrt{cx-1}\,\sqrt{cx+1}+bcd\tan^{-1}\left(\sqrt{cx-1}\,\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (d*(a + b*ArcCosh[c*x]))/x - c^2*d*x*(
a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) - (bc)\int \frac{d\left(-1-c^2x^2\right.}{x\sqrt{-1+cx}\,\sqrt{1}}$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) - (bcd)\int \frac{-1-c^2x}{x\sqrt{-1+cx}\,\sqrt{}}$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) +$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) +$$

$$= bcd\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{x} - c^2dx\left(a+b\cosh^{-1}(cx)\right) +$$

Mathematica [A] time = 0.19, size = 110, normalized size = 1.45

$$-ac^2dx-\frac{ad}{x}+\frac{bcd\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-bc^2dx\cosh^{-1}(cx)+bcd\sqrt{cx-1}\,\sqrt{cx+1}-\frac{bd\cosh^{-1}(cx)}{x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) - a*c^2*d*x + b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - (b*d*ArcCosh[
c*x])/x - b*c^2*d*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.51, size = 127, normalized size = 1.67

$$-\frac{ac^2dx^2-2\,bcdx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,bcdx-\left(bc^2+b\right)dx\log\left(-cx+\sqrt{c^2x^2-1}\right)+ad+\left(bc^2d\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -(a*c^2*d*x^2 - 2*b*c*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 -
 1)*b*c*d*x - (b*c^2 + b)*d*x*log(-c*x + sqrt(c^2*x^2 - 1)) + a*d + (b*c^2*
d*x^2 - (b*c^2 + b)*d*x + b*d)*log(c*x + sqrt(c^2*x^2 - 1)))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 100, normalized size = 1.32

$$-da\,c^2x-\frac{da}{x}-db\,\mathrm{arccosh}\,(cx)\,c^2x-\frac{db\,\mathrm{arccosh}\,(cx)}{x}+bcd\sqrt{cx-1}\,\sqrt{cx+1}-\frac{cdb\sqrt{cx-1}\,\sqrt{cx+1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-}}\right.}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -d*a*c^2*x-d*a/x-d*b*arccosh(c*x)*c^2*x-d*b*arccosh(c*x)/x+b*c*d*(c*x-1)^(1
/2)*(c*x+1)^(1/2)-c*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arcta
n(1/(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.45, size = 66, normalized size = 0.87

$$-ac^2dx-\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bcd-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -a*c^2*d*x - (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c*d - (c*arcsin(1/(c*
abs(x))) + arccosh(c*x)/x)*b*d - a*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int ac^2\,dx+\int\left(-\frac{a}{x^2}\right)\,dx+\int bc^2\,\mathrm{acosh}\,(cx)\,dx+\int\left(-\frac{b\,\mathrm{acosh}\,(cx)}{x^2}\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d*(Integral(a*c**2, x) + Integral(-a/x**2, x) + Integral(b*c**2*acosh(c*x)
, x) + Integral(-b*acosh(c*x)/x**2, x))
```

3.8 $\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=135

$$-\frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}bc^2d\mathrm{L}$$

```
[Out] -1/2*b*c^2*d*arccosh(c*x)-1/2*d*(-c^2*x^2+1)*(a+b*arccosh(c*x))/x^2-1/2*c^2
*d*(a+b*arccosh(c*x))^2/b-c^2*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)+1/2*b*c^2*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)+1/2*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x
```

Rubi [A] time = 0.13, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {5729, 97, 12, 52, 5660, 3718, 2190, 2279, 2391}

$$-\frac{1}{2}bc^2d\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-\frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)^2}{2b}-c^2d\log\left(e^{2\cosh^{-1}(cx)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*c^2*d*ArcCosh[c*x])/2 - (d*
(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*d*(a + b*ArcCosh[c*x])^2
)/(2*b) - c^2*d*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] - (b*c^2*d
*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2}\,dx -$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{1}{2}(bcd)\int$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2d\left(a+b\right.}{}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}\right.}{2x^2}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}\right.}{2x^2}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x} - \frac{1}{2}bc^2d\cosh^{-1}(cx) - \frac{d\left(1-c^2x^2\right)\left(a+b\cosh^{-1}\right.}{2x^2}$$

Mathematica [A] time = 0.09, size = 107, normalized size = 0.79

$$-ac^2d\log(x) - \frac{ad}{2x^2} - \frac{1}{2}bc^2d\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right) - \text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)\right) - \frac{bd\cosh}{2x}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/2*(a*d)/x^2 + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*d*ArcCosh[
c*x])/(2*x^2) - a*c^2*d*Log[x] - (b*c^2*d*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*L
og[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\,(cx)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.57, size = 140, normalized size = 1.04

$$-c^2da\ln(cx) - \frac{da}{2x^2} + \frac{c^2db\,\text{arccosh}\,(cx)^2}{2} + \frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{2x} - \frac{c^2db}{2} - \frac{db\,\text{arccosh}\,(cx)}{2x^2} - c^2db\,\text{arccosh}\,(cx)\ln\left(1 + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -c^2*d*a*ln(c*x)-1/2*d*a/x^2+1/2*c^2*d*b*arccosh(c*x)^2+1/2*b*c*d*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/x-1/2*c^2*d*b-1/2*d*b*arccosh(c*x)/x^2-c^2*d*b*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*c^2*d*b*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-bc^2d\int\frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{x}\,dx - ac^2d\log(x) + \frac{1}{2}\,bd\left(\frac{\sqrt{c^2x^2-1}\,c}{x} - \frac{\text{arcosh}\,(cx)}{x^2}\right) - \frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -b*c^2*d*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) - a*c^2*d*l
og(x) + 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a + b\,\text{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x^3}\right)dx+\int\frac{ac^2}{x}\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^3}\right)dx+\int\frac{bc^2\operatorname{acosh}(cx)}{x}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d*(Integral(-a/x**3, x) + Integral(a*c**2/x, x) + Integral(-b*acosh(c*x)/x
**3, x) + Integral(b*c**2*acosh(c*x)/x, x))
```

3.9 $\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=90

$$\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{5}{6}bc^3d\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

```
[Out] -1/3*d*(a+b*arccosh(c*x))/x^3+c^2*d*(a+b*arccosh(c*x))/x-5/6*b*c^3*d*arctan
((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A] time = 0.13, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {14, 5731, 12, 454, 92, 205}

$$\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{5}{6}bc^3d\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) + (c^2*d*(a + b*ArcCosh[c*x]))/x - (5*b*c^3*d*ArcTan[Sqrt[-1 + c*x]*Sq
rt[1 + c*x]])/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-(bc)\int \frac{d\left(-1+3\right.}{3x^3\sqrt{-1+c}}$$

$$= -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{3}(bcd)\int \frac{-1+}{x^3\sqrt{-1+}}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}$$

$$= \frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^2d\left(a+b\cosh^{-1}(cx)\right)}{x}$$

Mathematica [A] time = 0.26, size = 127, normalized size = 1.41

$$\frac{ac^2d}{x}-\frac{ad}{3x^3}+\frac{bc^2d\cosh^{-1}(cx)}{x}-\frac{5bc^3d\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{6\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd\cosh^{-1}(cx)}{3x^3}+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d)/x^3 + (a*c^2*d)/x + (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2)
 - (b*d*ArcCosh[c*x])/(3*x^3) + (b*c^2*d*ArcCosh[c*x])/x - (5*b*c^3*d*Sqrt[
-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.58, size = 146, normalized size = 1.62

$$-\frac{10\,bc^3dx^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-6\,ac^2dx^2+2\left(3\,bc^2-b\right)dx^3\log\left(-cx+\sqrt{c^2x^2-1}\right)-\sqrt{c^2x^2-1}\,bcdx+}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/6*(10*b*c^3*d*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 6*a*c^2*d*x^2 + 2*(
3*b*c^2 - b)*d*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*b*c*d*
x + 2*a*d - 2*(3*b*c^2*d*x^2 - (3*b*c^2 - b)*d*x^3 - b*d)*log(c*x + sqrt(c^
2*x^2 - 1)))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 108, normalized size = 1.20

$$\frac{c^2da}{x}-\frac{da}{3x^3}+\frac{c^2db\operatorname{arccosh}(cx)}{x}-\frac{db\operatorname{arccosh}(cx)}{3x^3}+\frac{5c^3db\sqrt{cx-1}\,\sqrt{cx+1}\,\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}}+\frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] c^2*d*a/x-1/3*d*a/x^3+c^2*d*b*arccosh(c*x)/x-1/3*d*b*arccosh(c*x)/x^3+5/6*c
^3*d*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(
1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

maxima [A] time = 0.49, size = 89, normalized size = 0.99

$$\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bc^2d-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\operatorname{arcosh}(cx)}{x^3}\right)bd+\frac{ac^2d}{x}-\frac{ad}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d - 1/6*((c^2*arcsin(1/(c*a
bs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d + a*c^2*d/x -
1/3*a*d/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{a}{x^4}\right)dx+\int\frac{ac^2}{x^2}\,dx+\int\left(-\frac{b\operatorname{acosh}(cx)}{x^4}\right)dx+\int\frac{bc^2\operatorname{acosh}(cx)}{x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d*(Integral(-a/x**4, x) + Integral(a*c**2/x**2, x) + Integral(-b*acosh(c*x
)/x**4, x) + Integral(b*c**2*acosh(c*x)/x**2, x))
```

3.10 $\int x^4 \left(d-c^2 d x^2\right)^2 \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=206

$$\frac{1}{9} c^4 d^2 x^9 \left(a+b \cosh^{-1}(cx)\right)-\frac{2}{7} c^2 d^2 x^7 \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{5} d^2 x^5 \left(a+b \cosh^{-1}(cx)\right)-\frac{b d^2 (cx-1)^{9/2} (cx+1)^{9/2}}{81 c^5}-$$

```
[Out] 4/945*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^5-1/525*b*d^2*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^5-10/441*b*d^2*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^5-1/81*b*d^2*(c*x-1
)^(9/2)*(c*x+1)^(9/2)/c^5+1/5*d^2*x^5*(a+b*arccosh(c*x))-2/7*c^2*d^2*x^7*(a
+b*arccosh(c*x))+1/9*c^4*d^2*x^9*(a+b*arccosh(c*x))-8/315*b*d^2*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c^5
```

Rubi [A] time = 0.29, antiderivative size = 264, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {270, 5731, 12, 520, 1251, 897, 1153}

$$\frac{1}{9} c^4 d^2 x^9 \left(a+b \cosh^{-1}(cx)\right)-\frac{2}{7} c^2 d^2 x^7 \left(a+b \cosh^{-1}(cx)\right)+\frac{1}{5} d^2 x^5 \left(a+b \cosh^{-1}(cx)\right)+\frac{b d^2 \left(1-c^2 x^2\right)^5}{81 c^5 \sqrt{cx-1} \sqrt{cx+1}}-\frac{}{44}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(945*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(525*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*d^2*(1 - c^2*x^2)^4)/(
441*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^5)/(81*c^5*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b*ArcCosh[c*x]))/5 - (2*c^2*d^2*
x^7*(a + b*ArcCosh[c*x]))/7 + (c^4*d^2*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{7}c^2d^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}c^4d^2x^9\left(a\right.$$

$$=\frac{8bd^2\left(1-c^2x^2\right)}{315c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{945c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c\right.}{525c^5\sqrt{-1+c}}$$

Mathematica [A] time = 0.21, size = 124, normalized size = 0.60

$$\frac{d^2\left(315ac^5x^5\left(35c^4x^4-90c^2x^2+63\right)+315bc^5x^5\left(35c^4x^4-90c^2x^2+63\right)\cosh^{-1}(cx)-b\sqrt{cx-1}\sqrt{cx+1}\left(1225c^8\right.\right.}{99225c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(315*a*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(2104 + 1052*c^2*x^2 + 789*c^4*x^4 - 2650*c^6*x^6 + 1225*c^8*x^8)
+ 315*b*c^5*x^5*(63 - 90*c^2*x^2 + 35*c^4*x^4)*ArcCosh[c*x]))/(99225*c^5)
```

fricas [A] time = 0.63, size = 165, normalized size = 0.80

$$\frac{11025\,ac^9d^2x^9 - 28350\,ac^7d^2x^7 + 19845\,ac^5d^2x^5 + 315\left(35\,bc^9d^2x^9 - 90\,bc^7d^2x^7 + 63\,bc^5d^2x^5\right)\log\left(cx + \sqrt{c^2x}\right.}{99225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*d^2*x^9 - 28350*a*c^7*d^2*x^7 + 19845*a*c^5*d^2*x^5 +
315*(35*b*c^9*d^2*x^9 - 90*b*c^7*d^2*x^7 + 63*b*c^5*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 - 1)) - (1225*b*c^8*d^2*x^8 - 2650*b*c^6*d^2*x^6 + 789*b*c^4*d^2*x
^4 + 1052*b*c^2*d^2*x^2 + 2104*b*d^2)*sqrt(c^2*x^2 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 128, normalized size = 0.62

$$\frac{d^2a\left(\frac{1}{9}c^9x^9 - \frac{2}{7}c^7x^7 + \frac{1}{5}c^5x^5\right) + d^2b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9} - \frac{2\operatorname{arccosh}(cx)c^7x^7}{7} + \frac{\operatorname{arccosh}(cx)c^5x^5}{5} - \frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1225c^8x^8-2650c^6\right.}{9922}\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(d^2*a*(1/9*c^9*x^9-2/7*c^7*x^7+1/5*c^5*x^5)+d^2*b*(1/9*arccosh(c*x)*
c^9*x^9-2/7*arccosh(c*x)*c^7*x^7+1/5*arccosh(c*x)*c^5*x^5-1/99225*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-2650*c^6*x^6+789*c^4*x^4+1052*c^2*x^2+2104
)))
```

maxima [A] time = 0.35, size = 319, normalized size = 1.55

$$\frac{1}{9}\,ac^4d^2x^9 - \frac{2}{7}\,ac^2d^2x^7 + \frac{1}{2835}\left(315\,x^9\operatorname{arcosh}(cx) - \left(\frac{35\,\sqrt{c^2x^2-1}\,x^8}{c^2} + \frac{40\,\sqrt{c^2x^2-1}\,x^6}{c^4} + \frac{48\,\sqrt{c^2x^2-1}\,x^4}{c^6} + \frac{6}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*c^4*d^2*x^9 - 2/7*a*c^2*d^2*x^7 + 1/2835*(315*x^9*arccosh(c*x) - (35*
sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
- 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c
)*b*c^4*d^2 + 1/5*a*d^2*x^5 - 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2
- 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 +
16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqr
t(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/
c^6)*c)*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \, (a + b \operatorname{acosh}(c\, x)) \, \left(d - c^2 \, d \, x^2\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 16.46, size = 236, normalized size = 1.15

$$\begin{cases} \frac{ac^4d^2x^9}{9} - \frac{2ac^2d^2x^7}{7} + \frac{ad^2x^5}{5} + \frac{bc^4d^2x^9 \operatorname{acosh}(cx)}{9} - \frac{bc^3d^2x^8\sqrt{c^2x^2-1}}{81} - \frac{2bc^2d^2x^7 \operatorname{acosh}(cx)}{7} + \frac{106bcd^2x^6\sqrt{c^2x^2-1}}{3969} + \frac{bd^2x^5 \operatorname{acosh}(cx)}{5} - \\ \frac{d^2x^5\left(a+\frac{i\pi b}{2}\right)}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**9/9 - 2*a*c**2*d**2*x**7/7 + a*d**2*x**5/5 + b*c*
*4*d**2*x**9*acosh(c*x)/9 - b*c**3*d**2*x**8*sqrt(c**2*x**2 - 1)/81 - 2*b*c
**2*d**2*x**7*acosh(c*x)/7 + 106*b*c*d**2*x**6*sqrt(c**2*x**2 - 1)/3969 + b
*d**2*x**5*acosh(c*x)/5 - 263*b*d**2*x**4*sqrt(c**2*x**2 - 1)/(33075*c) - 1
052*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(99225*c**3) - 2104*b*d**2*sqrt(c**2*x*
*2 - 1)/(99225*c**5), Ne(c, 0)), (d**2*x**5*(a + I*pi*b/2)/5, True))
```

3.11 $\int x^3 \left(d - c^2 d x^2\right)^2 \left(a + b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=200

$$\frac{1}{8}c^4d^2x^8\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{73bd^2\cosh^{-1}(cx)}{3072c^4}-\frac{1}{64}bc^3$$

```
[Out] -73/3072*b*d^2*arccosh(c*x)/c^4+1/4*d^2*x^4*(a+b*arccosh(c*x))-1/3*c^2*d^2*
x^6*(a+b*arccosh(c*x))+1/8*c^4*d^2*x^8*(a+b*arccosh(c*x))-73/3072*b*d^2*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-73/4608*b*d^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c+43/1152*b*c*d^2*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/64*b*c^3*d^2*x^7*(c*x
-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 284, normalized size of antiderivative = 1.42, number of steps used = 9, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 43, 5731, 12, 520, 1267, 459, 321, 217, 206}

$$\frac{1}{8}c^4d^2x^8\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{bc^3d^2x^7\left(1-c^2x^2\right)}{64\sqrt{cx-1}\sqrt{cx+1}}-\frac{43}{115}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (73*b*d^2*x*(1 - c^2*x^2))/(3072*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (73*b*
d^2*x^3*(1 - c^2*x^2))/(4608*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (43*b*c*d^2*
x^5*(1 - c^2*x^2))/(1152*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^2*x^7*(1
- c^2*x^2))/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c*x
]))/4 - (c^2*d^2*x^6*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^8*(a + b*ArcCosh[
c*x]))/8 - (73*b*d^2*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/
(3072*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}c^4d^2x^8$$

$$= \frac{bc^3d^2x^7\left(1-c^2x^2\right)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{3}c^2d^2x^6\left(a+b\right.$$

$$= -\frac{43bcd^2x^5\left(1-c^2x^2\right)}{1152\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^2x^7\left(1-c^2x^2\right)}{64\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\,\mathrm{c}\right.$$

$$= \frac{73bd^2x^3\left(1-c^2x^2\right)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{43bcd^2x^5\left(1-c^2x^2\right)}{1152\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3d^2x^7\left(1\right.}{64\sqrt{-1+cx}}$$

$$= \frac{73bd^2x\left(1-c^2x^2\right)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3\left(1-c^2x^2\right)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{43bcd^2x}{1152\sqrt{-1}}$$

$$= \frac{73bd^2x\left(1-c^2x^2\right)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3\left(1-c^2x^2\right)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{43bcd^2x}{1152\sqrt{-1}}$$

$$= \frac{73bd^2x\left(1-c^2x^2\right)}{3072c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{73bd^2x^3\left(1-c^2x^2\right)}{4608c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{43bcd^2x}{1152\sqrt{-1}}$$

Mathematica [A] time = 0.24, size = 194, normalized size = 0.97

$$\frac{d^2\left(1152ac^8x^8-3072ac^6x^6+2304ac^4x^4-144bc^7x^7\sqrt{cx-1}\sqrt{cx+1}+344bc^5x^5\sqrt{cx-1}\sqrt{cx+1}-146bc^3x^3\right.}{92}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(2304*a*c^4*x^4 - 3072*a*c^6*x^6 + 1152*a*c^8*x^8 - 219*b*c*x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x] - 146*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 344*b*c
^5*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 144*b*c^7*x^7*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x] + 384*b*c^4*x^4*(6 - 8*c^2*x^2 + 3*c^4*x^4)*ArcCosh[c*x] - 438*b*ArcT
anh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(9216*c^4)
```

fricas [A] time = 0.45, size = 161, normalized size = 0.80

$$\frac{1152\,ac^8d^2x^8-3072\,ac^6d^2x^6+2304\,ac^4d^2x^4+3\left(384\,bc^8d^2x^8-1024\,bc^6d^2x^6+768\,bc^4d^2x^4-73\,bd^2\right)\log\left(cx\right.}{9216\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*d^2*x^8 - 3072*a*c^6*d^2*x^6 + 2304*a*c^4*d^2*x^4 + 3*(3
84*b*c^8*d^2*x^8 - 1024*b*c^6*d^2*x^6 + 768*b*c^4*d^2*x^4 - 73*b*d^2)*log(c
*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*d^2*x^7 - 344*b*c^5*d^2*x^5 + 146*b*c^
3*d^2*x^3 + 219*b*c*d^2*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 230, normalized size = 1.15

$$\frac{c^4d^2a\,x^8}{8}-\frac{c^2d^2a\,x^6}{3}+\frac{d^2a\,x^4}{4}+\frac{c^4d^2b\,\mathrm{arccosh}\,(cx)\,x^8}{8}-\frac{c^2d^2b\,\mathrm{arccosh}\,(cx)\,x^6}{3}+\frac{d^2b\,\mathrm{arccosh}\,(cx)\,x^4}{4}-\frac{b\,c^3d^2x^7\sqrt{cx-1}}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*c^4*d^2*a*x^8-1/3*c^2*d^2*a*x^6+1/4*d^2*a*x^4+1/8*c^4*d^2*b*arccosh(c*x
)*x^8-1/3*c^2*d^2*b*arccosh(c*x)*x^6+1/4*d^2*b*arccosh(c*x)*x^4-1/64*b*c^3*
d^2*x^7*(c*x-1)^(1/2)*(c*x+1)^(1/2)+43/1152*b*c*d^2*x^5*(c*x-1)^(1/2)*(c*x+
1)^(1/2)-73/4608*b*d^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-73/3072*b*d^2*x*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-73/3072/c^4*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [B] time = 0.33, size = 346, normalized size = 1.73

$$\frac{1}{8}\,ac^4d^2x^8-\frac{1}{3}\,ac^2d^2x^6+\frac{1}{3072}\left(384\,x^8\,\mathrm{arcosh}\,(cx)-\left(\frac{48\,\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\,\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\,\sqrt{c^2x^2-1}\,x^3}{c^6}+\frac{105}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*c^4*d^2*x^8 - 1/3*a*c^2*d^2*x^6 + 1/3072*(384*x^8*arccosh(c*x) - (48*
sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
- 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x
^2 - 1)*c)/c^9)*c)*b*c^4*d^2 + 1/4*a*d^2*x^4 - 1/144*(48*x^6*arccosh(c*x) -
 (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*
x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d^2
+ 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2
- 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}\,(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 11.28, size = 224, normalized size = 1.12

$$\begin{cases} \frac{ac^4d^2x^8}{8}-\frac{ac^2d^2x^6}{3}+\frac{ad^2x^4}{4}+\frac{bc^4d^2x^8\,\mathrm{acosh}\,(cx)}{8}-\frac{bc^3d^2x^7\sqrt{c^2x^2-1}}{64}-\frac{bc^2d^2x^6\,\mathrm{acosh}\,(cx)}{3}+\frac{43bcd^2x^5\sqrt{c^2x^2-1}}{1152}+\frac{bd^2x^4\,\mathrm{acosh}\,(cx)}{4}-\frac{73b}{} \\ \frac{d^2x^4\left(a+\frac{i\pi b}{2}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**8/8 - a*c**2*d**2*x**6/3 + a*d**2*x**4/4 + b*c**4
*d**2*x**8*acosh(c*x)/8 - b*c**3*d**2*x**7*sqrt(c**2*x**2 - 1)/64 - b*c**2*
d**2*x**6*acosh(c*x)/3 + 43*b*c*d**2*x**5*sqrt(c**2*x**2 - 1)/1152 + b*d**2
*x**4*acosh(c*x)/4 - 73*b*d**2*x**3*sqrt(c**2*x**2 - 1)/(4608*c) - 73*b*d**
2*x*sqrt(c**2*x**2 - 1)/(3072*c**3) - 73*b*d**2*acosh(c*x)/(3072*c**4), Ne(
c, 0)), (d**2*x**4*(a + I*pi*b/2)/4, True))
```

3.12 $\int x^2 \left(d-c^2 d x^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=177

$$\frac{1}{7}c^4d^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(cx-1)^{7/2}(cx+1)^{7/2}}{49c^3}-\frac{bd}{}$$

```
[Out] 4/315*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-1/175*b*d^2*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^3-1/49*b*d^2*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/3*d^2*x^3*(a+b*ar
ccosh(c*x))-2/5*c^2*d^2*x^5*(a+b*arccosh(c*x))+1/7*c^4*d^2*x^7*(a+b*arccosh
(c*x))-8/105*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {270, 5731, 12, 520, 1251, 771}

$$\frac{1}{7}c^4d^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2\left(1-c^2x^2\right)^4}{49c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{b}{175c}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(105*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(
1 - c^2*x^2)^2)/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^
2)^3)/(175*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^2*(1 - c^2*x^2)^4)/(49*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 - (2*c
^2*d^2*x^5*(a + b*ArcCosh[c*x]))/5 + (c^4*d^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
```

```
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{5}c^2d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}c^4d^2x^7$$

$$=\frac{8bd^2\left(1-c^2x^2\right)}{105c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{315c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-\right.}{175c^3\sqrt{-1+}}$$

Mathematica [A] time = 0.18, size = 116, normalized size = 0.66

$$\frac{d^2\left(105ac^3x^3\left(15c^4x^4-42c^2x^2+35\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(225c^6x^6-612c^4x^4+409c^2x^2+818\right)+105bc^3x^3\left(1\right.\right.}{11025c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(105*a*c^3*x^3*(35 - 42*c^2*x^2 + 15*c^4*x^4) - b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(818 + 409*c^2*x^2 - 612*c^4*x^4 + 225*c^6*x^6) + 105*b*c^3*x^3*(3
5 - 42*c^2*x^2 + 15*c^4*x^4)*ArcCosh[c*x]))/(11025*c^3)
```

fricas [A] time = 0.50, size = 153, normalized size = 0.86

$$\frac{1575\,ac^7d^2x^7-4410\,ac^5d^2x^5+3675\,ac^3d^2x^3+105\left(15\,bc^7d^2x^7-42\,bc^5d^2x^5+35\,bc^3d^2x^3\right)\log\left(cx+\sqrt{c^2x^2-}\right.}{11025\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*d^2*x^7 - 4410*a*c^5*d^2*x^5 + 3675*a*c^3*d^2*x^3 + 105
*(15*b*c^7*d^2*x^7 - 42*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3)*log(c*x + sqrt(c^
```

```
2*x^2 - 1)) - (225*b*c^6*d^2*x^6 - 612*b*c^4*d^2*x^4 + 409*b*c^2*d^2*x^2 +
818*b*d^2)*sqrt(c^2*x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 120, normalized size = 0.68

$$\frac{d^2 a\left(\frac{1}{7}c^7x^7-\frac{2}{5}c^5x^5+\frac{1}{3}c^3x^3\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{2\operatorname{arccosh}(cx)c^5x^5}{5}+\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(225c^6x^6-612c^4x^4+40\right.}{11025}\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(d^2*a*(1/7*c^7*x^7-2/5*c^5*x^5+1/3*c^3*x^3)+d^2*b*(1/7*arccosh(c*x)*
c^7*x^7-2/5*arccosh(c*x)*c^5*x^5+1/3*c^3*x^3*arccosh(c*x)-1/11025*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*(225*c^6*x^6-612*c^4*x^4+409*c^2*x^2+818)))
```

maxima [A] time = 0.34, size = 261, normalized size = 1.47

$$\frac{1}{7}\,ac^4d^2x^7-\frac{2}{5}\,ac^2d^2x^5+\frac{1}{245}\left(35\,x^7\operatorname{arcosh}(cx)-\left(\frac{5\,\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\,\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\,\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\,\sqrt{c^2x^2-}}{c^8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*c^4*d^2*x^7 - 2/5*a*c^2*d^2*x^5 + 1/245*(35*x^7*arccosh(c*x) - (5*sqr
t(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*
x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^2 - 2/75*(15*x^5*arccosh(c*x
) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2
*x^2 - 1)/c^6)*c)*b*c^2*d^2 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(
sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\operatorname{acosh}(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 6.13, size = 209, normalized size = 1.18

$$\begin{cases}\frac{ac^4d^2x^7}{7}-\frac{2ac^2d^2x^5}{5}+\frac{ad^2x^3}{3}+\frac{bc^4d^2x^7\operatorname{acosh}(cx)}{7}-\frac{bc^3d^2x^6\sqrt{c^2x^2-1}}{49}-\frac{2bc^2d^2x^5\operatorname{acosh}(cx)}{5}+\frac{68bcd^2x^4\sqrt{c^2x^2-1}}{1225}+\frac{bd^2x^3\operatorname{acosh}(cx)}{3}- \\ \frac{d^2x^3\left(a+\frac{i\pi b}{2}\right)}{3}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**7/7 - 2*a*c**2*d**2*x**5/5 + a*d**2*x**3/3 + b*c*
*4*d**2*x**7*acosh(c*x)/7 - b*c**3*d**2*x**6*sqrt(c**2*x**2 - 1)/49 - 2*b*c
**2*d**2*x**5*acosh(c*x)/5 + 68*b*c*d**2*x**4*sqrt(c**2*x**2 - 1)/1225 + b*
d**2*x**3*acosh(c*x)/3 - 409*b*d**2*x**2*sqrt(c**2*x**2 - 1)/(11025*c) - 81
8*b*d**2*sqrt(c**2*x**2 - 1)/(11025*c**3), Ne(c, 0)), (d**2*x**3*(a + I*pi*
b/2)/3, True))
```

3.13 $\int x\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=136

$$-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}-\frac{bd^2x(cx-1)^{5/2}(cx+1)^{5/2}}{36c}+\frac{5bd^2x(cx-1)^{3/2}(cx+1)^{3/2}}{144c}-\frac{5b}{}$$

```
[Out] 5/144*b*d^2*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-1/36*b*d^2*x*(c*x-1)^(5/2)*(c*x
+1)^(5/2)/c+5/96*b*d^2*arccosh(c*x)/c^2-1/6*d^2*(-c^2*x^2+1)^3*(a+b*arccosh
(c*x))/c^2-5/96*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.07, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5716, 38, 52}

$$-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+\frac{5bd^2\cosh^{-1}(cx)}{96c^2}-\frac{bd^2x(cx-1)^{5/2}(cx+1)^{5/2}}{36c}+\frac{5bd^2x(cx-1)^{3/2}(cx+1)^{3/2}}{144c}-\frac{5b}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c) + (5*b*d^2*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/(144*c) - (b*d^2*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(
36*c) + (5*b*d^2*ArcCosh[c*x])/(96*c^2) - (d^2*(1 - c^2*x^2)^3*(a + b*ArcCo
sh[c*x]))/(6*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}-\frac{\left(bd^2\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}}{6c}$$

$$=-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-\frac{d^2\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6c^2}+$$

$$=\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-1+cx)^{5/2}(1+cx)^{5/2}}{36c}-\frac{d^2\left(1-\right.}{}$$

$$=-\frac{5bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-}{}$$

$$=-\frac{5bd^2x\sqrt{-1+cx}\sqrt{1+cx}}{96c}+\frac{5bd^2x(-1+cx)^{3/2}(1+cx)^{3/2}}{144c}-\frac{bd^2x(-}{}$$

Mathematica [A] time = 0.23, size = 126, normalized size = 0.93

$$\frac{d^2\left(cx\left(48acx\left(c^4x^4-3c^2x^2+3\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-8c^4x^4+26c^2x^2-33\right)\right)+48bc^2x^2\left(c^4x^4-3c^2x^2+3\right)c\right.}{288c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-33 + 26*c^2*x^2 - 8*c^4*x^4) +
48*a*c*x*(3 - 3*c^2*x^2 + c^4*x^4)) + 48*b*c^2*x^2*(3 - 3*c^2*x^2 + c^4*x^4
)*ArcCosh[c*x] - 66*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(288*c^2)
```

fricas [A] time = 0.64, size = 149, normalized size = 1.10

$$\frac{48\,ac^6d^2x^6-144\,ac^4d^2x^4+144\,ac^2d^2x^2+3\left(16\,bc^6d^2x^6-48\,bc^4d^2x^4+48\,bc^2d^2x^2-11\,bd^2\right)\log\left(cx+\sqrt{c^2x^2-}\right.}{288\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*d^2*x^6 - 144*a*c^4*d^2*x^4 + 144*a*c^2*d^2*x^2 + 3*(16*b*c
^6*d^2*x^6 - 48*b*c^4*d^2*x^4 + 48*b*c^2*d^2*x^2 - 11*b*d^2)*log(c*x + sqrt
(c^2*x^2 - 1)) - (8*b*c^5*d^2*x^5 - 26*b*c^3*d^2*x^3 + 33*b*c*d^2*x)*sqrt(c
^2*x^2 - 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 204, normalized size = 1.50

$$\frac{c^4d^2a\,x^6}{6}-\frac{c^2d^2a\,x^4}{2}+\frac{d^2a\,x^2}{2}+\frac{c^4d^2b\,\mathrm{arccosh}\,(cx)\,x^6}{6}-\frac{c^2d^2b\,\mathrm{arccosh}\,(cx)\,x^4}{2}+\frac{d^2b\,\mathrm{arccosh}\,(cx)\,x^2}{2}-\frac{c^3d^2b\sqrt{cx-1}}{3\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*c^4*d^2*a*x^6-1/2*c^2*d^2*a*x^4+1/2*d^2*a*x^2+1/6*c^4*d^2*b*arccosh(c*x
)*x^6-1/2*c^2*d^2*b*arccosh(c*x)*x^4+1/2*d^2*b*arccosh(c*x)*x^2-1/36*c^3*d^
2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5+13/144*c*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*x^3-11/96*b*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-11/96/c^2*d^2*b*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [B] time = 0.46, size = 287, normalized size = 2.11

$$\frac{1}{6}\,ac^4d^2x^6-\frac{1}{2}\,ac^2d^2x^4+\frac{1}{288}\left(48\,x^6\operatorname{arcosh}\left(cx\right)-\left(\frac{8\,\sqrt{c^2x^2-1}\,x^5}{c^2}+\frac{10\,\sqrt{c^2x^2-1}\,x^3}{c^4}+\frac{15\,\sqrt{c^2x^2-1}\,x}{c^6}+\frac{15\log\left(2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*c^4*d^2*x^6 - 1/2*a*c^2*d^2*x^4 + 1/288*(48*x^6*arccosh(c*x) - (8*sqr
t(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1
)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^4*d^2 - 1/16*
(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/
c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c^2*d^2 + 1/2*a*d^2*
x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 4.22, size = 197, normalized size = 1.45

$$\begin{cases} \frac{ac^4d^2x^6}{6}-\frac{ac^2d^2x^4}{2}+\frac{ad^2x^2}{2}+\frac{bc^4d^2x^6\operatorname{acosh}\left(cx\right)}{6}-\frac{bc^3d^2x^5\sqrt{c^2x^2-1}}{36}-\frac{bc^2d^2x^4\operatorname{acosh}\left(cx\right)}{2}+\frac{13bcd^2x^3\sqrt{c^2x^2-1}}{144}+\frac{bd^2x^2\operatorname{acosh}\left(cx\right)}{2}-\frac{11b}{} \\ \frac{d^2x^2\left(a+\frac{i\pi b}{2}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**6/6 - a*c**2*d**2*x**4/2 + a*d**2*x**2/2 + b*c**4
*d**2*x**6*acosh(c*x)/6 - b*c**3*d**2*x**5*sqrt(c**2*x**2 - 1)/36 - b*c**2*
d**2*x**4*acosh(c*x)/2 + 13*b*c*d**2*x**3*sqrt(c**2*x**2 - 1)/144 + b*d**2*
x**2*acosh(c*x)/2 - 11*b*d**2*x*sqrt(c**2*x**2 - 1)/(96*c) - 11*b*d**2*acos
h(c*x)/(96*c**2), Ne(c, 0)), (d**2*x**2*(a + I*pi*b/2)/2, True))
```

3.14 $\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=143

$$\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bd^2(cx-1)^{5/2}(cx+1)^{5/2}}{25c}+\frac{4b}{}$$

```
[Out] 4/45*b*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-1/25*b*d^2*(c*x-1)^(5/2)*(c*x+1)^(
5/2)/c+d^2*x*(a+b*arccosh(c*x))-2/3*c^2*d^2*x^3*(a+b*arccosh(c*x))+1/5*c^4*
d^2*x^5*(a+b*arccosh(c*x))-8/15*b*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.15, antiderivative size = 177, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {194, 5680, 12, 520, 1247, 698}

$$\frac{1}{5}c^4d^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{bd^2\left(1-c^2x^2\right)^3}{25c\sqrt{cx-1}\sqrt{cx+1}}+\frac{4bd}{45c\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*(1 - c^2*x^2))/(15*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*d^2*(1 -
 c^2*x^2)^2)/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*(1 - c^2*x^2)^3)/
(25*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) - (2*c^2*d
^2*x^3*(a + b*ArcCosh[c*x]))/3 + (c^4*d^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{2}{3}c^2d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}c^4d^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= \frac{8bd^2\left(1-c^2x^2\right)}{15c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bd^2\left(1-c^2x^2\right)^2}{45c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c^2x^2\right)^3}{25c\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.17, size = 99, normalized size = 0.69

$$\frac{d^2\left(15acx\left(3c^4x^4-10c^2x^2+15\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-9c^4x^4+38c^2x^2-149\right)+15bcx\left(3c^4x^4-10c^2x^2+15\right)\cos\right.}{225c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-149 + 38*c^2*x^2 - 9*c^4*x^4) + 15*a
*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 15*b*c*x*(15 - 10*c^2*x^2 + 3*c^4*x^4)
*ArcCosh[c*x]))/(225*c)
```

fricas [A] time = 0.64, size = 133, normalized size = 0.93

$$\frac{45\,ac^5d^2x^5-150\,ac^3d^2x^3+225\,acd^2x+15\left(3\,bc^5d^2x^5-10\,bc^3d^2x^3+15\,bcd^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4d^2x\right.}{225\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*d^2*x^5 - 150*a*c^3*d^2*x^3 + 225*a*c*d^2*x + 15*(3*b*c^5*d
^2*x^5 - 10*b*c^3*d^2*x^3 + 15*b*c*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (9
*b*c^4*d^2*x^4 - 38*b*c^2*d^2*x^2 + 149*b*d^2)*sqrt(c^2*x^2 - 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 102, normalized size = 0.71

$$\frac{d^2a\left(\frac{1}{5}c^5x^5-\frac{2}{3}c^3x^3+cx\right)+d^2b\left(\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{2c^3x^3\operatorname{arccosh}(cx)}{3}+cx\operatorname{arccosh}(cx)-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(9c^4x^4-38c^2x^2+149}{225}\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(d^2*a*(1/5*c^5*x^5-2/3*c^3*x^3+c*x)+d^2*b*(1/5*arccosh(c*x)*c^5*x^5-2/
3*c^3*x^3*arccosh(c*x)+c*x*arccosh(c*x)-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(
9*c^4*x^4-38*c^2*x^2+149)))
```

maxima [A] time = 0.37, size = 194, normalized size = 1.36

$$\frac{1}{5}\,ac^4d^2x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-1}}{c^6}\right)c\right)bc^4d^2-\frac{2}{3}\,ac^2d^2x^3-\frac{2}{9}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*c^4*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^
2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^4*d^2 - 2
/3*a*c^2*d^2*x^3 - 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 +
 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c
^2*x^2 - 1))*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 2.23, size = 172, normalized size = 1.20

$$\begin{cases}\frac{ac^4d^2x^5}{5}-\frac{2ac^2d^2x^3}{3}+ad^2x+\frac{bc^4d^2x^5\operatorname{acosh}(cx)}{5}-\frac{bc^3d^2x^4\sqrt{c^2x^2-1}}{25}-\frac{2bc^2d^2x^3\operatorname{acosh}(cx)}{3}+\frac{38bcd^2x^2\sqrt{c^2x^2-1}}{225}+bd^2x\operatorname{acosh}(c\\ d^2x\left(a+\frac{i\pi b}{2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*c**4*d**2*x**5/5 - 2*a*c**2*d**2*x**3/3 + a*d**2*x + b*c**4*d*
*2*x**5*acosh(c*x)/5 - b*c**3*d**2*x**4*sqrt(c**2*x**2 - 1)/25 - 2*b*c**2*d
**2*x**3*acosh(c*x)/3 + 38*b*c*d**2*x**2*sqrt(c**2*x**2 - 1)/225 + b*d**2*x
*acosh(c*x) - 149*b*d**2*sqrt(c**2*x**2 - 1)/(225*c), Ne(c, 0)), (d**2*x*(a
 + I*pi*b/2), True))
```

3.15 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=184

$$\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)^2}{2b}+d^2\log\left(e^{-2\cosh^{-1}(c\ldots\right.$$

```
[Out] -1/16*b*c*d^2*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)-11/32*b*d^2*arccosh(c*x)+1/2*d^
2*(-c^2*x^2+1)*(a+b*arccosh(c*x))+1/4*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))
+1/2*d^2*(a+b*arccosh(c*x))^2/b+d^2*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)+11/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\ldots\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (11*b*c*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (b*c*d^2*x*(-1 + c*x)^(3/2
)*(1 + c*x)^(3/2))/16 - (11*b*d^2*ArcCosh[c*x])/32 + (d^2*(1 - c^2*x^2)*(a
+ b*ArcCosh[c*x]))/2 + (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^2*
(a + b*ArcCosh[c*x])^2)/(2*b) + d^2*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCo
sh[c*x])] + (b*d^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx=\frac{1}{4}d^2\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+d\int\frac{\left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(c\right.}{x}$$

$$=-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{2}d^2\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)$$

$$=\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{2}d^2\,(1$$

$$=\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2$$

$$=\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2$$

$$=\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2$$

$$=\frac{11}{32}bcd^2x\sqrt{-1+cx}\,\sqrt{1+cx}-\frac{1}{16}bcd^2x(-1+cx)^{3/2}(1+cx)^{3/2}-\frac{11}{32}bd^2$$

Mathematica [A] time = 0.28, size = 162, normalized size = 0.88

$$\frac{1}{32}d^2\left(8ac^4x^4-32ac^2x^2+32a\log(x)-2bc^3x^3\sqrt{cx-1}\,\sqrt{cx+1}+8b\cosh^{-1}(cx)\left(c^4x^4-4c^2x^2+4\log\left(e^{-2\cosh}\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (d^2*(-32*a*c^2*x^2 + 8*a*c^4*x^4 + 13*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] -
 2*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 16*b*ArcCosh[c*x]^2 + 26*b*ArcT
```

```
anh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 8*b*ArcCosh[c*x]*(-4*c^2*x^2 + c^4*x^4 +
4*Log[1 + E^(-2*ArcCosh[c*x])]) + 32*a*Log[x] - 16*b*PolyLog[2, -E^(-2*ArcC
osh[c*x])]))/32
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\,(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.34, size = 201, normalized size = 1.09

$$\frac{d^2a\,c^4x^4}{4} - d^2a\,c^2x^2 + d^2a\ln(cx) + \frac{13b\,d^2\text{arccosh}\,(cx)}{32} + d^2b\,\text{arccosh}\,(cx)\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right) + \frac{d^2b\,\text{po}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/4*d^2*a*c^4*x^4-d^2*a*c^2*x^2+d^2*a*ln(c*x)+13/32*b*d^2*arccosh(c*x)+d^2*
b*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*d^2*b*polylog(
2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/16*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)*c^3*x^3+13/32*b*c*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/4*d^2*b*arccosh(c
*x)*c^4*x^4-d^2*b*arccosh(c*x)*c^2*x^2-1/2*d^2*b*arccosh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,ac^4d^2x^4 - ac^2d^2x^2 + ad^2\log(x) + \int bc^4d^2x^3\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right) - 2\,bc^2d^2x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right) + \frac{b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*c^4*d^2*x^4 - a*c^2*d^2*x^2 + a*d^2*log(x) + integrate(b*c^4*d^2*x^3*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*x*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\text{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a}{x}\,dx+\int\left(-2ac^2x\right)dx+\int ac^4x^3\,dx+\int\frac{b\operatorname{acosh}(cx)}{x}\,dx+\int\left(-2bc^2x\operatorname{acosh}(cx)\right)dx+\int bc^4x^3\operatorname{acos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] d**2*(Integral(a/x, x) + Integral(-2*a*c**2*x, x) + Integral(a*c**4*x**3, x
) + Integral(b*acosh(c*x)/x, x) + Integral(-2*b*c**2*x*acosh(c*x), x) + Int
egral(b*c**4*x**3*acosh(c*x), x))
```

3.16 $\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=135

$$\frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(cx)\right)-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{1}{9}bcd^2(cx-1)^{3/2}(cx+1)^{3/2}+\frac{5}{3}bcd^2\sqrt{}$$

```
[Out] -1/9*b*c*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)-d^2*(a+b*arccosh(c*x))/x-2*c^2*d^2
*x*(a+b*arccosh(c*x))+1/3*c^4*d^2*x^3*(a+b*arccosh(c*x))+b*c*d^2*arctan((c*
x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 182, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {270, 5731, 12, 520, 1251, 897, 1153, 205}

$$\frac{1}{3}c^4d^2x^3\left(a+b\cosh^{-1}(cx)\right)-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bcd^2\left(1-c^2\right.}{3\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-5*b*c*d^2*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 -
 c^2*x^2)^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/
x - 2*c^2*d^2*x*(a + b*ArcCosh[c*x]) + (c^4*d^2*x^3*(a + b*ArcCosh[c*x]))/3
 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
```

```
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-2c^2d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^2x^3\left(a+b\right.$$

$$= -\frac{5bcd^2\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}$$

$$= -\frac{5bcd^2\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}$$

Mathematica [A] time = 0.17, size = 131, normalized size = 0.97

$$\frac{d^2\left(3ac^4x^4-18ac^2x^2-9a-bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+3b\left(c^4x^4-6c^2x^2-3\right)\cosh^{-1}(cx)+16bcx\sqrt{cx-1}\sqrt{cx+1}\right.}{9x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*(-9*a - 18*a*c^2*x^2 + 3*a*c^4*x^4 + 16*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 +
c*x] - b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 3*b*(-3 - 6*c^2*x^2 + c^4*x
^4)*ArcCosh[c*x] - 9*b*c*x*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))/(9*x)
```

fricas [A] time = 0.64, size = 201, normalized size = 1.49

$$3\,ac^4d^2x^4-18\,ac^2d^2x^2+18\,bcd^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-3\left(bc^4-6\,bc^2-3\,b\right)d^2x\log\left(-cx+\sqrt{c^2x^2-1}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^4*d^2*x^4 - 18*a*c^2*d^2*x^2 + 18*b*c*d^2*x*arctan(-c*x + sqrt(c
^2*x^2 - 1)) - 3*(b*c^4 - 6*b*c^2 - 3*b)*d^2*x*log(-c*x + sqrt(c^2*x^2 - 1)
) - 9*a*d^2 + 3*(b*c^4*d^2*x^4 - 6*b*c^2*d^2*x^2 - (b*c^4 - 6*b*c^2 - 3*b)*
d^2*x - 3*b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^3*d^2*x^3 - 16*b*c*d^2
*x)*sqrt(c^2*x^2 - 1))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 167, normalized size = 1.24

$$\frac{d^2a\,c^4x^3}{3}-2d^2a\,c^2x-\frac{d^2a}{x}+\frac{d^2b\,\mathrm{arccosh}\,(cx)\,c^4x^3}{3}-2d^2b\,\mathrm{arccosh}\,(cx)\,c^2x-\frac{d^2b\,\mathrm{arccosh}\,(cx)}{x}-\frac{d^2b\sqrt{cx-1}\,\sqrt{cx+1}\,c}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/3*d^2*a*c^4*x^3-2*d^2*a*c^2*x-d^2*a/x+1/3*d^2*b*arccosh(c*x)*c^4*x^3-2*d^
2*b*arccosh(c*x)*c^2*x-d^2*b*arccosh(c*x)/x-1/9*d^2*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*c^3*x^2+16/9*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c*d^2*b*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.46, size = 143, normalized size = 1.06

$$\frac{1}{3}\,ac^4d^2x^3+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^4d^2-2\,ac^2d^2x-2\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*c^4*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
+ 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^2 - 2*a*c^2*d^2*x - 2*(c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*c*d^2 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b
*d^2 - a*d^2/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\left(-2ac^2\right)dx + \int\frac{a}{x^2}\,dx + \int ac^4x^2\,dx + \int\left(-2bc^2\,\mathrm{acosh}\,(cx)\right)dx + \int\frac{b\,\mathrm{acosh}\,(cx)}{x^2}\,dx + \int bc^4x^2\,\mathrm{acosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] d**2*(Integral(-2*a*c**2, x) + Integral(a/x**2, x) + Integral(a*c**4*x**2,
x) + Integral(-2*b*c**2*acosh(c*x), x) + Integral(b*acosh(c*x)/x**2, x) + I
ntegral(b*c**4*x**2*acosh(c*x), x))
```

3.17 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=200

$$-c^2d^2(1-c^2x^2)(a+b\cosh^{-1}(cx))-\frac{d^2(1-c^2x^2)^2(a+b\cosh^{-1}(cx))}{2x^2}-\frac{c^2d^2(a+b\cosh^{-1}(cx))^2}{b}-2c^2d^2\log\left(e^{-2c\cdots}\right.$$

```
[Out] -1/2*b*c*d^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/x-1/4*b*c^2*d^2*arccosh(c*x)-c^2*d
^2*(-c^2*x^2+1)*(a+b*arccosh(c*x))-1/2*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x)
)/x^2-c^2*d^2*(a+b*arccosh(c*x))^2/b-2*c^2*d^2*(a+b*arccosh(c*x))*ln(1+1/(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+b*c^2*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)+1/4*b*c^3*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5729, 97, 12, 38, 52, 5727, 5660, 3718, 2190, 2279, 2391}

$$-bc^2d^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-c^2d^2(1-c^2x^2)(a+b\cosh^{-1}(cx))-\frac{d^2(1-c^2x^2)^2(a+b\cosh^{-1}(cx))}{2x^2}+\frac{c^2d^2(a\cdots}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c^3*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/4 - (b*c*d^2*(-1 + c*x)^(3/2)*(1
 + c*x)^(3/2))/(2*x) - (b*c^2*d^2*ArcCosh[c*x])/4 - c^2*d^2*(1 - c^2*x^2)*(
a + b*ArcCosh[c*x]) - (d^2*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/(2*x^2) +
(c^2*d^2*(a + b*ArcCosh[c*x])^2)/b - 2*c^2*d^2*(a + b*ArcCosh[c*x])*Log[1 +
 E^(2*ArcCosh[c*x])] - b*c^2*d^2*PolyLog[2, -E^(2*ArcCosh[c*x])]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^3}dx = -\frac{d^2(1-c^2x^2)^2(a+b\cosh^{-1}(cx))}{2x^2} - (2c^2d)\int \frac{(d-c^2dx^2)(a+b\cosh^{-1}}{x}$$

$$= -\frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} - c^2d^2(1-c^2x^2)(a+b\cosh^{-1}(cx)) - \frac{d^2(1}{}$$

$$= -\frac{1}{2}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} - c^2d^2(1-c^2$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} + \frac{1}{2}bc^2d^2\cosh$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} - \frac{1}{4}bc^2d^2\cosh$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} - \frac{1}{4}bc^2d^2\cosh$$

$$= \frac{1}{4}bc^3d^2x\sqrt{-1+cx}\sqrt{1+cx} - \frac{bcd^2(-1+cx)^{3/2}(1+cx)^{3/2}}{2x} - \frac{1}{4}bc^2d^2\cosh$$

Mathematica [A] time = 0.27, size = 182, normalized size = 0.91

$$\frac{d^2\left(2ac^4x^4 - 8ac^2x^2\log(x) - 2a - bc^3x^3\sqrt{cx-1}\sqrt{cx+1} + 4bc^2x^2\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right) - 4bc^2x^2\cosh^{-1}(cx)^2 - 2bc\right.}{4x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*(-2*a + 2*a*c^4*x^4 + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - b*c^3*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^2*x^2*ArcCosh[c*x]^2 - 2*b*c^2*x^2*Ar
cTanh[Sqrt[(-1 + c*x)/(1 + c*x)]] + 2*b*ArcCosh[c*x]*(-1 + c^4*x^4 - 4*c^2*
x^2*Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*c^2*x^2*Log[x] + 4*b*c^2*x^2*PolyLo
g[2, -E^(-2*ArcCosh[c*x])]))/(4*x^2)
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.56, size = 220, normalized size = 1.10

$$\frac{c^4d^2a\,x^2}{2}-2c^2d^2a\ln(cx)-\frac{d^2a}{2x^2}+c^2d^2b\operatorname{arccosh}(cx)^2-\frac{b\,c^3d^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4}+\frac{c^4d^2b\operatorname{arccosh}(cx)\,x^2}{2}-\frac{b\,c^2d^2\mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/2*c^4*d^2*a*x^2-2*c^2*d^2*a*ln(c*x)-1/2*d^2*a/x^2+c^2*d^2*b*arccosh(c*x)^
2-1/4*b*c^3*d^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/2*c^4*d^2*b*arccosh(c*x)*x^
2-1/4*b*c^2*d^2*arccosh(c*x)-1/2*d^2*b*c^2+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/x-1/2*d^2*b*arccosh(c*x)/x^2-2*c^2*d^2*b*arccosh(c*x)*ln(1+(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)-c^2*d^2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}ac^4d^2x^2-2\,ac^2d^2\log(x)+\frac{1}{2}bd^2\left(\frac{\sqrt{c^2x^2-1}\,c}{x}-\frac{\operatorname{arcosh}(cx)}{x^2}\right)-\frac{ad^2}{2\,x^2}+\int bc^4d^2x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)-\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*c^4*d^2*x^2 - 2*a*c^2*d^2*log(x) + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x -
 arccosh(c*x)/x^2) - 1/2*a*d^2/x^2 + integrate(b*c^4*d^2*x*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) - 2*b*c^2*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int\frac{a}{x^3}\,dx+\int\left(-\frac{2ac^2}{x}\right)dx+\int ac^4x\,dx+\int\frac{b\operatorname{acosh}(cx)}{x^3}\,dx+\int\left(-\frac{2bc^2\operatorname{acosh}(cx)}{x}\right)dx+\int bc^4x\operatorname{acosh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] d**2*(Integral(a/x**3, x) + Integral(-2*a*c**2/x, x) + Integral(a*c**4*x, x
) + Integral(b*acosh(c*x)/x**3, x) + Integral(-2*b*c**2*acosh(c*x)/x, x) +
Integral(b*c**4*x*acosh(c*x), x))
```

3.18 $\int \frac{(d-c^2dx^2)^2(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=142

$$c^4d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-bc^3d^2\sqrt{cx-1}\,\sqrt{cx+1}-\frac{11}{6}bc^3d^2\tan^{-}$$

```
[Out] -1/3*d^2*(a+b*arccosh(c*x))/x^3+2*c^2*d^2*(a+b*arccosh(c*x))/x+c^4*d^2*x*(a
+b*arccosh(c*x))-11/6*b*c^3*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))-b*c^3*d
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A] time = 0.23, antiderivative size = 186, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {270, 5731, 12, 520, 1251, 897, 1157, 388, 205}

$$c^4d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\,\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c^3*d^2*(1 - c^2*x^2))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^
2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(
3*x^3) + (2*c^2*d^2*(a + b*ArcCosh[c*x]))/x + c^4*d^2*x*(a + b*ArcCosh[c*x]
) - (11*b*c^3*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
```

```
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^4}dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+c^4d^2x\left(a+b\cosh^{-1}\right.$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{2c^2d^2\left(a+b\cosh^{-1}(\right.}{x}$$

$$= \frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+$$

$$= \frac{bc^3d^2\left(1-c^2x^2\right)}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+$$

Mathematica [A] time = 0.17, size = 135, normalized size = 0.95

$$\frac{d^2\left(6ac^4x^4+12ac^2x^2-2a-6bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+11bc^3x^3\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+2b\left(3c^4x^4+6c^2x^2-1\right)\cosh\right.}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^2*(-2*a + 12*a*c^2*x^2 + 6*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x
] - 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(-1 + 6*c^2*x^2 + 3*c^4*
x^4)*ArcCosh[c*x] + 11*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]))
/(6*x^3)
```

fricas [A] time = 0.57, size = 213, normalized size = 1.50

$$6\,ac^4d^2x^4-22\,bc^3d^2x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+12\,ac^2d^2x^2-2\left(3\,bc^4+6\,bc^2-b\right)d^2x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c^4*d^2*x^4 - 22*b*c^3*d^2*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
12*a*c^2*d^2*x^2 - 2*(3*b*c^4 + 6*b*c^2 - b)*d^2*x^3*log(-c*x + sqrt(c^2*x^
2 - 1)) - 2*a*d^2 + 2*(3*b*c^4*d^2*x^4 + 6*b*c^2*d^2*x^2 - (3*b*c^4 + 6*b*c
^2 - b)*d^2*x^3 - b*d^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (6*b*c^3*d^2*x^3 -
b*c*d^2*x)*sqrt(c^2*x^2 - 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 167, normalized size = 1.18

$$c^4d^2ax+\frac{2c^2d^2a}{x}-\frac{d^2a}{3x^3}+c^4d^2b\,\mathrm{arccosh}\,(cx)\,x+\frac{2c^2d^2b\,\mathrm{arccosh}\,(cx)}{x}-\frac{d^2b\,\mathrm{arccosh}\,(cx)}{3x^3}-b\,c^3d^2\sqrt{cx-1}\,\sqrt{cx+1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] c^4*d^2*a*x+2*c^2*d^2*a/x-1/3*d^2*a/x^3+c^4*d^2*b*arccosh(c*x)*x+2*c^2*d^2*
b*arccosh(c*x)/x-1/3*d^2*b*arccosh(c*x)/x^3-b*c^3*d^2*(c*x-1)^(1/2)*(c*x+1)
^(1/2)+11/6*c^3*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(
1/(c^2*x^2-1)^(1/2))+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

maxima [A] time = 0.46, size = 137, normalized size = 0.96

$$ac^4d^2x+\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-1}\right)bc^3d^2+2\left(c\,\mathrm{arcsin}\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bc^2d^2-\frac{1}{6}\left(\left(c^2\,\mathrm{arcsin}\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] a*c^4*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*c^3*d^2 + 2*(c*arcsi
n(1/(c*abs(x))) + arccosh(c*x)/x)*b*c^2*d^2 - 1/6*((c^2*arcsin(1/(c*abs(x))
) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)/x^3)*b*d^2 + 2*a*c^2*d^2/x -
1/3*a*d^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int ac^4\,dx+\int\frac{a}{x^4}\,dx+\int\left(-\frac{2ac^2}{x^2}\right)dx+\int bc^4\,\mathrm{acosh}\,(cx)\,dx+\int\frac{b\,\mathrm{acosh}\,(cx)}{x^4}\,dx+\int\left(-\frac{2bc^2\,\mathrm{acosh}\,(cx)}{x^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] d**2*(Integral(a*c**4, x) + Integral(a/x**4, x) + Integral(-2*a*c**2/x**2,
x) + Integral(b*c**4*acosh(c*x), x) + Integral(b*acosh(c*x)/x**4, x) + Inte
gral(-2*b*c**2*acosh(c*x)/x**2, x))
```

3.19 $\int x^4 \left(d - c^2 d x^2\right)^3 \left(a + b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=256

$$-\frac{1}{11}c^6 d^3 x^{11} \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4 d^3 x^9 \left(a + b\cosh^{-1}(cx)\right) - \frac{3}{7}c^2 d^3 x^7 \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{5}d^3 x^5 \left(a + b\cosh^{-1}(cx\right.$$

```
[Out] 8/3465*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^5-2/1925*b*d^3*(c*x-1)^(5/2)*(c*
x+1)^(5/2)/c^5+1/1617*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^5+4/297*b*d^3*(c*
x-1)^(9/2)*(c*x+1)^(9/2)/c^5+1/121*b*d^3*(c*x-1)^(11/2)*(c*x+1)^(11/2)/c^5+
1/5*d^3*x^5*(a+b*arccosh(c*x))-3/7*c^2*d^3*x^7*(a+b*arccosh(c*x))+1/3*c^4*d
^3*x^9*(a+b*arccosh(c*x))-1/11*c^6*d^3*x^11*(a+b*arccosh(c*x))-16/1155*b*d^
3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5
```

Rubi [A] time = 0.44, antiderivative size = 326, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {270, 5731, 12, 1610, 1799, 1620}

$$-\frac{1}{11}c^6 d^3 x^{11} \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{3}c^4 d^3 x^9 \left(a + b\cosh^{-1}(cx)\right) - \frac{3}{7}c^2 d^3 x^7 \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{5}d^3 x^5 \left(a + b\cosh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(1155*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3
*(1 - c^2*x^2)^2)/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c
^2*x^2)^3)/(1925*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4
)/(1617*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*d^3*(1 - c^2*x^2)^5)/(297*
c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^6)/(121*c^5*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (3*c^2*d^3*x^7
*(a + b*ArcCosh[c*x]))/7 + (c^4*d^3*x^9*(a + b*ArcCosh[c*x]))/3 - (c^6*d^3*
x^11*(a + b*ArcCosh[c*x]))/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{7}c^2d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}c^4d^3x^9$$

$$=\frac{16bd^3\left(1-c^2x^2\right)}{1155c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{3465c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2bd^3}{1925c^5\sqrt{\cdot}}$$

Mathematica [A] time = 0.28, size = 147, normalized size = 0.57

$$-\frac{d^3\left(3465ac^5x^5\left(105c^6x^6-385c^4x^4+495c^2x^2-231\right)+3465bc^5x^5\left(105c^6x^6-385c^4x^4+495c^2x^2-231\right)\cosh}{4002}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/4002075*(d^3*(3465*a*c^5*x^5*(-231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6
*x^6) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(50488 + 25244*c^2*x^2 + 18933*c^4*x
^4 - 117625*c^6*x^6 + 111475*c^8*x^8 - 33075*c^10*x^10) + 3465*b*c^5*x^5*(-
231 + 495*c^2*x^2 - 385*c^4*x^4 + 105*c^6*x^6)*ArcCosh[c*x]))/c^5
```

fricas [A] time = 0.76, size = 201, normalized size = 0.79

$$-\frac{363825\,ac^{11}d^3x^{11}-1334025\,ac^9d^3x^9+1715175\,ac^7d^3x^7-800415\,ac^5d^3x^5+3465\left(105\,bc^{11}d^3x^{11}-385\,bc^9d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/4002075*(363825*a*c^11*d^3*x^11 - 1334025*a*c^9*d^3*x^9 + 1715175*a*c^7*
d^3*x^7 - 800415*a*c^5*d^3*x^5 + 3465*(105*b*c^11*d^3*x^11 - 385*b*c^9*d^3*
x^9 + 495*b*c^7*d^3*x^7 - 231*b*c^5*d^3*x^5)*log(c*x + sqrt(c^2*x^2 - 1)) -
 (33075*b*c^10*d^3*x^10 - 111475*b*c^8*d^3*x^8 + 117625*b*c^6*d^3*x^6 - 189
33*b*c^4*d^3*x^4 - 25244*b*c^2*d^3*x^2 - 50488*b*d^3)*sqrt(c^2*x^2 - 1))/c^
5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 158, normalized size = 0.62

$$\frac{-d^3a\left(\frac{1}{11}c^{11}x^{11}-\frac{1}{3}c^9x^9+\frac{3}{7}c^7x^7-\frac{1}{5}c^5x^5\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^{11}x^{11}}{11}-\frac{\operatorname{arccosh}(cx)c^9x^9}{3}+\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{\operatorname{arccosh}(cx)c^5x^5}{5}-\right.}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(-d^3*a*(1/11*c^11*x^11-1/3*c^9*x^9+3/7*c^7*x^7-1/5*c^5*x^5)-d^3*b*(1
/11*arccosh(c*x)*c^11*x^11-1/3*arccosh(c*x)*c^9*x^9+3/7*arccosh(c*x)*c^7*x^
7-1/5*arccosh(c*x)*c^5*x^5-1/4002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^1
0*x^10-111475*c^8*x^8+117625*c^6*x^6-18933*c^4*x^4-25244*c^2*x^2-50488)))
```

maxima [B] time = 0.36, size = 465, normalized size = 1.82

$$-\frac{1}{11}ac^6d^3x^{11}+\frac{1}{3}ac^4d^3x^9-\frac{3}{7}ac^2d^3x^7-\frac{1}{7623}\left(693\,x^{11}\operatorname{arcosh}(cx)-\left(\frac{63\sqrt{c^2x^2-1}\,x^{10}}{c^2}+\frac{70\sqrt{c^2x^2-1}\,x^8}{c^4}+\frac{80\sqrt{c^2}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/11*a*c^6*d^3*x^11 + 1/3*a*c^4*d^3*x^9 - 3/7*a*c^2*d^3*x^7 - 1/7623*(693*
x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 - 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x
^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6 + 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*
sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(c^2*x^2 - 1)/c^12)*c)*b*c^6*d^3 + 1/9
45*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2
- 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8
+ 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*c^4*d^3 + 1/5*a*d^3*x^5 - 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^2*d^3 + 1/7
5*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)
*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

sympy [A] time = 39.28, size = 296, normalized size = 1.16

$$\begin{cases} -\frac{ac^6d^3x^{11}}{11} + \frac{ac^4d^3x^9}{3} - \frac{3ac^2d^3x^7}{7} + \frac{ad^3x^5}{5} - \frac{bc^6d^3x^{11}\operatorname{acosh}(cx)}{11} + \frac{bc^5d^3x^{10}\sqrt{c^2x^2-1}}{121} + \frac{bc^4d^3x^9\operatorname{acosh}(cx)}{3} - \frac{91bc^3d^3x^8\sqrt{c^2x^2-1}}{3267} - \\ \frac{d^3x^5\left(a+\frac{i\pi b}{2}\right)}{5} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**11/11 + a*c**4*d**3*x**9/3 - 3*a*c**2*d**3*x**7/
7 + a*d**3*x**5/5 - b*c**6*d**3*x**11*acosh(c*x)/11 + b*c**5*d**3*x**10*sqr
t(c**2*x**2 - 1)/121 + b*c**4*d**3*x**9*acosh(c*x)/3 - 91*b*c**3*d**3*x**8*
sqrt(c**2*x**2 - 1)/3267 - 3*b*c**2*d**3*x**7*acosh(c*x)/7 + 4705*b*c*d**3*
x**6*sqrt(c**2*x**2 - 1)/160083 + b*d**3*x**5*acosh(c*x)/5 - 6311*b*d**3*x*
*4*sqrt(c**2*x**2 - 1)/(1334025*c) - 25244*b*d**3*x**2*sqrt(c**2*x**2 - 1)/
(4002075*c**3) - 50488*b*d**3*sqrt(c**2*x**2 - 1)/(4002075*c**5), Ne(c, 0))
, (d**3*x**5*(a + I*pi*b/2)/5, True))
```

3.20 $\int x^3 \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=230

$$-\frac{d^3(cx-1)^5(cx+1)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{d^3(cx-1)^4(cx+1)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{49bd^3\cosh^{-1}(cx)}{5120c^4}+\frac{bd^3x(cx-$$

```
[Out] 49/7680*b*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-49/9600*b*d^3*x*(c*x-1)^(5/
2)*(c*x+1)^(5/2)/c^3+7/1600*b*d^3*x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/100*b
*d^3*x*(c*x-1)^(9/2)*(c*x+1)^(9/2)/c^3+49/5120*b*d^3*arccosh(c*x)/c^4-1/8*d
^3*(c*x-1)^4*(c*x+1)^4*(a+b*arccosh(c*x))/c^4-1/10*d^3*(c*x-1)^5*(c*x+1)^5*
(a+b*arccosh(c*x))/c^4-49/5120*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.28, antiderivative size = 328, normalized size of antiderivative = 1.43, number of steps used = 11, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {266, 43, 5731, 12, 566, 21, 388, 195, 217, 206}

$$\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{7bd^3x\left(1-c^2x^2\right.}{1600c^3\sqrt{cx-1}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (49*b*d^3*x*(1 - c^2*x^2))/(5120*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*
d^3*x*(1 - c^2*x^2)^2)/(7680*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (49*b*d^3*
x*(1 - c^2*x^2)^3)/(9600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*d^3*x*(1
- c^2*x^2)^4)/(1600*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*x*(1 - c^2*x
^2)^5)/(100*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(1 - c^2*x^2)^4*(a + b
*ArcCosh[c*x]))/(8*c^4) + (d^3*(1 - c^2*x^2)^5*(a + b*ArcCosh[c*x]))/(10*c^
4) + (49*b*d^3*Sqrt[-1 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*
c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 566

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Dist[((e1 + f1*x^(n/2))^FracPart[r]*(e2 + f2*x^(n/2))^FracPart[r])/(e1*e2 +
 f1*f2*x^n)^FracPart[r], Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}$$

$$=-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}$$

$$=-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}$$

$$=-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3\left(1-c^2x^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10c^4}$$

$$=-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^4}+\frac{d^3(1}{}$$

$$=\frac{7bd^3x\left(1-c^2x^2\right)^4}{1600c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bd^3x\left(1-c^2x^2\right)^5}{100c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(1-c^2x^2\right)^4}{}$$

$$=\frac{49bd^3x\left(1-c^2x^2\right)^3}{9600c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7bd^3x\left(1-c^2x^2\right)^4}{1600c^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bd^3x(1}{100c^3\sqrt{-1+}}$$

$$=\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^3}{9600c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{7bd^3x(1}{1600c^3\sqrt{-1}}$$

$$=\frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x(}{9600c^3\sqrt{-1}}$$

$$=\frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x(}{9600c^3\sqrt{-1}}$$

$$=\frac{49bd^3x\left(1-c^2x^2\right)}{5120c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x\left(1-c^2x^2\right)^2}{7680c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{49bd^3x(}{9600c^3\sqrt{-1}}$$

Mathematica [A] time = 0.37, size = 162, normalized size = 0.70

$$-\frac{d^3\left(1920ac^4x^4\left(4c^6x^6-15c^4x^4+20c^2x^2-10\right)+1920bc^4x^4\left(4c^6x^6-15c^4x^4+20c^2x^2-10\right)\cosh^{-1}(cx)+bcx\sqrt{c}\right.}{76800c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/76800*(d^3*(1920*a*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 + 4*c^6*x^6) +
 b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1185 + 790*c^2*x^2 - 3208*c^4*x^4 + 27
36*c^6*x^6 - 768*c^8*x^8) + 1920*b*c^4*x^4*(-10 + 20*c^2*x^2 - 15*c^4*x^4 +
 4*c^6*x^6)*ArcCosh[c*x] + 2370*b*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^4
```

fricas [A] time = 0.59, size = 197, normalized size = 0.86

$$-\frac{7680\,ac^{10}d^3x^{10}-28800\,ac^8d^3x^8+38400\,ac^6d^3x^6-19200\,ac^4d^3x^4+15\left(512\,bc^{10}d^3x^{10}-1920\,bc^8d^3x^8+2560\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/76800*(7680*a*c^10*d^3*x^10 - 28800*a*c^8*d^3*x^8 + 38400*a*c^6*d^3*x^6
- 19200*a*c^4*d^3*x^4 + 15*(512*b*c^10*d^3*x^10 - 1920*b*c^8*d^3*x^8 + 2560
*b*c^6*d^3*x^6 - 1280*b*c^4*d^3*x^4 + 79*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)
) - (768*b*c^9*d^3*x^9 - 2736*b*c^7*d^3*x^7 + 3208*b*c^5*d^3*x^5 - 790*b*c^
3*d^3*x^3 - 1185*b*c*d^3*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 284, normalized size = 1.23

$$-\frac{c^6 d^3 a\, x^{10}}{10}+\frac{3 c^4 d^3 a\, x^8}{8}-\frac{c^2 d^3 a\, x^6}{2}+\frac{d^3 a\, x^4}{4}-\frac{c^6 d^3 b \,\mathrm{arccosh}\left(c x\right) x^{10}}{10}+\frac{3 c^4 d^3 b \,\mathrm{arccosh}\left(c x\right) x^8}{8}-\frac{c^2 d^3 b \,\mathrm{arccosh}\left(c x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/10*c^6*d^3*a*x^10+3/8*c^4*d^3*a*x^8-1/2*c^2*d^3*a*x^6+1/4*d^3*a*x^4-1/10
*c^6*d^3*b*arccosh(c*x)*x^10+3/8*c^4*d^3*b*arccosh(c*x)*x^8-1/2*c^2*d^3*b*a
rccosh(c*x)*x^6+1/4*d^3*b*arccosh(c*x)*x^4+1/100*c^5*d^3*b*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*x^9-57/1600*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^7+401/9600*
c*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5-79/7680/c*d^3*b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*x^3-79/5120*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-79/5120/c^4*d^
3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [B] time = 0.37, size = 501, normalized size = 2.18

$$-\frac{1}{10}\, ac^6 d^3 x^{10}+\frac{3}{8}\, ac^4 d^3 x^8-\frac{1}{2}\, ac^2 d^3 x^6-\frac{1}{12800}\left(1280\, x^{10}\,\mathrm{arcosh}\left(cx\right)-\left(\frac{128\,\sqrt{c^2 x^2-1}\, x^9}{c^2}+\frac{144\,\sqrt{c^2 x^2-1}\, x^7}{c^4}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/10*a*c^6*d^3*x^10 + 3/8*a*c^4*d^3*x^8 - 1/2*a*c^2*d^3*x^6 - 1/12800*(128
0*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sqrt(c^2*x^2 - 1
)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2 - 1)*x^3/c^8 +
 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^
11)*c)*b*c^6*d^3 + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7
/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sq
rt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*
c^4*d^3 + 1/4*a*d^3*x^4 - 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*
x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*lo
g(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^2*d^3 + 1/32*(8*x^4*arccosh(
c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c
^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

sympy [A] time = 28.39, size = 287, normalized size = 1.25

$$\begin{cases} -\frac{ac^6d^3x^{10}}{10} + \frac{3ac^4d^3x^8}{8} - \frac{ac^2d^3x^6}{2} + \frac{ad^3x^4}{4} - \frac{bc^6d^3x^{10}\operatorname{acosh}(cx)}{10} + \frac{bc^5d^3x^9\sqrt{c^2x^2-1}}{100} + \frac{3bc^4d^3x^8\operatorname{acosh}(cx)}{8} - \frac{57bc^3d^3x^7\sqrt{c^2x^2-1}}{1600} - \frac{bc^2d \\ \frac{d^3x^4\left(a+\frac{i\pi b}{2}\right)}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**10/10 + 3*a*c**4*d**3*x**8/8 - a*c**2*d**3*x**6/
2 + a*d**3*x**4/4 - b*c**6*d**3*x**10*acosh(c*x)/10 + b*c**5*d**3*x**9*sqrt
(c**2*x**2 - 1)/100 + 3*b*c**4*d**3*x**8*acosh(c*x)/8 - 57*b*c**3*d**3*x**7
*sqrt(c**2*x**2 - 1)/1600 - b*c**2*d**3*x**6*acosh(c*x)/2 + 401*b*c*d**3*x*
*5*sqrt(c**2*x**2 - 1)/9600 + b*d**3*x**4*acosh(c*x)/4 - 79*b*d**3*x**3*sqr
t(c**2*x**2 - 1)/(7680*c) - 79*b*d**3*x*sqrt(c**2*x**2 - 1)/(5120*c**3) - 7
9*b*d**3*acosh(c*x)/(5120*c**4), Ne(c, 0)), (d**3*x**4*(a + I*pi*b/2)/4, Tr
ue))
```

3.21 $\int x^2 \left(d-c^2 d x^2\right)^3 \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=227

$$-\frac{1}{9}c^6d^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(c\right.$$

```
[Out] 8/945*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c^3-2/525*b*d^3*(c*x-1)^(5/2)*(c*x+
1)^(5/2)/c^3+1/441*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c^3+1/81*b*d^3*(c*x-1)
^(9/2)*(c*x+1)^(9/2)/c^3+1/3*d^3*x^3*(a+b*arccosh(c*x))-3/5*c^2*d^3*x^5*(a+
b*arccosh(c*x))+3/7*c^4*d^3*x^7*(a+b*arccosh(c*x))-1/9*c^6*d^3*x^9*(a+b*arc
cosh(c*x))-16/315*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
```

Rubi [A] time = 0.40, antiderivative size = 285, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = $\{270, 5731, 12, 1610, 1799, 1620\}$

$$-\frac{1}{9}c^6d^3x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}c^4d^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(c\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*
(1 - c^2*x^2)^2)/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d^3*(1 - c^2
*x^2)^3)/(525*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(
441*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*d^3*(1 - c^2*x^2)^5)/(81*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (3*c^2*d^3*
x^5*(a + b*ArcCosh[c*x]))/5 + (3*c^4*d^3*x^7*(a + b*ArcCosh[c*x]))/7 - (c^6
*d^3*x^9*(a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right) - \frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{7}c^4d^3x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right) - \frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{7}c^4d^3x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right) - \frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{7}c^4d^3x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right) - \frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{7}c^4d^3x^7\left(a\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right) - \frac{3}{5}c^2d^3x^5\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{7}c^4d^3x^7\left(a\right.$$

$$= \frac{16bd^3\left(1-c^2x^2\right)}{315c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{8bd^3\left(1-c^2x^2\right)^2}{945c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bd^3\left(1-c\right.}{525c^3\sqrt{-1+c}}$$

Mathematica [A] time = 0.29, size = 139, normalized size = 0.61

$$-\frac{d^3\left(315ac^3x^3\left(35c^6x^6-135c^4x^4+189c^2x^2-105\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-1225c^8x^8+4675c^6x^6-6297c^4x^4+26\right.}{99225c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/99225*(d^3*(315*a*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^4 + 35*c^6*x^6
) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(5258 + 2629*c^2*x^2 - 6297*c^4*x^4 + 46
75*c^6*x^6 - 1225*c^8*x^8) + 315*b*c^3*x^3*(-105 + 189*c^2*x^2 - 135*c^4*x^
4 + 35*c^6*x^6)*ArcCosh[c*x]))/c^3
```

fricas [A] time = 0.55, size = 189, normalized size = 0.83

$$-\frac{11025\,ac^9d^3x^9-42525\,ac^7d^3x^7+59535\,ac^5d^3x^5-33075\,ac^3d^3x^3+315\left(35\,bc^9d^3x^9-135\,bc^7d^3x^7+189\,bc^5d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/99225*(11025*a*c^9*d^3*x^9 - 42525*a*c^7*d^3*x^7 + 59535*a*c^5*d^3*x^5 -
 33075*a*c^3*d^3*x^3 + 315*(35*b*c^9*d^3*x^9 - 135*b*c^7*d^3*x^7 + 189*b*c^
5*d^3*x^5 - 105*b*c^3*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^8*d
^3*x^8 - 4675*b*c^6*d^3*x^6 + 6297*b*c^4*d^3*x^4 - 2629*b*c^2*d^3*x^2 - 525
8*b*d^3)*sqrt(c^2*x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 150, normalized size = 0.66

$$\frac{-d^3a\left(\frac{1}{9}c^9x^9-\frac{3}{7}c^7x^7+\frac{3}{5}c^5x^5-\frac{1}{3}c^3x^3\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^9x^9}{9}-\frac{3\operatorname{arccosh}(cx)c^7x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}-\frac{c^3x^3\operatorname{arccosh}(cx)}{3}-\right.}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(-d^3*a*(1/9*c^9*x^9-3/7*c^7*x^7+3/5*c^5*x^5-1/3*c^3*x^3)-d^3*b*(1/9*
arccosh(c*x)*c^9*x^9-3/7*arccosh(c*x)*c^7*x^7+3/5*arccosh(c*x)*c^5*x^5-1/3*
c^3*x^3*arccosh(c*x)-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*x^8-4675
*c^6*x^6+6297*c^4*x^4-2629*c^2*x^2-5258)))
```

maxima [B] time = 0.37, size = 388, normalized size = 1.71

$$-\frac{1}{9}ac^6d^3x^9+\frac{3}{7}ac^4d^3x^7-\frac{1}{2835}\left(315\,x^9\operatorname{arcosh}(cx)-\left(\frac{35\sqrt{c^2x^2-1}\,x^8}{c^2}+\frac{40\sqrt{c^2x^2-1}\,x^6}{c^4}+\frac{48\sqrt{c^2x^2-1}\,x^4}{c^6}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/9*a*c^6*d^3*x^9 + 3/7*a*c^4*d^3*x^7 - 1/2835*(315*x^9*arccosh(c*x) - (35
*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2
 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*
c)*b*c^6*d^3 - 3/5*a*c^2*d^3*x^5 + 3/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2
*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c
^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^4*d^3 - 1/25*(15*x^5*arccosh(c*x) - (
3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
- 1)/c^6)*c)*b*c^2*d^3 + 1/3*a*d^3*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(
c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

sympy [A] time = 16.71, size = 272, normalized size = 1.20

$$\begin{cases} -\frac{ac^6d^3x^9}{9} + \frac{3ac^4d^3x^7}{7} - \frac{3ac^2d^3x^5}{5} + \frac{ad^3x^3}{3} - \frac{bc^6d^3x^9\operatorname{acosh}(cx)}{9} + \frac{bc^5d^3x^8\sqrt{c^2x^2-1}}{81} + \frac{3bc^4d^3x^7\operatorname{acosh}(cx)}{7} - \frac{187bc^3d^3x^6\sqrt{c^2x^2-1}}{3969} - \frac{3bc^2 \\ \frac{d^3x^3\left(a+\frac{i\pi b}{2}\right)}{3} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**9/9 + 3*a*c**4*d**3*x**7/7 - 3*a*c**2*d**3*x**5/
5 + a*d**3*x**3/3 - b*c**6*d**3*x**9*acosh(c*x)/9 + b*c**5*d**3*x**8*sqrt(c
**2*x**2 - 1)/81 + 3*b*c**4*d**3*x**7*acosh(c*x)/7 - 187*b*c**3*d**3*x**6*s
qrt(c**2*x**2 - 1)/3969 - 3*b*c**2*d**3*x**5*acosh(c*x)/5 + 2099*b*c*d**3*x
**4*sqrt(c**2*x**2 - 1)/33075 + b*d**3*x**3*acosh(c*x)/3 - 2629*b*d**3*x**2
*sqrt(c**2*x**2 - 1)/(99225*c) - 5258*b*d**3*sqrt(c**2*x**2 - 1)/(99225*c**
3), Ne(c, 0)), (d**3*x**3*(a + I*pi*b/2)/3, True))
```

3.22 $\int x\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=166

$$-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{35bd^3\cosh^{-1}(cx)}{1024c^2}+\frac{bd^3x(cx-1)^{7/2}(cx+1)^{7/2}}{64c}-\frac{7bd^3x(cx-1)^{5/2}(cx+1)^{5/2}}{384c}$$

```
[Out] 35/1536*b*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-7/384*b*d^3*x*(c*x-1)^(5/2)*(
c*x+1)^(5/2)/c+1/64*b*d^3*x*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c+35/1024*b*d^3*arc
cosh(c*x)/c^2-1/8*d^3*(-c^2*x^2+1)^4*(a+b*arccosh(c*x))/c^2-35/1024*b*d^3*x
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5716, 38, 52}

$$-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{35bd^3\cosh^{-1}(cx)}{1024c^2}+\frac{bd^3x(cx-1)^{7/2}(cx+1)^{7/2}}{64c}-\frac{7bd^3x(cx-1)^{5/2}(cx+1)^{5/2}}{384c}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-35*b*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1024*c) + (35*b*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/(1536*c) - (7*b*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(
5/2))/(384*c) + (b*d^3*x*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2))/(64*c) + (35*b*d
^3*ArcCosh[c*x])/(1024*c^2) - (d^3*(1 - c^2*x^2)^4*(a + b*ArcCosh[c*x]))/(8
*c^2)
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{\left(bd^3\right)\int(-1+cx)^{7/2}(1+cx)^{7/2}\,dx}{8c}$$
$$=\frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\frac{d^3\left(1-c^2x^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8c^2}-\frac{(7bd}{}$$
$$=-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-1+cx)^{7/2}(1+cx)^{7/2}}{64c}-\frac{d^3\left(1-\right.}{}$$
$$=\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-1+cx)^{5/2}(1+cx)^{5/2}}{384c}+\frac{bd^3x(-}{}$$
$$=-\frac{35bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-}{}$$
$$=-\frac{35bd^3x\sqrt{-1+cx}\sqrt{1+cx}}{1024c}+\frac{35bd^3x(-1+cx)^{3/2}(1+cx)^{3/2}}{1536c}-\frac{7bd^3x(-}{}$$

Mathematica [A] time = 0.39, size = 150, normalized size = 0.90

$$-\frac{d^3\left(cx\left(384acx\left(c^6x^6-4c^4x^4+6c^2x^2-4\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(-48c^6x^6+200c^4x^4-326c^2x^2+279\right)\right)+384bc^2\right.}{3072c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/3072*(d^3*(c*x*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(279 - 326*c^2*x^2 + 200*
c^4*x^4 - 48*c^6*x^6) + 384*a*c*x*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)) +
 384*b*c^2*x^2*(-4 + 6*c^2*x^2 - 4*c^4*x^4 + c^6*x^6)*ArcCosh[c*x] + 558*b*
ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2
```

fricas [A] time = 0.47, size = 185, normalized size = 1.11

$$-\frac{384\,ac^8d^3x^8-1536\,ac^6d^3x^6+2304\,ac^4d^3x^4-1536\,ac^2d^3x^2+3\left(128\,bc^8d^3x^8-512\,bc^6d^3x^6+768\,bc^4d^3x^4-512\right.}{3072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3072*(384*a*c^8*d^3*x^8 - 1536*a*c^6*d^3*x^6 + 2304*a*c^4*d^3*x^4 - 1536
*a*c^2*d^3*x^2 + 3*(128*b*c^8*d^3*x^8 - 512*b*c^6*d^3*x^6 + 768*b*c^4*d^3*x
^4 - 512*b*c^2*d^3*x^2 + 93*b*d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7
*d^3*x^7 - 200*b*c^5*d^3*x^5 + 326*b*c^3*d^3*x^3 - 279*b*c*d^3*x)*sqrt(c^2*
x^2 - 1))/c^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 258, normalized size = 1.55

$$-\frac{c^6 d^3 a\,x^8}{8}+\frac{c^4 d^3 a\,x^6}{2}-\frac{3c^2 d^3 a\,x^4}{4}+\frac{d^3 a\,x^2}{2}-\frac{c^6 d^3 b\,\mathrm{arccosh}\,(cx)\,x^8}{8}+\frac{c^4 d^3 b\,\mathrm{arccosh}\,(cx)\,x^6}{2}-\frac{3c^2 d^3 b\,\mathrm{arccosh}\,(cx)\,x}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*c^6*d^3*a*x^8+1/2*c^4*d^3*a*x^6-3/4*c^2*d^3*a*x^4+1/2*d^3*a*x^2-1/8*c^
6*d^3*b*arccosh(c*x)*x^8+1/2*c^4*d^3*b*arccosh(c*x)*x^6-3/4*c^2*d^3*b*arcco
sh(c*x)*x^4+1/2*d^3*b*arccosh(c*x)*x^2+1/64*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x^7-25/384*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5+163/1536*c*d^3*
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-93/1024*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c-93/1024/c^2*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x
+(c^2*x^2-1)^(1/2))
```

maxima [B] time = 0.37, size = 423, normalized size = 2.55

$$-\frac{1}{8}\,ac^6d^3x^8+\frac{1}{2}\,ac^4d^3x^6-\frac{1}{3072}\left(384\,x^8\,\mathrm{arcosh}\,(cx)-\left(\frac{48\,\sqrt{c^2x^2-1}\,x^7}{c^2}+\frac{56\,\sqrt{c^2x^2-1}\,x^5}{c^4}+\frac{70\,\sqrt{c^2x^2-1}\,x^3}{c^6}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/8*a*c^6*d^3*x^8 + 1/2*a*c^4*d^3*x^6 - 1/3072*(384*x^8*arccosh(c*x) - (48
*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2
 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*c)/c^9)*c)*b*c^6*d^3 - 3/4*a*c^2*d^3*x^4 + 1/96*(48*x^6*arccosh(c*
x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(
c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*c^4*
d^3 - 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*
x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*c^2*d^3 +
 1/2*a*d^3*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log
(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\,(c\,x)\right)\,{\left(d-c^2\,d\,x^2\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

sympy [A] time = 11.61, size = 260, normalized size = 1.57

$$\begin{cases} -\frac{ac^6d^3x^8}{8}+\frac{ac^4d^3x^6}{2}-\frac{3ac^2d^3x^4}{4}+\frac{ad^3x^2}{2}-\frac{bc^6d^3x^8\,\mathrm{acosh}\,(cx)}{8}+\frac{bc^5d^3x^7\sqrt{c^2x^2-1}}{64}+\frac{bc^4d^3x^6\,\mathrm{acosh}\,(cx)}{2}-\frac{25bc^3d^3x^5\sqrt{c^2x^2-1}}{384}-\frac{3bc}{} \\ \frac{d^3x^2\left(a+\frac{i\pi b}{2}\right)}{2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**8/8 + a*c**4*d**3*x**6/2 - 3*a*c**2*d**3*x**4/4
+ a*d**3*x**2/2 - b*c**6*d**3*x**8*acosh(c*x)/8 + b*c**5*d**3*x**7*sqrt(c**
2*x**2 - 1)/64 + b*c**4*d**3*x**6*acosh(c*x)/2 - 25*b*c**3*d**3*x**5*sqrt(c
**2*x**2 - 1)/384 - 3*b*c**2*d**3*x**4*acosh(c*x)/4 + 163*b*c*d**3*x**3*sqr
t(c**2*x**2 - 1)/1536 + b*d**3*x**2*acosh(c*x)/2 - 93*b*d**3*x*sqrt(c**2*x*
*2 - 1)/(1024*c) - 93*b*d**3*acosh(c*x)/(1024*c**2), Ne(c, 0)), (d**3*x**2*
(a + I*pi*b/2)/2, True))
```

3.23 $\int \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=191

$$-\frac{1}{7}c^6d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+$$

```
[Out] 8/105*b*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/c-6/175*b*d^3*(c*x-1)^(5/2)*(c*x+1)
^(5/2)/c+1/49*b*d^3*(c*x-1)^(7/2)*(c*x+1)^(7/2)/c+d^3*x*(a+b*arccosh(c*x))-
c^2*d^3*x^3*(a+b*arccosh(c*x))+3/5*c^4*d^3*x^5*(a+b*arccosh(c*x))-1/7*c^6*d
^3*x^7*(a+b*arccosh(c*x))-16/35*b*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.26, antiderivative size = 237, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {194, 5680, 12, 1610, 1799, 1850}

$$-\frac{1}{7}c^6d^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^3*(1 - c^2*x^2))/(35*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8*b*d^3*(1
 - c^2*x^2)^2)/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*d^3*(1 - c^2*x^2)
^3)/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^3*(1 - c^2*x^2)^4)/(49*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[c*x]) - c^2*d^3*x^3*(a +
 b*ArcCosh[c*x]) + (3*c^4*d^3*x^5*(a + b*ArcCosh[c*x]))/5 - (c^6*d^3*x^7*(a
 + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cos\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cos\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cos\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cos\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)-c^2d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}c^4d^3x^5\left(a+b\cos\right.$$

$$= \frac{16bd^3\left(1-c^2x^2\right)}{35c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{8bd^3\left(1-c^2x^2\right)^2}{105c\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{6bd^3\left(1-c^2x^2\right)^3}{175c\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [A] time = 0.27, size = 123, normalized size = 0.64

$$-\frac{d^3\left(105acx\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)+b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(-75c^6x^6+351c^4x^4-757c^2x^2+2161\right)+105bc\right.}{3675c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/3675*(d^3*(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2161 - 757*c^2*x^2 + 351*c^4*
x^4 - 75*c^6*x^6) + 105*a*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) +
 105*b*c*x*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6)*ArcCosh[c*x]))/c
```

fricas [A] time = 0.53, size = 169, normalized size = 0.88

$$-\frac{525\,ac^7d^3x^7-2205\,ac^5d^3x^5+3675\,ac^3d^3x^3-3675\,acd^3x+105\left(5\,bc^7d^3x^7-21\,bc^5d^3x^5+35\,bc^3d^3x^3-35\,bcd^3\right.}{3675\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/3675*(525*a*c^7*d^3*x^7 - 2205*a*c^5*d^3*x^5 + 3675*a*c^3*d^3*x^3 - 3675
*a*c*d^3*x + 105*(5*b*c^7*d^3*x^7 - 21*b*c^5*d^3*x^5 + 35*b*c^3*d^3*x^3 - 3
5*b*c*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*d^3*x^6 - 351*b*c^4*d
^3*x^4 + 757*b*c^2*d^3*x^2 - 2161*b*d^3)*sqrt(c^2*x^2 - 1))/c
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 132, normalized size = 0.69

$$\frac{-d^3a\left(\frac{1}{7}c^7x^7-\frac{3}{5}c^5x^5+c^3x^3-cx\right)-d^3b\left(\frac{\operatorname{arccosh}(cx)c^7x^7}{7}-\frac{3\operatorname{arccosh}(cx)c^5x^5}{5}+c^3x^3\operatorname{arccosh}(cx)-cx\operatorname{arccosh}(cx)-\right.}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-d^3*a*(1/7*c^7*x^7-3/5*c^5*x^5+c^3*x^3-c*x)-d^3*b*(1/7*arccosh(c*x)*c
^7*x^7-3/5*arccosh(c*x)*c^5*x^5+c^3*x^3*arccosh(c*x)-c*x*arccosh(c*x)-1/367
5*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*x^6-351*c^4*x^4+757*c^2*x^2-2161)))
```

maxima [A] time = 0.35, size = 302, normalized size = 1.58

$$-\frac{1}{7}ac^6d^3x^7+\frac{3}{5}ac^4d^3x^5-\frac{1}{245}\left(35x^7\operatorname{arcosh}(cx)-\left(\frac{5\sqrt{c^2x^2-1}\,x^6}{c^2}+\frac{6\sqrt{c^2x^2-1}\,x^4}{c^4}+\frac{8\sqrt{c^2x^2-1}\,x^2}{c^6}+\frac{16\sqrt{c^2}}{c}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*a*c^6*d^3*x^7 + 3/5*a*c^4*d^3*x^5 - 1/245*(35*x^7*arccosh(c*x) - (5*sq
rt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)
*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*c^6*d^3 + 1/25*(15*x^5*arccosh(c*
x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^
2*x^2 - 1)/c^6)*c)*b*c^4*d^3 - a*c^2*d^3*x^3 - 1/3*(3*x^3*arccosh(c*x) - c*
(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^2*d^3 + a*d^3*x
+ (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(d-c^2\,d\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3, x)
```

sympy [A] time = 6.32, size = 228, normalized size = 1.19

$$\begin{cases}-\frac{ac^6d^3x^7}{7}+\frac{3ac^4d^3x^5}{5}-ac^2d^3x^3+ad^3x-\frac{bc^6d^3x^7\operatorname{acosh}(cx)}{7}+\frac{bc^5d^3x^6\sqrt{c^2x^2-1}}{49}+\frac{3bc^4d^3x^5\operatorname{acosh}(cx)}{5}-\frac{117bc^3d^3x^4\sqrt{c^2x^2-1}}{1225}- \\ d^3x\left(a+\frac{i\pi b}{2}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((-a*c**6*d**3*x**7/7 + 3*a*c**4*d**3*x**5/5 - a*c**2*d**3*x**3 +
a*d**3*x - b*c**6*d**3*x**7*acosh(c*x)/7 + b*c**5*d**3*x**6*sqrt(c**2*x**2
```

```
- 1)/49 + 3*b*c**4*d**3*x**5*acosh(c*x)/5 - 117*b*c**3*d**3*x**4*sqrt(c**2*
x**2 - 1)/1225 - b*c**2*d**3*x**3*acosh(c*x) + 757*b*c*d**3*x**2*sqrt(c**2*
x**2 - 1)/3675 + b*d**3*x*acosh(c*x) - 2161*b*d**3*sqrt(c**2*x**2 - 1)/(367
5*c), Ne(c, 0)), (d**3*x*(a + I*pi*b/2), True))
```

3.24 $\int \frac{\left(d-c^2 d x^2\right)^3 \left(a+b \cosh^{-1}(cx)\right)}{x} \, dx$

Optimal. Leaf size=239

$$\frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^3\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{d^3}{}$$

```
[Out] -7/72*b*c*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/36*b*c*d^3*x*(c*x-1)^(5/2)*(c
*x+1)^(5/2)-19/48*b*d^3*arccosh(c*x)+1/2*d^3*(-c^2*x^2+1)*(a+b*arccosh(c*x)
)+1/4*d^3*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))+1/6*d^3*(-c^2*x^2+1)^3*(a+b*arc
cosh(c*x))+1/2*d^3*(a+b*arccosh(c*x))^2/b+d^3*(a+b*arccosh(c*x))*ln(1+1/(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)+19/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5727, 5660, 3718, 2190, 2279, 2391, 38, 52}

$$\frac{1}{2}bd^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)+\frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}d^3\left(\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (19*b*c*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/48 - (7*b*c*d^3*x*(-1 + c*x)^(3
/2)*(1 + c*x)^(3/2))/72 + (b*c*d^3*x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/36 -
 (19*b*d^3*ArcCosh[c*x])/48 + (d^3*(1 - c^2*x^2)*(a + b*ArcCosh[c*x]))/2 +
(d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 + (d^3*(1 - c^2*x^2)^3*(a + b*
ArcCosh[c*x]))/6 - (d^3*(a + b*ArcCosh[c*x])^2)/(2*b) + d^3*(a + b*ArcCosh[
c*x])*Log[1 + E^(2*ArcCosh[c*x])] + (b*d^3*PolyLog[2, -E^(2*ArcCosh[c*x])])
/2
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
 :> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{6}d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)+d\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh^{-1}(cx)\right.}{x}$$

$$= \frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}+\frac{1}{4}d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}$$

$$= -\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x(-1+cx)^{5/2}(1+cx)^{5/2}+\frac{1}{2}d^3$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x$$

$$= \frac{19}{48}bcd^3x\sqrt{-1+cx}\sqrt{1+cx}-\frac{7}{72}bcd^3x(-1+cx)^{3/2}(1+cx)^{3/2}+\frac{1}{36}bcd^3x$$

Mathematica [A] time = 0.39, size = 207, normalized size = 0.87

$$-\frac{1}{144}d^3\left(24ac^6x^6-108ac^4x^4+216ac^2x^2-144a\log(x)-4bc^5x^5\sqrt{cx-1}\sqrt{cx+1}+22bc^3x^3\sqrt{cx-1}\sqrt{cx+1}+1\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/144*(d^3*(216*a*c^2*x^2 - 108*a*c^4*x^4 + 24*a*c^6*x^6 - 75*b*c*x*Sqrt[-
1 + c*x]*Sqrt[1 + c*x] + 22*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 4*b*c^
5*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 72*b*ArcCosh[c*x]^2 - 150*b*ArcTanh[Sq
rt[(-1 + c*x)/(1 + c*x)]] + 12*b*ArcCosh[c*x]*(18*c^2*x^2 - 9*c^4*x^4 + 2*c
^6*x^6 - 12*Log[1 + E^(-2*ArcCosh[c*x])]) - 144*a*Log[x] + 72*b*PolyLog[2,
-E^(-2*ArcCosh[c*x])]))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{ac^6d^3x^6 - 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 - ad^3 + \left(bc^6d^3x^6 - 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 - bd^3\right)\operatorname{arcosh}(cx)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.47, size = 255, normalized size = 1.07

$$-\frac{d^3a\,c^6x^6}{6}+\frac{3d^3a\,c^4x^4}{4}-\frac{3d^3a\,c^2x^2}{2}+d^3a\ln(cx)+d^3b\operatorname{arccosh}(cx)\ln\left(1+\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2\right)+\frac{25b\,d^3\text{arc}}{48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x)
```

```
[Out] -1/6*d^3*a*c^6*x^6+3/4*d^3*a*c^4*x^4-3/2*d^3*a*c^2*x^2+d^3*a*ln(c*x)+d^3*b*
arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+25/48*b*d^3*arccosh(
c*x)+1/2*d^3*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/36*d^3*b*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^5-11/72*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
c^3*x^3+25/48*b*c*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)+3/4*d^3*b*arccosh(c*x)*
c^4*x^4-3/2*d^3*b*arccosh(c*x)*c^2*x^2-1/2*d^3*b*arccosh(c*x)^2-1/6*d^3*b*a
rccosh(c*x)*c^6*x^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}\,ac^6d^3x^6+\frac{3}{4}\,ac^4d^3x^4-\frac{3}{2}\,ac^2d^3x^2+ad^3\log(x)-\int bc^6d^3x^5\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)-3\,bc^4d^3x^3\log\left(cx+\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/6*a*c^6*d^3*x^6 + 3/4*a*c^4*d^3*x^4 - 3/2*a*c^2*d^3*x^2 + a*d^3*log(x) -
 integrate(b*c^6*d^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d
```

```
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) - b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x,
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-\frac{a}{x}\right)dx + \int 3ac^2x\,dx + \int\left(-3ac^4x^3\right)dx + \int ac^6x^5\,dx + \int\left(-\frac{b\,\mathrm{acosh}\,(cx)}{x}\right)dx + \int 3bc^2x\,\mathrm{acosh}\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] -d**3*(Integral(-a/x, x) + Integral(3*a*c**2*x, x) + Integral(-3*a*c**4*x**
3, x) + Integral(a*c**6*x**5, x) + Integral(-b*acosh(c*x)/x, x) + Integral(
3*b*c**2*x*acosh(c*x), x) + Integral(-3*b*c**4*x**3*acosh(c*x), x) + Integr
al(b*c**6*x**5*acosh(c*x), x))
```

3.25 $\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=180

$$-\frac{1}{5}c^6d^3x^5\left(a+b\cosh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)-3c^2d^3x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{2\cdots}$$

```
[Out] -1/5*b*c*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/25*b*c*d^3*(c*x-1)^(5/2)*(c*x+1)
^(5/2)-d^3*(a+b*arccosh(c*x))/x-3*c^2*d^3*x*(a+b*arccosh(c*x))+c^4*d^3*x^3*
(a+b*arccosh(c*x))-1/5*c^6*d^3*x^5*(a+b*arccosh(c*x))+b*c*d^3*arctan((c*x-1
)^(1/2)*(c*x+1)^(1/2))+11/5*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 239, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {270, 5731, 12, 1610, 1799, 1620, 63, 205}

$$-\frac{1}{5}c^6d^3x^5\left(a+b\cosh^{-1}(cx)\right)+c^4d^3x^3\left(a+b\cosh^{-1}(cx)\right)-3c^2d^3x\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{\cdots}{25\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (-11*b*c*d^3*(1 - c^2*x^2))/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
- c^2*x^2)^2)/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1 - c^2*x^2)^3)/
(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x - 3*c^2*d^
3*x*(a + b*ArcCosh[c*x]) + c^4*d^3*x^3*(a + b*ArcCosh[c*x]) - (c^6*d^3*x^5*
(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
```

```
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x} - 3c^2d^3x\left(a+b\cosh^{-1}(cx)\right) + c^4d^3x^3\left(a+b\cosh\right.$$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - $$

$$= -\frac{11bcd^3\left(1-c^2x^2\right)}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^2}{5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3\left(1-c^2x^2\right)^3}{25\sqrt{-1+cx}\sqrt{1+cx}} - $$

Mathematica [A] time = 0.31, size = 136, normalized size = 0.76

$$\frac{1}{25}d^3\left(-5ac^6x^5+25ac^4x^3-75ac^2x-\frac{25a}{x}+bc\sqrt{cx-1}\sqrt{cx+1}\left(c^4x^4-7c^2x^2+61\right)-\frac{5b\left(c^6x^6-5c^4x^4+15c^2x^2\right.}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^3*((-25*a)/x - 75*a*c^2*x + 25*a*c^4*x^3 - 5*a*c^6*x^5 + b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(61 - 7*c^2*x^2 + c^4*x^4) - (5*b*(5 + 15*c^2*x^2 - 5*c^4
*x^4 + c^6*x^6)*ArcCosh[c*x])/x - 25*b*c*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])]))/25
```

fricas [A] time = 0.55, size = 249, normalized size = 1.38

$$-\frac{5\,ac^{6}d^{3}x^{6}-25\,ac^{4}d^{3}x^{4}+75\,ac^{2}d^{3}x^{2}-50\,bcd^{3}x\arctan\left(-cx+\sqrt{c^{2}x^{2}-1}\right)-5\left(bc^{6}-5\,bc^{4}+15\,bc^{2}+5\,b\right)d^{3}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] -1/25*(5*a*c^6*d^3*x^6 - 25*a*c^4*d^3*x^4 + 75*a*c^2*d^3*x^2 - 50*b*c*d^3*x
*arctan(-c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^
3*x*log(-c*x + sqrt(c^2*x^2 - 1)) + 25*a*d^3 + 5*(b*c^6*d^3*x^6 - 5*b*c^4*d
^3*x^4 + 15*b*c^2*d^3*x^2 - (b*c^6 - 5*b*c^4 + 15*b*c^2 + 5*b)*d^3*x + 5*b*
d^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^5*d^3*x^5 - 7*b*c^3*d^3*x^3 + 61*b
*c*d^3*x)*sqrt(c^2*x^2 - 1))/x
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 219, normalized size = 1.22

$$-\frac{d^{3}a\,c^{6}x^{5}}{5}+d^{3}a\,c^{4}x^{3}-3d^{3}a\,c^{2}x-\frac{d^{3}a}{x}-\frac{d^{3}b\,\mathrm{arccosh}\left(cx\right)c^{6}x^{5}}{5}+d^{3}b\,\mathrm{arccosh}\left(cx\right)c^{4}x^{3}-3d^{3}b\,\mathrm{arccosh}\left(cx\right)c^{2}x-\frac{d^{3}b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -1/5*d^3*a*c^6*x^5+d^3*a*c^4*x^3-3*d^3*a*c^2*x-d^3*a/x-1/5*d^3*b*arccosh(c*
x)*c^6*x^5+d^3*b*arccosh(c*x)*c^4*x^3-3*d^3*b*arccosh(c*x)*c^2*x-d^3*b*arcc
osh(c*x)/x+1/25*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5*x^4-7/25*d^3*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*c^3*x^2+61/25*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-c*d
^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/
2))
```

maxima [A] time = 0.47, size = 231, normalized size = 1.28

$$-\frac{1}{5}\,ac^{6}d^{3}x^{5}-\frac{1}{75}\left(15\,x^{5}\,\mathrm{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{2}}+\frac{4\,\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{4}}+\frac{8\,\sqrt{c^{2}x^{2}-1}}{c^{6}}\right)c\right)bc^{6}d^{3}+ac^{4}d^{3}x^{3}+\frac{1}{3}\left(3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -1/5*a*c^6*d^3*x^5 - 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c
^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*c^6*d^3 +
a*c^4*d^3*x^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*
sqrt(c^2*x^2 - 1)/c^4))*b*c^4*d^3 - 3*a*c^2*d^3*x - 3*(c*x*arccosh(c*x) - s
qrt(c^2*x^2 - 1))*b*c*d^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d^3
 - a*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int 3ac^2\,dx + \int\left(-\frac{a}{x^2}\right)dx + \int\left(-3ac^4x^2\right)dx + \int ac^6x^4\,dx + \int 3bc^2\,\mathrm{acosh}\,(cx)\,dx + \int\left(-\frac{b\,\mathrm{acosh}\,(cx)}{x^2}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] -d**3*(Integral(3*a*c**2, x) + Integral(-a/x**2, x) + Integral(-3*a*c**4*x*
*2, x) + Integral(a*c**6*x**4, x) + Integral(3*b*c**2*acosh(c*x), x) + Inte
gral(-b*acosh(c*x)/x**2, x) + Integral(-3*b*c**4*x**2*acosh(c*x), x) + Inte
gral(b*c**6*x**4*acosh(c*x), x))
```

3.26 $\int \frac{(d-c^2dx^2)^3(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=267

$$-\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{2}c^2d^3\left(1-c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] -7/16*b*c^3*d^3*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)+1/2*b*c*d^3*(c*x-1)^(5/2)*(c*
x+1)^(5/2)/x+3/32*b*c^2*d^3*arccosh(c*x)-3/2*c^2*d^3*(-c^2*x^2+1)*(a+b*arcc
osh(c*x))-3/4*c^2*d^3*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))-1/2*d^3*(-c^2*x^2+1
)^3*(a+b*arccosh(c*x))/x^2-3/2*c^2*d^3*(a+b*arccosh(c*x))^2/b-3*c^2*d^3*(a+
b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3/2*b*c^2*d^3*p
olylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/32*b*c^3*d^3*x*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5729, 97, 12, 38, 52, 5727, 5660, 3718, 2190, 2279, 2391}

$$-\frac{3}{2}bc^2d^3\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)-\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right)-\frac{3}{2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-3*b*c^3*d^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/32 - (7*b*c^3*d^3*x*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2))/16 + (b*c*d^3*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(2
*x) + (3*b*c^2*d^3*ArcCosh[c*x])/32 - (3*c^2*d^3*(1 - c^2*x^2)*(a + b*ArcCo
sh[c*x]))/2 - (3*c^2*d^3*(1 - c^2*x^2)^2*(a + b*ArcCosh[c*x]))/4 - (d^3*(1
- c^2*x^2)^3*(a + b*ArcCosh[c*x]))/(2*x^2) + (3*c^2*d^3*(a + b*ArcCosh[c*x]
)^2)/(2*b) - 3*c^2*d^3*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])] - (
3*b*c^2*d^3*PolyLog[2, -E^(2*ArcCosh[c*x])])/2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(1-c^2x^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \left(3c^2d\right)\int \frac{\left(d-c^2dx^2\right)^2\left(a+b\cosh\right.}{x}$$

$$= \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x} - \frac{3}{4}c^2d^3\left(1-c^2x^2\right)^2\left(a+b\cosh^{-1}(cx)\right) -$$

$$= \frac{3}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bcd^3(-1+cx)^{5/2}(1+cx)^{5/2}}{2x} - \frac{3}{2}c^2d$$

$$= -\frac{33}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bc}{}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bc}{}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bc}{}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bc}{}$$

$$= -\frac{3}{32}bc^3d^3x\sqrt{-1+cx}\,\sqrt{1+cx} - \frac{7}{16}bc^3d^3x(-1+cx)^{3/2}(1+cx)^{3/2} + \frac{bc}{}$$

Mathematica [A] time = 0.37, size = 226, normalized size = 0.85

$$-\frac{d^3\left(8ac^6x^6 - 48ac^4x^4 + 96ac^2x^2\log(x) + 16a - 2bc^5x^5\sqrt{cx-1}\,\sqrt{cx+1} + 21bc^3x^3\sqrt{cx-1}\,\sqrt{cx+1} - 48bc^2x\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -1/32*(d^3*(16*a - 48*a*c^4*x^4 + 8*a*c^6*x^6 - 16*b*c*x*Sqrt[-1 + c*x]*Sqr
t[1 + c*x] + 21*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^5*x^5*Sqrt[-
1 + c*x]*Sqrt[1 + c*x] + 48*b*c^2*x^2*ArcCosh[c*x]^2 + 42*b*c^2*x^2*ArcTanh
[Sqrt[(-1 + c*x)/(1 + c*x)]] + 8*b*ArcCosh[c*x]*(2 - 6*c^4*x^4 + c^6*x^6 +
12*c^2*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 96*a*c^2*x^2*Log[x] - 48*b*c^2*x
^2*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/x^2
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{ac^6d^3x^6 - 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 - ad^3 + \left(bc^6d^3x^6 - 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 - bd^3\right)\mathrm{arcosh}\left(cx\right)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))/x^3,
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.78, size = 275, normalized size = 1.03

$$-\frac{c^6 d^3 a x^4}{4}+\frac{3c^4 d^3 a x^2}{2}-3c^2 d^3 a \ln(cx)-\frac{d^3 a}{2x^2}-\frac{d^3 b c^2}{2}-\frac{21 b c^2 d^3 \operatorname{arccosh}(cx)}{32}+\frac{c d^3 b\sqrt{cx+1}\,\sqrt{cx-1}}{2x}+\frac{c^5 d^3 b\sqrt{cx-1}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/4*c^6*d^3*a*x^4+3/2*c^4*d^3*a*x^2-3*c^2*d^3*a*ln(c*x)-1/2*d^3*a/x^2-1/2*
d^3*b*c^2-21/32*b*c^2*d^3*arccosh(c*x)+1/2*c*d^3*b/x*(c*x+1)^(1/2)*(c*x-1)^
(1/2)+1/16*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3-21/32*b*c^3*d^3*x*(c*x
-1)^(1/2)*(c*x+1)^(1/2)-1/4*c^6*d^3*b*arccosh(c*x)*x^4+3/2*c^4*d^3*b*arccos
h(c*x)*x^2-1/2*d^3*b*arccosh(c*x)/x^2-3/2*c^2*d^3*b*polylog(2,-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)+3/2*c^2*d^3*b*arccosh(c*x)^2-3*c^2*d^3*b*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}ac^6d^3x^4+\frac{3}{2}ac^4d^3x^2-3ac^2d^3\log(x)+\frac{1}{2}bd^3\left(\frac{\sqrt{c^2x^2-1}\,c}{x}-\frac{\operatorname{arcosh}(cx)}{x^2}\right)-\frac{ad^3}{2x^2}-\int bc^6d^3x^3\log\left(cx+\sqrt{cx+1}\,\sqrt{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*c^6*d^3*x^4 + 3/2*a*c^4*d^3*x^2 - 3*a*c^2*d^3*log(x) + 1/2*b*d^3*(sq
rt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) - 1/2*a*d^3/x^2 - integrate(b*c^6*d
^3*x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) - 3*b*c^4*d^3*x*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + 3*b*c^2*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-\frac{a}{x^3}\right)dx+\int\frac{3ac^2}{x}\,dx+\int\left(-3ac^4x\right)dx+\int ac^6x^3\,dx+\int\left(-\frac{b\,\mathrm{acosh}\,(cx)}{x^3}\right)dx+\int\frac{3bc^2\,\mathrm{acosh}\,(cx)}{x}\,d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] -d**3*(Integral(-a/x**3, x) + Integral(3*a*c**2/x, x) + Integral(-3*a*c**4*
x, x) + Integral(a*c**6*x**3, x) + Integral(-b*acosh(c*x)/x**3, x) + Integr
al(3*b*c**2*acosh(c*x)/x, x) + Integral(-3*b*c**4*x*acosh(c*x), x) + Integr
al(b*c**6*x**3*acosh(c*x), x))
```

3.27 $\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=195

$$-\frac{1}{3}c^6d^3x^3\left(a+b\cosh^{-1}(cx)\right)+3c^4d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{1}{9}$$

```
[Out] 1/9*b*c^3*d^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)-1/3*d^3*(a+b*arccosh(c*x))/x^3+3*
c^2*d^3*(a+b*arccosh(c*x))/x+3*c^4*d^3*x*(a+b*arccosh(c*x))-1/3*c^6*d^3*x^3
*(a+b*arccosh(c*x))-17/6*b*c^3*d^3*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))-8/3*
b*c^3*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/x^2
```

Rubi [A] time = 0.39, antiderivative size = 252, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {270, 5731, 12, 1610, 1799, 1621, 897, 1153, 205}

$$-\frac{1}{3}c^6d^3x^3\left(a+b\cosh^{-1}(cx)\right)+3c^4d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{b}{9}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (8*b*c^3*d^3*(1 - c^2*x^2))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^3*(1
 - c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d^3*(1 - c^2*x^2)
^2)/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]))/(3*x^3) +
 (3*c^2*d^3*(a + b*ArcCosh[c*x]))/x + 3*c^4*d^3*x*(a + b*ArcCosh[c*x]) - (c
^6*d^3*x^3*(a + b*ArcCosh[c*x]))/3 - (17*b*c^3*d^3*Sqrt[-1 + c^2*x^2]*ArcTa
n[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3c^4d^3x\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh\right.}{x}$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh\right.}{x}$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{3c^2d^3\left(a+b\cosh\right.}{x}$$

$$= \frac{8bc^3d^3\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^3\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^3\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{8bc^3d^3\left(1-c^2x^2\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcd^3\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3d^3\left(1-c^2x^2\right)^2}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.31, size = 142, normalized size = 0.73

$$\frac{d^3\left(-6ac^6x^6+54ac^4x^4+54ac^2x^2-6a+51bc^3x^3\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\,\sqrt{cx+1}}\right)+bcx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2c^4x^4-50c^2x^2\right.\right.}{18x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (d^3*(-6*a + 54*a*c^2*x^2 + 54*a*c^4*x^4 - 6*a*c^6*x^6 + b*c*x*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(3 - 50*c^2*x^2 + 2*c^4*x^4) - 6*b*(1 - 9*c^2*x^2 - 9*c^4*
x^4 + c^6*x^6)*ArcCosh[c*x] + 51*b*c^3*x^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x])]))/(18*x^3)
```

fricas [A] time = 0.66, size = 253, normalized size = 1.30

$$-\frac{6\,ac^6d^3x^6-54\,ac^4d^3x^4+102\,bc^3d^3x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)-54\,ac^2d^3x^2-6\left(bc^6-9\,bc^4-9\,bc^2+b\right)d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] -1/18*(6*a*c^6*d^3*x^6 - 54*a*c^4*d^3*x^4 + 102*b*c^3*d^3*x^3*arctan(-c*x +
 sqrt(c^2*x^2 - 1)) - 54*a*c^2*d^3*x^2 - 6*(b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*
d^3*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + 6*a*d^3 + 6*(b*c^6*d^3*x^6 - 9*b*c^
4*d^3*x^4 - 9*b*c^2*d^3*x^2 - (b*c^6 - 9*b*c^4 - 9*b*c^2 + b)*d^3*x^3 + b*d
^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^5*d^3*x^5 - 50*b*c^3*d^3*x^3 + 3*
b*c*d^3*x)*sqrt(c^2*x^2 - 1))/x^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 223, normalized size = 1.14

$$-\frac{c^6d^3a\,x^3}{3}+3c^4d^3ax+\frac{3c^2d^3a}{x}-\frac{d^3a}{3x^3}-\frac{c^6d^3b\,\mathrm{arccosh}\,(cx)\,x^3}{3}+3c^4d^3b\,\mathrm{arccosh}\,(cx)\,x+\frac{3c^2d^3b\,\mathrm{arccosh}\,(cx)}{x}-\frac{d^3b\,\mathrm{arcc}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -1/3*c^6*d^3*a*x^3+3*c^4*d^3*a*x+3*c^2*d^3*a/x-1/3*d^3*a/x^3-1/3*c^6*d^3*b*
arccosh(c*x)*x^3+3*c^4*d^3*b*arccosh(c*x)*x+3*c^2*d^3*b*arccosh(c*x)/x-1/3*
d^3*b*arccosh(c*x)/x^3+1/9*c^5*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-25/9*b
*c^3*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+17/6*c^3*d^3*b*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*c*d^3*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/x^2
```

maxima [A] time = 0.56, size = 208, normalized size = 1.07

$$-\frac{1}{3}\,ac^6d^3x^3-\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bc^6d^3+3\,ac^4d^3x+3\left(cx\,\mathrm{arcosh}\,(cx)-\sqrt{c^2x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/3*a*c^6*d^3*x^3 - 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2
 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*c^6*d^3 + 3*a*c^4*d^3*x + 3*(c*x*arccosh(c*x
) - sqrt(c^2*x^2 - 1))*b*c^3*d^3 + 3*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)
/x)*b*c^2*d^3 - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c +
 2*arccosh(c*x)/x^3)*b*d^3 + 3*a*c^2*d^3/x - 1/3*a*d^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-3ac^4\right)dx+\int\left(-\frac{a}{x^4}\right)dx+\int\frac{3ac^2}{x^2}\,dx+\int ac^6x^2\,dx+\int\left(-3bc^4\operatorname{acosh}(cx)\right)dx+\int\left(-\frac{b\operatorname{acosh}(cx}{x^4}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] -d**3*(Integral(-3*a*c**4, x) + Integral(-a/x**4, x) + Integral(3*a*c**2/x*
*2, x) + Integral(a*c**6*x**2, x) + Integral(-3*b*c**4*acosh(c*x), x) + Int
egral(-b*acosh(c*x)/x**4, x) + Integral(3*b*c**2*acosh(c*x)/x**2, x) + Inte
gral(b*c**6*x**2*acosh(c*x), x))
```

3.28 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=158

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^5d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{b\text{Li}_2\left(\right.}{}$$

```
[Out] -x*(a+b*arccosh(c*x))/c^4/d-1/3*x^3*(a+b*arccosh(c*x))/c^2/d+2*(a+b*arccosh
(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d+b*polylog(2,-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/c^5/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
/c^5/d+11/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d+1/9*b*x^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^3/d
```

Rubi [A] time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5766, 100, 12, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^5d}-\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d}+\frac{2\tanh^{-1}\left(e^{\text{cc}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^5*d) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(9*c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^4*d) - (x^3*(a + b*ArcCos
h[c*x]))/(3*c^2*d) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^5*
d) + (b*PolyLog[2, -E^ArcCosh[c*x]])/(c^5*d) - (b*PolyLog[2, E^ArcCosh[c*x]
])/(c^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d} + \frac{\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2} + \frac{b\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3cd}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d} + \int \frac{a+b\,c\cdots}{d\cdots}$$

$$= \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a+\cdots\right.}{\cdots}$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a\cdots\right.}{\cdots}$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a\cdots\right.}{\cdots}$$

$$= \frac{11b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^5d} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3d} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d} - \frac{x^3\left(a\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.34, size = 227, normalized size = 1.44

$$-\frac{6ac^3x^3+18acx+9a\log(1-cx)-9a\log(cx+1)+6bc^3x^3\cosh^{-1}(cx)-2bc^2x^2\sqrt{cx-1}\,\sqrt{cx+1}+18b\,\mathrm{Li}_2\left(-\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -1/18*(18*a*c*x + 6*a*c^3*x^3 - 18*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 18*b*c*x*
Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*x^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 18*b*c*x*ArcCosh[c*x] + 6*b*c^3*x^3*ArcCosh
[c*x] - 9*b*ArcCosh[c*x]^2 - 18*b*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
 18*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 9*a*Log[1 - c*x] - 9*a*Log[1 +
 c*x] + 18*b*PolyLog[2, -E^(-ArcCosh[c*x])] + 18*b*PolyLog[2, E^ArcCosh[c*x
]])/(c^5*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^2*d*x^2 - d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 263, normalized size = 1.66

$$-\frac{a\,x^3}{3c^2d}-\frac{ax}{c^4d}-\frac{a\ln(cx-1)}{2c^5d}+\frac{a\ln(cx+1)}{2c^5d}+\frac{11b\sqrt{cx-1}\,\sqrt{cx+1}}{9c^5d}+\frac{b\,\mathrm{polylog}\left(2,-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{c^5d}-\frac{b\,\mathrm{polylo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/3/c^2*a/d*x^3-1/c^4*a/d*x-1/2/c^5*a/d*ln(c*x-1)+1/2/c^5*a/d*ln(c*x+1)+11
/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d+b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/c^5/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d+1/9*b*x^
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d-1/3/c^2*b/d*arccosh(c*x)*x^3-1/c^4*b/d*
arccosh(c*x)*x-1/c^5*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
+1/c^5*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{72}\left(4\,c^4\left(\frac{2\left(c^2x^3+3\,x\right)}{c^8d}-\frac{3\,\log(cx+1)}{c^9d}+\frac{3\,\log(cx-1)}{c^9d}\right)+36\,c^2\left(\frac{2\,x}{c^6d}-\frac{\log(cx+1)}{c^7d}+\frac{\log(cx-1)}{c^7d}\right)+648\,c\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/72*(4*c^4*(2*(c^2*x^3 + 3*x)/(c^8*d) - 3*log(c*x + 1)/(c^9*d) + 3*log(c*x
 - 1)/(c^9*d)) + 36*c^2*(2*x/(c^6*d) - log(c*x + 1)/(c^7*d) + log(c*x - 1)/
(c^7*d)) + 648*c*integrate(1/12*x*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x) - 3*
(4*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + 3*log(c*x + 1)^2 + 6*log(c*x + 1)*log(c*x - 1))/(c^5
```

```
*d) + 72*integrate(-1/6*(2*c^3*x^3 + 6*c*x - 3*log(c*x + 1) + 3*log(c*x - 1
))/(c^7*d*x^3 - c^5*d*x + (c^6*d*x^2 - c^4*d)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x) - 216*integrate(1/12*log(c*x - 1)/(c^6*d*x^2 - c^4*d), x))*b - 1/6*a*(2
*(c^2*x^3 + 3*x)/(c^4*d) - 3*log(c*x + 1)/(c^5*d) + 3*log(c*x - 1)/(c^5*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^{4}}{c^{2}x^{2}-1}\, dx + \int \frac{bx^{4}\operatorname{acosh}{\left(cx \right)}}{c^{2}x^{2}-1}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**4/(c**2*x**2 - 1), x) + Integral(b*x**4*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

3.29 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=140

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}-\frac{b\mathrm{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^4d}+\frac{b\cosh^{-1}}{4c^4}$$

```
[Out] 1/4*b*arccosh(c*x)/c^4/d-1/2*x^2*(a+b*arccosh(c*x))/c^2/d+1/2*(a+b*arccosh(
c*x))^2/b/c^4/d-(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)/c^4/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/c^4/d+1/4*b*x*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d
```

Rubi [A] time = 0.20, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5766, 90, 52, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b\mathrm{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d}-\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d) + (b*ArcCosh[c*x])/(4*c^4*d) -
 (x^2*(a + b*ArcCosh[c*x]))/(2*c^2*d) + (a + b*ArcCosh[c*x])^2/(2*b*c^4*d)
- ((a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d) - (b*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*c^4*d)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} + \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2} + \frac{b\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2cd}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d} - \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} - \frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x\right.}{c^4d}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\cosh^{-1}(\right.}{2bc^4d}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\cosh^{-1}(\right.}{2bc^4d}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\cosh^{-1}(\right.}{2bc^4d}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{4c^3d} + \frac{b\cosh^{-1}(cx)}{4c^4d} - \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d} + \frac{\left(a+b\cosh^{-1}(\right.}{2bc^4d}$$

Mathematica [A] time = 0.31, size = 151, normalized size = 1.08

$$-\frac{2c^2x^2\left(a+b\cosh^{-1}(cx)\right) - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{b} + 4\log\left(1-e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right) + 4\log\left(e^{\cosh^{-1}(cx)}+1\right)}{4c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] -1/4*(2*c^2*x^2*(a + b*ArcCosh[c*x]) - (2*(a + b*ArcCosh[c*x])^2)/b - b*(c*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]) + 4
*(a + b*ArcCosh[c*x])*Log[1 - E^ArcCosh[c*x]] + 4*(a + b*ArcCosh[c*x])*Log[
1 + E^ArcCosh[c*x]] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*PolyLog[2, E^Ar
cCosh[c*x]])/(c^4*d)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^3\,\text{arcosh}\,(cx) + ax^3}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^3*arccosh(c*x) + a*x^3)/(c^2*d*x^2 - d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.29, size = 244, normalized size = 1.74

$$-\frac{a\,x^2}{2c^2d}-\frac{a\ln\left(cx-1\right)}{2c^4d}-\frac{a\ln\left(cx+1\right)}{2c^4d}+\frac{b\text{arccosh}\left(cx\right)^2}{2c^4d}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{4c^3d}-\frac{b\,\text{arccosh}\left(cx\right)x^2}{2c^2d}+\frac{b\,\text{arccosh}\left(cx\right)}{4d\,c^4}-\frac{b\,\text{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2/c^2*a/d*x^2-1/2/c^4*a/d*ln(c*x-1)-1/2/c^4*a/d*ln(c*x+1)+1/2/c^4*b/d*ar
ccosh(c*x)^2+1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d-1/2/c^2*b/d*arccosh(
c*x)*x^2+1/4*b*arccosh(c*x)/d/c^4-1/c^4*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))-1/c^4*b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1
/c^4*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^4*b/d*polyl
og(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{x^2}{c^2d}+\frac{\log\left(c^2x^2-1\right)}{c^4d}\right)+\frac{1}{8}b\left(\frac{2\,c^2x^2-4\left(c^2x^2+\log\left(cx+1\right)+\log\left(cx-1\right)\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+2\left(\right.}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(x^2/(c^2*d) + log(c^2*x^2 - 1)/(c^4*d)) + 1/8*b*((2*c^2*x^2 - 4*(c^
2*x^2 + log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
 + 2*(log(c*x - 1) + 1)*log(c*x + 1) + log(c*x + 1)^2 + log(c*x - 1)^2 + 2*
log(c*x - 1))/(c^4*d) - 8*integrate(1/2*(c^2*x^2 + log(c*x + 1) + log(c*x -
 1))/(c^6*d*x^3 - c^4*d*x + (c^5*d*x^2 - c^3*d)*e^(1/2*log(c*x + 1) + 1/2*l
og(c*x - 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^3}{c^2x^2-1}\,dx + \int \frac{bx^3\,\mathrm{acosh}\,(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**3/(c**2*x**2 - 1), x) + Integral(b*x**3*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

3.30 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=102

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{c^3d}+\frac{b\sqrt{cx-1}\,\sqrt{\ldots}}{c^3d}$$

```
[Out] -x*(a+b*arccosh(c*x))/c^2/d+2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))/c^3/d+b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d-b*
polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d+b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c^3/d
```

Rubi [A] time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5766, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(\ldots\right)}{c^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d) - (x*(a + b*ArcCosh[c*x]))/(c^2*d)
 + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3*d) + (b*PolyLog[2,
 -E^ArcCosh[c*x]])/(c^3*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c^3*d)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{c^2}+\frac{b\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{cd}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,c\right.}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cos}\right.}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cos}\right.}{c^3d}$$
$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cos}\right.}{c^3d}$$

Mathematica [A] time = 0.16, size = 155, normalized size = 1.52

$$\frac{-2acx-a\log(1-cx)+a\log(cx+1)-2b\text{Li}_2\left(-e^{-\cosh^{-1}(cx)}\right)-2b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)+2b\sqrt{\frac{cx-1}{cx+1}}+2bcx\sqrt{\frac{cx-1}{cx+1}}+}{2c^3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (-2*a*c*x + 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*b*c*x*Sqrt[(-1 + c*x)/(1 + c
*x)] - 2*b*c*x*ArcCosh[c*x] + b*ArcCosh[c*x]^2 + 2*b*ArcCosh[c*x]*Log[1 + E
^(-ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] - a*Log[1 - c*
x] + a*Log[1 + c*x] - 2*b*PolyLog[2, -E^(-ArcCosh[c*x])] - 2*b*PolyLog[2, E
^ArcCosh[c*x]])/(2*c^3*d)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^2\operatorname{arcosh}(cx)+ax^2}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\operatorname{arcosh}(cx)+a)x^2}{c^2dx^2-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d), x)
```

maple [A] time = 0.27, size = 208, normalized size = 2.04

$$-\frac{ax}{c^2d}-\frac{a\ln(cx-1)}{2c^3d}+\frac{a\ln(cx+1)}{2c^3d}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^3d}-\frac{b\operatorname{arccosh}(cx)\ln\left(1-cx-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{c^3d}+\frac{b\operatorname{arccosh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/c^2*a/d*x-1/2/c^3*a/d*ln(c*x-1)+1/2/c^3*a/d*ln(c*x+1)+b*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c^3/d-1/c^3*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))+1/c^3*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*
arccosh(c*x)*x-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d+b*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(4\,c^2\left(\frac{2\,x}{c^4d}-\frac{\log(cx+1)}{c^5d}+\frac{\log(cx-1)}{c^5d}\right)+24\,c\int\frac{x\log(cx-1)}{4\left(c^4dx^2-c^2d\right)}\,dx-\frac{4\left(2\,cx-\log(cx+1)+\log(cx-1)\right)\log}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(4*c^2*(2*x/(c^4*d) - log(c*x + 1)/(c^5*d) + log(c*x - 1)/(c^5*d)) + 24
*c*integrate(1/4*x*log(c*x - 1)/(c^4*d*x^2 - c^2*d), x) - (4*(2*c*x - log(c
*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + log(c*x +
1)^2 + 2*log(c*x + 1)*log(c*x - 1))/(c^3*d) + 8*integrate(-1/2*(2*c*x - log
(c*x + 1) + log(c*x - 1))/(c^5*d*x^3 - c^3*d*x + (c^4*d*x^2 - c^2*d)*sqrt(c
*x + 1)*sqrt(c*x - 1)), x) - 8*integrate(1/4*log(c*x - 1)/(c^4*d*x^2 - c^2*
d), x))*b - 1/2*a*(2*x/(c^2*d) - log(c*x + 1)/(c^3*d) + log(c*x - 1)/(c^3*d
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{ax^2}{c^2x^2-1}\,dx+\int\frac{bx^2\operatorname{acosh}(cx)}{c^2x^2-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x**2/(c**2*x**2 - 1), x) + Integral(b*x**2*acosh(c*x)/(c**2*x*
*2 - 1), x))/d
```

3.31 $\int \frac{x(a+b\cosh^{-1}(cx))}{d-c^2dx^2}\,dx$

Optimal. Leaf size=74

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^2d}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^2/b/c^2/d-(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/c^2/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2)/c^2/d
```

Rubi [A] time = 0.12, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {5715, 3716, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^2d}+\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d}-\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(2*b*c^2*d) - ((a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*c^2*d)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\coth(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1-e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{c^2d}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\,\text{Subst}\left(\int \log\left(1\right.\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\right.}{2c}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-e^{2\cosh^{-1}(cx)}\right)}{c^2d} - \frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^2d}$$

Mathematica [A] time = 0.09, size = 85, normalized size = 1.15

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-2b^2\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)-2}{2bc^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2
*b*Log[1 + E^ArcCosh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*Pol
yLog[2, E^ArcCosh[c*x]])/(2*b*c^2*d)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx\,\text{arcosh}\,(cx)+ax}{c^2dx^2-d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*x*arccosh(c*x) + a*x)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\,\text{arcosh}\,(cx)+a)x}{c^2dx^2-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d), x)
```

maple [A] time = 0.09, size = 179, normalized size = 2.42

$$-\frac{a\ln(cx-1)}{2c^2d}-\frac{a\ln(cx+1)}{2c^2d}+\frac{b\text{arccosh}\,(cx)^2}{2c^2d}-\frac{b\,\text{arccosh}\,(cx)\ln\left(1+cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{c^2d}-\frac{b\,\text{polylog}\left(2,-cx-\right.}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2/c^2*a/d*ln(c*x-1)-1/2/c^2*a/d*ln(c*x+1)+1/2/c^2*b/d*arccosh(c*x)^2-1/c
^2*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))-1/c^2*b/d*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,b\left(\frac{4\left(\log\left(cx+1\right)+\log\left(cx-1\right)\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)-\log\left(cx+1\right)^{2}-2\,\log\left(cx+1\right)\log\left(cx-1\right)-}{c^{2}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/8*b*((4*(log(c*x + 1) + log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1) - log(c*x - 1)^2)/(c^2*
d) + 8*integrate(1/2*(log(c*x + 1) + log(c*x - 1))/(c^4*d*x^3 - c^2*d*x + (
c^3*d*x^2 - c*d)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) - 1/2*a*log(
c^2*d*x^2 - d)/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{c^{2}x^{2}-1}\,dx+\int \frac{bx\,\mathrm{acosh}\,(cx)}{c^{2}x^{2}-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a*x/(c**2*x**2 - 1), x) + Integral(b*x*acosh(c*x)/(c**2*x**2 - 1
), x))/d
```

3.32 $\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=59

$$\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d+b*polylog
(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d-b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))/c/d
```

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{cd}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{cd}+\frac{2\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
```

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d) + (b*PolyLog[2, -E^A
rcCosh[c*x]])/(c*d) - (b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx, x, \cosh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\,\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \cosh^{-1}(cx)\right)}{cd}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(cx)}\right)}{cd} - \frac{b\,\text{S}}{}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{cd} + \frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{cd} - \frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

Mathematica [A] time = 0.07, size = 64, normalized size = 1.08

$$-\frac{\left(\left(\log\left(1-e^{\cosh^{-1}(cx)}\right)-\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)\left(a+b\cosh^{-1}(cx)\right)\right)+b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)-b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{cd}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2),x]
[Out] (-((a + b*ArcCosh[c*x])*(Log[1 - E^ArcCosh[c*x]] - Log[1 + E^ArcCosh[c*x]])
) + b*PolyLog[2, -E^ArcCosh[c*x]] - b*PolyLog[2, E^ArcCosh[c*x]])/(c*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^2dx^2-d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{arcosh}\,(cx)+a}{c^2dx^2-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

maple [C] time = 0.61, size = 326, normalized size = 5.53

$$\frac{a\,\text{arctanh}\,(cx)}{cd}+\frac{b\,\text{arctanh}\,(cx)\,\text{arccosh}\,(cx)}{cd}+\frac{2ib\sqrt{-c^2x^2+1}\,\sqrt{\frac{1}{2}+\frac{cx}{2}}\,\sqrt{-\frac{1}{2}+\frac{cx}{2}}\,\text{arctanh}\,(cx)\ln\left(1+\frac{i(cx+1)}{\sqrt{-c^2x^2+}}\right.}{cd\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
[Out] 1/c*a/d*arctanh(c*x)+1/c*b/d*arctanh(c*x)*arccosh(c*x)+2*I/c*b/d*(-c^2*x^2+
1)^(1/2)*(1/2+1/2*c*x)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c^2*x^2-1)*arctanh(c*x)*
ln(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I/c*b/d*(-c^2*x^2+1)^(1/2)*(1/2+1/2*c*
```

```
x)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c^2*x^2-1)*arctanh(c*x)*ln(1-I*(c*x+1)/(-c^2
*x^2+1)^(1/2))+2*I/c*b/d*(-c^2*x^2+1)^(1/2)*(1/2+1/2*c*x)^(1/2)*(-1/2+1/2*c
*x)^(1/2)/(c^2*x^2-1)*dilog(1+I*(c*x+1)/(-c^2*x^2+1)^(1/2))-2*I/c*b/d*(-c^2
*x^2+1)^(1/2)*(1/2+1/2*c*x)^(1/2)*(-1/2+1/2*c*x)^(1/2)/(c^2*x^2-1)*dilog(1-
I*(c*x+1)/(-c^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}b\left(\frac{4\left(\log\left(cx+1\right)-\log\left(cx-1\right)\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)-\log\left(cx+1\right)^{2}-2\log\left(cx+1\right)\log\left(cx-1\right)}{cd}+8\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*b*((4*(log(c*x + 1) - log(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1)) - log(c*x + 1)^2 - 2*log(c*x + 1)*log(c*x - 1))/(c*d) + 8*integrate(1/4
*(3*c*x - 1)*log(c*x - 1)/(c^2*d*x^2 - d), x) + 8*integrate(1/2*(log(c*x +
1) - log(c*x - 1))/(c^3*d*x^3 - c*d*x + (c^2*d*x^2 - d)*sqrt(c*x + 1)*sqrt(
c*x - 1)), x)) + 1/2*a*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^{2}x^{2}-1}\,dx+\int \frac{b\operatorname{acosh}\left(cx\right)}{c^{2}x^{2}-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**2 - 1), x) + Integral(b*acosh(c*x)/(c**2*x**2 - 1), x
))/d
```

3.33 $\int \frac{a+b\cosh^{-1}(cx)}{x(d-c^2dx^2)}\,dx$

Optimal. Leaf size=61

$$\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*p
olylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)/d
```

Rubi [A] time = 0.13, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5721, 5461, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*PolyLog[2, -E^(
2*ArcCosh[c*x])])/(2*d) - (b*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx = -\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,\cosh^{-1}(cx)\right)}{d}$$
$$= -\frac{2\,\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx,x,\cosh^{-1}(cx)\right)}{d}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\,\text{Subst}\left(\int \log\left(1-e^{2x}\right)\,dx,x,\cosh^{-1}(cx)\right)}{d}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2\cosh^{-1}(cx)}\right)}{2d} - \frac{b\,\text{S}}{}$$
$$= \frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d} + \frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d} - \frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}$$

Mathematica [A] time = 0.16, size = 93, normalized size = 1.52

$$-\frac{a\log\left(1-c^2x^2\right)}{2d}+\frac{a\log(x)}{d}-\frac{b\left(\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-\text{Li}_2\left(e^{-2\cosh^{-1}(cx)}\right)+2\cosh^{-1}(cx)\left(\log\left(1-e^{-2\cosh^{-1}(cx)}\right)-\text{l}\right.\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)),x]
```

```
[Out] (a*Log[x])/d - (a*Log[1 - c^2*x^2])/(2*d) - (b*(2*ArcCosh[c*x]*(Log[1 - E^(
-2*ArcCosh[c*x])] - Log[1 + E^(-2*ArcCosh[c*x])]) + PolyLog[2, -E^(-2*ArcCo
sh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x), x)
```

maple [A] time = 0.14, size = 91, normalized size = 1.49

$$\frac{a\ln(cx)}{d}-\frac{a\ln(cx-1)}{2d}-\frac{a\ln(cx+1)}{2d}-\frac{b\,\text{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^2}\right)}{d}+\frac{b\,\text{dilog}\left(\frac{1}{\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)^4}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x)
```

```
[Out] a/d*ln(c*x)-1/2*a/d*ln(c*x-1)-1/2*a/d*ln(c*x+1)-b/d*dilog(1/(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)+1/4*b/d*dilog(1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{\log\left(cx+1\right)}{d}+\frac{\log\left(cx-1\right)}{d}-\frac{2\log(x)}{d}\right)-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^2dx^3-dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(c*x + 1)/d + log(c*x - 1)/d - 2*log(x)/d) - b*integrate(log(c*x
 + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^3 - d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x\,\left(d-c^2\,d\,x^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{a}{c^2x^3-x}\,dx+\int\frac{b\,\mathrm{acosh}\,(cx)}{c^2x^3-x}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**3 - x), x) + Integral(b*acosh(c*x)/(c**2*x**3 - x), x
))/d
```

3.34 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)}\,dx$

Optimal. Leaf size=95

$$-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{2c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}+\frac{bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{d}+\frac{bc\tan^{-1}(}{}$$

```
[Out] (-a-b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*c*(a+b*
arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+b*c*polylog(2,-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/d
```

Rubi [A] time = 0.14, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5746, 92, 205, 5694, 4182, 2279, 2391}

$$\frac{bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{2c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(c\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (2*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d + (b*c*PolyLog[2,
-E^ArcCosh[c*x]])/d - (b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{dx}+c^2\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx+\frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}-\frac{c\,\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\cosh^{-1}(cx)\right)}{d}+\frac{\left(bc^2\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx}+\frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d}+\frac{2c\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{d}$$

Mathematica [A] time = 0.30, size = 132, normalized size = 1.39

$$\frac{-\frac{a+b\cosh^{-1}(cx)}{x}-c\log\left(1-e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)+c\log\left(e^{\cosh^{-1}(cx)}+1\right)\left(a+b\cosh^{-1}(cx)\right)+\frac{bc\sqrt{c^2x^2-1}}{\sqrt{cx}}}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])/x) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x
^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - c*(a + b*ArcCosh[c*x])*Log[1 - E^Arc
Cosh[c*x]] + c*(a + b*ArcCosh[c*x])*Log[1 + E^ArcCosh[c*x]] + b*c*PolyLog[2
, -E^ArcCosh[c*x]] - b*c*PolyLog[2, E^ArcCosh[c*x]])/d
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^4 - d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{(c^2dx^2 - d)x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^2), x)
```

maple [A] time = 0.27, size = 161, normalized size = 1.69

$$-\frac{a}{dx} - \frac{ca\ln(cx-1)}{2d} + \frac{ca\ln(cx+1)}{2d} - \frac{b\operatorname{arccosh}(cx)}{dx} + \frac{2cb\arctan\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)}{d} + \frac{cb\operatorname{dilog}\left(cx+\sqrt{cx-1}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x)
```

```
[Out] -a/d/x-1/2*c*a/d*ln(c*x-1)+1/2*c*a/d*ln(c*x+1)-b/d*arccosh(c*x)/x+2*c*b/d*a
rctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c*b/d*dilog(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))+c*b/d*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c*b/d*arccosh(c*x)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(24\,c^3\int \frac{x\log(cx-1)}{4\left(c^2dx^2-d\right)}\,dx - 4\,c^2\left(\frac{\log(cx+1)}{cd} - \frac{\log(cx-1)}{cd}\right) - 8\,c^2\int \frac{\log(cx-1)}{4\left(c^2dx^2-d\right)}\,dx - \frac{cx\log(cx+1)^2 + 2\,c}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/8*(24*c^3*integrate(1/4*x*log(c*x - 1)/(c^2*d*x^2 - d), x) - 4*c^2*(log(c
*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 8*c^2*integrate(1/4*log(c*x - 1)/(c^2
*d*x^2 - d), x) - (c*x*log(c*x + 1)^2 + 2*c*x*log(c*x + 1)*log(c*x - 1) - 4
*(c*x*log(c*x + 1) - c*x*log(c*x - 1) - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)))/(d*x) + 8*integrate(1/2*(c^2*x*log(c*x + 1) - c^2*x*log(c*x - 1) -
2*c)/(c^3*d*x^4 - c*d*x^2 + (c^2*d*x^3 - d*x)*sqrt(c*x + 1)*sqrt(c*x - 1)),
 x))*b + 1/2*a*(c*log(c*x + 1)/d - c*log(c*x - 1)/d - 2/(d*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,\left(d - c^2\,d\,x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^4 - x^2}\, dx + \int \frac{b\operatorname{acosh}(cx)}{c^2x^4 - x^2}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**4 - x**2), x) + Integral(b*acosh(c*x)/(c**2*x**4 - x*
*2), x))/d
```

3.35 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)}\,dx$

Optimal. Leaf size=118

$$\frac{2c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{bc^2\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{b\ldots}{\ldots}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d/x^2+2*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)/d-1/2*b*c^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d+1/2*
b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x
```

Rubi [A] time = 0.20, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5746, 95, 5721, 5461, 4182, 2279, 2391}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d}+\frac{2c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{a+\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (2*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d + (b*c^2*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(2*d) - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])
/(2*d)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)}\,dx &= -\frac{a+b\cosh^{-1}(cx)}{2dx^2}+c^2\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx+\frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d}\\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{c^2\operatorname{Subst}\left(\int (a+bx)\operatorname{csch}(x)\operatorname{sech}(x)\,dx,x,\mathrm{cc}\right.}{d}\\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}-\frac{\left(2c^2\right)\operatorname{Subst}\left(\int (a+bx)\operatorname{csch}(2x)\,dx,x,\cosh\right.}{d}\\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}\\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}\\ &= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx}-\frac{a+b\cosh^{-1}(cx)}{2dx^2}+\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)}{d}\end{aligned}$$

Mathematica [A] time = 0.57, size = 144, normalized size = 1.22

$$-\frac{ac^2\log\left(1-c^2x^2\right)-2ac^2\log(x)+\frac{a}{x^2}+bc^2\left(\frac{\cosh^{-1}(cx)}{c^2x^2}+\operatorname{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-\operatorname{Li}_2\left(e^{-2\cosh^{-1}(cx)}\right)-\frac{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}{cx}+\right.}{2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)),x]
```

```
[Out] -1/2*(a/x^2 - 2*a*c^2*Log[x] + a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + ArcCosh[c*x]/(c^2*x^2) + 2*ArcCosh[c*
x]*Log[1 - E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x]
)] + PolyLog[2, -E^(-2*ArcCosh[c*x])] - PolyLog[2, E^(-2*ArcCosh[c*x])]))/d
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{b\operatorname{arcosh}(cx)+a}{c^2dx^5-dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{\left(c^2dx^2 - d\right)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^3), x)
```

maple [B] time = 0.28, size = 301, normalized size = 2.55

$$-\frac{a}{2d\,x^2}+\frac{c^2 a \ln(cx)}{d}-\frac{c^2 a \ln(cx-1)}{2d}-\frac{c^2 a \ln(cx+1)}{2d}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2dx}-\frac{c^2 b}{2d}-\frac{b \operatorname{arccosh}(cx)}{2d\,x^2}+\frac{c^2 b \operatorname{arccosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x)
```

```
[Out] -1/2*a/d/x^2+c^2*a/d*ln(c*x)-1/2*c^2*a/d*ln(c*x-1)-1/2*c^2*a/d*ln(c*x+1)+1/
2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x-1/2*c^2*b/d-1/2*b/d*arccosh(c*x)/x^2+
c^2*b/d*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*c^2*po
lylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-c^2*b/d*arccosh(c*x)*ln(1+c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*b/d*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-c^2*b/d*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-c^2*b/d
*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{c^2 \log(cx+1)}{d}+\frac{c^2 \log(cx-1)}{d}-\frac{2\,c^2 \log(x)}{d}+\frac{1}{dx^2}\right)a-b\int \frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{c^2dx^5-dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*log(c*x + 1)/d + c^2*log(c*x - 1)/d - 2*c^2*log(x)/d + 1/(d*x^2))
*a - b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^2*d*x^5 - d*x^3)
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b \operatorname{acosh}(c\,x)}{x^3\,\left(d-c^2\,d\,x^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^2x^5-x^3}\, dx+\int \frac{b \operatorname{acosh}(cx)}{c^2x^5-x^3}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**5 - x**3), x) + Integral(b*acosh(c*x)/(c**2*x**5 - x*
*3), x))/d
```

3.36 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d-c^2dx^2)}\,dx$

Optimal. Leaf size=157

$$\frac{2c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}+\frac{bc^3\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc^3}{}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3-c^2*(a+b*arccosh(c*x))/d/x+7/6*b*c^3*arctan((
c*x-1)^(1/2)*(c*x+1)^(1/2))/d+2*c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/d+b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d-
b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d+1/6*b*c*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d/x^2
```

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5746, 103, 12, 92, 205, 5694, 4182, 2279, 2391}

$$\frac{bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{d}-\frac{bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{d}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{2c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) - (c^2*(a + b*ArcCosh[c*x]))/(d*x) + (7*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(6*d) + (2*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d
+ (b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/d - (b*c^3*PolyLog[2, E^ArcCosh[c*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3dx^3}+c^2\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)}\,dx+\frac{(bc)\int \frac{1}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+c^4\int \frac{a+b\cosh^{-1}(}{d-c^2dx^2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}-\frac{c^3\,\mathrm{Subst}\left(\int(a+bx\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{bc^3\tan^{-1}\left(\sqrt{-1+c\right.}{d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{7bc^3\tan^{-1}\left(\sqrt{-1+\right.}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2}-\frac{a+b\cosh^{-1}(cx)}{3dx^3}-\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{dx}+\frac{7bc^3\tan^{-1}\left(\sqrt{-1+\right.}{6d}$$

Mathematica [A] time = 0.36, size = 223, normalized size = 1.42

$$\frac{-6ac^3\log\left(1-e^{\cosh^{-1}(cx)}\right)+6ac^3\log\left(e^{\cosh^{-1}(cx)}+1\right)-\frac{6ac^2}{x}-\frac{2a}{x^3}+6bc^3\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)-6bc^3\mathrm{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)),x]
```

```
[Out] ((-2*a)/x^3 - (6*a*c^2)/x + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/x^2 - (2*b*A
rcCosh[c*x])/x^3 - (6*b*c^2*ArcCosh[c*x])/x + (7*b*c^3*Sqrt[-1 + c^2*x^2]*A
rcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*a*c^3*Log[1 -
 E^ArcCosh[c*x]] - 6*b*c^3*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 6*a*c^3*L
og[1 + E^ArcCosh[c*x]] + 6*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 6*b
*c^3*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(6*d
)
```

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\operatorname{arcosh}(cx)+a}{c^2dx^6-dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^2*d*x^6 - d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\operatorname{arcosh}(cx)+a}{\left(c^2dx^2-d\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)*x^4), x)
```

maple [A] time = 0.35, size = 225, normalized size = 1.43

$$-\frac{a}{3d\,x^3}-\frac{c^2a}{dx}-\frac{c^3a\ln(cx-1)}{2d}+\frac{c^3a\ln(cx+1)}{2d}-\frac{c^2b\operatorname{arccosh}(cx)}{dx}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,x^2}-\frac{b\operatorname{arccosh}(cx)}{3d\,x^3}+\frac{7c^3b\operatorname{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x)
```

```
[Out] -1/3*a/d/x^3-c^2*a/d/x-1/2*c^3*a/d*ln(c*x-1)+1/2*c^3*a/d*ln(c*x+1)-c^2*b/d*
arccosh(c*x)/x+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2-1/3*b/d*arccosh(c*
x)/x^3+7/3*c^3*b/d*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c^3*b/d*dilog(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+c^3*b/d*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))+c^3*b/d*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(\frac{3\,c^3\log(cx+1)}{d}-\frac{3\,c^3\log(cx-1)}{d}-\frac{2\left(3\,c^2x^2+1\right)}{dx^3}\right)a+\frac{1}{24}\left(216\,c^5\int\frac{x^3\log(cx-1)}{12\left(c^2dx^4-dx^2\right)}\,dx-12\,c^4\left(\frac{\log(cx+}{cd}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/6*(3*c^3*log(c*x + 1)/d - 3*c^3*log(c*x - 1)/d - 2*(3*c^2*x^2 + 1)/(d*x^3
))*a + 1/24*(216*c^5*integrate(1/12*x^3*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x
) - 12*c^4*(log(c*x + 1)/(c*d) - log(c*x - 1)/(c*d)) - 72*c^4*integrate(1/1
2*x^2*log(c*x - 1)/(c^2*d*x^4 - d*x^2), x) - 4*c^2*(c*log(c*x + 1)/d - c*lo
g(c*x - 1)/d - 2/(d*x)) - (3*c^3*x^3*log(c*x + 1)^2 + 6*c^3*x^3*log(c*x + 1
)*log(c*x - 1) - 4*(3*c^3*x^3*log(c*x + 1) - 3*c^3*x^3*log(c*x - 1) - 6*c^2
*x^2 - 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(d*x^3) + 24*integrate(1/
6*(3*c^4*x^3*log(c*x + 1) - 3*c^4*x^3*log(c*x - 1) - 6*c^3*x^2 - 2*c)/(c^3*
d*x^6 - c*d*x^4 + (c^2*d*x^5 - d*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,\left(d-c^2\,d\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^{2}x^{6}-x^{4}}\, dx + \int \frac{b\,\mathrm{acosh}\,(cx)}{c^{2}x^{6}-x^{4}}\, dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d),x)
```

```
[Out] -(Integral(a/(c**2*x**6 - x**4), x) + Integral(b*acosh(c*x)/(c**2*x**6 - x*
*4), x))/d
```

3.37 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=177

$$-\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^5d^2}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{3b\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}$$

```
[Out] 3/2*x*(a+b*arccosh(c*x))/c^4/d^2+1/2*x^3*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x
^2+1)-3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2
-3/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2+3/2*b*polylog(2,
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2-1/2*b*x^2/c^3/d^2/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2
```

Rubi [A] time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5750, 98, 21, 74, 5766, 5694, 4182, 2279, 2391}

$$-\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^5d^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -(b*x^2)/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])/(2*c^5*d^2) + (3*x*(a + b*ArcCosh[c*x]))/(2*c^4*d^2) + (x^3*(a +
b*ArcCosh[c*x]))/(2*c^2*d^2*(1 - c^2*x^2)) - (3*(a + b*ArcCosh[c*x])*ArcTan
h[E^ArcCosh[c*x]])/(c^5*d^2) - (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^5*d^2
) + (3*b*PolyLog[2, E^ArcCosh[c*x]])/(2*c^5*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4 \left(a + b \cosh^{-1}(cx)\right)}{\left(d - c^2 dx^2\right)^2} dx = \frac{x^3 \left(a + b \cosh^{-1}(cx)\right)}{2c^2 d^2 \left(1 - c^2 x^2\right)} + \frac{b \int \frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}} dx}{2cd^2} - \frac{3 \int \frac{x^2 \left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2} dx}{2c^2 d}$$

$$= -\frac{bx^2}{2c^3 d^2 \sqrt{-1+cx}\sqrt{1+cx}} + \frac{3x \left(a + b \cosh^{-1}(cx)\right)}{2c^4 d^2} + \frac{x^3 \left(a + b \cosh^{-1}(cx)\right)}{2c^2 d^2 \left(1 - c^2 x^2\right)} - \frac{b}{}$$

$$= -\frac{bx^2}{2c^3 d^2 \sqrt{-1+cx}\sqrt{1+cx}} - \frac{3b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5 d^2} + \frac{3x \left(a + b \cosh^{-1}(cx)\right)}{2c^4 d^2} + \frac{x^3}{}$$

$$= -\frac{bx^2}{2c^3 d^2 \sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5 d^2} + \frac{3x \left(a + b \cosh^{-1}(cx)\right)}{2c^4 d^2} + \frac{x^3}{}$$

$$= -\frac{bx^2}{2c^3 d^2 \sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5 d^2} + \frac{3x \left(a + b \cosh^{-1}(cx)\right)}{2c^4 d^2} + \frac{x^3}{}$$

$$= -\frac{bx^2}{2c^3 d^2 \sqrt{-1+cx}\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{2c^5 d^2} + \frac{3x \left(a + b \cosh^{-1}(cx)\right)}{2c^4 d^2} + \frac{x^3}{}$$

Mathematica [A] time = 1.09, size = 244, normalized size = 1.38

$$-\frac{2acx}{c^2x^2-1} + 4acx + 3a\log(1-cx) - 3a\log(cx+1) - 6b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right) + 6b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right) - 4bcx\sqrt{\frac{cx-1}{cx+1}} + \frac{bcx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (4*a*c*x - 3*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*b*c*x*Sqrt[(-1 + c*x)/(1 + c*
x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + (b*c*x*Sqrt[(-1 + c*x)/(1
+ c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + 4*b*c*x*ArcCosh[c*x] + (b*A
rcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 6*b*ArcCosh[c*x]*Log[
1 - E^ArcCosh[c*x]] - 6*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + 3*a*Log[1
- c*x] - 3*a*Log[1 + c*x] - 6*b*PolyLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2
, E^ArcCosh[c*x]])/(4*c^5*d^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{c^4 d^2 x^4 - 2\,c^2 d^2 x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.61, size = 300, normalized size = 1.69

$$\frac{ax}{c^4d^2}-\frac{a}{4c^5d^2(cx-1)}+\frac{3a\ln(cx-1)}{4c^5d^2}-\frac{a}{4c^5d^2(cx+1)}-\frac{3a\ln(cx+1)}{4c^5d^2}+\frac{b\,\mathrm{arccosh}(cx)\,x}{c^4d^2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{c^5d^2}-\frac{b\,\mathrm{arcc}}{2c^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^4*a/d^2*x-1/4/c^5*a/d^2/(c*x-1)+3/4/c^5*a/d^2*ln(c*x-1)-1/4/c^5*a/d^2/(
c*x+1)-3/4/c^5*a/d^2*ln(c*x+1)+1/c^4*b/d^2*arccosh(c*x)*x-b*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^5/d^2-1/2/c^4*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c^5*b/d^2
/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-3/2/c^5*b/d^2*arccosh(c*x)*ln(1+c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(
1/2))/c^5/d^2+3/2/c^5*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))+3/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{64}\left(16\,c^4\left(\frac{2\,x}{c^{10}d^2x^2-c^8d^2}-\frac{4\,x}{c^8d^2}+\frac{3\,\log(cx+1)}{c^9d^2}-\frac{3\,\log(cx-1)}{c^9d^2}\right)-576\,c^3\int\frac{x^3\log(cx-1)}{8\left(c^8d^2x^4-2\,c^6d^2x^2+c^4d^2\right)}\,dx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(16*c^4*(2*x/(c^10*d^2*x^2 - c^8*d^2) - 4*x/(c^8*d^2) + 3*log(c*x + 1)
/(c^9*d^2) - 3*log(c*x - 1)/(c^9*d^2)) - 576*c^3*integrate(1/8*x^3*log(c*x
- 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) - 24*c^2*(2*x/(c^8*d^2*x^2
 - c^6*d^2) + log(c*x + 1)/(c^7*d^2) - log(c*x - 1)/(c^7*d^2)) + 192*c^2*in
tegrate(1/8*x^2*log(c*x - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x) -
9*(c*(2/(c^8*d^2*x - c^7*d^2) - log(c*x + 1)/(c^7*d^2) + log(c*x - 1)/(c^7*
d^2)) + 4*log(c*x - 1)/(c^8*d^2*x^2 - c^6*d^2))*c + 4*(3*(c^2*x^2 - 1)*log(
c*x + 1)^2 + 6*(c^2*x^2 - 1)*log(c*x + 1)*log(c*x - 1) + 4*(4*c^3*x^3 - 6*c
*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x - 1))*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^2*x^2 - c^5*d^2) - 64*integrate(-1/4*
(4*c^3*x^3 - 6*c*x - 3*(c^2*x^2 - 1)*log(c*x + 1) + 3*(c^2*x^2 - 1)*log(c*x
 - 1))/(c^9*d^2*x^5 - 2*c^7*d^2*x^3 + c^5*d^2*x + (c^8*d^2*x^4 - 2*c^6*d^2*
x^2 + c^4*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 192*integrate(1/8*log(c*x
 - 1)/(c^8*d^2*x^4 - 2*c^6*d^2*x^2 + c^4*d^2), x))*b - 1/4*a*(2*x/(c^6*d^2*
x^2 - c^4*d^2) - 4*x/(c^4*d^2) + 3*log(c*x + 1)/(c^5*d^2) - 3*log(c*x - 1)/
(c^5*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^4}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{bx^4\,\mathrm{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**4/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**4*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.38 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=179

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2c^4d^2}-\frac{b\sqrt{}}{2c^4d^2}$$

```
[Out] 1/2*b*arccosh(c*x)/c^4/d^2+1/2*x^2*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-
1/2*(a+b*arccosh(c*x))^2/b/c^4/d^2+(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)/c^4/d^2+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)/c^4/d^2-1/2*b/c^4/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*(c*x-1)^(1/2
)/c^4/d^2/(c*x+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5750, 89, 12, 78, 52, 5715, 3716, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2c^4d^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2}+\frac{\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c^4*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*Sqrt[-1 + c*x])/(2*c^4*d^2
*Sqrt[1 + c*x]) + (b*ArcCosh[c*x])/(2*c^4*d^2) + (x^2*(a + b*ArcCosh[c*x]))
/(2*c^2*d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*c^4*d^2) + ((a + b
*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c^4*d^2) + (b*PolyLog[2, E^(2*
ArcCosh[c*x])])/(2*c^4*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
```

```
+ 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx = \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2} - \frac{\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{c^2d}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} + \frac{\text{Subst}\left(\int(a+bx)\coth(x)\,dx,\right.}{c^4d^2}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^4d^2} - \frac{2\,\text{Subs}}{}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)} - \frac{\left(a+b\cosh\right.}{2bc^4d}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right.}{2c^2d^2\left(1-c^2x^2\right)}$$

$$= -\frac{b}{2c^4d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\sqrt{-1+cx}}{2c^4d^2\sqrt{1+cx}} + \frac{b\cosh^{-1}(cx)}{2c^4d^2} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right.}{2c^2d^2\left(1-c^2x^2\right)}$$

Mathematica [A] time = 0.68, size = 209, normalized size = 1.17

$$\frac{-\frac{2a}{c^2x^2-1} + 2a\log\left(1-c^2x^2\right) + 4b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right) + 4b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right) - b\sqrt{\frac{cx-1}{cx+1}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}}{1-cx} + \frac{bcx\sqrt{\frac{cx-1}{cx+1}}}{1-cx} - 2b\cosh^{-}}{4c^4d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (-(b*Sqrt[(-1 + c*x)/(1 + c*x)]) + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x)
 + (b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a)/(-1 + c^2*x^2) + (b
*ArcCosh[c*x])/(1 - c*x) + (b*ArcCosh[c*x])/(1 + c*x) - 2*b*ArcCosh[c*x]^2
+ 4*b*ArcCosh[c*x]*Log[1 - E^ArcCosh[c*x]] + 4*b*ArcCosh[c*x]*Log[1 + E^Arc
Cosh[c*x]] + 2*a*Log[1 - c^2*x^2] + 4*b*PolyLog[2, -E^ArcCosh[c*x]] + 4*b*P
olyLog[2, E^ArcCosh[c*x]])/(4*c^4*d^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3\,\text{arcosh}\,(cx)+ax^3}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.61, size = 309, normalized size = 1.73

$$-\frac{a}{4c^4d^2(cx-1)}+\frac{a\ln(cx-1)}{2c^4d^2}+\frac{a}{4c^4d^2(cx+1)}+\frac{a\ln(cx+1)}{2c^4d^2}-\frac{b\operatorname{arccosh}(cx)^2}{2c^4d^2}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,x}{2c^3d^2\left(c^2x^2-1\right)}+\frac{b\,x^2}{2c^2d^2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c^4*a/d^2/(c*x-1)+1/2/c^4*a/d^2*ln(c*x-1)+1/4/c^4*a/d^2/(c*x+1)+1/2/c^
4*a/d^2*ln(c*x+1)-1/2/c^4*b/d^2*arccosh(c*x)^2-1/2/c^3*b/d^2/(c^2*x^2-1)*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/2/c^2*b/d^2/(c^2*x^2-1)*x^2-1/2/c^4*b/d^2/(c^
2*x^2-1)*arccosh(c*x)-1/2/c^4*b/d^2/(c^2*x^2-1)+1/c^4*b/d^2*arccosh(c*x)*ln
(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/c^4*b/d^2*polylog(2,-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))+1/c^4*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))+1/c^4*b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{\left(c^2x^2-1\right)\log(cx+1)^2+2\left(c^2x^2-1\right)\log(cx+1)\log(cx-1)+\left(c^2x^2-1\right)\log(cx-1)^2-4\left(\left(c^2x^2-1\right)\mathrm{l}\right.}{c^6d^2x^2-c^4d^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c*
x - 1) + (c^2*x^2 - 1)*log(c*x - 1)^2 - 4*((c^2*x^2 - 1)*log(c*x + 1) + (c^
2*x^2 - 1)*log(c*x - 1) - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 2)/(c
^6*d^2*x^2 - c^4*d^2) - 8*integrate(1/2*((c^2*x^2 - 1)*log(c*x + 1) + (c^2*
x^2 - 1)*log(c*x - 1) - 1)/(c^8*d^2*x^5 - 2*c^6*d^2*x^3 + c^4*d^2*x + (c^7*
d^2*x^4 - 2*c^5*d^2*x^2 + c^3*d^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))
, x)) - 1/2*a*(1/(c^6*d^2*x^2 - c^4*d^2) - log(c^2*x^2 - 1)/(c^4*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^3}{c^4x^4-2c^2x^2+1}\, dx + \int \frac{bx^3\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\, dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**3/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**3*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.39 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=124

$$-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^3d^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}-\frac{}{2c^3d^2\sqrt{cx}}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-(a+b*arccosh(c*x))*arctanh(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2-1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/c^3/d^2+1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/
d^2-1/2*b/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5750, 74, 5694, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2c^3d^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(c\right.}{c^3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c^3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*c
^2*d^2*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^3
*d^2) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c^3*d^2) + (b*PolyLog[2, E^ArcCo
sh[c*x]])/(2*c^3*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx=\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}-\frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2c^2d}$$

$$=-\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{\operatorname{Subst}\left(\int(a+bx)\operatorname{csch}(x)\,d\right.}{2c^3d^2}$$

$$=-\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{c^3d^2}$$

$$=-\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{c^3d^2}$$

$$=-\frac{b}{2c^3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(\right.}{c^3d^2}$$

Mathematica [A] time = 0.76, size = 206, normalized size = 1.66

$$\frac{-\frac{2acx}{c^2x^2-1}+a\log(1-cx)-a\log(cx+1)-2b\operatorname{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)+2b\operatorname{Li}_2\left(e^{\cosh^{-1}(cx)}\right)+\frac{bcx\sqrt{\frac{cx-1}{cx+1}}}{1-cx}+\frac{b\sqrt{\frac{cx-1}{cx+1}}}{1-cx}+b\sqrt{\frac{cx}{cx}}}{4c^3d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (b*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) +
(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c*x)/(-1 + c^2*x^2) + (
b*ArcCosh[c*x])/(1 - c*x) - (b*ArcCosh[c*x])/(1 + c*x) + 2*b*ArcCosh[c*x]*L
og[1 - E^ArcCosh[c*x]] - 2*b*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] + a*Log[1
 - c*x] - a*Log[1 + c*x] - 2*b*PolyLog[2, -E^ArcCosh[c*x]] + 2*b*PolyLog[2,
 E^ArcCosh[c*x]])/(4*c^3*d^2)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{bx^2\operatorname{arcosh}(cx)+ax^2}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{(c^2dx^2 - d)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^2, x)
```

maple [A] time = 0.31, size = 255, normalized size = 2.06

$$-\frac{a}{4c^3d^2(cx-1)}+\frac{a\ln(cx-1)}{4c^3d^2}-\frac{a}{4c^3d^2(cx+1)}-\frac{a\ln(cx+1)}{4c^3d^2}-\frac{b\operatorname{arccosh}(cx)x}{2c^2d^2(c^2x^2-1)}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2c^3d^2(c^2x^2-1)}-\frac{b\operatorname{arccosh}(cx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c^3*a/d^2/(c*x-1)+1/4/c^3*a/d^2*ln(c*x-1)-1/4/c^3*a/d^2/(c*x+1)-1/4/c^
3*a/d^2*ln(c*x+1)-1/2/c^2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c^3*b/d^2/(c
^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/2/c^3*b/d^2*arccosh(c*x)*ln(1+c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/c^3/d^2+1/2/c^3*b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))
+1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{64}\left(192\,c^3\int \frac{x^3\log(cx-1)}{8\left(c^6d^2x^4-2\,c^4d^2x^2+c^2d^2\right)}\,dx+8\,c^2\left(\frac{2\,x}{c^6d^2x^2-c^4d^2}+\frac{\log(cx+1)}{c^5d^2}-\frac{\log(cx-1)}{c^5d^2}\right)-64\,c^2\int \frac{}{8(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2
+ c^2*d^2), x) + 8*c^2*(2*x/(c^6*d^2*x^2 - c^4*d^2) + log(c*x + 1)/(c^5*d^2
) - log(c*x - 1)/(c^5*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^6*d^
2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x) + 3*(c*(2/(c^6*d^2*x - c^5*d^2) - log(
c*x + 1)/(c^5*d^2) + log(c*x - 1)/(c^5*d^2)) + 4*log(c*x - 1)/(c^6*d^2*x^2
- c^4*d^2))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x +
 1)*log(c*x - 1) - 4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*lo
g(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^5*d^2*x^2 - c^3*d^2)
 + 64*integrate(1/4*(2*c*x + (c^2*x^2 - 1)*log(c*x + 1) - (c^2*x^2 - 1)*log
(c*x - 1))/(c^7*d^2*x^5 - 2*c^5*d^2*x^3 + c^3*d^2*x + (c^6*d^2*x^4 - 2*c^4*
d^2*x^2 + c^2*d^2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(
c*x - 1)/(c^6*d^2*x^4 - 2*c^4*d^2*x^2 + c^2*d^2), x))*b - 1/4*a*(2*x/(c^4*d
^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax^2}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx^2\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x**2/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x**2*acosh(
c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.40 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=61

$$\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{bx}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/c^2/d^2/(-c^2*x^2+1)-1/2*b*x/c/d^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {5716, 39}

$$\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}-\frac{bx}{2cd^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] -(b*x)/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*c^2
*d^2*(1 - c^2*x^2))
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}+\frac{b\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2cd^2}\\ &= -\frac{bx}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2c^2d^2\left(1-c^2x^2\right)}\end{aligned}$$

Mathematica [A] time = 0.14, size = 53, normalized size = 0.87

$$\frac{a+bcx\sqrt{cx-1}\sqrt{cx+1}+b\cosh^{-1}(cx)}{2c^2d^2-2c^4d^2x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] (a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*ArcCosh[c*x])/(2*c^2*d^2 - 2*c^
4*d^2*x^2)
```

fricas [A] time = 0.74, size = 65, normalized size = 1.07

$$-\frac{ac^2x^2 + \sqrt{c^2x^2-1}\,bcx + b\log\left(cx + \sqrt{c^2x^2-1}\right)}{2\left(c^4d^2x^2 - c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a*c^2*x^2 + sqrt(c^2*x^2 - 1)*b*c*x + b*log(c*x + sqrt(c^2*x^2 - 1)))
/(c^4*d^2*x^2 - c^2*d^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x}{\left(c^2dx^2 - d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^2, x)
```

maple [A] time = 0.02, size = 64, normalized size = 1.05

$$\frac{-\frac{a}{2d^2\left(c^2x^2-1\right)} + \frac{b\left(-\frac{\operatorname{arccosh}(cx)}{2\left(c^2x^2-1\right)} - \frac{cx}{2\sqrt{cx-1}\,\sqrt{cx+1}}\right)}{d^2}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] 1/c^2*(-1/2*a/d^2/(c^2*x^2-1)+b/d^2*(-1/2/(c^2*x^2-1)*arccosh(c*x)-1/2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*c*x))
```

maxima [B] time = 0.37, size = 134, normalized size = 2.20

$$-\frac{1}{4}\left(\left(\frac{\sqrt{c^2x^2-1}\,c^2d^2}{c^7d^4x + c^6d^4} + \frac{\sqrt{c^2x^2-1}\,c^2d^2}{c^7d^4x - c^6d^4}\right)c^2 + \frac{2\operatorname{arcosh}(cx)}{c^4d^2x^2 - c^2d^2}\right)b - \frac{a}{2\left(c^4d^2x^2 - c^2d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((sqrt(c^2*x^2 - 1)*c^2*d^2/(c^7*d^4*x + c^6*d^4) + sqrt(c^2*x^2 - 1)*
c^2*d^2/(c^7*d^4*x - c^6*d^4))*c^2 + 2*arccosh(c*x)/(c^4*d^2*x^2 - c^2*d^2)
)*b - 1/2*a/(c^4*d^2*x^2 - c^2*d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\left(a + b\,\mathrm{acosh}\left(c\,x\right)\right)}{\left(d - c^2\,d\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{ax}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{bx\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*x/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*x*acosh(c*x)/(
c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.41 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=120

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd^2}+\frac{b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{}{2cd^2\sqrt{cx}}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+(a+b*arccosh(c*x))*arctanh(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2+1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/c/d^2-1/2*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2-1/2*b/
c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2cd^2}-\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2cd^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] -b/(2*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(2*d^2
*(1 - c^2*x^2)) + ((a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c*d^2) +
(b*PolyLog[2, -E^ArcCosh[c*x]])/(2*c*d^2) - (b*PolyLog[2, E^ArcCosh[c*x]])/
(2*c*d^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{(bc)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{d-c^2dx^2}\,dx}{2d}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\,dx,x,\text{cosh}\ldots\right)}{2cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(c\ldots}\right.}{cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(c\ldots}\right.}{cd^2}\\
&= -\frac{b}{2cd^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\frac{\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(c\ldots}\right.}{cd^2}
\end{aligned}$$

Mathematica [A] time = 1.41, size = 189, normalized size = 1.58

$$\frac{\frac{ac^2x^2\log(cx+1)+\left(a-ac^2x^2\right)\log(1-cx)-2acx-a\log(cx+1)-2b\cosh^{-1}(cx)\left(\left(c^2x^2-1\right)\log\left(1-e^{\cosh^{-1}(cx)}\right)+\left(1-c^2x^2\right)\log\left(e^{\cosh^{-1}(cx)}+1\right)+cx\right)-2bcx\sqrt{\frac{cx-1}{cx+1}}\ldots}{c^2x^2-1}\ldots}{4cd^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((-2*a*c*x - 2*b*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*b*c*x*Sqrt[(-1 + c*x)/(1 +
c*x)] - 2*b*ArcCosh[c*x]*(c*x + (-1 + c^2*x^2)*Log[1 - E^ArcCosh[c*x]] + (1
 - c^2*x^2)*Log[1 + E^ArcCosh[c*x]]) + (a - a*c^2*x^2)*Log[1 - c*x] - a*Log
[1 + c*x] + a*c^2*x^2*Log[1 + c*x])/(-1 + c^2*x^2) + 2*b*PolyLog[2, -E^ArcC
osh[c*x]] - 2*b*PolyLog[2, E^ArcCosh[c*x]])/(4*c*d^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(c^2 dx^2 - d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^2, x)
```

maple [A] time = 0.10, size = 252, normalized size = 2.10

$$-\frac{a}{4c\,d^2\,(cx-1)}-\frac{a\ln(cx-1)}{4c\,d^2}-\frac{a}{4c\,d^2\,(cx+1)}+\frac{a\ln(cx+1)}{4c\,d^2}-\frac{b\,\mathrm{arccosh}(cx)\,x}{2d^2\left(c^2x^2-1\right)}-\frac{b\sqrt{cx+1}\,\sqrt{cx-1}}{2c\,d^2\left(c^2x^2-1\right)}+\frac{b\,\mathrm{arccosh}(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/4/c*a/d^2/(c*x-1)-1/4/c*a/d^2*ln(c*x-1)-1/4/c*a/d^2/(c*x+1)+1/4/c*a/d^2*
ln(c*x+1)-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*x-1/2/c*b/d^2/(c^2*x^2-1)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)+1/2/c*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))+1/2*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2-1/2/c*b
/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/2*b*polylog(2,c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{64}\left(192\,c^3\int\frac{x^3\log(cx-1)}{8\left(c^4d^2x^4-2\,c^2d^2x^2+d^2\right)}\,dx-8\,c^2\left(\frac{2\,x}{c^4d^2x^2-c^2d^2}+\frac{\log(cx+1)}{c^3d^2}-\frac{\log(cx-1)}{c^3d^2}\right)-64\,c^2\int\frac{}{8\left(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(192*c^3*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 +
 d^2), x) - 8*c^2*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - l
og(c*x - 1)/(c^3*d^2)) - 64*c^2*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x^4
 - 2*c^2*d^2*x^2 + d^2), x) + 3*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/
(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2
))*c - 4*((c^2*x^2 - 1)*log(c*x + 1)^2 + 2*(c^2*x^2 - 1)*log(c*x + 1)*log(c
*x - 1) + 4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^3*d^2*x^2 - c*d^2) + 64*integ
rate(-1/4*(2*c*x - (c^2*x^2 - 1)*log(c*x + 1) + (c^2*x^2 - 1)*log(c*x - 1))
/(c^5*d^2*x^5 - 2*c^3*d^2*x^3 + c*d^2*x + (c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^
2)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + 64*integrate(1/8*log(c*x - 1)/(c^4*d^
2*x^4 - 2*c^2*d^2*x^2 + d^2), x))*b - 1/4*a*(2*x/(c^2*d^2*x^2 - d^2) - log(
c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}(c\,x)}{\left(d - c^2\,d\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^4-2c^2x^2+1}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*acosh(c*x)/(c**4
*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.42 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=116

$$\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{b\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{b\text{Li}_2\left(e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{}{2d^2\sqrt{cx}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+2*(a+b*arccosh(c*x))*arctanh((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)/d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-
1/2*b*c*x/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {5754, 5721, 5461, 4182, 2279, 2391, 39}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^2}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c*x)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(2*d^2
*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2
+ (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2) - (b*PolyLog[2, E^(2*ArcCosh[
c*x])])/(2*d^2)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
```

```
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx &= \frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{(bc)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)}\,dx}{d}\\
&= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{\text{Subst}\left(\int (a+bx)\text{csch}(x)\text{sech}(x)\,dx,x,c\right.}{d^2}\\
&= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}-\frac{2\,\text{Subst}\left(\int (a+bx)\text{csch}(2x)\,dx,x,\cosh^{-}\right.}{d^2}\\
&= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right.}{d^2}\\
&= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right.}{d^2}\\
&= -\frac{bcx}{2d^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{2d^2\left(1-c^2x^2\right)}+\frac{2\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right.}{d^2}
\end{aligned}$$

Mathematica [A] time = 0.81, size = 149, normalized size = 1.28

$$\frac{\frac{a}{1-c^2x^2}-a\log\left(1-c^2x^2\right)+2a\log(x)+b\left(\frac{\cosh^{-1}(cx)}{1-c^2x^2}-\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+\text{Li}_2\left(e^{-2\cosh^{-1}(cx)}\right)+\frac{cx\sqrt{\frac{cx-1}{cx+1}}}{1-cx}-2\cosh^{-}\right.}{2d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a/(1 - c^2*x^2) + 2*a*Log[x] - a*Log[1 - c^2*x^2] + b*((c*x*Sqrt[(-1 + c*x
)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 2*ArcCosh[c*x]*Log[1
 - E^(-2*ArcCosh[c*x])] + 2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, E^(-2*ArcCosh[c*x])]))/(2*d^2)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\left(cx\right) + a}{c^4d^2x^5 - 2\,c^2d^2x^3 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\left(cx\right) + a}{\left(c^2dx^2 - d\right)^2 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x), x)
```

maple [B] time = 0.28, size = 339, normalized size = 2.92

$$\frac{a\ln\left(cx\right)}{d^2} - \frac{a}{4d^2\left(cx-1\right)} - \frac{a\ln\left(cx-1\right)}{2d^2} + \frac{a}{4d^2\left(cx+1\right)} - \frac{a\ln\left(cx+1\right)}{2d^2} - \frac{b\sqrt{cx+1}\,\sqrt{cx-1}\,cx}{2d^2\left(c^2x^2-1\right)} + \frac{b\,c^2x^2}{2d^2\left(c^2x^2-1\right)} - \frac{b\,\text{arc}}{2d^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)-1/4*a/d^2/(c*x-1)-1/2*a/d^2*ln(c*x-1)+1/4*a/d^2/(c*x+1)-1/2*a
/d^2*ln(c*x+1)-1/2*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x+1/2*b/
d^2/(c^2*x^2-1)*c^2*x^2-1/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)-1/2*b/d^2/(c^2*x
^2-1)+b/d^2*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*po
lylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-b/d^2*arccosh(c*x)*ln(1+c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^
(1/2))-b/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^2*polyl
og(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{1}{c^2d^2x^2 - d^2} + \frac{\log\left(cx+1\right)}{d^2} + \frac{\log\left(cx-1\right)}{d^2} - \frac{2\,\log(x)}{d^2}\right) + b\int \frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{c^4d^2x^5 - 2\,c^2d^2x^3 + d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(1/(c^2*d^2*x^2 - d^2) + log(c*x + 1)/d^2 + log(c*x - 1)/d^2 - 2*log
(x)/d^2) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^4*d^2*x^5
- 2*c^2*d^2*x^3 + d^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\text{acosh}\left(c\,x\right)}{x\left(d - c^2\,d\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^5-2c^2x^3+x}\,dx + \int \frac{b\operatorname{acosh}(cx)}{c^4x^5-2c^2x^3+x}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**5 - 2*c**2*x**3 + x), x) + Integral(b*acosh(c*x)/(c**4
*x**5 - 2*c**2*x**3 + x), x))/d**2
```

3.43 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=170

$$\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)}+\frac{3c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{3bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{2d^2}-$$

```
[Out] (-a-b*arccosh(c*x))/d^2/x/(-c^2*x^2+1)+3/2*c^2*x*(a+b*arccosh(c*x))/d^2/(-c
^2*x^2+1)+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+3*c*(a+b*arccosh(c*x)
)*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+3/2*b*c*polylog(2,-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/d^2-3/2*b*c*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))/d^2-1/2*b*c/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5746, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{3bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)}+\frac{3c\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^2*x*(
1 - c^2*x^2)) + (3*c^2*x*(a + b*ArcCosh[c*x]))/(2*d^2*(1 - c^2*x^2)) + (b*c
*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (3*c*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/d^2 + (3*b*c*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (3
*b*c*PolyLog[2, E^ArcCosh[c*x]])/(2*d^2)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
```

```
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx - \frac{(bc)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2}$$

$$= \frac{bc}{d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{b\int \frac{c+c^2x}{x\sqrt{-1+cx}\,(1}}{d^2}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} - \frac{(3c)\,\text{Subst}}{}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{3c\left(a+b\,c\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan^{-1}\left(\right.}{}$$

$$= -\frac{bc}{2d^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{d^2x\left(1-c^2x^2\right)} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)} + \frac{bc\tan^{-1}\left(\right.}{}$$

Mathematica [A] time = 0.72, size = 283, normalized size = 1.66

$$-\frac{2ac^2x}{c^2x^2-1} - 3ac\log(1-cx) + 3ac\log(cx+1) - \frac{4a}{x} + \frac{4bc\sqrt{c^2x^2-1}\,\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}} + \frac{bc^2x\sqrt{\frac{cx-1}{cx+1}}}{1-cx} + 6bc\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^2),x]
```

```
[Out] ((-4*a)/x + b*c*Sqrt[(-1 + c*x)/(1 + c*x)] + (b*c*Sqrt[(-1 + c*x)/(1 + c*x)
])/(1 - c*x) + (b*c^2*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (2*a*c^2*x)
/(-1 + c^2*x^2) - (4*b*ArcCosh[c*x])/x + (b*c*ArcCosh[c*x])/(1 - c*x) - (b*
c*ArcCosh[c*x])/(1 + c*x) + (4*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*
x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - 6*b*c*ArcCosh[c*x]*Log[1 - E^ArcCos
h[c*x]] + 6*b*c*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] - 3*a*c*Log[1 - c*x] +
 3*a*c*Log[1 + c*x] + 6*b*c*PolyLog[2, -E^ArcCosh[c*x]] - 6*b*c*PolyLog[2,
E^ArcCosh[c*x]])/(4*d^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^2), x)
```

maple [A] time = 0.36, size = 259, normalized size = 1.52

$$-\frac{a}{d^2x}-\frac{ca}{4d^2(cx-1)}-\frac{3ca\ln(cx-1)}{4d^2}-\frac{ca}{4d^2(cx+1)}+\frac{3ca\ln(cx+1)}{4d^2}-\frac{3b\,\mathrm{arccosh}(cx)\,c^2x}{2d^2\left(c^2x^2-1\right)}-\frac{cb\sqrt{cx+1}\,\sqrt{cx-1}}{2d^2\left(c^2x^2-1\right)}+\frac{b\,a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/4*c*a/d^2/(c*x-1)-3/4*c*a/d^2*ln(c*x-1)-1/4*c*a/d^2/(c*x+1)+3/4*
c*a/d^2*ln(c*x+1)-3/2*b/d^2/(c^2*x^2-1)*arccosh(c*x)*c^2*x-1/2*c*b/d^2/(c^2
*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+b/d^2/x/(c^2*x^2-1)*arccosh(c*x)+2*c*b/
d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*c*b/d^2*dilog(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))+3/2*c*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/
2*c*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{64}\left(576\,c^5\int\frac{x^3\log(cx-1)}{8\left(c^4d^2x^4-2\,c^2d^2x^2+d^2\right)}\,dx-24\,c^4\left(\frac{2\,x}{c^4d^2x^2-c^2d^2}+\frac{\log(cx+1)}{c^3d^2}-\frac{\log(cx-1)}{c^3d^2}\right)-192\,c^4\int\frac{}{8\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(576*c^5*integrate(1/8*x^3*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 +
 d^2), x) - 24*c^4*(2*x/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) -
log(c*x - 1)/(c^3*d^2)) - 192*c^4*integrate(1/8*x^2*log(c*x - 1)/(c^4*d^2*x
^4 - 2*c^2*d^2*x^2 + d^2), x) + 9*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1
)/(c^3*d^2) + log(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d
^2))*c^3 + 16*c^2*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x
 - 1)/(c*d^2)) + 192*c^2*integrate(1/8*log(c*x - 1)/(c^4*d^2*x^4 - 2*c^2*d^
2*x^2 + d^2), x) - 4*(3*(c^3*x^3 - c*x)*log(c*x + 1)^2 + 6*(c^3*x^3 - c*x)*
log(c*x + 1)*log(c*x - 1) + 4*(6*c^2*x^2 - 3*(c^3*x^3 - c*x)*log(c*x + 1) +
 3*(c^3*x^3 - c*x)*log(c*x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
)/(c^2*d^2*x^3 - d^2*x) + 64*integrate(-1/4*(6*c^3*x^2 - 3*(c^4*x^3 - c^2*x
)*log(c*x + 1) + 3*(c^4*x^3 - c^2*x)*log(c*x - 1) - 4*c)/(c^5*d^2*x^6 - 2*c
^3*d^2*x^4 + c*d^2*x^2 + (c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(c*x + 1
)*sqrt(c*x - 1)), x))*b - 1/4*a*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) -
3*c*log(c*x + 1)/d^2 + 3*c*log(c*x - 1)/d^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^4x^6-2c^2x^4+x^2}\,dx+\int\frac{b\,\mathrm{acosh}(cx)}{c^4x^6-2c^2x^4+x^2}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**6 - 2*c**2*x**4 + x**2), x) + Integral(b*acosh(c*x)/(c
**4*x**6 - 2*c**2*x**4 + x**2), x))/d**2
```

3.44 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d-c^2dx^2)^2}\,dx$

Optimal. Leaf size=152

$$\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}+\frac{bc^2\text{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc^2}{}$$

```
[Out] c^2*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+1/2*(-a-b*arccosh(c*x))/d^2/x^2/(-c
^2*x^2+1)+4*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)/d^2+b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-b*c^2*po
lylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*b*c/d^2/x/(c*x-1)^(1/2
)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5746, 103, 12, 39, 5754, 5721, 5461, 4182, 2279, 2391}

$$\frac{bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2}-\frac{bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d^2}+\frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)}+\frac{4c^2\tanh^{-1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c)/(2*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*(a + b*ArcCosh[c*x]))/
(d^2*(1 - c^2*x^2)) - (a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c^2*x^2)) + (4*c
^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d^2 + (b*c^2*PolyLog[2
, -E^(2*ArcCosh[c*x])])/d^2 - (b*c^2*PolyLog[2, E^(2*ArcCosh[c*x])])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} + \left(2c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx - \frac{(bc)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} - \frac{(bc)\int \frac{2c}{(-1+cx)^{3/2}}}{2d^2}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3x}{d^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh}{2d^2x^2\left(1-\right.}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} - \left(4c^2\right)\operatorname{Subst}\left(\int\right.$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} + \frac{4c^2\left(a+b\cosh\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} + \frac{4c^2\left(a+b\cosh\right.}{}$$

$$= -\frac{bc}{2d^2x\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2\left(1-c^2x^2\right)} + \frac{4c^2\left(a+b\cosh\right.}{}$$

Mathematica [B] time = 0.60, size = 319, normalized size = 2.10

$$\frac{4ac^4x^4\log(x) - 2ac^2x^2 - 4ac^2x^2\log(x) + 2ac^2x^2\log\left(1-c^2x^2\right) - 2ac^4x^4\log\left(1-c^2x^2\right) + a - 4bc^4x^4\cosh^{-1}(cx)\,\mathrm{l}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^2),x]
```

```
[Out] (a - 2*a*c^2*x^2 - b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)] + b*ArcCosh[c*x] - 2*b*c^2*x^2*ArcCosh[c*x] + 4*b*c^2*x^2*A
rcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 -
 E^(-2*ArcCosh[c*x])] - 4*b*c^2*x^2*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x]
)] + 4*b*c^4*x^4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 4*a*c^2*x^2*Lo
g[x] + 4*a*c^4*x^4*Log[x] + 2*a*c^2*x^2*Log[1 - c^2*x^2] - 2*a*c^4*x^4*Log[
1 - c^2*x^2] - 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, -E^(-2*ArcCosh[c*x])]
+ 2*b*c^2*x^2*(-1 + c^2*x^2)*PolyLog[2, E^(-2*ArcCosh[c*x])])/(2*d^2*x^2*(-
1 + c^2*x^2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{b\operatorname{arcosh}(cx)+a}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\left(c^2dx^2-d\right)^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^3), x)
```

maple [A] time = 0.34, size = 371, normalized size = 2.44

$$-\frac{a}{2d^2x^2}+\frac{2c^2a\ln(cx)}{d^2}-\frac{c^2a}{4d^2(cx-1)}-\frac{c^2a\ln(cx-1)}{d^2}+\frac{c^2a}{4d^2(cx+1)}-\frac{c^2a\ln(cx+1)}{d^2}-\frac{c^2b\,\mathrm{arccosh}(cx)}{d^2\left(c^2x^2-1\right)}-\frac{cb\sqrt{cx+}}{2d^2x\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2+2*c^2*a/d^2*ln(c*x)-1/4*c^2*a/d^2/(c*x-1)-c^2*a/d^2*ln(c*x-1
)+1/4*c^2*a/d^2/(c*x+1)-c^2*a/d^2*ln(c*x+1)-c^2*b/d^2/(c^2*x^2-1)*arccosh(c
*x)-1/2*c*b/d^2/x/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/2*b/d^2/x^2/(c^
2*x^2-1)*arccosh(c*x)+2*c^2*b/d^2*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)+b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-2*c
^2*b/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*arccosh(c*x)*ln(1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*c^2*b/d^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{2c^2\log(cx+1)}{d^2}+\frac{2c^2\log(cx-1)}{d^2}-\frac{4c^2\log(x)}{d^2}+\frac{2c^2x^2-1}{c^2d^2x^4-d^2x^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^4d^2x^7-2c^2d^2x^5+d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(2*c^2*log(c*x + 1)/d^2 + 2*c^2*log(c*x - 1)/d^2 - 4*c^2*log(x)/d^2
+ (2*c^2*x^2 - 1)/(c^2*d^2*x^4 - d^2*x^2)) + b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{acosh}(c\,x)}{x^3\left(d-c^2\,d\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{a}{c^4x^7-2c^2x^5+x^3}\,dx+\int\frac{b\,\mathrm{acosh}(cx)}{c^4x^7-2c^2x^5+x^3}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**7 - 2*c**2*x**5 + x**3), x) + Integral(b*acosh(c*x)/(c
**4*x**7 - 2*c**2*x**5 + x**3), x))/d**2
```

3.45 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=248

$$\frac{5c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}-\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)}+\frac{5c^4x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}+\cdots$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d^2/x^3/(-c^2*x^2+1)-5/3*c^2*(a+b*arccosh(c*x))/d^2
/x/(-c^2*x^2+1)+5/2*c^4*x*(a+b*arccosh(c*x))/d^2/(-c^2*x^2+1)+13/6*b*c^3*ar
ctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+5*c^3*(a+b*arccosh(c*x))*arctanh(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2+5/2*b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/d^2-5/2*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-1/
3*b*c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c/d^2/x^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {5746, 103, 12, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{5bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{2d^2}-\frac{5bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{2d^2}+\frac{5c^4x\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(1-c^2x^2\right)}-\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)}-\frac{a}{3d}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

```
[Out] -(b*c^3)/(3*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c)/(6*d^2*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(3*d^2*x^3*(1 - c^2*x^2)) - (5*c
^2*(a + b*ArcCosh[c*x]))/(3*d^2*x*(1 - c^2*x^2)) + (5*c^4*x*(a + b*ArcCosh[
c*x]))/(2*d^2*(1 - c^2*x^2)) + (13*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x
]])/(6*d^2) + (5*c^3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/d^2 + (5
*b*c^3*PolyLog[2, -E^ArcCosh[c*x]])/(2*d^2) - (5*b*c^3*PolyLog[2, E^ArcCosh
[c*x]])/(2*d^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} + \frac{1}{3}\left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^2}\,dx - \frac{(bc)\int \frac{1}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2}$$

$$= -\frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^2x\left(1-c^2x^2\right)} + \left(5c^4\right)\int \frac{a+}{(d}$$

$$= \frac{5bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cosh\right.}{3d^2x\left(1-c\right.}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cos\right.}{3d^2x\left(1-\right.}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cos\right.}{3d^2x\left(1-\right.}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cos\right.}{3d^2x\left(1-\right.}$$

$$= -\frac{bc^3}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc}{6d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3\left(1-c^2x^2\right)} - \frac{5c^2\left(a+b\cos\right.}{3d^2x\left(1-\right.}$$

Mathematica [A] time = 1.70, size = 377, normalized size = 1.52

$$-\frac{15ac^3\log(1-cx)-15ac^3\log(cx+1)+\frac{24ac^2}{x}+\frac{6ac^4x}{c^2x^2-1}+\frac{4a}{x^3}+\frac{3bc^4x\sqrt{\frac{cx-1}{cx+1}}}{cx-1}-30bc^3\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)+30bc^3\text{Li}_2\left(e^{c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^2),x]
```

```
[Out] -1/12*((4*a)/x^3 + (24*a*c^2)/x - 3*b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)] + (3*b
*c^3*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) + (3*b*c^4*x*Sqrt[(-1 + c*x)/(1
 + c*x)])/(-1 + c*x) - (2*b*c^3)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c)/(
x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*a*c^4*x)/(-1 + c^2*x^2) + (4*b*ArcCo
sh[c*x])/x^3 + (24*b*c^2*ArcCosh[c*x])/x + (3*b*c^3*ArcCosh[c*x])/(-1 + c*x
```

```
) + (3*b*c^3*ArcCosh[c*x])/(1 + c*x) - (26*b*c^3*Sqrt[-1 + c^2*x^2]*ArcTan[
Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 30*b*c^3*ArcCosh[c*x]
*Log[1 - E^ArcCosh[c*x]] - 30*b*c^3*ArcCosh[c*x]*Log[1 + E^ArcCosh[c*x]] +
15*a*c^3*Log[1 - c*x] - 15*a*c^3*Log[1 + c*x] - 30*b*c^3*PolyLog[2, -E^ArcC
osh[c*x]] + 30*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/d^2
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{c^4d^2x^8-2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^2*x^4), x)
```

maple [A] time = 0.47, size = 352, normalized size = 1.42

$$-\frac{a}{3d^2x^3}-\frac{2c^2a}{d^2x}-\frac{c^3a}{4d^2\,(cx-1)}-\frac{5c^3a\ln(cx-1)}{4d^2}-\frac{c^3a}{4d^2\,(cx+1)}+\frac{5c^3a\ln(cx+1)}{4d^2}-\frac{5c^4b\,\text{arccosh}\,(cx)\,x}{2d^2\left(c^2x^2-1\right)}-\frac{c^3b\sqrt{cx+1}}{3d^2\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x)
```

```
[Out] -1/3*a/d^2/x^3-2*c^2*a/d^2/x-1/4*c^3*a/d^2/(c*x-1)-5/4*c^3*a/d^2*ln(c*x-1)-
1/4*c^3*a/d^2/(c*x+1)+5/4*c^3*a/d^2*ln(c*x+1)-5/2*c^4*b/d^2/(c^2*x^2-1)*arc
cosh(c*x)*x-1/3*c^3*b/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+5/3*c^2*b
/d^2/x/(c^2*x^2-1)*arccosh(c*x)-1/6*c*b/d^2/x^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)+1/3*b/d^2/x^3/(c^2*x^2-1)*arccosh(c*x)+13/3*c^3*b/d^2*arctan(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*c^3*b/d^2*dilog(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))+5/2*c^3*b/d^2*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/2*c^3*b/
d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{12}\left(\frac{15\,c^3\log(cx+1)}{d^2}-\frac{15\,c^3\log(cx-1)}{d^2}-\frac{2\left(15\,c^4x^4-10\,c^2x^2-2\right)}{c^2d^2x^5-d^2x^3}\right)a+\frac{1}{192}\left(8640\,c^7\int\frac{x^5\log(cx-}{24\left(c^4d^2x^6-2\,c^2d^2x\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(15*c^3*log(c*x + 1)/d^2 - 15*c^3*log(c*x - 1)/d^2 - 2*(15*c^4*x^4 - 1
0*c^2*x^2 - 2)/(c^2*d^2*x^5 - d^2*x^3))*a + 1/192*(8640*c^7*integrate(1/24*
x^5*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x) - 120*c^6*(2*x
/(c^4*d^2*x^2 - c^2*d^2) + log(c*x + 1)/(c^3*d^2) - log(c*x - 1)/(c^3*d^2))
 - 2880*c^6*integrate(1/24*x^4*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 +
d^2*x^2), x) + 45*(c*(2/(c^4*d^2*x - c^3*d^2) - log(c*x + 1)/(c^3*d^2) + lo
```

```
g(c*x - 1)/(c^3*d^2)) + 4*log(c*x - 1)/(c^4*d^2*x^2 - c^2*d^2))*c^5 + 80*c^
4*(2*x/(c^2*d^2*x^2 - d^2) - log(c*x + 1)/(c*d^2) + log(c*x - 1)/(c*d^2)) +
 2880*c^4*integrate(1/24*x^2*log(c*x - 1)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^
2*x^2), x) + 16*c^2*(2*(3*c^2*x^2 - 2)/(c^2*d^2*x^3 - d^2*x) - 3*c*log(c*x
+ 1)/d^2 + 3*c*log(c*x - 1)/d^2) - 4*(15*(c^5*x^5 - c^3*x^3)*log(c*x + 1)^2
 + 30*(c^5*x^5 - c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 20*c^
2*x^2 - 15*(c^5*x^5 - c^3*x^3)*log(c*x + 1) + 15*(c^5*x^5 - c^3*x^3)*log(c*
x - 1) - 4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^2*d^2*x^5 - d^2*x^3)
 + 192*integrate(-1/12*(30*c^5*x^4 - 20*c^3*x^2 - 15*(c^6*x^5 - c^4*x^3)*lo
g(c*x + 1) + 15*(c^6*x^5 - c^4*x^3)*log(c*x - 1) - 4*c)/(c^5*d^2*x^8 - 2*c^
3*d^2*x^6 + c*d^2*x^4 + (c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3)*sqrt(c*x +
1)*sqrt(c*x - 1)), x))*b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a}{c^4x^8-2c^2x^6+x^4}\,dx+\int \frac{b\,\mathrm{acosh}\,(cx)}{c^4x^8-2c^2x^6+x^4}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a/(c**4*x**8 - 2*c**2*x**6 + x**4), x) + Integral(b*acosh(c*x)/(c
**4*x**8 - 2*c**2*x**6 + x**4), x))/d**2
```

3.46 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=249

$$\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4c^5d^3}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}+\frac{3b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}-$$

```
[Out] 1/12*b*x^3/c^2/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-1/12*b*(c*x-1)^(3/2)/c^5/d^3
/(c*x+1)^(3/2)+1/4*x^3*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)^2-3/8*x*(a+b
*arccosh(c*x))/c^4/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3+3/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/c^5/d^3-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3+1/
4*b/c^5/d^3/(c*x+1)^(3/2)/(c*x-1)^(1/2)+3/8*b/c^5/d^3/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5750, 94, 89, 21, 37, 74, 5694, 4182, 2279, 2391}

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^5d^3}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}+\frac{3\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c^2*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^5*d^3*Sqrt[-
1 + c*x]*(1 + c*x)^(3/2)) - (b*(-1 + c*x)^(3/2))/(12*c^5*d^3*(1 + c*x)^(3/2
)) + (3*b)/(8*c^5*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x^3*(a + b*ArcCosh[c
*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) - (3*x*(a + b*ArcCosh[c*x]))/(8*c^4*d^3*(
1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*c^5*d^3
) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c^5*d^3) - (3*b*PolyLog[2, E^ArcCo
sh[c*x]])/(8*c^5*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3} - \frac{3\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8c^4d^3\left(1-c^2x^2\right)}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{3b}{8c^5d^3\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}} + \frac{3b}{8c^5d^3\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}} - \frac{b(-1+cx)^{3/2}}{12c^5d^3(1+cx)^{3/2}}$$

$$= \frac{bx^3}{12c^2d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{4c^5d^3\sqrt{-1+cx}\,(1+cx)^{3/2}} - \frac{b(-1+cx)^{3/2}}{12c^5d^3(1+cx)^{3/2}}$$

Mathematica [A] time = 1.79, size = 287, normalized size = 1.15

$$\frac{30acx}{c^2x^2-1} + \frac{12acx}{\left(c^2x^2-1\right)^2} - 9a\log(1-cx) + 9a\log(cx+1) + 18b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right) - 18b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right) + \frac{b\sqrt{cx-1}\,(cx+2)}{(cx+1)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (30*a*c*x)/(-1 + c^2*
x^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 1
5*b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 15*b*(Sqrt
[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) + (9*b*ArcCosh[c*x]*(ArcCo
sh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 - (9*b*ArcCosh[c*x]*(ArcCosh[c*x] -
 4*Log[1 + E^ArcCosh[c*x]]))/2 - 9*a*Log[1 - c*x] + 9*a*Log[1 + c*x] + 18*b
*PolyLog[2, -E^ArcCosh[c*x]] - 18*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^5*d^3
)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^4\,\text{arcosh}\,(cx) + ax^4}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\,\text{arcosh}\,(cx) + a)x^4}{\left(c^2dx^2 - d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^4/(c^2*d*x^2 - d)^3, x)
```

maple [A] time = 0.84, size = 383, normalized size = 1.54

$$\frac{a}{16c^5d^3(cx-1)^2}+\frac{5a}{16c^5d^3(cx-1)}-\frac{3a\ln(cx-1)}{16c^5d^3}-\frac{a}{16c^5d^3(cx+1)^2}+\frac{5a}{16c^5d^3(cx+1)}+\frac{3a\ln(cx+1)}{16c^5d^3}+\frac{5b\operatorname{arccos}}{8c^2d^3\left(c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c^5*a/d^3/(c*x-1)^2+5/16/c^5*a/d^3/(c*x-1)-3/16/c^5*a/d^3*ln(c*x-1)-1/
16/c^5*a/d^3/(c*x+1)^2+5/16/c^5*a/d^3/(c*x+1)+3/16/c^5*a/d^3*ln(c*x+1)+5/8/
c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3+5/8/c^3*b/d^3/(c^4*x^4-2*c
^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-3/8/c^4*b/d^3/(c^4*x^4-2*c^2*x^2+
1)*arccosh(c*x)*x-13/24/c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-
1)^(1/2)+3/8/c^5*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3
/8*b*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^5/d^3-3/8/c^5*b/d^3*arcc
osh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/8*b*polylog(2,c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))/c^5/d^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3
*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 80*c^4*(2*(5*c^2*x^3 - 3*x)/(c^12*d^3
*x^4 - 2*c^10*d^3*x^2 + c^8*d^3) + 3*log(c*x + 1)/(c^9*d^3) - 3*log(c*x - 1
)/(c^9*d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^10*d^3*x^6 - 3*c
^8*d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6
)/(c^12*d^3*x^3 - c^11*d^3*x^2 - c^10*d^3*x + c^9*d^3) - 5*log(c*x + 1)/(c^
9*d^3) + 5*log(c*x - 1)/(c^9*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^12*
d^3*x^4 - 2*c^10*d^3*x^2 + c^8*d^3))*c^3 - 48*c^2*(2*(c^2*x^3 + x)/(c^10*d^
3*x^4 - 2*c^8*d^3*x^2 + c^6*d^3) - log(c*x + 1)/(c^7*d^3) + log(c*x - 1)/(c
^7*d^3)) + 12288*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*
d^3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 3*log(c*x + 1)/(c^7*d^3)
 + 3*log(c*x - 1)/(c^7*d^3)) - 16*log(c*x - 1)/(c^10*d^3*x^4 - 2*c^8*d^3*x^
2 + c^6*d^3))*c - 32*(3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x
^4 - 2*c^2*x^2 + 1)*log(c*x + 1)*log(c*x - 1) - 4*(10*c^3*x^3 - 6*c*x + 3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x
 - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^9*d^3*x^4 - 2*c^7*d^3*x^2
 + c^5*d^3) + 2048*integrate(1/16*(10*c^3*x^3 - 6*c*x + 3*(c^4*x^4 - 2*c^2*
x^2 + 1)*log(c*x + 1) - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^11*d^3
*x^7 - 3*c^9*d^3*x^5 + 3*c^7*d^3*x^3 - c^5*d^3*x + (c^10*d^3*x^6 - 3*c^8*d^
3*x^4 + 3*c^6*d^3*x^2 - c^4*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*in
tegrate(1/32*log(c*x - 1)/(c^10*d^3*x^6 - 3*c^8*d^3*x^4 + 3*c^6*d^3*x^2 - c
^4*d^3), x))*b + 1/16*a*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
 c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^4}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^4\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**4/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**4*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

3.47 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=136

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{b\sqrt{cx-1}}{4c^4d^3\sqrt{cx+1}}+\frac{b}{4c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

```
[Out] 1/12*b*x^3/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)-1/4*b*arccosh(c*x)/c^4/d^3+1/4
*x^4*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+1/4*b/c^4/d^3/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+1/4*b*(c*x-1)^(1/2)/c^4/d^3/(c*x+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5722, 98, 21, 89, 12, 78, 52}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{b\sqrt{cx-1}}{4c^4d^3\sqrt{cx+1}}+\frac{b}{4c^4d^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\cosh^{-1}(cx)}{4c^4d^3}+\frac{bx^3}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x^3)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(4*c^4*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (b*Sqrt[-1 + c*x])/(4*c^4*d^3*Sqrt[1 + c*x]) - (b*A
rcCosh[c*x])/(4*c^4*d^3) + (x^4*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
```

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 5722

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2*d + e,
0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx=\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{(bc)\int\frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{b\int\frac{x^2(-3-3cx)}{(-1+cx)^{3/2}(1+cx)^{5/2}}\,dx}{12cd^3}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{b\int\frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{4cd^3}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}}+\frac{x^4(}{}$$

$$=\frac{bx^3}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{b}{4c^4d^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b\sqrt{-1+cx}}{4c^4d^3\sqrt{1+cx}}-\frac{b\,\mathrm{cc}}{}$$

Mathematica [A] time = 0.25, size = 83, normalized size = 0.61

$$\frac{a\left(6c^2x^2-3\right)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(4c^2x^2-3\right)+3b\left(2c^2x^2-1\right)\cosh^{-1}(cx)}{12c^4d^3\left(c^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + 4*c^2*x^2) + a*(-3 + 6*c^2*x^2) +
 3*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x])/(12*c^4*d^3*(-1 + c^2*x^2)^2)
```

fricas [A] time = 0.72, size = 101, normalized size = 0.74

$$\frac{3\,ac^4x^4+3\left(2\,bc^2x^2-b\right)\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(4\,bc^3x^3-3\,bcx\right)\sqrt{c^2x^2-1}}{12\left(c^8d^3x^4-2\,c^6d^3x^2+c^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*c^4*x^4 + 3*(2*b*c^2*x^2 - b)*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b
*c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d
^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 136, normalized size = 1.00

$$\frac{-\frac{a\left(-\frac{1}{16(cx-1)^2}-\frac{3}{16(cx-1)}-\frac{1}{16(cx+1)^2}+\frac{3}{16(cx+1)}\right)}{d^3}-\frac{b\left(-\frac{\operatorname{arccosh}(cx)}{16(cx-1)^2}-\frac{3\operatorname{arccosh}(cx)}{16(cx-1)}-\frac{\operatorname{arccosh}(cx)}{16(cx+1)^2}+\frac{3\operatorname{arccosh}(cx)}{16(cx+1)}-\frac{cx\left(4c^2x^2-3\right)}{12(cx+1)^{\frac{3}{2}}(cx-1)^{\frac{3}{2}}}\right)}{d^3}}{c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^4*(-a/d^3*(-1/16/(c*x-1)^2-3/16/(c*x-1)-1/16/(c*x+1)^2+3/16/(c*x+1))-b/
d^3*(-1/16*arccosh(c*x)/(c*x-1)^2-3/16*arccosh(c*x)/(c*x-1)-1/16*arccosh(c*
x)/(c*x+1)^2+3/16*arccosh(c*x)/(c*x+1)-1/12*c*x*(4*c^2*x^2-3)/(c*x+1)^(3/2)
/(c*x-1)^(3/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}\,b\left(\frac{4\,c^2x^2+4\left(2\,c^2x^2-1\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)-3}{c^8d^3x^4-2\,c^6d^3x^2+c^4d^3}+16\int\frac{}{4\left(c^{10}d^3x^7-3\,c^8d^3x^5+3\,c^6d^3x^3-c^4d^3x+\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b*((4*c^2*x^2 + 4*(2*c^2*x^2 - 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) - 3)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 16*integrate(1/4*(2*c^2*x
^2 - 1)/(c^10*d^3*x^7 - 3*c^8*d^3*x^5 + 3*c^6*d^3*x^3 - c^4*d^3*x + (c^9*d^
3*x^6 - 3*c^7*d^3*x^4 + 3*c^5*d^3*x^2 - c^3*d^3)*e^(1/2*log(c*x + 1) + 1/2*
log(c*x - 1))), x)) + 1/4*(2*c^2*x^2 - 1)*a/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 +
c^4*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^3}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx + \int \frac{bx^3\,\mathrm{acosh}\left(cx\right)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**3/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**3*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

3.48 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=186

$$-\frac{\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4c^3d^3}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b\mathrm{Li}_2\left(e\right.}{8}$$

```
[Out] 1/12*b/c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*x*(a+b*arccosh(c*x))/c^2/d^3
/(-c^2*x^2+1)^2-1/8*x*(a+b*arccosh(c*x))/c^2/d^3/(-c^2*x^2+1)-1/4*(a+b*arcc
osh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3-1/8*b*polylog(2,
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3+1/8*b*polylog(2,c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))/c^3/d^3+1/8*b/c^3/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5750, 74, 5689, 5694, 4182, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}+\frac{b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8c^3d^3}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{\tanh^{-1}\left(e^{\cos\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c^3*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + b/(8*c^3*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*c^2*d^3*(1 - c^2*x^2)^2) -
(x*(a + b*ArcCosh[c*x]))/(8*c^2*d^3*(1 - c^2*x^2)) - ((a + b*ArcCosh[c*x])*
ArcTanh[E^ArcCosh[c*x]])/(4*c^3*d^3) - (b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c
^3*d^3) + (b*PolyLog[2, E^ArcCosh[c*x]])/(8*c^3*d^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{b\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx}{4c^2d}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2} - \frac{x\left(a+b\cosh^{-1}(cx)\right)}{8c^2d^3\left(1-c^2x^2\right)} - $$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}$$

$$= \frac{b}{12c^3d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{b}{8c^3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4c^2d^3\left(1-c^2x^2\right)^2}$$

Mathematica [A] time = 1.67, size = 287, normalized size = 1.54

$$\frac{6acx}{c^2x^2-1} + \frac{12acx}{\left(c^2x^2-1\right)^2} + 3a\log(1-cx) - 3a\log(cx+1) - 6b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right) + 6b\text{Li}_2\left(e^{\cosh^{-1}(cx)}\right) + \frac{b\sqrt{cx-1}\,(cx+2)}{(cx+1)^{3/2}} -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (-((b*(-2 + c*x)*Sqrt[1 + c*x])/(-1 + c*x)^(3/2)) + (b*Sqrt[-1 + c*x]*(2 +
c*x))/(1 + c*x)^(3/2) + (12*a*c*x)/(-1 + c^2*x^2)^2 + (6*a*c*x)/(-1 + c^2*x
^2) + (3*b*ArcCosh[c*x])/(-1 + c*x)^2 - (3*b*ArcCosh[c*x])/(1 + c*x)^2 - 3*
b*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)) - 3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)) - (3*b*ArcCosh[c*x]*(ArcCosh[
c*x] - 4*Log[1 - E^ArcCosh[c*x]]))/2 + (3*b*ArcCosh[c*x]*(ArcCosh[c*x] - 4*
Log[1 + E^ArcCosh[c*x]]))/2 + 3*a*Log[1 - c*x] - 3*a*Log[1 + c*x] - 6*b*Pol
yLog[2, -E^ArcCosh[c*x]] + 6*b*PolyLog[2, E^ArcCosh[c*x]])/(48*c^3*d^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{bx^2 \operatorname{arcosh}(cx) + ax^2}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2
*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b \operatorname{arcosh}(cx) + a)x^2}{\left(c^2dx^2 - d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x^2/(c^2*d*x^2 - d)^3, x)
```

maple [A] time = 0.58, size = 380, normalized size = 2.04

$$\frac{a}{16c^3d^3\,(cx-1)^2}+\frac{a}{16c^3d^3\,(cx-1)}+\frac{a\ln(cx-1)}{16c^3d^3}-\frac{a}{16c^3d^3\,(cx+1)^2}+\frac{a}{16c^3d^3\,(cx+1)}-\frac{a\ln(cx+1)}{16c^3d^3}+\frac{b\operatorname{arccosh}(c}{8d^3\left(c^4x^4-2c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c^3*a/d^3/(c*x-1)^2+1/16/c^3*a/d^3/(c*x-1)+1/16/c^3*a/d^3*ln(c*x-1)-1/
16/c^3*a/d^3/(c*x+1)^2+1/16/c^3*a/d^3/(c*x+1)-1/16/c^3*a/d^3*ln(c*x+1)+1/8*
b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3+1/8/c*b/d^3/(c^4*x^4-2*c^2*x^2
+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/8/c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arc
cosh(c*x)*x-1/24/c^3*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)-1/8/c^3*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8*b*po
lylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/c^3/d^3+1/8/c^3*b/d^3*arccosh(c*x
)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))/c^3/d^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/2048*(6144*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*
x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) - 16*c^4*(2*(5*c^2*x^3 - 3*x)/(c^10*d^3*
x^4 - 2*c^8*d^3*x^2 + c^6*d^3) + 3*log(c*x + 1)/(c^7*d^3) - 3*log(c*x - 1)/
(c^7*d^3)) - 2048*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*
d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x) + 6*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c
^10*d^3*x^3 - c^9*d^3*x^2 - c^8*d^3*x + c^7*d^3) - 5*log(c*x + 1)/(c^7*d^3)
 + 5*log(c*x - 1)/(c^7*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^10*d^3*x^
4 - 2*c^8*d^3*x^2 + c^6*d^3))*c^3 - 16*c^2*(2*(c^2*x^3 + x)/(c^8*d^3*x^4 -
2*c^6*d^3*x^2 + c^4*d^3) - log(c*x + 1)/(c^5*d^3) + log(c*x - 1)/(c^5*d^3))
 + 4096*c^2*integrate(1/32*x^2*log(c*x - 1)/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 +
3*c^4*d^3*x^2 - c^2*d^3), x) + 3*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^8*d^3*x^3
 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 3*log(c*x + 1)/(c^5*d^3) + 3*log(c*
x - 1)/(c^5*d^3)) - 16*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)
)*c - 32*((c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 2*(c^4*x^4 - 2*c^2*x^2
 + 1)*log(c*x + 1)*log(c*x - 1) + 4*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x
^2 + 1)*log(c*x + 1) + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)))/(c^7*d^3*x^4 - 2*c^5*d^3*x^2 + c^3*d^3) + 2048*
integrate(-1/16*(2*c^3*x^3 + 2*c*x - (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)
 + (c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^9*d^3*x^7 - 3*c^7*d^3*x^5 + 3
*c^5*d^3*x^3 - c^3*d^3*x + (c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c
^2*d^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - 2048*integrate(1/32*log(c*x - 1)
/(c^8*d^3*x^6 - 3*c^6*d^3*x^4 + 3*c^4*d^3*x^2 - c^2*d^3), x))*b + 1/16*a*(2
*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*
d^3) + log(c*x - 1)/(c^3*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax^2}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx^2\operatorname{acosh}(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x**2/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integra
l(b*x**2*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

3.49 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=91

$$\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{bx}{6cd^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

```
[Out] 1/12*b*x/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*(a+b*arccosh(c*x))/c^2/d^3/(
-c^2*x^2+1)^2-1/6*b*x/c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {5716, 40, 39}

$$\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{bx}{6cd^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx}{12cd^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (b*x)/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (b*x)/(6*c*d^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*c^2*d^3*(1 - c^2*x^2)^2)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx &= \frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}-\frac{b\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4cd^3}\\
&= \frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}+\frac{b\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{6cd^3}\\
&= \frac{bx}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{bx}{6cd^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{a+b\cosh^{-1}(cx)}{4c^2d^3\left(1-c^2x^2\right)^2}
\end{aligned}$$

Mathematica [A] time = 0.22, size = 64, normalized size = 0.70

$$\frac{3a+bcx\sqrt{cx-1}\sqrt{cx+1}\left(3-2c^2x^2\right)+3b\cosh^{-1}(cx)}{12c^2d^3\left(c^2x^2-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] (3*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - 2*c^2*x^2) + 3*b*ArcCosh[c*x
])/(12*c^2*d^3*(-1 + c^2*x^2)^2)
```

fricas [A] time = 0.62, size = 98, normalized size = 1.08

$$-\frac{3\,ac^4x^4-6\,ac^2x^2-3\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(2\,bc^3x^3-3\,bcx\right)\sqrt{c^2x^2-1}}{12\left(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*c^4*x^4 - 6*a*c^2*x^2 - 3*b*log(c*x + sqrt(c^2*x^2 - 1)) + (2*b*
c^3*x^3 - 3*b*c*x)*sqrt(c^2*x^2 - 1))/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^
3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\,\mathrm{arcosh}\,(cx)+a)x}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)*x/(c^2*d*x^2 - d)^3, x)
```

maple [A] time = 0.01, size = 86, normalized size = 0.95

$$\frac{\frac{a}{4d^3\left(c^2x^2-1\right)^2}-\frac{b\left(-\frac{\mathrm{arccosh}(cx)}{4\left(c^2x^2-1\right)^2}+\frac{cx\left(2c^2x^2-3\right)}{12\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)}\right)}{d^3}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/c^2*(1/4*a/d^3/(c^2*x^2-1)^2-b/d^3*(-1/4/(c^2*x^2-1)^2*arccosh(c*x)+1/12/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*c*x*(2*c^2*x^2-3)/(c^2*x^2-1)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{16}b\left(\frac{4\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+1}{c^6d^3x^4-2\,c^4d^3x^2+c^2d^3}+16\int\frac{1}{4\left(c^8d^3x^7-3\,c^6d^3x^5+3\,c^4d^3x^3-c^2d^3x+\left(c^7d^3x^6-3\,c^5d^3x^4\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/16*b*((4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 1)/(c^6*d^3*x^4 - 2*c^4
*d^3*x^2 + c^2*d^3) + 16*integrate(1/4/(c^8*d^3*x^7 - 3*c^6*d^3*x^5 + 3*c^4
*d^3*x^3 - c^2*d^3*x + (c^7*d^3*x^6 - 3*c^5*d^3*x^4 + 3*c^3*d^3*x^2 - c*d^3
)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)) + 1/4*a/(c^6*d^3*x^4 - 2*c^
4*d^3*x^2 + c^2*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{ax}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{bx\,\mathrm{acosh}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a*x/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b
*x*acosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

3.50 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=180

$$\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4cd^3}+\frac{3b\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\cdots$$

```
[Out] 1/12*b/c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*x*(a+b*arccosh(c*x))/d^3/(-c^2
*x^2+1)^2+3/8*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+3/4*(a+b*arccosh(c*x))*
arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3+3/8*b*polylog(2,-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))/c/d^3-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)/c/d^3-3/8*b/c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3b\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8cd^3}-\frac{3b\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8cd^3}+\frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{3\tanh\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

```
[Out] b/(12*c*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (3*b)/(8*c*d^3*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (3*x
*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2)) + (3*(a + b*ArcCosh[c*x])*ArcT
anh[E^ArcCosh[c*x]])/(4*c*d^3) + (3*b*PolyLog[2, -E^ArcCosh[c*x]])/(8*c*d^3
) - (3*b*PolyLog[2, E^ArcCosh[c*x]])/(8*c*d^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^2}\,dx}{4d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)} + \frac{(3bc)\int}{}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a\right.}{8d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a\right.}{8d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a\right.}{8d}$$

$$= \frac{b}{12cd^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{3b}{8cd^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2} + \frac{3x\left(a\right.}{8d}$$

Mathematica [A] time = 1.17, size = 316, normalized size = 1.76

$$\frac{-\frac{6ax}{c^2x^2-1} + \frac{4ax}{\left(c^2x^2-1\right)^2} - \frac{3a\log(1-cx)}{c} + \frac{3a\log(cx+1)}{c} - \frac{3b\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-4\text{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)\right)}{2c} + \frac{3b\left(\cosh^{-1}(cx\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((4*a*x)/(-1 + c^2*x^2)^2 - (6*a*x)/(-1 + c^2*x^2) + (b*(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(3*c*(1 + c*x)^2) + (b*((2 - c*x)*S
qrt[-1 + c*x]*Sqrt[1 + c*x] + 3*ArcCosh[c*x]))/(3*c*(-1 + c*x)^2) + (3*b*(-
(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + ArcCosh[c*x]/(1 - c*x)))/c + (3*b*(Sqrt[(-
1 + c*x)/(1 + c*x)] - ArcCosh[c*x]/(1 + c*x)))/c - (3*a*Log[1 - c*x])/c + (
```

```
3*a*Log[1 + c*x])/c - (3*b*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCos
h[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/(2*c) + (3*b*(ArcCosh[c*x]*(ArcC
osh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(2*c
))/(16*d^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b \operatorname{arcosh}(cx) + a}{c^6 d^3 x^6 - 3\, c^4 d^3 x^4 + 3\, c^2 d^3 x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2
 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{\left(c^2 d x^2 - d\right)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/(c^2*d*x^2 - d)^3, x)
```

maple [A] time = 0.16, size = 378, normalized size = 2.10

$$\frac{a}{16c\,d^3\,(cx-1)^2} - \frac{3a}{16c\,d^3\,(cx-1)} - \frac{3a\ln(cx-1)}{16c\,d^3} - \frac{a}{16c\,d^3\,(cx+1)^2} - \frac{3a}{16c\,d^3\,(cx+1)} + \frac{3a\ln(cx+1)}{16c\,d^3} - \frac{3c^2 b \operatorname{arccos}}{8d^3\left(c^4x^4 - \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)
```

```
[Out] 1/16/c*a/d^3/(c*x-1)^2-3/16/c*a/d^3/(c*x-1)-3/16/c*a/d^3*ln(c*x-1)-1/16/c*a
/d^3/(c*x+1)^2-3/16/c*a/d^3/(c*x+1)+3/16/c*a/d^3*ln(c*x+1)-3/8*c^2*b/d^3/(c
^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^3-3/8*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^2+5/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x+
11/24/c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+3/8/c*b/d^3
*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/8*b*polylog(2,-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3-3/8/c*b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^
(1/2)*(c*x+1)^(1/2))-3/8*b*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/c/d^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(18432*c^5*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 48*c^4*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4 -
 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5*
d^3)) - 6144*c^4*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x
^4 + 3*c^2*d^3*x^2 - d^3), x) + 18*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3*x
^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*log(
c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^6*
d^3*x^2 + c^4*d^3))*c^3 + 80*c^2*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d^3*
```

```
x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 12288*c
^2*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3
*x^2 - d^3), x) + 9*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^3*x^
2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c^3*d
^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c - 32*(3*(
c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1)^2 + 6*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c
*x + 1)*log(c*x - 1) + 4*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*
log(c*x + 1) + 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)))/(c^5*d^3*x^4 - 2*c^3*d^3*x^2 + c*d^3) + 2048*integrat
e(-1/16*(6*c^3*x^3 - 10*c*x - 3*(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x + 1) + 3*
(c^4*x^4 - 2*c^2*x^2 + 1)*log(c*x - 1))/(c^7*d^3*x^7 - 3*c^5*d^3*x^5 + 3*c^
3*d^3*x^3 - c*d^3*x + (c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3)*s
qrt(c*x + 1)*sqrt(c*x - 1)), x) - 6144*integrate(1/32*log(c*x - 1)/(c^6*d^3
*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x))*b - 1/16*a*(2*(3*c^2*x^3 -
 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(
c*x - 1)/(c*d^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx+\int \frac{b\,\mathrm{acosh}\,(cx)}{c^6x^6-3c^4x^4+3c^2x^2-1}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x) + Integral(b*a
cosh(c*x)/(c**6*x**6 - 3*c**4*x**4 + 3*c**2*x**2 - 1), x))/d**3
```

3.51 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=171

$$\frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)}+\frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^3}+\frac{b\mathrm{Li}_2\left(-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{b\mathrm{Li}_2\left(e^{2\cdots}\right)}{2\cdots}$$

```
[Out] 1/12*b*c*x/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/4*(a+b*arccosh(c*x))/d^3/(-c^2
*x^2+1)^2+1/2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+2*(a+b*arccosh(c*x))*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+1/2*b*polylog(2,-(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)/d^3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2)/d^3-2/3*b*c*x/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5754, 5721, 5461, 4182, 2279, 2391, 39, 40}

$$\frac{b\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{b\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)}+\frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2}+\frac{2\tanh^{-1}\left(e^{2\cosh\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c*x)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c*x)/(3*d^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (a + b*ArcCosh[c*x])/(4*d^3*(1 - c^2*x^2)^2) + (a
 + b*ArcCosh[c*x])/(2*d^3*(1 - c^2*x^2)) + (2*(a + b*ArcCosh[c*x])*ArcTanh[
E^(2*ArcCosh[c*x])])/d^3 + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (b
*PolyLog[2, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx = \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} - \frac{(bc)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3} + \frac{\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^2}\,dx}{d}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^3\left(1-c^2x^2\right)} + \frac{(bc)\int \frac{1}{(-1+cx)^{3/2}(1+}}{6d^3}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-}}{2d^3\left(1-c^2\right.}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-}}{2d^3\left(1-c^2\right.}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-}}{2d^3\left(1-c^2\right.}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-}}{2d^3\left(1-c^2\right.}$$

$$= \frac{bcx}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}} - \frac{2bcx}{3d^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{a+b\cosh^{-1}(cx)}{4d^3\left(1-c^2x^2\right)^2} + \frac{a+b\cosh^{-}}{2d^3\left(1-c^2\right.}$$

Mathematica [A] time = 1.54, size = 210, normalized size = 1.23

$$-\frac{\frac{6a}{c^2x^2-1}-\frac{3a}{\left(c^2x^2-1\right)^2}+6a\log\left(1-c^2x^2\right)-12a\log(x)+b\left(\frac{6\cosh^{-1}(cx)}{c^2x^2-1}-\frac{3\cosh^{-1}(cx)}{\left(c^2x^2-1\right)^2}+6\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)-6\text{Li}_2\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^3),x]
```

```
[Out] -1/12*((-3*a)/(-1 + c^2*x^2)^2 + (6*a)/(-1 + c^2*x^2) - 12*a*Log[x] + 6*a*L
og[1 - c^2*x^2] + b*((8*c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(-1 + c*x) - (c*x*(
(-1 + c*x)/(1 + c*x))^(3/2))/(-1 + c*x)^3 - (3*ArcCosh[c*x])/(-1 + c^2*x^2)
^2 + (6*ArcCosh[c*x])/(-1 + c^2*x^2) + 12*ArcCosh[c*x]*Log[1 - E^(-2*ArcCos
h[c*x])] - 12*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 6*PolyLog[2, -E^(
-2*ArcCosh[c*x])] - 6*PolyLog[2, E^(-2*ArcCosh[c*x])]))/d^3
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3
 - d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\,\text{arcosh}\,(cx)+a}{\left(c^2dx^2-d\right)^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x), x)
```

maple [B] time = 0.52, size = 508, normalized size = 2.97

$$\frac{a\ln(cx)}{d^3}+\frac{a}{16d^3(cx-1)^2}-\frac{5a}{16d^3(cx-1)}-\frac{a\ln(cx-1)}{2d^3}+\frac{a}{16d^3(cx+1)^2}+\frac{5a}{16d^3(cx+1)}-\frac{a\ln(cx+1)}{2d^3}-\frac{2b\sqrt{cx+1}}{3d^3\left(c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x)
```

```
[Out] a/d^3*ln(c*x)+1/16*a/d^3/(c*x-1)^2-5/16*a/d^3/(c*x-1)-1/2*a/d^3*ln(c*x-1)+1
/16*a/d^3/(c*x+1)^2+5/16*a/d^3/(c*x+1)-1/2*a/d^3*ln(c*x+1)-2/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^3+2/3*b/d^3/(c^4*x^4-2*c^
2*x^2+1)*c^4*x^4-1/2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^2*x^2+3/4*b
/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c*x-4/3*b/d^3/(c^4*x
^4-2*c^2*x^2+1)*c^2*x^2+3/4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2/3*b/
d^3/(c^4*x^4-2*c^2*x^2+1)+b/d^3*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2)+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-b/d^3
*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^3*polylog(2,-c*x-(c
*x-1)^(1/2)*(c*x+1)^(1/2))-b/d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))-b/d^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a\left(\frac{2c^2x^2-3}{c^4d^3x^4-2c^2d^3x^2+d^3}+\frac{2\log(cx+1)}{d^3}+\frac{2\log(cx-1)}{d^3}-\frac{4\log(x)}{d^3}\right)-b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{c^6d^3x^7-3c^4d^3x^5+3c^2d^3x^3-\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((2*c^2*x^2 - 3)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) + 2*log(c*x + 1
)/d^3 + 2*log(c*x - 1)/d^3 - 4*log(x)/d^3) - b*integrate(log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\,(c\,x)}{x\left(d-c^2\,d\,x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx+\int \frac{b\,\mathrm{acosh}\,(cx)}{c^6x^7-3c^4x^5+3c^2x^3-x}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x) + Integral(b*a
cosh(c*x)/(c**6*x**7 - 3*c**4*x**5 + 3*c**2*x**3 - x), x))/d**3
```

3.52 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=230

$$\frac{15c^2x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{15c\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(c\right.}{4d^3}$$

```
[Out] 1/12*b*c/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+(-a-b*arccosh(c*x))/d^3/x/(-c^2*x^
2+1)^2+5/4*c^2*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+15/8*c^2*x*(a+b*arcc
osh(c*x))/d^3/(-c^2*x^2+1)+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+15/4
*c*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+15/8*b*c
*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3-15/8*b*c*polylog(2,c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))/d^3-7/8*b*c/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5746, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{15bc\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{15bc\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{15c^2x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{5c^2x\left(a+b\cosh^{-1}(cx\right.}{4d^3\left(1-c^2x^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (7*b*c)/(8*d^3*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (a + b*ArcCosh[c*x])/(d^3*x*(1 - c^2*x^2)^2) + (5*c^2
*x*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) + (15*c^2*x*(a + b*ArcCosh
[c*x]))/(8*d^3*(1 - c^2*x^2)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d^3 + (15*c*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (15*b*c
*PolyLog[2, -E^ArcCosh[c*x]])/(8*d^3) - (15*b*c*PolyLog[2, E^ArcCosh[c*x]])
/(8*d^3)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\left(5c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^3}$$

$$= -\frac{bc}{3d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{b\int \frac{}{x(-1}}{}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}+\frac{15c^2x\left(\right.}{8c}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+\right.}{4d^3\left(\right.}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+\right.}{4d^3\left(\right.}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+\right.}{4d^3\left(\right.}$$

$$= \frac{bc}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{7bc}{8d^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{a+b\cosh^{-1}(cx)}{d^3x\left(1-c^2x^2\right)^2}+\frac{5c^2x\left(a+\right.}{4d^3\left(\right.}$$

Mathematica [A] time = 1.78, size = 362, normalized size = 1.57

$$-\frac{84ac^2x}{c^2x^2-1}+\frac{24ac^2x}{\left(c^2x^2-1\right)^2}-90ac\log(1-cx)+90ac\log(cx+1)-\frac{96a}{x}+\frac{96bc\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-45bc\left(\cosh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-96*a)/x + (24*a*c^2*x)/(-1 + c^2*x^2)^2 - (84*a*c^2*x)/(-1 + c^2*x^2) -
(2*b*c*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCosh[c*x]))/(-1 + c*
x)^2 + (2*b*c*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3*ArcCosh[c*x]))/(1
 + c*x)^2 - (96*b*ArcCosh[c*x])/x + 42*b*c*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)])
 + ArcCosh[c*x]/(1 - c*x)) + 42*b*c*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c
*x]/(1 + c*x)) + (96*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - 90*a*c*Log[1 - c*x] + 90*a*c*Log[1 + c*x] - 4
5*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^ArcCosh[c*x]]) - 4*PolyLog[
2, -E^ArcCosh[c*x]]) + 45*b*c*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 - E^Arc
Cosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]]))/(96*d^3)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\,\text{arcosh}\,(cx)+a}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\operatorname{arcosh}(cx)+a}{\left(c^2dx^2-d\right)^3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^2), x)
```

maple [A] time = 0.45, size = 392, normalized size = 1.70

$$-\frac{a}{d^3x}+\frac{ca}{16d^3(cx-1)^2}-\frac{7ca}{16d^3(cx-1)}-\frac{15ca\ln(cx-1)}{16d^3}-\frac{ca}{16d^3(cx+1)^2}-\frac{7ca}{16d^3(cx+1)}+\frac{15ca\ln(cx+1)}{16d^3}-\frac{15b\operatorname{arcc}}{8d^3\left(c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x)
```

```
[Out] -a/d^3/x+1/16*c*a/d^3/(c*x-1)^2-7/16*c*a/d^3/(c*x-1)-15/16*c*a/d^3*ln(c*x-1
)-1/16*c*a/d^3/(c*x+1)^2-7/16*c*a/d^3/(c*x+1)+15/16*c*a/d^3*ln(c*x+1)-15/8*
b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*c^4*x^3-7/8*b/d^3/(c^4*x^4-2*c^2*x
^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3*x^2+25/8*b/d^3/(c^4*x^4-2*c^2*x^2+1)*
arccosh(c*x)*c^2*x+23/24*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1
)^(1/2)-b/d^3/x/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+2*c*b/d^3*arctan(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))+15/8*c*b/d^3*dilog(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))+15/8*c*b/d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+15/8*c*b/d^3*arcco
sh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/2048*(92160*c^7*integrate(1/32*x^5*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*
x^4 + 3*c^2*d^3*x^2 - d^3), x) - 240*c^6*(2*(5*c^2*x^3 - 3*x)/(c^8*d^3*x^4
- 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x - 1)/(c^5
*d^3)) - 30720*c^6*integrate(1/32*x^4*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3
*x^4 + 3*c^2*d^3*x^2 - d^3), x) + 90*(c*(2*(5*c^2*x^2 + 3*c*x - 6)/(c^8*d^3
*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^5*d^3) + 5*lo
g(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d^3*x^4 - 2*c^
6*d^3*x^2 + c^4*d^3))*c^5 + 400*c^4*(2*(c^2*x^3 + x)/(c^6*d^3*x^4 - 2*c^4*d
^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^3*d^3)) + 6144
0*c^4*integrate(1/32*x^2*log(c*x - 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*
d^3*x^2 - d^3), x) + 45*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/(c^6*d^3*x^3 - c^5*d^
3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3) + 3*log(c*x - 1)/(c
^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3))*c^3 + 1
28*c^2*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x^2 + d^3) - 3*log(c*x
 + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 30720*c^2*integrate(1/32*log(c*x
- 1)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x) - 32*(15*(c^5*
x^5 - 2*c^3*x^3 + c*x)*log(c*x + 1)^2 + 30*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(
c*x + 1)*log(c*x - 1) + 4*(30*c^4*x^4 - 50*c^2*x^2 - 15*(c^5*x^5 - 2*c^3*x^
3 + c*x)*log(c*x + 1) + 15*(c^5*x^5 - 2*c^3*x^3 + c*x)*log(c*x - 1) + 16)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 + d^3*x
```

```
) + 2048*integrate(-1/16*(30*c^5*x^4 - 50*c^3*x^2 - 15*(c^6*x^5 - 2*c^4*x^3
 + c^2*x)*log(c*x + 1) + 15*(c^6*x^5 - 2*c^4*x^3 + c^2*x)*log(c*x - 1) + 16
*c)/(c^7*d^3*x^8 - 3*c^5*d^3*x^6 + 3*c^3*d^3*x^4 - c*d^3*x^2 + (c^6*d^3*x^7
 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))
*b - 1/16*a*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2*c^2*d^3*x^3 +
 d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,{\left(d-c^2\,d\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^8-3c^4x^6+3c^2x^4-x^2}\,dx+\int \frac{b\,\mathrm{acosh}\,(cx)}{c^6x^8-3c^4x^6+3c^2x^4-x^2}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x) + Integral(
b*acosh(c*x)/(c**6*x**8 - 3*c**4*x**6 + 3*c**2*x**4 - x**2), x))/d**3
```

3.53 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=250

$$\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^3\left(1-c^2x^2\right)}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{6c^2\tanh^{-1}\left(e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^3}-$$

```
[Out] 1/2*b*c/d^3/x/(c*x-1)^(3/2)/(c*x+1)^(3/2)-5/12*b*c^3*x/d^3/(c*x-1)^(3/2)/(c
*x+1)^(3/2)+3/4*c^2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)^2+1/2*(-a-b*arccosh
(c*x))/d^3/x^2/(-c^2*x^2+1)^2+3/2*c^2*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+6
*c^2*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+3/
2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-3/2*b*c^2*polyl
og(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-2/3*b*c^3*x/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {5746, 103, 12, 40, 39, 5754, 5721, 5461, 4182, 2279, 2391}

$$\frac{3bc^2\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{2d^3}-\frac{3bc^2\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{2d^3}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^3\left(1-c^2x^2\right)}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

```
[Out] (b*c)/(2*d^3*x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - (5*b*c^3*x)/(12*d^3*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)) - (2*b*c^3*x)/(3*d^3*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*c^2*(a + b*ArcCosh[c*x]))/(4*d^3*(1 - c^2*x^2)^2) - (a + b*ArcCo
sh[c*x])/(2*d^3*x^2*(1 - c^2*x^2)^2) + (3*c^2*(a + b*ArcCosh[c*x]))/(2*d^3*
(1 - c^2*x^2)) + (6*c^2*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/d
^3 + (3*b*c^2*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3) - (3*b*c^2*PolyLog[2
, E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
```

IntegerQ[p]

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\left(3c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{2d^3}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{2d^3x^2\left(1-c^2x^2\right)^2}+\frac{(bc)\int \frac{}{(-1+}}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc^3x}{4d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{4d^3\left(1-c^2x^2\right)^2}-\frac{a}{2d}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3c^2}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3c}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3c}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3c}{}$$

$$= \frac{bc}{2d^3x(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{5bc^3x}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{2bc^3x}{3d^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3c}{}$$

Mathematica [A] time = 3.03, size = 273, normalized size = 1.09

$$-\frac{\frac{12ac^2}{c^2x^2-1}-\frac{3ac^2}{\left(c^2x^2-1\right)^2}+18ac^2\log\left(1-c^2x^2\right)-36ac^2\log(x)+\frac{6a}{x^2}+bc^2\left(\frac{12\cosh^{-1}(cx)}{c^2x^2-1}-\frac{3\cosh^{-1}(cx)}{\left(c^2x^2-1\right)^2}+\frac{6\cosh^{-1}(cx)}{c^2x^2}+18\mathrm{Li}_2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^3),x]
```

```
[Out] -1/12*((6*a)/x^2 - (3*a*c^2)/(-1 + c^2*x^2)^2 + (12*a*c^2)/(-1 + c^2*x^2) -
 36*a*c^2*Log[x] + 18*a*c^2*Log[1 - c^2*x^2] + b*c^2*(-((c*x)/(((-1 + c*x)/
(1 + c*x))^(3/2)*(1 + c*x)^3)) + (14*c*x)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)) - (6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*ArcCosh[c*x])/(
c^2*x^2) - (3*ArcCosh[c*x])/(-1 + c^2*x^2)^2 + (12*ArcCosh[c*x])/(-1 + c^2*
x^2) + 36*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c*x])] - 36*ArcCosh[c*x]*Log[1
 + E^(-2*ArcCosh[c*x])] + 18*PolyLog[2, -E^(-2*ArcCosh[c*x])] - 18*PolyLog[
2, E^(-2*ArcCosh[c*x])]))/d^3
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{b\,\mathrm{arcosh}\,(cx)+a}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5
 - d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b \operatorname{arcosh}(cx) + a}{\left(c^2dx^2 - d\right)^3 x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^3), x)
```

maple [B] time = 0.65, size = 641, normalized size = 2.56

$$-\frac{a}{2d^3x^2}+\frac{3c^2a\ln(cx)}{d^3}+\frac{c^2a}{16d^3(cx-1)^2}-\frac{9c^2a}{16d^3(cx-1)}-\frac{3c^2a\ln(cx-1)}{2d^3}+\frac{c^2a}{16d^3(cx+1)^2}+\frac{9c^2a}{16d^3(cx+1)}-\frac{3c^2a\ln(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x)
```

```
[Out] -1/2*a/d^3/x^2+3*c^2*a/d^3*ln(c*x)+1/16*c^2*a/d^3/(c*x-1)^2-9/16*c^2*a/d^3/
(c*x-1)-3/2*c^2*a/d^3*ln(c*x-1)+1/16*c^2*a/d^3/(c*x+1)^2+9/16*c^2*a/d^3/(c*
x+1)-3/2*c^2*a/d^3*ln(c*x+1)-2/3*c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3+2/3*c^6*b/d^3/(c^4*x^4-2*c^2*x^2+1)*x^4-3/2*c^4*b/d^3
/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x^2+1/4*c^3*b/d^3/(c^4*x^4-2*c^2*x^2+1)
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-4/3*c^4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*x^2+9/4*
c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)+1/2*c*b/d^3/(c^4*x^4-2*c^2*x^2
+1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)+2/3*c^2*b/d^3/(c^4*x^4-2*c^2*x^2+1)-1/2*b
/d^3/(c^4*x^4-2*c^2*x^2+1)/x^2*arccosh(c*x)+3*c^2*b/d^3*arccosh(c*x)*ln(1+(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+3/2*b*c^2*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/d^3-3*c^2*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))-3*c^2*b/d^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-3*c^2*b/
d^3*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-3*c^2*b/d^3*polylog(
2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a\left(\frac{6c^4x^4-9c^2x^2+2}{c^4d^3x^6-2c^2d^3x^4+d^3x^2}+\frac{6c^2\log(cx+1)}{d^3}+\frac{6c^2\log(cx-1)}{d^3}-\frac{12c^2\log(x)}{d^3}\right)-b\int\frac{\log\left(cx+\sqrt{cx}\right.}{c^6d^3x^9-3c^4d^3x^7-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*c^4*x^4 - 9*c^2*x^2 + 2)/(c^4*d^3*x^6 - 2*c^2*d^3*x^4 + d^3*x^2)
 + 6*c^2*log(c*x + 1)/d^3 + 6*c^2*log(c*x - 1)/d^3 - 12*c^2*log(x)/d^3) - b
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^6*d^3*x^9 - 3*c^4*d^3*
x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx+\int \frac{b\operatorname{acosh}(cx)}{c^6x^9-3c^4x^7+3c^2x^5-x^3}\,dx}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x) + Integral(
b*acosh(c*x)/(c**6*x**9 - 3*c**4*x**7 + 3*c**2*x**5 - x**3), x))/d**3
```

3.54 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=310

$$\frac{35c^3\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{4d^3}-\frac{7c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^3x\left(1-c^2x^2\right)^2}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{35c^4x\left(a+b\cosh^{-1}(c\right.}{8d^3\left(1-c^2x^2\right)}$$

```
[Out] -1/12*b*c^3/d^3/(c*x-1)^(3/2)/(c*x+1)^(3/2)+1/6*b*c/d^3/x^2/(c*x-1)^(3/2)/(
c*x+1)^(3/2)+1/3*(-a-b*arccosh(c*x))/d^3/x^3/(-c^2*x^2+1)^2-7/3*c^2*(a+b*ar
ccosh(c*x))/d^3/x/(-c^2*x^2+1)^2+35/12*c^4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x
^2+1)^2+35/8*c^4*x*(a+b*arccosh(c*x))/d^3/(-c^2*x^2+1)+19/6*b*c^3*arctan((c
*x-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/4*c^3*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/d^3+35/8*b*c^3*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/d^3-35/8*b*c^3*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/d^3-29/2
4*b*c^3/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {5746, 103, 12, 104, 21, 92, 205, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{35bc^3\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{8d^3}-\frac{35bc^3\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{8d^3}+\frac{35c^4x\left(a+b\cosh^{-1}(cx)\right)}{8d^3\left(1-c^2x^2\right)}+\frac{35c^4x\left(a+b\cosh^{-1}\right.}{12d^3\left(1-c^2x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

```
[Out] -(b*c^3)/(12*d^3*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) + (b*c)/(6*d^3*x^2*(-1 +
 c*x)^(3/2)*(1 + c*x)^(3/2)) - (29*b*c^3)/(24*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (a + b*ArcCosh[c*x])/(3*d^3*x^3*(1 - c^2*x^2)^2) - (7*c^2*(a + b*Arc
Cosh[c*x]))/(3*d^3*x*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(12
*d^3*(1 - c^2*x^2)^2) + (35*c^4*x*(a + b*ArcCosh[c*x]))/(8*d^3*(1 - c^2*x^2
)) + (19*b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(6*d^3) + (35*c^3*(a +
 b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(4*d^3) + (35*b*c^3*PolyLog[2, -E
^ArcCosh[c*x]])/(8*d^3) - (35*b*c^3*PolyLog[2, E^ArcCosh[c*x]])/(8*d^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
```

```
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}+\frac{1}{3}\left(7c^2\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^3}\,dx+\frac{(bc)\int \frac{1}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{3d^3}$$

$$= \frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c^2\left(a+b\cosh^{-1}(cx)\right)}{3d^3x\left(1-c^2x^2\right)^2}+\frac{1}{3}\left(35c^4\right.$$

$$= -\frac{7bc^3}{9d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7c}{}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3d^3x^3\left(1-c^2x^2\right)^2}-\frac{7}{}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{49bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -\frac{bc^3}{12d^3(-1+cx)^{3/2}(1+cx)^{3/2}}+\frac{bc}{6d^3x^2(-1+cx)^{3/2}(1+cx)^{3/2}}-\frac{29bc^3}{24d^3\sqrt{-1+cx}\sqrt{1+c}}$$

Mathematica [A] time = 1.93, size = 471, normalized size = 1.52

$$-105ac^3\log(1-cx)+105ac^3\log(cx+1)-\frac{144ac^2}{x}-\frac{66ac^4x}{c^2x^2-1}+\frac{12ac^4x}{\left(c^2x^2-1\right)^2}-\frac{16a}{x^3}-\frac{105}{2}bc^3\left(\cosh^{-1}(cx)\left(\cosh^{-1}(cx)-4\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^3),x]
```

```
[Out] ((-16*a)/x^3 - (144*a*c^2)/x + (12*a*c^4*x)/(-1 + c^2*x^2)^2 - (66*a*c^4*x)
/(-1 + c^2*x^2) - (b*c^3*((-2 + c*x)*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*ArcCo
sh[c*x]))/(-1 + c*x)^2 + (b*c^3*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c*x) - 3
*ArcCosh[c*x]))/(1 + c*x)^2 + 33*b*c^3*(-(1/Sqrt[(-1 + c*x)/(1 + c*x)]) + A
rcCosh[c*x]/(1 - c*x)) + 33*b*c^3*(Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x
]/(1 + c*x)) + 144*b*c^2*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]*ArcTan[
Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (8*b*(-2*ArcCosh[c*x
] + (c*x*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^
2]]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x^3 - 105*a*c^3*Log[1 - c*x] + 105*a
*c^3*Log[1 + c*x] - (105*b*c^3*(ArcCosh[c*x]*(ArcCosh[c*x] - 4*Log[1 + E^Ar
cCosh[c*x]]) - 4*PolyLog[2, -E^ArcCosh[c*x]]))/2 + (105*b*c^3*(ArcCosh[c*x]
*(ArcCosh[c*x] - 4*Log[1 - E^ArcCosh[c*x]]) - 4*PolyLog[2, E^ArcCosh[c*x]])
)/2)/(48*d^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{b\operatorname{arcosh}(cx)+a}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral(-(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^
6 - d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{b\operatorname{arcosh}(cx)+a}{\left(c^2dx^2-d\right)^3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate(-(b*arccosh(c*x) + a)/((c^2*d*x^2 - d)^3*x^4), x)
```

maple [A] time = 0.58, size = 504, normalized size = 1.63

$$-\frac{a}{3d^3x^3}-\frac{3c^2a}{d^3x}+\frac{c^3a}{16d^3\left(cx-1\right)^2}-\frac{11c^3a}{16d^3\left(cx-1\right)}-\frac{35c^3a\ln\left(cx-1\right)}{16d^3}-\frac{c^3a}{16d^3\left(cx+1\right)^2}-\frac{11c^3a}{16d^3\left(cx+1\right)}+\frac{35c^3a\ln\left(cx+1\right)}{16d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x)
```

```
[Out] -1/3*a/d^3/x^3-3*c^2*a/d^3/x+1/16*c^3*a/d^3/(c*x-1)^2-11/16*c^3*a/d^3/(c*x-
1)-35/16*c^3*a/d^3*ln(c*x-1)-1/16*c^3*a/d^3/(c*x+1)^2-11/16*c^3*a/d^3/(c*x+
1)+35/16*c^3*a/d^3*ln(c*x+1)-35/8*c^6*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c
*x)*x^3-29/24*c^5*b/d^3/(c^4*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
```

```
^2+175/24*c^4*b/d^3/(c^4*x^4-2*c^2*x^2+1)*arccosh(c*x)*x+9/8*c^3*b/d^3/(c^4
*x^4-2*c^2*x^2+1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)-7/3*c^2*b/d^3/x/(c^4*x^4-2*c^
2*x^2+1)*arccosh(c*x)+1/6*c*b/d^3/(c^4*x^4-2*c^2*x^2+1)/x^2*(c*x+1)^(1/2)*(
c*x-1)^(1/2)-1/3*b/d^3/(c^4*x^4-2*c^2*x^2+1)/x^3*arccosh(c*x)+19/3*c^3*b/d^
3*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+35/8*c^3*b/d^3*dilog(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))+35/8*c^3*b/d^3*dilog(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)+35/8*c^3*b/d^3*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/6144*(1935360*c^9*integrate(1/96*x^7*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) - 1680*c^8*(2*(5*c^2*x^3 - 3*x)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3) + 3*log(c*x + 1)/(c^5*d^3) - 3*log(c*x -
1)/(c^5*d^3)) - 645120*c^8*integrate(1/96*x^6*log(c*x - 1)/(c^6*d^3*x^8 - 3
*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 630*(c*(2*(5*c^2*x^2 + 3*c*x
- 6)/(c^8*d^3*x^3 - c^7*d^3*x^2 - c^6*d^3*x + c^5*d^3) - 5*log(c*x + 1)/(c^
5*d^3) + 5*log(c*x - 1)/(c^5*d^3)) + 16*(2*c^2*x^2 - 1)*log(c*x - 1)/(c^8*d
^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3))*c^7 + 2800*c^6*(2*(c^2*x^3 + x)/(c^6*d^3
*x^4 - 2*c^4*d^3*x^2 + c^2*d^3) - log(c*x + 1)/(c^3*d^3) + log(c*x - 1)/(c^
3*d^3)) + 1290240*c^6*integrate(1/96*x^4*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*
d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x) + 315*(c*(2*(3*c^2*x^2 - 3*c*x - 2)/
(c^6*d^3*x^3 - c^5*d^3*x^2 - c^4*d^3*x + c^3*d^3) - 3*log(c*x + 1)/(c^3*d^3
) + 3*log(c*x - 1)/(c^3*d^3)) - 16*log(c*x - 1)/(c^6*d^3*x^4 - 2*c^4*d^3*x^
2 + c^2*d^3))*c^5 + 896*c^4*(2*(3*c^2*x^3 - 5*x)/(c^4*d^3*x^4 - 2*c^2*d^3*x
^2 + d^3) - 3*log(c*x + 1)/(c*d^3) + 3*log(c*x - 1)/(c*d^3)) - 645120*c^4*i
ntegrate(1/96*x^2*log(c*x - 1)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4
 - d^3*x^2), x) + 128*c^2*(2*(15*c^4*x^4 - 25*c^2*x^2 + 8)/(c^4*d^3*x^5 - 2
*c^2*d^3*x^3 + d^3*x) - 15*c*log(c*x + 1)/d^3 + 15*c*log(c*x - 1)/d^3) - 32
*(105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1)^2 + 210*(c^7*x^7 - 2*c^5
*x^5 + c^3*x^3)*log(c*x + 1)*log(c*x - 1) + 4*(210*c^6*x^6 - 350*c^4*x^4 +
112*c^2*x^2 - 105*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x + 1) + 105*(c^7*x
^7 - 2*c^5*x^5 + c^3*x^3)*log(c*x - 1) + 16)*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)))/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3) + 6144*integrate(-1/48*(2
10*c^7*x^6 - 350*c^5*x^4 + 112*c^3*x^2 - 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3
)*log(c*x + 1) + 105*(c^8*x^7 - 2*c^6*x^5 + c^4*x^3)*log(c*x - 1) + 16*c)/(
c^7*d^3*x^10 - 3*c^5*d^3*x^8 + 3*c^3*d^3*x^6 - c*d^3*x^4 + (c^6*d^3*x^9 - 3
*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x))*b
 + 1/48*a*(105*c^3*log(c*x + 1)/d^3 - 105*c^3*log(c*x - 1)/d^3 - 2*(105*c^6
*x^6 - 175*c^4*x^4 + 56*c^2*x^2 + 8)/(c^4*d^3*x^7 - 2*c^2*d^3*x^5 + d^3*x^3
))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a}{c^{6} x^{10} - 3 c^{4} x^{8} + 3 c^{2} x^{6} - x^{4}}\, dx + \int \frac{b \operatorname{acosh}{\left(c x \right)}}{c^{6} x^{10} - 3 c^{4} x^{8} + 3 c^{2} x^{6} - x^{4}}\, dx}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**3,x)
```

```
[Out] -(Integral(a/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x) + Integral
(b*acosh(c*x)/(c**6*x**10 - 3*c**4*x**8 + 3*c**2*x**6 - x**4), x))/d**3
```

3.55 $\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2} dx$

Optimal. Leaf size=53

$$\frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

```
[Out] 2*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+polylog(2,-a*x-
(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
/a/c
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5694, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]
]/(a*c) - PolyLog[2, E^ArcCosh[a*x]]/(a*c)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{\text{Subst}\left(\int \log\right.}{}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx, x, e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx\right.}{ac}$$
$$= \frac{2\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}$$

Mathematica [A] time = 0.05, size = 77, normalized size = 1.45

$$\frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{\cosh^{-1}(ax)\log\left(1-e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{\cosh^{-1}(ax)\log\left(e^{\cosh^{-1}(ax)}+1\right)}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2),x]
```

```
[Out] -((ArcCosh[a*x]*Log[1 - E^ArcCosh[a*x]])/(a*c)) + (ArcCosh[a*x]*Log[1 + E^A
rcCosh[a*x]])/(a*c) + PolyLog[2, -E^ArcCosh[a*x]]/(a*c) - PolyLog[2, E^ArcC
osh[a*x]]/(a*c)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{arcosh}\,(ax)}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{arcosh}\,(ax)}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c), x)
```

maple [C] time = 0.02, size = 309, normalized size = 5.83

$$\frac{\text{arctanh}\,(ax)\,\text{arccosh}\,(ax)}{ac} + \frac{2i\sqrt{-a^2x^2+1}\,\sqrt{\frac{1}{2}+\frac{ax}{2}}\,\sqrt{-\frac{1}{2}+\frac{ax}{2}}\,\text{arctanh}\,(ax)\ln\left(1+\frac{i(ax+1)}{\sqrt{-a^2x^2+1}}\right)}{ac\left(a^2x^2-1\right)} - \frac{2i\sqrt{-a^2x^2+1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c),x)
```

```
[Out] 1/a/c*arctanh(a*x)*arccosh(a*x)+2*I/a/c*(-a^2*x^2+1)^(1/2)*(1/2+1/2*a*x)^(1
/2)*(-1/2+1/2*a*x)^(1/2)/(a^2*x^2-1)*arctanh(a*x)*ln(1+I*(a*x+1)/(-a^2*x^2+
```

```
1)^(1/2))-2*I/a/c*(-a^2*x^2+1)^(1/2)*(1/2+1/2*a*x)^(1/2)*(-1/2+1/2*a*x)^(1/
2)/(a^2*x^2-1)*arctanh(a*x)*ln(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))+2*I/a/c*(-a^
2*x^2+1)^(1/2)*(1/2+1/2*a*x)^(1/2)*(-1/2+1/2*a*x)^(1/2)/(a^2*x^2-1)*dilog(1
+I*(a*x+1)/(-a^2*x^2+1)^(1/2))-2*I/a/c*(-a^2*x^2+1)^(1/2)*(1/2+1/2*a*x)^(1/
2)*(-1/2+1/2*a*x)^(1/2)/(a^2*x^2-1)*dilog(1-I*(a*x+1)/(-a^2*x^2+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{4\left(\log\left(ax+1\right)-\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)-\log\left(ax+1\right)^{2}-2\,\log\left(ax+1\right)\log\left(ax-1\right)+\log}{8\,ac}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/8*(4*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))
 - log(a*x + 1)^2 - 2*log(a*x + 1)*log(a*x - 1) + log(a*x - 1)^2)/(a*c) + 1
/2*(log(a*x - 1)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c) + integr
ate(1/2*(log(a*x + 1) - log(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*
sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\mathrm{acosh}\left(a\,x\right)}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\mathrm{acosh}\left(ax\right)}{a^{2}x^{2}-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)/(a**2*x**2 - 1), x)/c
```

3.56 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^2}\,dx$

Optimal. Leaf size=109

$$\frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)}+\frac{\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{1}{2ac^2\sqrt{ax-1}\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

```
[Out] 1/2*x*arccosh(a*x)/c^2/(-a^2*x^2+1)+arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2))/a/c^2+1/2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-
1/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-1/2/a/c^2/(a*x-1)^(1/2
)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}+\frac{x\cosh^{-1}(ax)}{2c^2(1-a^2x^2)}-\frac{1}{2ac^2\sqrt{ax-1}\sqrt{ax+1}}+\frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] -1/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x])/(2*c^2*(1 - a^
2*x^2)) + (ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) + PolyLog[2, -E^Ar
cCosh[a*x]]/(2*a*c^2) - PolyLog[2, E^ArcCosh[a*x]]/(2*a*c^2)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
```

```
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} - \frac{\text{Subst}\left(\int x\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{2ac^2}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{1}{2ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\text{Li}_2\left(-e^{\text{co}}\right.}{2a}$$

Mathematica [A] time = 0.89, size = 120, normalized size = 1.10

$$\frac{-\frac{2\left(\cosh^{-1}(ax)\left(\left(a^2x^2-1\right)\log\left(1-e^{\cosh^{-1}(ax)}\right)+\left(1-a^2x^2\right)\log\left(e^{\cosh^{-1}(ax)}+1\right)+ax\right)+\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\right)}{a^2x^2-1} + 2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) - 2\text{Li}_2\left(e^{\cosh^{-1}(\right.}{4ac^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^2,x]
```

```
[Out] ((-2*(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x) + ArcCosh[a*x]*(a*x + (-1 + a^2*
x^2)*Log[1 - E^ArcCosh[a*x]] + (1 - a^2*x^2)*Log[1 + E^ArcCosh[a*x]])))/(-1
 + a^2*x^2) + 2*PolyLog[2, -E^ArcCosh[a*x]] - 2*PolyLog[2, E^ArcCosh[a*x]])
/(4*a*c^2)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}\,(ax)}{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{(a^2cx^2-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.10, size = 184, normalized size = 1.69

$$-\frac{x\operatorname{arccosh}(ax)}{2(a^2x^2-1)c^2}-\frac{\sqrt{ax-1}\,\sqrt{ax+1}}{2a(a^2x^2-1)c^2}+\frac{\operatorname{arccosh}(ax)\ln\left(1+ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2a\,c^2}+\frac{\operatorname{polylog}\left(2,-ax-\sqrt{ax-1}\right.}{2a\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)/c^2*x*arccosh(a*x)-1/2/a/(a^2*x^2-1)/c^2*(a*x-1)^(1/2)*(a*
x+1)^(1/2)+1/2/a/c^2*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+1/2
*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-1/2/a/c^2*arccosh(a*x)*l
n(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-1/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(a^2x^2-1)\log(ax+1)^2+2(a^2x^2-1)\log(ax+1)\log(ax-1)-(a^2x^2-1)\log(ax-1)^2+4\,ax+4\left(2\,ax-\left(a^2\right.\right.}{16\left(a^3c^2x^2-a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*((a^2*x^2 - 1)*log(a*x + 1)^2 + 2*(a^2*x^2 - 1)*log(a*x + 1)*log(a*x
 - 1) - (a^2*x^2 - 1)*log(a*x - 1)^2 + 4*a*x + 4*(2*a*x - (a^2*x^2 - 1)*log(
a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1
)) - 2*(a^2*x^2 - 1)*log(a*x - 1))/(a^3*c^2*x^2 - a*c^2) + 1/4*(log(a*x - 1
)*log(1/2*a*x + 1/2) + dilog(-1/2*a*x + 1/2))/(a*c^2) - 1/8*log(a*x + 1)/(a
*c^2) + integrate(-1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*
log(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4 - 2*a^2
*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{(c-a^2\,c\,x^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.57 $\int \frac{\cosh^{-1}(ax)}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=164

$$\frac{3x\cosh^{-1}(ax)}{8c^3(1-a^2x^2)}+\frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)^2}+\frac{3\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3}{8ac^3\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{12ac^3(ax-1)^{3/2}(\ldots}$$

```
[Out] 1/12/a/c^3/(a*x-1)^(3/2)/(a*x+1)^(3/2)+1/4*x*arccosh(a*x)/c^3/(-a^2*x^2+1)^
2+3/8*x*arccosh(a*x)/c^3/(-a^2*x^2+1)+3/4*arccosh(a*x)*arctanh(a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))/a/c^3+3/8*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/
a/c^3-3/8*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3/(a*x-1
)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5689, 74, 5694, 4182, 2279, 2391}

$$\frac{3\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{3\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}+\frac{3x\cosh^{-1}(ax)}{8c^3(1-a^2x^2)}+\frac{x\cosh^{-1}(ax)}{4c^3(1-a^2x^2)^2}-\frac{3}{8ac^3\sqrt{ax-1}\sqrt{ax+1}}+\frac{\ldots}{1\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(12*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - 3/(8*a*c^3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (x*ArcCosh[a*x])/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x
])/(8*c^3*(1 - a^2*x^2)) + (3*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^
3) + (3*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) - (3*PolyLog[2, E^ArcCosh[a*
x]])/(8*a*c^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)}{8c^3\left(1-a^2x^2\right)} + \frac{(3a)\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/}}}{8c^3}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(a}{8c^3\left(1-a^2x\right.}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(a}{8c^3\left(1-a^2x\right.}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(a}{8c^3\left(1-a^2x\right.}$$

$$= \frac{1}{12ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3}{8ac^3\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(a}{8c^3\left(1-a^2x\right.}$$

Mathematica [A] time = 2.49, size = 223, normalized size = 1.36

$$36\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) - 36\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right) - \frac{2\sqrt{ax+1}\,(ax-2)}{(ax-1)^{3/2}} + \frac{2\sqrt{ax-1}\,(ax+2)}{(ax+1)^{3/2}} + \frac{6\cosh^{-1}(ax)}{(ax-1)^2} - \frac{6\cosh^{-1}(ax)}{(ax+1)^2} + 18\left(\frac{\cosh^{-1}(ax}{1-ax}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^3,x]
```

```
[Out] ((-2*(-2 + a*x)*Sqrt[1 + a*x])/(-1 + a*x)^(3/2) + (2*Sqrt[-1 + a*x]*(2 + a*
x))/(1 + a*x)^(3/2) + (6*ArcCosh[a*x])/(-1 + a*x)^2 - (6*ArcCosh[a*x])/(1 +
 a*x)^2 + 18*(-(1/Sqrt[(-1 + a*x)/(1 + a*x)]) + ArcCosh[a*x]/(1 - a*x)) + 1
8*(Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x]/(1 + a*x)) + 9*ArcCosh[a*x]*(A
rcCosh[a*x] - 4*Log[1 - E^ArcCosh[a*x]]) - 9*ArcCosh[a*x]*(ArcCosh[a*x] - 4
*Log[1 + E^ArcCosh[a*x]]) + 36*PolyLog[2, -E^ArcCosh[a*x]] - 36*PolyLog[2,
E^ArcCosh[a*x]])/(96*a*c^3)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\operatorname{arcosh}(ax)}{a^6c^3x^6 - 3a^4c^3x^4 + 3a^2c^3x^2 - c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arcosh}(ax)}{(a^2cx^2 - c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.30, size = 276, normalized size = 1.68

$$-\frac{3a^2x^3\operatorname{arccosh}(ax)}{8\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} - \frac{3a\sqrt{ax+1}\,\sqrt{ax-1}\,x^2}{8\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} + \frac{5x\operatorname{arccosh}(ax)}{8\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} + \frac{11\sqrt{ax-1}\,\sqrt{ax+1}}{24a\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} + \frac{3\operatorname{arccosl}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*x^3*arccosh(a*x)-3/8*a/(a^4*x^4-2*a^2*x^
2+1)/c^3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^2+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*x*ar
ccosh(a*x)+11/24/a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3/
8/a/c^3*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/8*polylog(2,-a
*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3*arccosh(a*x)*ln(1-a*x-(a*x-
1)^(1/2)*(a*x+1)^(1/2))-3/8*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^
3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{10a^3x^3 + 3\left(a^4x^4 - 2a^2x^2 + 1\right)\log(ax+1)^2 + 6\left(a^4x^4 - 2a^2x^2 + 1\right)\log(ax+1)\log(ax-1) - 3\left(a^4x^4 - 2a^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/64*(10*a^3*x^3 + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1)^2 + 6*(a^4*x^4
 - 2*a^2*x^2 + 1)*log(a*x + 1)*log(a*x - 1) - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*l
og(a*x - 1)^2 - 14*a*x + 4*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a
*x + 1)*sqrt(a*x - 1)) - 7*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))/(a^5*c^3
*x^4 - 2*a^3*c^3*x^2 + a*c^3) + 3/16*(log(a*x - 1)*log(1/2*a*x + 1/2) + dil
og(-1/2*a*x + 1/2))/(a*c^3) - 7/64*log(a*x + 1)/(a*c^3) + integrate(-1/16*(
6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a^4*x^4
- 2*a^2*x^2 + 1)*log(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3
 - a*c^3*x + (a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x +
 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{\left(c-a^2\,c\,x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)/(c - a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**3
```

3.58 $\int x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=278

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{32bc^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-c}}{}$$

```
[Out] -1/16*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4-1/24*x^3*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/6*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)+1/32*b*x^2*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/96*b*x
^4*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/36*b*c*x^6*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)/b/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5743, 5759, 5676, 30}

$$\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^4}-\frac{\sqrt{d-c^2}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^4*
Sqrt[d - c^2*d*x^2])/(96*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^6*Sqrt[d
- c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(16*c^4) - (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(24*c^2) + (x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int x^4\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{6}x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\int\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{6\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{24c^2}+\frac{1}{6}x^5\sqrt{}$$

$$=\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\left(a+\right.}{16c^4}$$

$$=\frac{bx^2\sqrt{d-c^2dx^2}}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^4\sqrt{d-c^2dx^2}}{96c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^6\sqrt{d-c^2d}}{36\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 1.22, size = 198, normalized size = 0.71

$$\frac{-144a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+48acx\left(8c^4x^4-2c^2x^2-3\right)\sqrt{d-c^2dx^2}+\frac{b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2+18\cosh\left(2\cosh^{-1}(\right.\right.}{}}{2304c^5}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*x*Sqrt[d - c^2*d*x^2]*(-3 - 2*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt[d]*
ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^
2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c
*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x)))/(2304*c^5)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4\,\text{arcosh}\,(cx)+ax^4\right)\sqrt{-c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^4, x)
```

maple [A] time = 0.81, size = 449, normalized size = 1.62

$$-\frac{a\,x^3\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{6c^2d}-\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{8c^4d}+\frac{ax\sqrt{-c^2d\,x^2+d}}{16c^4}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16c^4\sqrt{c^2d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\operatorname{arccosh}(c}{32\sqrt{cx-1}\,\sqrt{cx+1}\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/6*a*x^3*(-c^2*d*x^2+d)^(3/2)/c^2/d-1/8*a/c^4*x*(-c^2*d*x^2+d)^(3/2)/d+1/
16*a/c^4*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^4*d/(c^2*d)^(1/2)*arctan((c^2*d)^(
1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)/c^5*arccosh(c*x)^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*
x-1)*arccosh(c*x)*x^7-5/24*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh
(c*x)*x^5-1/48*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^
3+1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)*x-25/2304*
b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c^5/(c*x-1)^(1/2)-1/36*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^6+1/96*b*(-d*(c^2*x^2-1))^(1/2)
/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^4+1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1
/2)/c^3/(c*x-1)^(1/2)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{48}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}{c^2d}-\frac{3\sqrt{-c^2dx^2+d}\,x}{c^4}+\frac{6\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^4d}-\frac{3\sqrt{d}\,\arcsin(cx)}{c^5}\right)a+b\int\sqrt{-c^2dx^2+d}\,x^4\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/48*(8*(-c^2*d*x^2 + d)^(3/2)*x^3/(c^2*d) - 3*sqrt(-c^2*d*x^2 + d)*x/c^4
+ 6*(-c^2*d*x^2 + d)^(3/2)*x/(c^4*d) - 3*sqrt(d)*arcsin(c*x)/c^5)*a + b*int
egrate(sqrt(-c^2*d*x^2 + d)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.59 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=201

$$-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx^2\sqrt{}}{16c\sqrt{cx}}$$

```
[Out] -1/8*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)+1/16*b*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/16*b*c*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/
16*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A] time = 0.58, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5798, 5743, 5759, 5676, 30}

$$\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^4\sqrt{d}}{16\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x^2*Sqrt[d - c^2*d*x^2])/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^4*
Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/4 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\int\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}}{4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^2}+\frac{1}{4}x^3\sqrt{d}$$

$$=\frac{bx^2\sqrt{d-c^2dx^2}}{16c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}-\frac{x\sqrt{d-c^2dx^2}\left(a+\right.}{8c^2}$$

Mathematica [A] time = 1.10, size = 151, normalized size = 0.75

$$-\frac{-16acx\left(2c^2x^2-1\right)\sqrt{d-c^2dx^2}+16a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{b\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cosh^{-1}(cx\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}}{128c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/128*(-16*a*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 16*a*Sqrt[d]*ArcTa
n[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*Sqrt[d - c^2*d*x
^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCos
h[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/c^3
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(bx^2\,\text{arcosh}\,(cx)+ax^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^2dx^2 + d}\,(b\operatorname{arcosh}(cx) + a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*x^2, x)
```

maple [B] time = 0.38, size = 346, normalized size = 1.72

$$-\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{4c^2d}+\frac{ax\sqrt{-c^2d\,x^2+d}}{8c^2}+\frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8c^2\sqrt{c^2d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)^2}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c^3}+\frac{b\sqrt{-d\left(c^2x^2-1\right.}}{4\,(cx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*a/
c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/16*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2+1/4*b*(-
d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-3/8*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-1/128*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1
/2)/c^3/(c*x-1)^(1/2)-1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)
^(1/2)*x^4+1/16*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}a\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2}-\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2d}+\frac{\sqrt{d}\,\arcsin(cx)}{c^3}\right)+b\int\sqrt{-c^2dx^2+d}\,x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/8*a*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) + sq
rt(d)*arcsin(c*x)/c^3) + b*integrate(sqrt(-c^2*d*x^2 + d)*x^2*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\operatorname{acosh}(cx)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.60 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=124

$$\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] 1/2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-1/4*b*c*x^2*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/
2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5713, 5683, 5676, 30}

$$\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*c*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2)/(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(bc}{2}$$

$$= -\frac{bcx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}}{4bc\sqrt{-}}$$

Mathematica [A] time = 0.61, size = 144, normalized size = 1.16

$$\frac{1}{8}\left(4ax\sqrt{d-c^2dx^2} - \frac{4a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}(c^2x^2-1)}\right)}{c} - \frac{b\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-2\cosh^{-1}(\right.}{c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (4*a*x*Sqrt[d - c^2*d*x^2] - (4*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/
(Sqrt[d]*(-1 + c^2*x^2))])/c - (b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + C
osh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)))/8
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.24, size = 239, normalized size = 1.93

$$\frac{ax\sqrt{-c^2d\,x^2+d}}{2} + \frac{ad\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\text{arccosh}\,(cx)^2}{4\sqrt{cx-1}\,\sqrt{cx+1}\,c} + \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,c^2\text{arccosh}\,(cx)\,x^3}{2\,(cx+1)\,(cx-1)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a*x*(-c^2*d*x^2+d)^(1/2)+1/2*a*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(
-c^2*d*x^2+d)^(1/2))-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/c*arccosh(c*x)^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*c^2*arccos
h(c*x)*x^3-1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1
/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x+1/8*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{-c^2dx^2+d}\,x+\frac{\sqrt{d}\,\arcsin\left(cx\right)}{c}\right)a+b\int\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a + b*integrate(sqrt(-
c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\left(c\,x\right))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.61 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=118

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x+1/2*c*(a+b*arccosh(c*x))^2*(-c^2
*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*ln(x)*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5798, 5738, 29, 5676}

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x) + (c*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d - c^
2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-\right.}{}$$
$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{c\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.47, size = 137, normalized size = 1.16

$$-\frac{a\sqrt{d-c^2dx^2}}{x}+ac\sqrt{d}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,\left(c^2x^2-1\right)}\right)+\frac{1}{2}bc\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}-\frac{2\cosh^{-1}(cx)}{cx}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*Sqrt[d - c^2*d*x^2])/x) + a*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])
/(Sqrt[d]*(-1 + c^2*x^2))] + (b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c
*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
)/2
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.49, size = 286, normalized size = 2.42

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{dx}-a\,c^2x\sqrt{-c^2d\,x^2+d}-\frac{a\,c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}\,(cx)^2\,c}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{-d\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(3/2)-a*c^2*x*(-c^2*d*x^2+d)^(1/2)-a*c^2*d/(c^2*d)^(1
/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c-b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c
*x)/(c*x+1)/(c*x-1)*x*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*
x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c\sqrt{d}\arcsin\left(cx\right)+\frac{\sqrt{-c^2dx^2+d}}{x}\right)a+b\int\frac{\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxima"
)
```

```
[Out] -(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a + b*integrate(sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**2, x)
```

3.62 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^3\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^3-1/6*b*c*(-c^2*d*x^2+d)^(
1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c^3*ln(x)*(-c^2*d*x^2+d)^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 127, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5798, 5724, 14}

$$-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^3\log(x)\sqrt{d-c^2dx^2}}{3\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x
)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (b*c^3*Sqrt
[d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^4}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{-1+}{x}}{3\sqrt{-1+cx}\sqrt{1+c}}$$
$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int\left(-\right.}{3\sqrt{-1+cx}\sqrt{1-}}$$
$$= -\frac{bc\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}$$

Mathematica [A] time = 0.13, size = 88, normalized size = 0.74

$$\frac{\sqrt{d-c^2dx^2}\left(\frac{(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{1}{3}bc\left(c^2\log(x)+\frac{1}{2x^2}\right)\right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]
))/(3*x^3) - (b*c*(1/(2*x^2) + c^2*Log[x]))/3))/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

fricas [B] time = 0.57, size = 462, normalized size = 3.88

$$\left[\frac{2\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\,\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\,\sqrt{}}{c^2x^4-x^2}\right.}{6\left(c^2x^5-x^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="fricas"
)
```

```
[Out] [1/6*(2*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) + (b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) +
 2*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3), -1/
6*(2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x
^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(b*c^4*x^4
 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - sqr
t(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(a*c^4*x^4 - 2*a*
c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^5 - x^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.73, size = 1017, normalized size = 8.55

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{3d\,x^3}+\frac{2b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}\left(cx\right)c^3}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,x^4\operatorname{arccosh}\left(cx\right)c^7}{\left(3c^4x^4-3c^2x^2+1\right)\sqrt{cx+1}\,\sqrt{cx-1}}+\frac{b\sqrt{-d\left(c^2x^2-\right.}}{\left(3c^4x^4-3c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(3/2)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)*arccosh(c*x)*c^3-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*
x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+b*(-d*(c^2*x^2-1))^
(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-1/6*b*(-
d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*c^6+1/6*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^8+b*(-d*(c^2*x^2-1))^(
1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c
^5-3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^6+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c^4-
1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^5-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x
+1)/(c*x-1)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4
*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+1/2*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+1/6*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-5/3*b*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^3/(c*x+
1)/(c*x-1)*arccosh(c*x)-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

maxima [C] time = 0.49, size = 150, normalized size = 1.26

$$\frac{\left(c^4d^2\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right)+i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+\frac{\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,d}{x^2}\right)bc}{6\,d}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b\,a}{3\,dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/6*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d
)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*
d/x^2)*b*c/d - 1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(d*x^3) - 1/3*(-c^
2*d*x^2 + d)^(3/2)*a/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\,\mathrm{acosh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**4, x)
```

3.63 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=199

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{5dx^5}-\frac{2c^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{15dx^3}-\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc^5\log(x)}{15\sqrt{cx-}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^5-2/15*c^2*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccosh(c*x))/d/x^3-1/20*b*c*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+1/30*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-2/15*b*c^5*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 226, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 97, 12, 103, 95, 5733, 14}

$$\frac{2c^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{bc^3}{30x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(15*x^3) + (2*c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*x
) - (2*b*c^5*Sqrt[d - c^2*d*x^2]*Log[x])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2}{\cdots}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2}{\cdots}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15x^3} + \frac{2}{\cdots}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cdots\right.}{5x^5}$$

Mathematica [A] time = 0.22, size = 128, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(8c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+12(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(8c^{4}\cdots\right.\right.}{60x^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 8*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) - b*c
*x*(3 - 2*c^2*x^2 + 8*c^4*x^4*Log[x])))/(60*x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

fricas [A] time = 0.58, size = 549, normalized size = 2.76

$$\left[\frac{4\left(2\,bc^6x^6-bc^4x^4-4\,bc^2x^2+3\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+4\left(bc^7x^7-bc^5x^5\right)\sqrt{-d}\,\log\left(\frac{c^2dx^6+c^2dx}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="fricas"
)
```

```
[Out] [1/60*(4*(2*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(b*c^7*x^7 - b*c^5*x^5)*sqrt(-d)*log((c^2
*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 -
1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 -
3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(2*a*c^6*x^6 - a*c^4*x^
4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5), -1/60*(8*(b*c
^7*x^7 - b*c^5*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(
x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(2*b*c^6*x^6 - b*c^
4*x^4 - 4*b*c^2*x^2 + 3*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
) - (2*b*c^3*x^3 - (2*b*c^3 - 3*b*c)*x^5 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 4*(2*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 3*a)*sqrt(-c^2
*d*x^2 + d))/(c^2*x^7 - x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.90, size = 1741, normalized size = 8.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x)
```

```
[Out] 2/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^7*c^12-
1/4*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*c^5+4/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*arccosh(c*x)*c^5-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^5-2*b*(-d*(c^2*x^2-1))^(1/
2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcc
osh(c*x)*c^11+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2
+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9+2*b*(-d*(c^2*x^2-1))^(
1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*ar
ccosh(c*x)*c^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+
9)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(15*c
^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10-17/3*b
*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c*x+1)/(c*
x-1)*arccosh(c*x)*c^8+98/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-
15*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+12/5*b*(-d*(c^2*x^2-1))^(1
/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-
```

```
27/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^3/(c*x+
1)/(c*x-1)*arccosh(c*x)*c^2-1/5*a/d/x^5*(-c^2*d*x^2+d)^(3/2)-2/15*b*(-d*(c^
2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5*c^10-3/10*b*(-d*(c^
2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3*c^8+3/10*b*(-d*(c^2
*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x*c^6-2/15*a*c^2/d/x^3*(
-c^2*d*x^2+d)^(3/2)-9/20*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*
c^2*x^2+9)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(
15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-6/5*
b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*arccosh(c*x)*c^5-11/12*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5
*c^4*x^4-15*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+21/20*b*(-d*(c^2
*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)/x^2/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*c^3-2/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+
9)*x^9/(c*x+1)/(c*x-1)*c^14+4/15*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4
*x^4-15*c^2*x^2+9)*x^7/(c*x+1)/(c*x-1)*c^12+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(1
5*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*c^10-3/5*b*(-d*(c^2*x
^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*c^8+3/
10*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)*x/(c*x+1)/(
c*x-1)*c^6+9/5*b*(-d*(c^2*x^2-1))^(1/2)/(15*c^6*x^6-5*c^4*x^4-15*c^2*x^2+9)
/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)
```

maxima [A] time = 0.45, size = 146, normalized size = 0.73

$$-\frac{1}{60}\left(8\,c^4\sqrt{-d}\,\log(x)-\frac{2\,c^2\sqrt{-d}\,x^2-3\,\sqrt{-d}}{x^4}\right)bc-\frac{1}{15}\,b\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{dx^3}+\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^5}\right)\operatorname{arcosh}(cx)-\frac{1}{15}\,a\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/60*(8*c^4*sqrt(-d)*log(x) - (2*c^2*sqrt(-d)*x^2 - 3*sqrt(-d))/x^4)*b*c -
 1/15*b*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(3/2)/(d
*x^5))*arccosh(c*x) - 1/15*a*(2*(-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^3) + 3*(-c^
2*d*x^2 + d)^(3/2)/(d*x^5))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\operatorname{acosh}(cx)\right)}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**6,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**6, x)
```

3.64 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=279

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}-\frac{4c^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{35dx^5}-\frac{8c^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx\right.}{105dx^3}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/d/x^7-4/35*c^2*(-c^2*d*x^2+d)^
(3/2)*(a+b*arccosh(c*x))/d/x^5-8/105*c^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(
c*x))/d/x^3-1/42*b*c*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/140*b*c^3*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/105*b*c^5
*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/105*b*c^7*ln(x)*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 303, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 97, 12, 103, 95, 5733, 14}

$$\frac{8c^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x}+\frac{4c^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^3}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}-\frac{\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(140*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^5*Sqrt[
d - c^2*d*x^2])/(105*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(7*x^7) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(35*x^5) + (4*c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*x^3
) + (8*c^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*x) - (8*b*c^7*Sqr
t[d - c^2*d*x^2]*Log[x])/(105*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
```

```
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} + 4$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} + 4$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} + \frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} + 4$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{140x^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^5\sqrt{d-c^2d}}{105x^2\sqrt{-1+cx}}$$

Mathematica [A] time = 0.27, size = 146, normalized size = 0.52

$$\frac{\sqrt{d-c^2dx^2}\left(16c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(2c^2x^2+3\right)\left(a+b\cosh^{-1}(cx)\right)+60(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(c\right.\right.}{420x^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(60*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*
x]) + 16*c^2*x^2*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(3 + 2*c^2*x^2)*(a + b*Ar
cCosh[c*x]) - b*c*x*(10 - 3*c^2*x^2 - 8*c^4*x^4 + 32*c^6*x^6*Log[x])))/(420
*x^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.75, size = 615, normalized size = 2.20

$$\left[\frac{4\left(8\,bc^8x^8-4\,bc^6x^6-bc^4x^4-18\,bc^2x^2+15\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+16\left(bc^9x^9-bc^7x^7\right)\sqrt{-d}\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/420*(4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 16*(b*c^9*x^9 - b*c^7*x^7)
*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (8*b*c^5*x^5 - (8*b*c
^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b*c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1) + 4*(8*a*c^8*x^8 - 4*a*c^6*x^6 - a*c^4*x^4 - 18*a*c^2*x^2
+ 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7), -1/420*(32*(b*c^9*x^9 - b*c^
7*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt
(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 4*(8*b*c^8*x^8 - 4*b*c^6*x^6 - b*c
^4*x^4 - 18*b*c^2*x^2 + 15*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (8*b*c^5*x^5 - (8*b*c^5 + 3*b*c^3 - 10*b*c)*x^7 + 3*b*c^3*x^3 - 10*b
*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(8*a*c^8*x^8 - 4*a*c^6*x^6
 - a*c^4*x^4 - 18*a*c^2*x^2 + 15*a)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.93, size = 2534, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x)
```

```
[Out] 225/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*
x^2+225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)-128/105*b*(-d*(c^2*x^2-1))^(1/2)/
(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^13/(c*x+1)/(c*x-1)*c
^20+16/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315
*c^2*x^2+225)*x^11/(c*x+1)/(c*x-1)*c^18+40/21*b*(-d*(c^2*x^2-1))^(1/2)/(280
*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9/(c*x+1)/(c*x-1)*c^16+2
14/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2
*x^2+225)*x^7/(c*x+1)/(c*x-1)*c^14-152/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^
8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5/(c*x+1)/(c*x-1)*c^12-30/7
*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+2
25)*x^3/(c*x+1)/(c*x-1)*c^10+20/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105
```

```
*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c*x+1)/(c*x-1)*c^8+16/3*b*(-d*(c^2*
x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*c^13-469/60*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-1
05*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+
71/28*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*
x^2+225)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+255/28*b*(-d*(c^2*x^2-1))^(1/2
)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^4/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c^3-120/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*
c^4*x^4-315*c^2*x^2+225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-75/14
*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+2
25)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-1/7*a/d/x^7*(-c^2*d*x^2+d)^(3/2)-8/10
5*a*c^4/d/x^3*(-c^2*d*x^2+d)^(3/2)+16/15*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*
x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^9*c^16-10/7*b*(-d*(c^2*x^2-1)
)^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3*c^10+20/7*
b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+22
5)*x*c^8+128/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x
^4-315*c^2*x^2+225)*x^11*c^18-88/105*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-
105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7*c^14-302/105*b*(-d*(c^2*x^2-1))
^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^5*c^12+64/3*b
*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225
)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-56/3*b*(-d*(c^2*x^2-1))^(1/2)/(280*
c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^7/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^14-4/15*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^
4-315*c^2*x^2+225)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-351/5*b*(-d*(c^2*x
^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^3/(c*x+
1)/(c*x-1)*arccosh(c*x)*c^10+3057/35*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-
105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-
594/35*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2
*x^2+225)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+342/7*b*(-d*(c^2*x^2-1))^(1/2)
/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)/x^3/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^4-585/7*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*
c^4*x^4-315*c^2*x^2+225)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-4/35*a*c^2/d/
x^5*(-c^2*d*x^2+d)^(3/2)+8/5*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*
x^6-21*c^4*x^4-315*c^2*x^2+225)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x
)*c^11+24*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*
c^2*x^2+225)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-64/3*b*(-d*(c
^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^8/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15+8*b*(-d*(c^2*x^2-1))^(1/2)/(28
0*c^8*x^8-105*c^6*x^6-21*c^4*x^4-315*c^2*x^2+225)*x^6/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*arccosh(c*x)*c^13-73/20*b*(-d*(c^2*x^2-1))^(1/2)/(280*c^8*x^8-105*c^
6*x^6-21*c^4*x^4-315*c^2*x^2+225)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+16/105*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^7-8/105*b
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c^7
```

maxima [A] time = 0.44, size = 207, normalized size = 0.74

$$-\frac{1}{420}\left(32\,c^{6}\sqrt{-d}\,\log(x)-\frac{8\,c^{4}\sqrt{-d}\,x^{4}+3\,c^{2}\sqrt{-d}\,x^{2}-10\,\sqrt{-d}}{x^{6}}\right)bc-\frac{1}{105}\left(\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{4}}{dx^{3}}+\frac{12\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}}{dx^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^8,x, algorithm="maxima"
)
```

```
[Out] -1/420*(32*c^6*sqrt(-d)*log(x) - (8*c^4*sqrt(-d)*x^4 + 3*c^2*sqrt(-d)*x^2 -
 10*sqrt(-d))/x^6)*b*c - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(
-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*b*ar
ccosh(c*x) - 1/105*(8*(-c^2*d*x^2 + d)^(3/2)*c^4/(d*x^3) + 12*(-c^2*d*x^2 +
 d)^(3/2)*c^2/(d*x^5) + 15*(-c^2*d*x^2 + d)^(3/2)/(d*x^7))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^8, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\,\mathrm{acosh}\,(cx))}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**8,x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**8, x)
```

3.65 $\int x^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=272

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d}-\frac{bcx}{49\sqrt{c}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^6/d+2/5*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^6
/d^3+8/105*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/315*b
*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/175*b*x^5*(-c^2
*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*c*x^7*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 302, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 100, 12, 74, 5733}

$$-\frac{x^4(1-cx)(cx+1)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{4x^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{8(1-cx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*x*Sqrt[d - c^2*d*x^2])/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b*x
^3*Sqrt[d - c^2*d*x^2])/(315*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^5*Sqr
t[d - c^2*d*x^2])/(175*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^7*Sqrt[d -
c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*(1 - c*x)*(1 + c*x)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(105*c^6) - (4*x^2*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (x^4*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
```

```
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int x^5\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)(1+$$

$$=-\frac{8(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105c^6}-\frac{4x^2(1-cx)(1+$$

$$=\frac{8bx\sqrt{d-c^2dx^2}}{105c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{4bx^3\sqrt{d-c^2dx^2}}{315c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx^5\sqrt{d-}{175c\sqrt{-1+c}$$

Mathematica [A] time = 0.37, size = 152, normalized size = 0.56

$$\frac{\sqrt{d-c^2dx^2}\left(15c^3x^4(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{3/2}(cx+1)^{3/2}\left(3c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}+b\left(-\frac{15}{7}c^6\right.}{105c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (15*c^6*x^7)
/7) + 15*c^3*x^4*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (4
*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(2 + 3*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))
/(105*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.52, size = 203, normalized size = 0.75

$$\frac{105\left(15\,bc^8x^8-18\,bc^6x^6-bc^4x^4-4\,bc^2x^2+8\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(225\,bc^7x^7-63\,bc^5x^5-\right.}{11025\left(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/11025*(105*(15*b*c^8*x^8 - 18*b*c^6*x^6 - b*c^4*x^4 - 4*b*c^2*x^2 + 8*b)*
sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (225*b*c^7*x^7 - 63*b*c
^5*x^5 - 140*b*c^3*x^3 - 840*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
```

```
+ 105*(15*a*c^8*x^8 - 18*a*c^6*x^6 - a*c^4*x^4 - 4*a*c^2*x^2 + 8*a)*sqrt(-c
^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.68, size = 988, normalized size = 3.63

$$a\left(-\frac{x^{4}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{7c^{2}d}+\frac{-\frac{4x^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{35c^{2}d}-\frac{8\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{105d\,c^{4}}}{c^{2}}\right)+b\left(\frac{\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\left(64c^{8}x^{8}-144c^{6}x^{6}+64\sqrt{cx+1}\,\sqrt{c\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/7*x^4*(-c^2*d*x^2+d)^(3/2)/c^2/d+4/7/c^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/
2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2)))+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)
*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4
-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))/(c
*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/
c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccos
h(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^6/(c*x-1)-5/128*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1
)*(1+3*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+3/3200*(-d*(c^2*x^2-1))^(1/2)*(-16
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5
*arccosh(c*x))/(c*x+1)/c^6/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))/(c*x+1)/c^6/(
c*x-1))
```

maxima [A] time = 0.45, size = 205, normalized size = 0.75

$$-\frac{1}{105}\left(\frac{15\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x^{4}}{c^{2}d}+\frac{12\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x^{2}}{c^{4}d}+\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}{c^{6}d}\right)b\,\mathrm{arcosh}\left(cx\right)-\frac{1}{105}\left(\frac{15\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x^{4}}{c^{2}d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/105*(15*(-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*b*arccosh(c*x) - 1/105*(15*(
-c^2*d*x^2 + d)^(3/2)*x^4/(c^2*d) + 12*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(3/2)/(c^6*d))*a - 1/11025*(225*c^6*sqrt(-d)*x^7 - 63*c
^4*sqrt(-d)*x^5 - 140*c^2*sqrt(-d)*x^3 - 840*sqrt(-d)*x)*b/c^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\sqrt{-d\,(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.66 $\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=195

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^4d}-\frac{bcx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{cx-1}\sqrt{cx+1}}+$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^4/d+1/5*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/15*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+1/45*b*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-1/25*b*c*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 214, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 100, 12, 74, 5733}

$$-\frac{x^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}-\frac{2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4}-\frac{bcx^5\sqrt{d-}}{25\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*x*Sqrt[d - c^2*d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x^3*
Sqrt[d - c^2*d*x^2])/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^5*Sqrt[d
- c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*c^4) - (x^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
```

```
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4} - \frac{x^2(1-cx)(1+c$$

$$= -\frac{2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4} - \frac{x^2(1-cx)(1+c$$

$$= \frac{2bx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx^3\sqrt{d-c^2dx^2}}{45c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^5\sqrt{d-c^2d}{25\sqrt{-1+cx}\sqrt{1$$

Mathematica [A] time = 0.19, size = 128, normalized size = 0.66

$$\frac{\sqrt{d-c^2dx^2}\left(3c^2x^2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{3/2}(cx+1)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{15}bcx}{15c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*((b*c*x*(30 + 5*c^2*x^2 - 9*c^4*x^4))/15 + 2*(-1 + c*x
)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + 3*c^2*x^2*(-1 + c*x)^(3/2)*(
1 + c*x)^(3/2)*(a + b*ArcCosh[c*x])))/(15*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.51, size = 176, normalized size = 0.90

$$\frac{15\left(3\,bc^6x^6-4\,bc^4x^4-bc^2x^2+2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^5x^5-5\,bc^3x^3-30\,bcx\right)\sqrt{-c^2dx^2}}{225\left(c^6x^2-c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/225*(15*(3*b*c^6*x^6 - 4*b*c^4*x^4 - b*c^2*x^2 + 2*b)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 - 5*b*c^3*x^3 - 30*b*c*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 - 4*a*c^4*x^4 - a*c^2
*x^2 + 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 640, normalized size = 3.28

$$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{5c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{15d\,c^4}\right)+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(16c^6x^6-28c^4x^4+16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c^5+13\right.}{800\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b*(
1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(
-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/
(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^
2-1)*(-1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-
1)+1/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c
^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c
*x+1)/c^4/(c*x-1)+1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c
^4/(c*x-1))
```

maxima [A] time = 0.51, size = 144, normalized size = 0.74

$$-\frac{1}{15}b\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)-\frac{1}{15}a\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)-\frac{\left(9c^4\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/15*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c
^4*d))*arccosh(c*x) - 1/15*a*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^
2*d*x^2 + d)^(3/2)/(c^4*d)) - 1/225*(9*c^4*sqrt(-d)*x^5 - 5*c^2*sqrt(-d)*x^
3 - 30*sqrt(-d)*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\,\mathrm{acosh}\,(cx))\,\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.67 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=118

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}+\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^2/d+1/3*b*x*(-c^2*d*x^2+d)^(
1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5798, 5718}

$$-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*x*Sqrt[d - c^2*d*x^2])/(3*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^3*Sqr
t[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c^2)
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2}-\frac{\left(b\sqrt{d-c^2dx^2}\right)\int (-1}{3c\sqrt{-1+cx}\sqrt{1}}$$

$$=\frac{bx\sqrt{d-c^2dx^2}}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(1-cx)(1+cx)\sqrt{d-c^2d}}{3c^2}$$

Mathematica [A] time = 0.14, size = 98, normalized size = 0.83

$$\frac{\sqrt{d-c^2dx^2}\left(3a\left(c^2x^2-1\right)^2+bcx\sqrt{cx-1}\sqrt{cx+1}\left(3-c^2x^2\right)+3b\left(c^2x^2-1\right)^2\cosh^{-1}(cx)\right)}{9c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*
a*(-1 + c^2*x^2)^2 + 3*b*(-1 + c^2*x^2)^2*ArcCosh[c*x]))/(9*c^2*(-1 + c^2*x
^2))
```

fricas [A] time = 0.83, size = 142, normalized size = 1.20

$$\frac{3\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3-3\,bcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}+3\left(ac^4x^4\right.}{9\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(3*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^
2*x^2 - 1)) - (b*c^3*x^3 - 3*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
+ 3*(a*c^4*x^4 - 2*a*c^2*x^2 + a)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.29, size = 356, normalized size = 3.02

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\sqrt{cx+1}\,\sqrt{cx-1}\,xc+1\right)(}{72\,(cx+1)\,c^2\,(cx-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/c^2/d*(-c^2*d*x^2+d)^(3/2)+b*(1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4
-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/(c*x+1)/c^
2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*
x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*(-d*(c^2*x^2-1))^(1/2)*(-4
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1))
```

maxima [A] time = 0.34, size = 81, normalized size = 0.69

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b\,\mathrm{arcosh}\,(cx)}{3\,c^2d}-\frac{\left(c^2\sqrt{-d}\,dx^3-3\,\sqrt{-d}\,dx\right)b}{9\,cd}-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}a}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(-c^2*d*x^2 + d)^(3/2)*b*arccosh(c*x)/(c^2*d) - 1/9*(c^2*sqrt(-d)*d*x^
3 - 3*sqrt(-d)*d*x)*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.68 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=213

$$\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{2\sqrt{d-c^2dx^2}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{ib\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-b*c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b*polylog(2,-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5743, 5761, 4180, 2279, 2391, 8}

$$\frac{ib\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ib\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -((b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])
*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*Sqrt[d - c^2
*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
I*b*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(bc\sqrt{\right.}{\sqrt{-}}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{\sqrt{d-c^2dx^2}\,\text{Sub}}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a\right.}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a\right.}{}$$

$$= -\frac{bcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{2\sqrt{d-c^2dx^2}\left(a\right.}{}$$

Mathematica [A] time = 0.91, size = 233, normalized size = 1.09

$$a\sqrt{d-c^2dx^2} - a\sqrt{d}\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) + a\sqrt{d}\log(x) + \frac{b\sqrt{d-c^2dx^2}\left(i\text{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right) - i\text{Li}_2\left(ie^{-\cosh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*Sqrt[d - c^2*d*x^2] + a*Sqrt[d]*Log[x] - a*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d
 - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)
]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*
x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I
*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.43, size = 394, normalized size = 1.85

$$-\sqrt{d}\ \ln\left(\frac{2d+2\sqrt{d}\ \sqrt{-c^2d\,x^2+d}}{x}\right)a+a\sqrt{-c^2d\,x^2+d}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\ \text{arccosh}\,(cx)\,x^2c^2}{(cx+1)\,(cx-1)}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}}{\sqrt{cx+1}\ \sqrt{cx-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] -d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)*a+a*(-c^2*d*x^2+d)^(1/2
)+b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2*c^2-b*(-d*(c^2*
x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-b*(-d*(c^2*x^2-1))^(1/2)/(c*x
+1)/(c*x-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^
2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\,\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\sqrt{-c^2dx^2+d}\right)a+b\int\frac{\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c
^2*d*x^2 + d))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\,\mathrm{acosh}\,(cx))}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x,x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x, x)
```

3.69 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=235

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{c^2\sqrt{d-c^2dx^2}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibc^2\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(-i\right.}{2\sqrt{cx-1}\sqrt{cx+}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2-1/2*b*c*(-c^2*d*x^2+d)^(1/
2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+c^2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I
*b*c^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5738, 30, 5761, 4180, 2279, 2391}

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2
)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((I/2)*b*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[
c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d-c\right.}{2\sqrt{}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(c^2\sqrt{d-c^2dx\right.}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{c^2\sqrt{d-c^2dx^2}}{}$$

Mathematica [A] time = 1.10, size = 307, normalized size = 1.31

$$\frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{x^2} + ac^2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right) - ac^2\sqrt{d}\,\log(x) + \frac{bd(cx+1)\left(ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\,\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(cx}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*Log[
d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c
*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
 c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.71, size = 438, normalized size = 1.86

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{2d\,x^2}+\frac{a\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)c^2}{2}-\frac{a\sqrt{-c^2d\,x^2+d}\,c^2}{2}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,c^2}{2\,(cx+1)\,(cx-1)}-\frac{b\sqrt{}}{2x\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/2*a*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x
^2+d)^(1/2))/x)*c^2-1/2*a*(-c^2*d*x^2+d)^(1/2)*c^2-1/2*b*(-d*(c^2*x^2-1))^(
1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/x/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/x^2/(c*x+1)/(c*x-1)*ar
ccosh(c*x)-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-
1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(c^2\sqrt{d}\,\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\sqrt{-c^2dx^2+d}\,c^2-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^2}\right)a+b\int\frac{\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{}\right.}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a + b*integrate(
sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\sqrt{d - c^2\,d\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a + b\,\mathrm{acosh}\,(cx)\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**3, x)
```

3.70 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=315

$$\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4}+\frac{c^4\sqrt{d-c^2dx^2}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cos\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^4+1/8*c^2*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)/x^2-1/12*b*c*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(
c*x+1)^(1/2)+1/8*b*c^3*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/4*c^4*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x
^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*I*b*c^4*polylog(2,-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/8*I*b*c^4*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.74, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5738, 30, 5748, 5761, 4180, 2279, 2391}

$$-\frac{ibc^4\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{ibc^4\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2\sqrt{d-c^2dx^2}\left(a+b\,\text{co}\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -(b*c*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*S
qrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(4*x^4) + (c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x]))/(8*x^2) + (c^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^Arc
Cosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/8)*b*c^4*Sqrt[d - c^2*d*
x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((I/
8)*b*c^4*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*
Sqrt[1 + c*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{1}{x^4}\,dx}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(c^2\sqrt{d\right.}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{c^2\sqrt{d-}}{}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\right.}{4x^4}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\right.}{4x^4}$$

Mathematica [A] time = 1.05, size = 290, normalized size = 0.92

$$\frac{1}{24}\left(-3ac^4\sqrt{d}\log(x) + \frac{3a\left(c^2x^2-2\right)\sqrt{d-c^2dx^2}}{x^4} + 3ac^4\sqrt{d}\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) + \frac{b\sqrt{d-c^2dx^2}\left(-3ic^4x\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] ((3*a*(-2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^4 - 3*a*c^4*Sqrt[d]*Log[x] + 3*
a*c^4*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*Sqrt[d - c^2*d*x^2]
*(-2*c*x + 3*c^3*x^3 - 6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
+ 3*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - (3*I)*c^4*x
^4*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) - (
3*I)*c^4*x^4*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]
])))/(x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.75, size = 541, normalized size = 1.72

$$-\frac{a\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4d\,x^{4}}-\frac{a\,c^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{8d\,x^{2}}+\frac{a\,c^{4}\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^{2}d\,x^{2}+d}}{x}\right)}{8}-\frac{a\,c^{4}\sqrt{-c^{2}d\,x^{2}+d}}{8}+\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\mathrm{arc}}{8\left(cx+1\right)\left(cx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(3/2)-1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(3/2)+1/8*
a*c^4*d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-1/8*a*c^4*(-c^2*d*
x^2+d)^(1/2)+1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+
1/8*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-3/8*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x+1)/x^4/(c*x-1)*arccosh(c*x)-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4-1/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4+1
/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(c^{4}\sqrt{d}\,\log\left(\frac{2\sqrt{-c^{2}dx^{2}+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\sqrt{-c^{2}dx^{2}+d}\,c^{4}-\frac{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}}{dx^{2}}-\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}{dx^{4}}\right)a+b\int\frac{\sqrt{-c^{2}d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^5,x, algorithm="maxima"
)
```

```
[Out] 1/8*(c^4*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^4 - (-c^2*d*x^2 + d)^(3/2)*c^2/(d*x^2) - 2*(-c^2*d*x
^2 + d)^(3/2)/(d*x^4))*a + b*integrate(sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{d-c^{2}\,d\,x^{2}}}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{x^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(-c**2*d*x**2+d)**(1/2)/x**5,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))/x**5, x)
```

3.71 $\int x^4 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=360

$$\frac{1}{8}x^5\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{64c^2}$$

```
[Out] 1/8*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-3/128*d*x*(a+b*arccosh(c*x)
)*(-c^2*d*x^2+d)^(1/2)/c^4-1/64*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/
2)/c^2+1/16*d*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/256*b*d*x^2*(-c
^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/256*b*d*x^4*(-c^2*d*x^2
+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*b*c*d*x^6*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/64*b*c^3*d*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-3/256*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^5
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 372, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14}

$$\frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^5(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx^3\sqrt{d-c^2dx^2}\left(a+b\text{c}\right.}{64c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d*x^2*Sqrt[d - c^2*d*x^2])/(256*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d*x^4*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*x
^6*Sqrt[d - c^2*d*x^2])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^8*Sq
rt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d*x*Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^4) - (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(64*c^2) + (d*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
16 + (d*x^5*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8
 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(256*b*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^4(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d-c^2dx\right.}{}$$

$$= \frac{1}{16}dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^5(1-cx)(1+cx)\sqrt{d-c^2}$$

$$= -\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2dx^2}}{64\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{dx^3\sqrt{d-c^2dx^2}\,(a}{64c}$$

$$= \frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c^2dx^2}}{32\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^8\sqrt{d-c^2d}}{64\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{3bdx^2\sqrt{d-c^2dx^2}}{256c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^4\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{bcdx^6\sqrt{d-c}}{32\sqrt{-1+cx}\,\sqrt{}}$$

Mathematica [A] time = 4.82, size = 337, normalized size = 0.94

$$\frac{d\left(-1728a\sqrt{d}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-576acx\left(16c^6x^6-24c^4x^4+2c^2x^2+3\right)\sqrt{d-c^2dx^2}+\frac{32b\sqrt{d-c^2dx^2}\left(-72\cosh^{-1}(cx)^2\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-576*a*c*x*Sqrt[d - c^2*d*x^2]*(3 + 2*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
) - 1728*a*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2)
)] + (32*b*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]
] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*S
inh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqr
t[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d - c^2*d*x^2]*(1440*ArcCosh[c
*x]^2 - 576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*Arc
Cosh[c*x]] + 9*Cosh[8*ArcCosh[c*x]] - 24*ArcCosh[c*x]*(-48*Sinh[2*ArcCosh[c
*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8*ArcCosh
[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/(73728*c^5)
```

fricas [F] time = 1.07, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(ac^2dx^6-adx^4+\left(bc^2dx^6-bdx^4\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^6 - a*d*x^4 + (b*c^2*d*x^6 - b*d*x^4)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^4, x)
```

maple [A] time = 0.62, size = 561, normalized size = 1.56

$$-\frac{a\,x^{3}\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{8c^{2}d}-\frac{ax\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{16c^{4}d}+\frac{ax\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{64c^{4}}+\frac{3adx\sqrt{-c^{2}d\,x^{2}+d}}{128c^{4}}+\frac{3a\,d^{2}\arctan\left(\frac{\sqrt{c^{2}d}\,x}{\sqrt{-c^{2}d\,x^{2}+d}}\right)}{128c^{4}\sqrt{c^{2}d}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*a*x^3*(-c^2*d*x^2+d)^(5/2)/c^2/d-1/16*a/c^4*x*(-c^2*d*x^2+d)^(5/2)/d+1
/64*a/c^4*x*(-c^2*d*x^2+d)^(3/2)+3/128*a/c^4*d*x*(-c^2*d*x^2+d)^(1/2)+3/128
*a/c^4*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/64*
b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^8-1/32*b*(-d*(
c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^6+1/256*b*(-d*(c^2*x^2-
1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^4+3/256*b*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*x^2-3/256*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*d-1/8*b*(-d*(c^2*x^2-1))^(1/2)*d
/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^9+5/16*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+
1)*c^2/(c*x-1)*arccosh(c*x)*x^7-13/64*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c
*x-1)*arccosh(c*x)*x^5-1/128*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)
*arccosh(c*x)*x^3+3/128*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/c^4/(c*x-1)*arcc
osh(c*x)*x-15/8192*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^5/(c*x-1)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{128}\left(\frac{16\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{3}}{c^{2}d}-\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x}{c^{4}}+\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x}{c^{4}d}-\frac{3\sqrt{-c^{2}dx^{2}+d}\,dx}{c^{4}}-\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c^{5}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/128*(16*(-c^2*d*x^2 + d)^(5/2)*x^3/(c^2*d) - 2*(-c^2*d*x^2 + d)^(3/2)*x/
c^4 + 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^4*d) - 3*sqrt(-c^2*d*x^2 + d)*d*x/c^4 -
 3*d^(3/2)*arcsin(c*x)/c^5)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*x^4*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{4} \left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

3.72 $\int x^2 \left(d - c^2 d x^2\right)^{3/2} \left(a + b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=281

$$-\frac{dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c^2}+\frac{1}{6}x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.$$

```
[Out] 1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/16*d*x*(a+b*arccosh(c*x))
*(-c^2*d*x^2+d)^(1/2)/c^2+1/8*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
+1/32*b*d*x^2*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/96*b*c*d
*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*c^3*d*x^6*(-c^
2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*(a+b*arccosh(c*x))^2*(-
c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 293, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14}

$$\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}dx^3(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{dx\sqrt{d-c^2dx^2}\left(a+b\right.}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x^2*Sqrt[d - c^2*d*x^2])/(32*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c*
d*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^6
*Sqrt[d - c^2*d*x^2])/(36*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/8 + (d*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x]))/6 - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
```

```
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{\left(d\sqrt{d-c^2dx^2}\right.}{}$$

$$= \frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2d}$$

$$= -\frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+cx}} - \frac{dx\sqrt{d-c^2dx^2}\left(a\right.}{16c}$$

$$= \frac{bdx^2\sqrt{d-c^2dx^2}}{32c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{7bcdx^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}}{36\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 1.99, size = 270, normalized size = 0.96

$$\frac{d\left(-144a\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-48acx\left(8c^4x^4-14c^2x^2+3\right)\sqrt{d-c^2dx^2}-\frac{18b\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}\right.\right.}{\sqrt{\frac{cx-1}{cx+1}}\left(c\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d*(-48*a*c*x*Sqrt[d - c^2*d*x^2]*(3 - 14*c^2*x^2 + 8*c^4*x^4) - 144*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - (18*b*Sqrt
[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*S
inh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*Sqrt[d -
c^2*d*x^2]*(72*ArcCosh[c*x]^2 - 18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[
c*x]] + 2*Cosh[6*ArcCosh[c*x]] - 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] +
 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))))/(2304*c^3)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dx^4-adx^2+\left(bc^2dx^4-bdx^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^4 - a*d*x^2 + (b*c^2*d*x^4 - b*d*x^2)*arccosh(c*x))*sq
rt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*x^2, x)
```

maple [A] time = 0.46, size = 456, normalized size = 1.62

$$-\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{6c^2d}+\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{24c^2}+\frac{adx\sqrt{-c^2d\,x^2+d}}{16c^2}+\frac{a\,d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16c^2\sqrt{c^2d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\text{arcco}}{32\sqrt{cx-1}\,\sqrt{cx+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/6*a*x*(-c^2*d*x^2+d)^(5/2)/c^2/d+1/24*a/c^2*x*(-c^2*d*x^2+d)^(3/2)+1/16*
a/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a/c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d+1/36*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(
1/2)*c^3/(c*x-1)^(1/2)*x^6-7/96*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c/
(c*x-1)^(1/2)*x^4+1/32*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(
1/2)*x^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^
7+11/24*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-17/
48*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/16*b*(-d*(
```

```
c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+7/2304*b*(-d*(c^2*x^
2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}a\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2}-\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^2d}+\frac{3\sqrt{-c^2dx^2+d}\,dx}{c^2}+\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c^3}\right)+b\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2\log\Big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] 1/48*a*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*d)
 + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + b*integrat
e((-c^2*d*x^2 + d)^(3/2)*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2}\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

3.73 $\int \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=200

$$\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+3/8*d*x*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)-5/16*b*c*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/16*b*c^3*d*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3
/16*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(
1/2)
```

Rubi [A] time = 0.32, antiderivative size = 212, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5713, 5685, 5683, 5676, 30, 14}

$$\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{16bc\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*b*c*d*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (d*x*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (3*d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^2)/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
```

```
&& GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{\left(3d\sqrt{d-c^2dx^2}\right)}{}$$

$$= \frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\ ($$

$$= -\frac{5bcdx^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bc^3dx^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3}{8}dx\sqrt{d-c^2dx^2}\ (a+$$

Mathematica [A] time = 1.27, size = 235, normalized size = 1.18

$$-\frac{3ad^{3/2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{8c}-\frac{1}{8}adx\left(2c^2x^2-5\right)\sqrt{d-c^2dx^2}-\frac{bd\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)\right.}{8c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(a*d*x*(-5 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (3*a*d^(3/2)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/(8*c) - (b*d*Sqrt[d - c^2
*d*x^2]*(2*ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*Ar
cCosh[c*x]]))/(8*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b*d*Sqrt[d - c^
2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*A
rcCosh[c*x]]))/(128*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dx^2-ad+\left(bc^2dx^2-bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.25, size = 344, normalized size = 1.72

$$\frac{ax\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{4}+\frac{3adx\sqrt{-c^2 d\,x^2+d}}{8}+\frac{3a\,d^2\arctan\left(\frac{\sqrt{c^2 d}\,x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8\sqrt{c^2 d}}-\frac{3b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}\left(cx\right)^2 d}{16\sqrt{cx-1}\,\sqrt{cx+1}\,c}+\frac{b\sqrt{-d}}{16\sqrt{}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*x*(-c^2*d*x^2+d)^(3/2)+3/8*a*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a*d^2/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/16*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d+1/16*b*(-d*(c^2*x^2
-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-5/16*b*(-d*(c^2*x^2-1))^(1
/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x
+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+7/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*
x-1)*c^2*arccosh(c*x)*x^3-5/8*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*ar
ccosh(c*x)*x+17/128*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/
c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x+3\sqrt{-c^2dx^2+d}\,dx+\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c}\right)a+b\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

3.74 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=197

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}$$

```
[Out] -(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x-3/2*c^2*d*x*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)+1/4*b*c^3*d*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+3/4*c*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+b*c*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 209, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5740, 5683, 5676, 30, 14}

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{3cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}-\frac{d(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a\ \cdots\right.}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*c^3*d*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2
*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (d*(1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (3*c*d*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])^2)/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*Sqrt[d - c
^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)}{\sqrt{-1+cx}\,\sqrt{1}}$$
$$= -\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{x}$$
$$= \frac{bc^3dx^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx}{}$$

Mathematica [A] time = 1.08, size = 223, normalized size = 1.13

$$\frac{1}{8}\left(12acd^{3/2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - \frac{4ad\left(c^2x^2+2\right)\sqrt{d-c^2dx^2}}{x} + 4bcd\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)} - \right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-4*a*d*(2 + c^2*x^2)*Sqrt[d - c^2*d*x^2])/x + 12*a*c*d^(3/2)*ArcTan[(c*x*
Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 4*b*c*d*Sqrt[d - c^2*d*x^2
]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))) + (b*c*d*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[c*x]^2 + Co
sh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)))/8
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ac^2dx^2-ad+\left(bc^2dx^2-bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.52, size = 427, normalized size = 2.17

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{dx}-a\,c^2x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}-\frac{3a\,c^2dx\sqrt{-c^2d\,x^2+d}}{2}-\frac{3a\,c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+\frac{3b\sqrt{-d\left(c^2x^2-\right.}}{4\sqrt{cx-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(5/2)-a*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a*c^2*d*x*(-c^
2*d*x^2+d)^(1/2)-3/2*a*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))+3/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arc
cosh(c*x)^2*c*d-1/2*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(
c*x)*x^3+1/4*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2
-b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-1/2*
b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x-1/8*b*(-d*(c^
2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*d/(c*x+1)/(c*x-1)/x+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\sqrt{-c^2dx^2+d}\,c^2dx+3\,cd^{\frac{3}{2}}\arcsin\left(cx\right)+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{x}\right)a+b\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\log\left(cx+\sqrt{cx+1}\sqrt{cx-}\right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*
x^2 + d)^(3/2)/x)*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**2, x)
```

3.75 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=203

$$\frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}-\cdots$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3+c^2*d*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/x-1/6*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/2*c^3*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-4/3*b*c^3*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 215, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5740, 5738, 29, 5676, 14}

$$-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cdots\right.}{3x^3}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x - (d*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (c^3*d*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^3*d*Sqrt[
d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
```

```
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)}{3\sqrt{-1+cx}\,\sqrt{}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+\right.}{3x^3}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} - \frac{d(1-c}{}$$

Mathematica [A] time = 0.84, size = 259, normalized size = 1.28

$$\frac{d^2\left(-2a\sqrt{\frac{cx-1}{cx+1}}\left(4c^4x^4-5c^2x^2+1\right)+8bc^3x^3(cx-1)\log(cx)+bcx(cx-1)\right)-6ac^3d^{3/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan^{-1}}{6x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4)*ArcCosh[c*
x] + 3*b*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 6*a*c^3*d^(3/2)*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))] + d^2*(b*c*x*(-1 + c*x) - 2*a*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 - 5*c^2*x^2 + 4*c^4*x^4) + 8*b*c^3*x^3*(-1 + c*x)*Log[c*x]))/(6*x
^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 1181, normalized size = 5.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(5/2)+2/3*a*
c^4*x*(-c^2*d*x^2+d)^(3/2)+a*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a*c^4*d^2/(c^2*d)
^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*d*c^3+8/3*b*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^3-32*b*(-d*(c^2*x^2-1
))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^7+32*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^8-8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^3*c^6+8/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5
/(c*x+1)/(c*x-1)*c^8+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)
*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-52*b*(-d*(c^2*x^2-1))^(1/
2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+2/3*b*(-
d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*c^4-4*b*(-d*(c^2*x^2-1))^
(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-10/3*b
*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6-
4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)*c^3+73/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+3/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(
24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+2/3*b*(-d*(c^2*x^2-
1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-14/3*b*(-d*(c^2*
x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^3
/(c*x+1)/(c*x-1)*arccosh(c*x)-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(3\sqrt{-c^2dx^2 + d}\,c^4dx + 3\,c^3d^{\frac{3}{2}}\arcsin\left(cx\right) + \frac{2\left(-c^2dx^2 + d\right)^{\frac{3}{2}}c^2}{x} - \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}{dx^3}\right)a + b\int\frac{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a + b*integrate((-c
^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}\,(a + b\,\mathrm{acosh}\,(cx))}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**4, x)
```

3.76 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=166

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5dx^5}-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^5d\log(x)\sqrt{d-c^2dx^2}}{5\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5-1/20*b*c*d*(-c^2*d*x^2+d
)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5*b*c^5*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 179, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$-\frac{d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc^5d\log(x}{5\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3
*d*Sqrt[d - c^2*d*x^2])/(5*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)
^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (b*c^5*d
*Sqrt[d - c^2*d*x^2]*Log[x])/(5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right.}{5\sqrt{-1+c}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right.}{10\sqrt{}}$$

$$= -\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd\sqrt{d-c^2dx^2}\right.}{1}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{5x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{d(1-cx)^2(1+cx)}{}$$

Mathematica [A] time = 0.08, size = 94, normalized size = 0.57

$$-\frac{d\sqrt{d-c^2dx^2}\left(4(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+bcx\left(-4c^4x^4\log(x)-4c^2x^2+1\right)\right)}{20x^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -1/20*(d*Sqrt[d - c^2*d*x^2]*(4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*Arc
Cosh[c*x]) + b*c*x*(1 - 4*c^2*x^2 - 4*c^4*x^4*Log[x])))/(x^5*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

fricas [A] time = 0.62, size = 572, normalized size = 3.45

$$\left[-\frac{4\left(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-2\left(bc^7dx^7-bc^5dx^5\right)\sqrt{-d}\,\log\left(\frac{c^2dx^6}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] [-1/20*(4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(-
d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 -
 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) - (4*b*c^3*d*x^3 - (4*b*c^3 -
b*c)*d*x^5 - b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 4*(a*c^6*d*x
^6 - 3*a*c^4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 -
x^5), 1/20*(4*(b*c^7*d*x^7 - b*c^5*d*x^5)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 +
d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
 4*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x^2 + d)
```

```
*log(c*x + sqrt(c^2*x^2 - 1)) + (4*b*c^3*d*x^3 - (4*b*c^3 - b*c)*d*x^5 - b*
c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(a*c^6*d*x^6 - 3*a*c^4*d*
x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^7 - x^5)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.88, size = 2171, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x)
```

```
[Out] 3/10*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)*x^3*c^8-1/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)*x*c^6+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10
*c^4*x^4-5*c^2*x^2+1)*x^7*c^12-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-1
0*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5*c^10+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^
8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*ar
ccosh(c*x)*c^13-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x
^4-5*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+2*b*(-d*(
c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^4/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x
^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*c^7-56/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+28/5*b*(-d*(c^2*x^2-1))^(
1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arcc
osh(c*x)*c^4-8/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^
4-5*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-b*(-d*(c^2*x^2-1))^(1/2
)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^14+1/5*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d*c^5-2/5*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^5-3/2*b*(-d*(c^2*x^2-1))^(1/2)*d/
(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^5-1/5*a/d/x^5*(-c^2*d*x^2+d)^(5/2)+b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-1
0*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11-9/4*
b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^
4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8
-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+9/2
0*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/
x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*
x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/
5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/
(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+7/20*b*(-d*(c^2*x^2-1))^(1/2)*
d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^8-1/2
0*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*
x/(c*x+1)/(c*x-1)*c^6+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+
10*c^4*x^4-5*c^2*x^2+1)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)-1/5*b*(-d*(c^2*x^2
-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^9/(c*x+1)/(c*x
-1)*c^14+13/20*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-
```

```
5*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*c^12-3/4*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8
*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c^10+5*b*(-d*(c
^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^7/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^12-11*b*(-d*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6
*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+14*b*(-d
*(c^2*x^2-1))^(1/2)*d/(5*c^8*x^8-10*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*x^3/(c*
x+1)/(c*x-1)*arccosh(c*x)*c^8
```

maxima [C] time = 0.46, size = 189, normalized size = 1.14

$$-\frac{\left(2\,c^{6}d^{3}\sqrt{-\frac{1}{c^{4}d}}\,\log\left(x^{2}-\frac{1}{c^{2}}\right)+2i\,(-1)^{-2\,c^{2}dx^{2}+2\,d}\,c^{4}d^{\frac{5}{2}}\log\left(-2\,c^{2}d+\frac{2\,d}{x^{2}}\right)+\frac{3\,\sqrt{-c^{4}dx^{4}+2\,c^{2}dx^{2}-d}\,c^{2}d^{2}}{x^{2}}-\frac{\sqrt{-c^{4}dx^{4}+2\,c^{2}dx^{2}-}}{x^{4}}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/20*(2*c^6*d^3*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 2*I*(-1)^(-2*c^2*d*x^2
 + 2*d)*c^4*d^(5/2)*log(-2*c^2*d + 2*d/x^2) + 3*sqrt(-c^4*d*x^4 + 2*c^2*d*x
^2 - d)*c^2*d^2/x^2 - sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^2/x^4)*b*c/d - 1
/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(d*x^5) - 1/5*(-c^2*d*x^2 + d)^(5/
2)*a/(d*x^5)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^{2}\,d\,x^{2}\right)^{3/2}}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^6, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^{6}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**6, x)
```

3.77 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=247

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{35dx^5}-\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^7d\log(x}{35\sqrt{cx-}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^7-2/35*c^2*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))/d/x^5-1/42*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+2/35*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/70*b*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+2/35*b*c^7*d*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 322, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5798, 97, 12, 103, 95, 5733, 446, 76}

$$-\frac{2c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x}-\frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^3}+\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(35*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(70*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*x^5) - (c^4*d*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(35*x^3) - (2*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(35*x) - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x]))/(7*x^7) + (2*b*c^7*d*Sqrt[d - c^2*d*x^2]*Log[x])/(35*Sqrt[-1 + c*x]
*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
```

```
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{35x^3}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35x^5} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{35x^3}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3d\sqrt{d-c^2dx^2}}{35x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d\sqrt{d-}}{70x^2\sqrt{-1+c}}$$

Mathematica [A] time = 0.15, size = 136, normalized size = 0.55

$$-\frac{d\sqrt{d-c^2dx^2}\left(12c^2x^2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+30(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+b\right.}{210x^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -1/210*(d*Sqrt[d - c^2*d*x^2]*(30*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*A
rcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c
*x]) + b*c*x*(5 - 12*c^2*x^2 + 3*c^4*x^4 - 12*c^6*x^6*Log[x])))/(x^7*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 1.46, size = 648, normalized size = 2.62

$$\left[-\frac{6\left(2\,bc^8dx^8-bc^6dx^6-9\,bc^4dx^4+13\,bc^2dx^2-5\,bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-6\left(bc^9dx^9-bc^7dx^7\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [-1/210*(6*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 -
5*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(b*c^9*d*x^9 -
 b*c^7*d*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (3*b*c^5
*d*x^5 - (3*b*c^5 - 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*
a*c^4*d*x^4 + 13*a*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)
, 1/210*(12*(b*c^9*d*x^9 - b*c^7*d*x^7)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)
*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 6
*(2*b*c^8*d*x^8 - b*c^6*d*x^6 - 9*b*c^4*d*x^4 + 13*b*c^2*d*x^2 - 5*b*d)*sqr
```

```
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - (3*b*c^5
- 12*b*c^3 + 5*b*c)*d*x^7 - 12*b*c^3*d*x^3 + 5*b*c*d*x)*sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1) - 6*(2*a*c^8*d*x^8 - a*c^6*d*x^6 - 9*a*c^4*d*x^4 + 13*a
*c^2*d*x^2 - 5*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.90, size = 3144, normalized size = 12.73

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x)
```

```
[Out] -116/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154
*c^4*x^4-105*c^2*x^2+25)*x^5*c^12+359/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^1
0*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^7-4/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arc
cosh(c*x)*d*c^7+2/35*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d*c^7-2/35*a*c^2/d/x^5*(-c^2*d*x^2
+d)^(5/2)-25/42*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*
x^6+154*c^4*x^4-105*c^2*x^2+25)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+2/35*b*(-
d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*
c^2*x^2+25)*x^13/(c*x+1)/(c*x-1)*c^20-9/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c
^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^11/(c*x+1)/(c*
x-1)*c^18-1/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x
^6+154*c^4*x^4-105*c^2*x^2+25)*x^9/(c*x+1)/(c*x-1)*c^16+142/105*b*(-d*(c^2*
x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2
+25)*x^7/(c*x+1)/(c*x-1)*c^14-72/35*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^1
0-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/(c*x+1)/(c*x-1)*c^1
2+25/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*
c^4*x^4-105*c^2*x^2+25)*x^3/(c*x+1)/(c*x-1)*c^10-5/21*b*(-d*(c^2*x^2-1))^(1
/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x/(c*
x+1)/(c*x-1)*c^8+25/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-7
0*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)-1/2*
b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-
105*c^2*x^2+25)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15+5/2*b*(-d*(c^2*x^2-1))
^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^
6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^13-11/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10
*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c*x+1)^(1/2)/(
c*x-1)^(1/2)*c^11-161/30*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^
8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^
9+10/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c
^4*x^4-105*c^2*x^2+25)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-421/42*
b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-
105*c^2*x^2+25)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+55/14*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x
^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^
10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^10/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*arccosh(c*x)*c^17-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*
```

```
c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*arccosh(c*x)*c^15-4*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8
-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arc
cosh(c*x)*c^13+44/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*
c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh
(c*x)*c^11-6*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6
+154*c^4*x^4-105*c^2*x^2+25)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c
^9-2*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4
*x^4-105*c^2*x^2+25)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18+3*b*(-d*(c^2*x^
2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+2
5)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16+12*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*
c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7/(c*x+1)/(c*
x-1)*arccosh(c*x)*c^14-164/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^
8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^5/(c*x+1)/(c*x-1)*arccosh(c*
x)*c^12+52/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6
+154*c^4*x^4-105*c^2*x^2+25)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+1966/35*
b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-
105*c^2*x^2+25)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-3272/35*b*(-d*(c^2*x^2-1
))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/
x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+472/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^1
0*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^3/(c*x+1)/(c*x-1
)*arccosh(c*x)*c^4-170/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^
8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^2-1/7*a/d/x^7*(-c^2*d*x^2+d)^(5/2)+26/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c
^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^7*c^14+20/21*b
*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-1
05*c^2*x^2+25)*x^3*c^10-5/21*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^
8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x*c^8-2/35*b*(-d*(c^2*x^2-1))^
(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+154*c^4*x^4-105*c^2*x^2+25)*x^1
1*c^18+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(35*c^10*x^10-35*c^8*x^8-70*c^6*x^6+1
54*c^4*x^4-105*c^2*x^2+25)*x^9*c^16
```

maxima [A] time = 0.47, size = 163, normalized size = 0.66

$$\frac{1}{210}\left(12\,c^6\sqrt{-d}\,d\log(x)-\frac{3\,c^4\sqrt{-d}\,dx^4-12\,c^2\sqrt{-d}\,dx^2+5\,\sqrt{-d}\,d}{x^6}\right)bc-\frac{1}{35}\,b\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^5}+\frac{5\left(-c^2dx^2+\right.}{dx^7}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] 1/210*(12*c^6*sqrt(-d)*d*log(x) - (3*c^4*sqrt(-d)*d*x^4 - 12*c^2*sqrt(-d)*d
*x^2 + 5*sqrt(-d)*d)/x^6)*b*c - 1/35*b*(2*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^5
) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))*arccosh(c*x) - 1/35*a*(2*(-c^2*d*x^2
+ d)^(5/2)*c^2/(d*x^5) + 5*(-c^2*d*x^2 + d)^(5/2)/(d*x^7))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**8,x)
```

```
[Out] Timed out
```

3.78 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx$

Optimal. Leaf size=328

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{4c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{8c^4\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{315dx^5}$$

```
[Out] -1/9*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^9-4/63*c^2*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))/d/x^7-8/315*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(
c*x))/d/x^5-1/72*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)
+5/189*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/420*b
*c^5*d*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/315*b*c^7*d*(
-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/315*b*c^9*d*ln(x)*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.51, antiderivative size = 401, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1251, 893}

$$-\frac{8c^8d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315x}-\frac{4c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{315x^3}-\frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
*Sqrt[d - c^2*d*x^2])/(420*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^7*d*S
qrt[d - c^2*d*x^2])/(315*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(21*x^7) - (c^4*d*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(105*x^5) - (4*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x]))/(315*x^3) - (8*c^8*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(315
*x) - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*x
^9) + (8*b*c^9*d*Sqrt[d - c^2*d*x^2]*Log[x])/(315*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{105x^5}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^7} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{105x^5}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d\sqrt{d}}{420x^4\sqrt{-1+}}$$

Mathematica [A] time = 0.33, size = 154, normalized size = 0.47

$$-\frac{d\sqrt{d-c^2dx^2}\left(96c^2x^2(cx-1)^{5/2}\left(2c^2x^2+5\right)(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+840(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\,c\right.}{7560x^9\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -1/7560*(d*Sqrt[d - c^2*d*x^2]*(840*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b
*ArcCosh[c*x]) + 96*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(5 + 2*c^2*x^2
)*(a + b*ArcCosh[c*x]) + b*c*x*(105 - 200*c^2*x^2 + 18*c^4*x^4 + 48*c^6*x^6
 - 192*c^8*x^8*Log[x])))/(x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.67, size = 720, normalized size = 2.20

$$\left[-\frac{24\left(8\,bc^{10}dx^{10}-4\,bc^8dx^8-bc^6dx^6-53\,bc^4dx^4+85\,bc^2dx^2-35\,bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-9\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [-1/7560*(24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x^
4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
1)) - 96*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2
- d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c
^2*x^4 - x^2)) + (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 -
200*b*c^3 + 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2
 + d)*sqrt(c^2*x^2 - 1) + 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6
 - 53*a*c^4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^1
1 - x^9), 1/7560*(192*(b*c^11*d*x^11 - b*c^9*d*x^9)*sqrt(d)*arctan(sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x
```

```
^2 + d)) - 24*(8*b*c^10*d*x^10 - 4*b*c^8*d*x^8 - b*c^6*d*x^6 - 53*b*c^4*d*x
^4 + 85*b*c^2*d*x^2 - 35*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (48*b*c^7*d*x^7 + 18*b*c^5*d*x^5 - (48*b*c^7 + 18*b*c^5 - 200*b*c^3
+ 105*b*c)*d*x^9 - 200*b*c^3*d*x^3 + 105*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 24*(8*a*c^10*d*x^10 - 4*a*c^8*d*x^8 - a*c^6*d*x^6 - 53*a*c^
4*d*x^4 + 85*a*c^2*d*x^2 - 35*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.04, size = 4259, normalized size = 12.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x)
```

```
[Out] -35/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2
730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x*c^10-128/315*b*(-d*(c^2*x^2-1
))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x
^4-4725*c^2*x^2+1225)*x^15*c^24-16/45*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*
x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)
*x^13*c^22+1384/945*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10
+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^11*c^20+2306/94
5*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*
c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^9*c^18-40/63*b*(-d*(c^2*x^2-1))^(
1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4
725*c^2*x^2+1225)*x^7*c^16-2189/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^
12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x
^5*c^14+350/27*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*
c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3*c^12-16/315*b*(-d*
(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^9+8/315*b*(
-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2)*d*c^9-30055/504*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-
945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*c^9-43264/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x
^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*
x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^10+113594/63*b*(-d*(c^2*x^2-1))^(1/2)*d/(8
40*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x
^2+1225)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-174520/63*b*(-d*(c^2*x^2-1))^(1
/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-47
25*c^2*x^2+1225)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+19540/9*b*(-d*(c^2*x^
2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^
4*x^4-4725*c^2*x^2+1225)/x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-7700/9*b*(-d*
(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+
6210*c^4*x^4-4725*c^2*x^2+1225)/x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-4/63*a
*c^2/d/x^7*(-c^2*d*x^2+d)^(5/2)-829/56*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225
)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^11+25915/126*b*(-d*(c^2*x^2-1))^(1/2)*d
/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^
2*x^2+1225)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+1225/9*b*(-d*(c^2*x^2-1))^(
```

```
1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4
725*c^2*x^2+1225)/x^9/(c*x+1)/(c*x-1)*arccosh(c*x)-1225/72*b*(-d*(c^2*x^2-1
))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x
^4-4725*c^2*x^2+1225)/x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-16/3*b*(-d*(c^2*x^2
-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4
*x^4-4725*c^2*x^2+1225)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^19+4*b*(-d*(c^2*
x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*
c^4*x^4-4725*c^2*x^2+1225)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^17+4189/180*b*
(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*
x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^15-11
87/60*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2
730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*c^13-1285/6*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^
8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^4/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*c^5+21175/216*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^
10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/x^6/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*c^3+280/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^
10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9+128/315*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*
c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+
1225)*x^17/(c*x+1)/(c*x-1)*c^26-16/315*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225
)*x^15/(c*x+1)/(c*x-1)*c^24-344/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^
12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x
^13/(c*x+1)/(c*x-1)*c^22-922/945*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-
945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^11
/(c*x+1)/(c*x-1)*c^20+2906/945*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-94
5*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^9/(c
*x+1)/(c*x-1)*c^18+2069/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c
^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^7/(c*x+
1)/(c*x-1)*c^16-4639/189*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10
*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5/(c*x+1)/
(c*x-1)*c^14+455/27*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10
+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x+1)/(c*x-
1)*c^12-35/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^
8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x/(c*x+1)/(c*x-1)*c^10+6
4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-273
0*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^12/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
arccosh(c*x)*c^21-24*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^1
0+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^10/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*arccosh(c*x)*c^19+24/5*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^1
2*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+122
5)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17-208/3*b*(-d*(c^2*x^2-1
))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x
^4-4725*c^2*x^2+1225)*x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15+110
4/7*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-273
0*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*a
rccosh(c*x)*c^13-120*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^1
0+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^2/(c*x+1)^(1/2
)/(c*x-1)^(1/2)*arccosh(c*x)*c^11-64/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12
*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225
)*x^13/(c*x+1)/(c*x-1)*arccosh(c*x)*c^22+104/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(
840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*
x^2+1225)*x^11/(c*x+1)/(c*x-1)*arccosh(c*x)*c^20-212/15*b*(-d*(c^2*x^2-1))^
(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210*c^4*x^4-
4725*c^2*x^2+1225)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18+3151/15*b*(-d*(c^2
*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c^6*x^6+6210
*c^4*x^4-4725*c^2*x^2+1225)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-60632/105
*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8*x^8-2730*c
```

```
^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^1
4+59884/105*b*(-d*(c^2*x^2-1))^(1/2)*d/(840*c^12*x^12-945*c^10*x^10+189*c^8
*x^8-2730*c^6*x^6+6210*c^4*x^4-4725*c^2*x^2+1225)*x^3/(c*x+1)/(c*x-1)*arcco
sh(c*x)*c^12-8/315*a*c^4/d/x^5*(-c^2*d*x^2+d)^(5/2)-1/9*a/d/x^9*(-c^2*d*x^2
+d)^(5/2)
```

maxima [A] time = 0.73, size = 225, normalized size = 0.69

$$\frac{1}{7560}\left(192\,c^{8}\sqrt{-d}\,d\log(x)-\frac{48\,c^{6}\sqrt{-d}\,dx^{6}+18\,c^{4}\sqrt{-d}\,dx^{4}-200\,c^{2}\sqrt{-d}\,dx^{2}+105\,\sqrt{-d}\,d}{x^{8}}\right)bc-\frac{1}{315}\,b\left(\frac{8\left(-c^{2}dx^{2}+\right.}{dx^{5}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] 1/7560*(192*c^8*sqrt(-d)*d*log(x) - (48*c^6*sqrt(-d)*d*x^6 + 18*c^4*sqrt(-d
)*d*x^4 - 200*c^2*sqrt(-d)*d*x^2 + 105*sqrt(-d)*d)/x^8)*b*c - 1/315*b*(8*(-
c^2*d*x^2 + d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) +
35*(-c^2*d*x^2 + d)^(5/2)/(d*x^9))*arccosh(c*x) - 1/315*a*(8*(-c^2*d*x^2 +
d)^(5/2)*c^4/(d*x^5) + 20*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^7) + 35*(-c^2*d*x
^2 + d)^(5/2)/(d*x^9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^{2}\,d\,x^{2}\right)^{3/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^10, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**10,x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx$

Optimal. Leaf size=409

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{11dx^{11}}-\frac{2c^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{33dx^9}-\frac{16c^6\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(c\right.}{1155dx^5}$$

```
[Out] -1/11*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^11-2/33*c^2*(-c^2*d*x^2+d
)^(5/2)*(a+b*arccosh(c*x))/d/x^9-8/231*c^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccos
h(c*x))/d/x^7-16/1155*c^6*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/d/x^5-1/1
10*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/66*b*c^3*d
*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/1386*b*c^5*d*(-c^2*
d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/770*b*c^7*d*(-c^2*d*x^2+d)
^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/1155*b*c^9*d*(-c^2*d*x^2+d)^(1/2)/
x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+16/1155*b*c^11*d*ln(x)*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 480, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1799, 1620}

$$-\frac{16c^{10}d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155x}-\frac{8c^8d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{1155x^3}-\frac{2c^6d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{385x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c
^3*d*Sqrt[d - c^2*d*x^2])/(66*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d*
Sqrt[d - c^2*d*x^2])/(1386*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^7*d*Sqr
t[d - c^2*d*x^2])/(770*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c^9*d*Sqrt[
d - c^2*d*x^2])/(1155*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(33*x^9) - (c^4*d*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x]))/(231*x^7) - (2*c^6*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*
x]))/(385*x^5) - (8*c^8*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*x
^3) - (16*c^10*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*x) - (d*(1
 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(11*x^11) + (16
*b*c^11*d*Sqrt[d - c^2*d*x^2]*Log[x])/(1155*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{231x^7}$$

$$= \frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33x^9} - \frac{c^4d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{231x^7}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d\sqrt{d-c^2dx^2}}{66x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d\sqrt{}}{1386x^6\sqrt{-1}}$$

Mathematica [A] time = 0.45, size = 170, normalized size = 0.42

$$-\frac{d\sqrt{d-c^2dx^2}\left(12c^2x^2(cx-1)^{5/2}\left(8c^4x^4+20c^2x^2+35\right)(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+630(cx-1)^{5/2}(cx+1)\right.}{6930x^{11}\sqrt{cx-1}\,\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -1/6930*(d*Sqrt[d - c^2*d*x^2]*(630*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b
*ArcCosh[c*x]) + 12*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(35 + 20*c^2*x
^2 + 8*c^4*x^4)*(a + b*ArcCosh[c*x]) + b*c*x*(63 - 105*c^2*x^2 + 5*c^4*x^4
+ 9*c^6*x^6 + 24*c^8*x^8 - 96*c^10*x^10*Log[x])))/(x^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

fricas [A] time = 0.65, size = 792, normalized size = 1.94

$$\left[-\frac{6\left(16\,bc^{12}dx^{12}-8\,bc^{10}dx^{10}-2\,bc^8dx^8-bc^6dx^6-145\,bc^4dx^4+245\,bc^2dx^2-105\,bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [-1/6930*(6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x
^6 - 145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(
c*x + sqrt(c^2*x^2 - 1)) - 48*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(-d)*log(
(c^2*d*x^6 + c^2*d*x^2 - d*x^4 - sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^
4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (
24*b*c^9 + 9*b*c^7 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 -
 105*b*c^3*d*x^3 + 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 6*(
16*a*c^12*d*x^12 - 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^
4*d*x^4 + 245*a*c^2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11
```

```
), 1/6930*(96*(b*c^13*d*x^13 - b*c^11*d*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d
)) - 6*(16*b*c^12*d*x^12 - 8*b*c^10*d*x^10 - 2*b*c^8*d*x^8 - b*c^6*d*x^6 -
145*b*c^4*d*x^4 + 245*b*c^2*d*x^2 - 105*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) - (24*b*c^9*d*x^9 + 9*b*c^7*d*x^7 - (24*b*c^9 + 9*b*c^7
 + 5*b*c^5 - 105*b*c^3 + 63*b*c)*d*x^11 + 5*b*c^5*d*x^5 - 105*b*c^3*d*x^3 +
 63*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 6*(16*a*c^12*d*x^12 -
 8*a*c^10*d*x^10 - 2*a*c^8*d*x^8 - a*c^6*d*x^6 - 145*a*c^4*d*x^4 + 245*a*c^
2*d*x^2 - 105*a*d)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.24, size = 5518, normalized size = 13.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.71, size = 287, normalized size = 0.70

$$\frac{1}{6930}\left(96\,c^{10}\sqrt{-d}\,d\log(x)-\frac{24\,c^{8}\sqrt{-d}\,dx^{8}+9\,c^{6}\sqrt{-d}\,dx^{6}+5\,c^{4}\sqrt{-d}\,dx^{4}-105\,c^{2}\sqrt{-d}\,dx^{2}+63\,\sqrt{-d}\,d}{x^{10}}\right)bc-\frac{1}{1155}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] 1/6930*(96*c^10*sqrt(-d)*d*log(x) - (24*c^8*sqrt(-d)*d*x^8 + 9*c^6*sqrt(-d)
*d*x^6 + 5*c^4*sqrt(-d)*d*x^4 - 105*c^2*sqrt(-d)*d*x^2 + 63*sqrt(-d)*d)/x^1
0)*b*c - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*c^6/(d*x^5) + 40*(-c^2*d*x^2 + d
)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(5/2)*c^2/(d*x^9) + 105*(-c^2*d*x
^2 + d)^(5/2)/(d*x^11))*b*arccosh(c*x) - 1/1155*(16*(-c^2*d*x^2 + d)^(5/2)*
c^6/(d*x^5) + 40*(-c^2*d*x^2 + d)^(5/2)*c^4/(d*x^7) + 70*(-c^2*d*x^2 + d)^(
5/2)*c^2/(d*x^9) + 105*(-c^2*d*x^2 + d)^(5/2)/(d*x^11))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^12, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**12,x)
[Out] Timed out
```

3.80 $\int x^7 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=399

$$\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^8d^4}-\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^8d^3}+\frac{3\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^8d^2}-\frac{(d-c}{}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^8/d+3/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^8/d^2-1/3*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^8
/d^3+1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c^8/d^4+16/1155*b*d*x*(-
c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3465*b*d*x^3*(-c^2*d*x
^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/1925*b*d*x^5*(-c^2*d*x^2+d)^(
1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1617*b*d*x^7*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/297*b*c*d*x^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+1/121*b*c^3*d*x^11*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 460, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5798, 100, 12, 74, 5733, 1810}

$$-\frac{dx^6(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{11c^2}-\frac{2dx^4(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{33c^4}-\frac{8d}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d*x*Sqrt[d - c^2*d*x^2])/(1155*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (8
*b*d*x^3*Sqrt[d - c^2*d*x^2])/(3465*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*
b*d*x^5*Sqrt[d - c^2*d*x^2])/(1925*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d
*x^7*Sqrt[d - c^2*d*x^2])/(1617*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*c*d*
x^9*Sqrt[d - c^2*d*x^2])/(297*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^11
*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*d*(1 - c*x)^
2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(1155*c^8) - (8*d*x
^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*c
^6) - (2*d*x^4*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(33*c^4) - (d*x^6*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x]))/(11*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
```

```
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^7 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int x^7(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right) d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-}{}$$

$$= -\frac{16d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{1155c^8} - \frac{8dx^2(1-}{}$$

$$= \frac{16bdx\sqrt{d-c^2dx^2}}{1155c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{8bdx^3\sqrt{d-c^2dx^2}}{3465c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bd}{1925c^3}$$

Mathematica [A] time = 0.23, size = 182, normalized size = 0.46

$$-\frac{d\sqrt{d-c^2dx^2}\left(105c^5x^6(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right) + \frac{2(cx-1)^{5/2}(cx+1)^{5/2}\left(35c^4x^4+20c^2x^2+8\right)\left(a+b\cosh^{-1}(cx)\right)}{c}\right.}{1155c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/1155*(d*Sqrt[d - c^2*d*x^2]*(-(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 +
 (5*c^6*x^7)/7 - (140*c^8*x^9)/9 + (105*c^10*x^11)/11)) + 105*c^5*x^6*(-1 +
```

```
c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x)^(5/2)*(1 +
c*x)^(5/2)*(8 + 20*c^2*x^2 + 35*c^4*x^4)*(a + b*ArcCosh[c*x]))/c))/(c^7*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.69, size = 275, normalized size = 0.69

$$-\frac{3465\left(105\,bc^{12}dx^{12}-245\,bc^{10}dx^{10}+145\,bc^{8}dx^{8}+bc^{6}dx^{6}+2\,bc^{4}dx^{4}+8\,bc^{2}dx^{2}-16\,bd\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/4002075*(3465*(105*b*c^12*d*x^12 - 245*b*c^10*d*x^10 + 145*b*c^8*d*x^8 +
b*c^6*d*x^6 + 2*b*c^4*d*x^4 + 8*b*c^2*d*x^2 - 16*b*d)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) - (33075*b*c^11*d*x^11 - 53900*b*c^9*d*x^9 +
2475*b*c^7*d*x^7 + 4158*b*c^5*d*x^5 + 9240*b*c^3*d*x^3 + 55440*b*c*d*x)*sqr
t(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(105*a*c^12*d*x^12 - 245*a*c^10*
d*x^10 + 145*a*c^8*d*x^8 + a*c^6*d*x^6 + 2*a*c^4*d*x^4 + 8*a*c^2*d*x^2 - 16
*a*d)*sqrt(-c^2*d*x^2 + d))/(c^10*x^2 - c^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.00, size = 1846, normalized size = 4.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/11*x^6*(-c^2*d*x^2+d)^(5/2)/c^2/d+6/11/c^2*(-1/9*x^4*(-c^2*d*x^2+d)^(
5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*
x^2+d)^(5/2))))+b*(-1/247808*(-d*(c^2*x^2-1))^(1/2)*(1-3328*c^10*x^10+4096*
c^8*x^8-2352*c^6*x^6+620*c^4*x^4-61*c^2*x^2+1024*x^12*c^12+2816*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-11*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+1024*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9)
*(-1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*(c^2*x^2-1))^(1/2)*
(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c^6*
x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)
+1/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25
*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1
))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/3072*(-d*(c^2*x^2-1))^(
```

```
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*
(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcc
osh(c*x))*d/(c*x+1)/c^8/(c*x-1)-7/1024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)+1/30
72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4
+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1
)/c^8/(c*x-1)+11/51200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/
c^8/(c*x-1)+1/100352*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1)-1/55296*(-d*
(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+
576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^
4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/
(c*x+1)/c^8/(c*x-1)-1/247808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^11*c^11+1024*x^12*c^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c
^9-3328*c^10*x^10-2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+123
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1
)*(1+11*arccosh(c*x))*d/(c*x+1)/c^8/(c*x-1))
```

maxima [A] time = 0.93, size = 285, normalized size = 0.71

$$-\frac{1}{1155}\left(\frac{105\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^6}{c^2d}+\frac{70\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^4d}+\frac{40\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^6d}+\frac{16\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^8d}\right)b\operatorname{arcosh}(cx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2*d) + 70*(-c^2*d*x^2 + d)^(5/2)
*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^
(5/2)/(c^8*d))*b*arccosh(c*x) - 1/1155*(105*(-c^2*d*x^2 + d)^(5/2)*x^6/(c^2
*d) + 70*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^4*d) + 40*(-c^2*d*x^2 + d)^(5/2)*x^2
/(c^6*d) + 16*(-c^2*d*x^2 + d)^(5/2)/(c^8*d))*a + 1/4002075*(33075*c^10*sqr
t(-d)*d*x^11 - 53900*c^8*sqrt(-d)*d*x^9 + 2475*c^6*sqrt(-d)*d*x^7 + 4158*c^
4*sqrt(-d)*d*x^5 + 9240*c^2*sqrt(-d)*d*x^3 + 55440*sqrt(-d)*d*x)*b/c^7
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.81 $\int x^5 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=321

$$-\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^6d}-\frac{10bc}{441\sqrt{}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^6/d+2/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/9*(-c^2*d*x^2+d)^(9/2)*(a+b*arccosh(c*x))/c^6
/d^3+8/315*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/945
*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*d*x^5
*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-10/441*b*c*d*x^7*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*c^3*d*x^9*(-c^2*d*x^2+d)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 366, normalized size of antiderivative = 1.14, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5798, 100, 12, 74, 5733, 1153}

$$-\frac{dx^4(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9c^2}-\frac{4dx^2(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{8d}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d*x*Sqrt[d - c^2*d*x^2])/(315*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b
*d*x^3*Sqrt[d - c^2*d*x^2])/(945*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x
^5*Sqrt[d - c^2*d*x^2])/(525*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (10*b*c*d*x^
7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^9*Sq
rt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*d*(1 - c*x)^2*(1
+ c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(315*c^6) - (4*d*x^2*(1
- c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*c^4) - (
d*x^4*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*
c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^5 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right) \int x^5(-1+cx)^{3/2}(1+cx)^{3/2} \left(a+b\cosh^{-1}(cx)\right) d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-}{}$$

$$= -\frac{8d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{315c^6} - \frac{4dx^2(1-}{}$$

$$= \frac{8bdx\sqrt{d-c^2dx^2}}{315c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4bdx^3\sqrt{d-c^2dx^2}}{945c^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bdx^5\sqrt{}}{525c\sqrt{-1}}$$

Mathematica [A] time = 0.18, size = 164, normalized size = 0.51

$$-\frac{d\sqrt{d-c^2dx^2}\left(35c^3x^4(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{5/2}(cx+1)^{5/2}\left(5c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}-b\left(\frac{35c^8}{9}\right.\right.}{315c^5\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/315*(d*Sqrt[d - c^2*d*x^2]*(-(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (
50*c^6*x^7)/7 + (35*c^8*x^9)/9)) + 35*c^3*x^4*(-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(2 + 5*c^2*x
^2)*(a + b*ArcCosh[c*x]))/c))/(c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.65, size = 245, normalized size = 0.76

$$-\frac{315\left(35\,bc^{10}dx^{10}-85\,bc^{8}dx^{8}+53\,bc^{6}dx^{6}+bc^{4}dx^{4}+4\,bc^{2}dx^{2}-8\,bd\right)\sqrt{-c^{2}dx^{2}+d}\,\log\left(cx+\sqrt{c^{2}x^{2}-1}\right)-\left(122\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/99225*(315*(35*b*c^10*d*x^10 - 85*b*c^8*d*x^8 + 53*b*c^6*d*x^6 + b*c^4*d
*x^4 + 4*b*c^2*d*x^2 - 8*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 -
 1)) - (1225*b*c^9*d*x^9 - 2250*b*c^7*d*x^7 + 189*b*c^5*d*x^5 + 420*b*c^3*d
*x^3 + 2520*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 315*(35*a*c^1
0*d*x^10 - 85*a*c^8*d*x^8 + 53*a*c^6*d*x^6 + a*c^4*d*x^4 + 4*a*c^2*d*x^2 -
8*a*d)*sqrt(-c^2*d*x^2 + d))/(c^8*x^2 - c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.65, size = 1376, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/9*x^4*(-c^2*d*x^2+d)^(5/2)/c^2/d+4/9/c^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/
2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2)))+b*(-1/41472*(-d*(c^2*x^2-1))^(1/
2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c
^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^4+432*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+
9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*d/(c*x+1)/c^6/(c*x
-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-
25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1
))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c
^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(
1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arcco
sh(c*x))*d/(c*x+1)/c^6/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)+1/1152
*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/
c^6/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^6/
(c*x-1)-1/25088*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
```

```
*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^6/(c*x-1)-1/41472*(-d*(c^2*
x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4
-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d/(c*x+
1)/c^6/(c*x-1))
```

maxima [A] time = 1.36, size = 223, normalized size = 0.69

$$-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}+\frac{20\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^6d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{315}\left(\frac{35\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}{c^2d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/315*(35*(-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*b*arccosh(c*x) - 1/315*(35*(
-c^2*d*x^2 + d)^(5/2)*x^4/(c^2*d) + 20*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(5/2)/(c^6*d))*a + 1/99225*(1225*c^8*sqrt(-d)*d*x^9 - 2
250*c^6*sqrt(-d)*d*x^7 + 189*c^4*sqrt(-d)*d*x^5 + 420*c^2*sqrt(-d)*d*x^3 +
2520*sqrt(-d)*d*x)*b/c^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.82 $\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=243

$$\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d^2}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^4d}-\frac{8bcdx^5\sqrt{d-c^2dx^2}}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^4/d+1/7*(-c^2*d*x^2+d)^(7/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/35*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+1/105*b*d*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-8/175*b*c*d*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4
9*b*c^3*d*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 272, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5798, 100, 12, 74, 5733, 373}

$$-\frac{dx^2(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{2d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}+\frac{bc^3d}{49\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d*x*Sqrt[d - c^2*d*x^2])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*
x^3*Sqrt[d - c^2*d*x^2])/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*b*c*d*x^
5*Sqrt[d - c^2*d*x^2])/(175*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^7*Sq
rt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*d*(1 - c*x)^2*(1
+ c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*c^4) - (d*x^2*(1 - c
*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1-c}{}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1-c}{}$$

$$=-\frac{2d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35c^4}-\frac{dx^2(1-c}{}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bdx^3\sqrt{d-c^2dx^2}}{105c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{8bcdx^5\sqrt{d}}{175\sqrt{-1+c}}$$

Mathematica [A] time = 0.24, size = 150, normalized size = 0.62

$$-\frac{d\sqrt{d-c^2dx^2}\left(5c^2x^2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{5/2}(cx+1)^{5/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{7}b\right.}{35c^4\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/35*(d*Sqrt[d - c^2*d*x^2]*((-5*b*c*x*(-1 + c^2*x^2)^3)/7 - (19*b*c*(x -
(2*c^2*x^3)/3 + (c^4*x^5)/5))/7 + 2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b
*ArcCosh[c*x]) + 5*c^2*x^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[
c*x])))/(c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.63, size = 215, normalized size = 0.88

$$-\frac{105\left(5\,bc^8dx^8-13\,bc^6dx^6+9\,bc^4dx^4+bc^2dx^2-2\,bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^7dx^7-168\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] -1/3675*(105*(5*b*c^8*d*x^8 - 13*b*c^6*d*x^6 + 9*b*c^4*d*x^4 + b*c^2*d*x^2
- 2*b*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^7*d*x^
7 - 168*b*c^5*d*x^5 + 35*b*c^3*d*x^3 + 210*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) + 105*(5*a*c^8*d*x^8 - 13*a*c^6*d*x^6 + 9*a*c^4*d*x^4 + a*c
^2*d*x^2 - 2*a*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.58, size = 966, normalized size = 3.98

$$a\left(-\frac{x^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{7c^{2}d}-\frac{2\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}}{35d\,c^{4}}\right)+b\left(-\frac{\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\left(64c^{8}x^{8}-144c^{6}x^{6}+64\sqrt{cx+1}\,\sqrt{cx-1}\,x^{7}c^{7}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b*(
-1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c
^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+1)*(-1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(
1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(-1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c
^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*
x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(
c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c
*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^
5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1
)-1/6272*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64
*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c
^2*x^2+1)*(1+7*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

maxima [A] time = 1.09, size = 161, normalized size = 0.66

$$-\frac{1}{35}\left(\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{2}}{c^{2}d}+\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{c^{4}d}\right)b\,\mathrm{arcosh}\left(cx\right)-\frac{1}{35}\left(\frac{5\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{2}}{c^{2}d}+\frac{2\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{c^{4}d}\right)a+\frac{\left(75\,c^{6}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4
*d))*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*
d*x^2 + d)^(5/2)/(c^4*d))*a + 1/3675*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(
-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3} \left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

3.83 $\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=165

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}+\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/c^2/d+1/5*b*d*x*(-c^2*d*x^2+d)
^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/15*b*c*d*x^3*(-c^2*d*x^2+d)^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+1/25*b*c^3*d*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 178, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5798, 5718, 194}

$$-\frac{d(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{bc^3dx^5\sqrt{d-c^2dx^2}}{25\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{cx-1}\,\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d*x*Sqrt[d - c^2*d*x^2])/(5*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*x
^3*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*x^5*Sq
rt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)^2*(1 +
c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c^2)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{\left(bd\sqrt{d-c^2\right.}{5c\sqrt{}}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2}+\frac{\left(bd\sqrt{d-c^2\right.}{5c}$$

$$=\frac{bdx\sqrt{d-c^2dx^2}}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2bcdx^3\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bc^3dx^5\sqrt{d-c^2dx}}{25\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [A] time = 0.22, size = 107, normalized size = 0.65

$$-\frac{d\sqrt{d-c^2dx^2}\left(15a\left(c^2x^2-1\right)^3+15b\left(c^2x^2-1\right)^3\cosh^{-1}(cx)+bcx\sqrt{cx-1}\,\sqrt{cx+1}\left(-3c^4x^4+10c^2x^2-15\right)\right)}{75c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/75*(d*Sqrt[d - c^2*d*x^2]*(15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(-15 + 10*c^2*x^2 - 3*c^4*x^4) + 15*b*(-1 + c^2*x^2)^3*ArcCos
h[c*x]))/(c^2*(-1 + c^2*x^2))
```

fricas [A] time = 0.57, size = 185, normalized size = 1.12

$$-\frac{15\left(bc^6dx^6-3\,bc^4dx^4+3\,bc^2dx^2-bd\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(3\,bc^5dx^5-10\,bc^3dx^3+15\,bcdx\right.}{75\left(c^4x^2-c^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] -1/75*(15*(b*c^6*d*x^6 - 3*b*c^4*d*x^4 + 3*b*c^2*d*x^2 - b*d)*sqrt(-c^2*d*x
^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (3*b*c^5*d*x^5 - 10*b*c^3*d*x^3 + 15
*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(a*c^6*d*x^6 - 3*a*c^
4*d*x^4 + 3*a*c^2*d*x^2 - a*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.31, size = 620, normalized size = 3.76

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{5c^2d}+b\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(16c^6x^6-28c^4x^4+16\sqrt{cx+1}\,\sqrt{cx-1}\,x^5c^5+13c^2x^2-20\sqrt{cx+1}\right.}{800\,(cx+1)\,c^2\,(cx-1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/5*a/c^2/d*(-c^2*d*x^2+d)^(5/2)+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*
x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arc
cosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c
^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2
)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c
^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)
*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(
c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

maxima [A] time = 0.77, size = 102, normalized size = 0.62

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}b\operatorname{arcosh}\left(cx\right)}{5\,c^2d}-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}a}{5\,c^2d}+\frac{\left(3\,c^4\sqrt{-d}\,d^2x^5-10\,c^2\sqrt{-d}\,d^2x^3+15\,\sqrt{-d}\,d^2x\right)b}{75\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(
5/2)*a/(c^2*d) + 1/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 1
5*sqrt(-d)*d^2*x)*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x)), x)
```

3.84 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=292

$$\frac{1}{3}\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{2d\sqrt{d-c^2dx^2}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cdots\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+d*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)-4/3*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9
*b*c^3*d*x^3*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d*(a+b*arcc
osh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)+I*b*d*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d*polylog(2,I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A] time = 0.79, antiderivative size = 304, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5798, 5745, 5743, 5761, 4180, 2279, 2391, 8}

$$\frac{ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-4*b*c*d*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*
d*x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]) + (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/3 - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Arc
Tan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d*Sqrt[d - c^2*d
*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*
b*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
```

```
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{\left(d\sqrt{d-c^2dx^2}\right)}{\sqrt{}}$$

$$= -\frac{bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} + d\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} + d\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} + d\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} + d\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= -\frac{4bcdx\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}} + d\sqrt{d-c^2dx^2}\left(a+b\right.$$

Mathematica [A] time = 1.25, size = 336, normalized size = 1.15

$$-ad^{3/2}\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)-\frac{1}{3}ad\left(c^2x^2-4\right)\sqrt{d-c^2dx^2}+ad^{3/2}\log(x)+\frac{bd\sqrt{d-c^2dx^2}\left(i\text{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -1/3*(a*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b*d*Sqrt[d - c^2*d*x^2]*(9
*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*Ar
cCosh[c*x]]))/(36*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a*d^(3/2)*Log[x]
- a*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d*Sqrt[d - c^2*d*x^2]
*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCos
h[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*Po
lyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ac^2dx^2-ad+\left(bc^2dx^2-bd\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.48, size = 499, normalized size = 1.71

$$\frac{\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}} a}{3}-a\,d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)+a\sqrt{-c^2 d\,x^2+d}\,d-\frac{4b\sqrt{-d\left(c^2x^2-1\right)}\,d\,\mathrm{arccosh}\left(cx\right)}{3\left(cx+1\right)\left(cx-1\right)}+\frac{ib\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*a-a*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2)
)/x)+a*(-c^2*d*x^2+d)^(1/2)*d-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1
)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d+I*b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)*d-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*d-1/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-
1)*arccosh(c*x)*x^4*c^4+5/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arcc
osh(c*x)*x^2*c^2+1/9*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)
*x^3*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-I*b
*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\left(-c^2dx^2+d\right)^{\frac{3}{2}}-3\sqrt{-c^2dx^2+d}\,d\right)a+b\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}\log\left(cx+\sqrt{c}\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (
-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a + b*integrate((-c^2*d*x
^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x, x)
```

3.85 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=311

$$-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3c^2d\sqrt{d-c^2dx^2}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2-3/2*c^2*d*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)-1/2*b*c*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+b*c^3*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*c^2*
d*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*c^2*d*polylog(2,-I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*
I*b*c^2*d*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5743, 5761, 4180, 2279, 2391, 8, 14}

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)
/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + (((3*I)/2)*b*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*
E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right.}{2\sqrt{-1+cx}}$$

$$= -\frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{2x^2}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a\right.$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3}{2}c^2d\sqrt{d-c^2dx^2}\left(a\right.$$

Mathematica [A] time = 1.58, size = 500, normalized size = 1.61

$$\frac{1}{2}\left(3ac^2d^{3/2}\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) - 3ac^2d^{3/2}\log(x) - \frac{ad\left(2c^2x^2+1\right)\sqrt{d-c^2dx^2}}{x^2} + \frac{bd^2(cx+1)\left(ic^2x^2\sqrt{\frac{cx-}{cx+}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-((a*d*(1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/x^2) - 3*a*c^2*d^(3/2)*Log[x]
+ 3*a*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*Sqrt[d
- c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(
-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[
c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) + (b*d^2*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - ArcCosh[c*x] + c*
x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 -
I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1
 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)
/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcC
osh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.64, size = 542, normalized size = 1.74

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{2d\,x^2}-\frac{a\,c^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{2}+\frac{3a\,c^2d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2}-\frac{3a\,c^2\sqrt{-c^2d\,x^2+d}\,d}{2}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}}{(cx+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(5/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(3/2)+3/2*a*c^2*
d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-3/2*a*c^2*(-c^2*d*x^2+d)
^(1/2)*d-b*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2+b*
(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/2*b*(-d*(c^2*x
^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arccosh(c*x)-1/2*b*(-d*(c^2*x^2-1))^(1/2
)*d/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d/x^2/(c*x
+1)/(c*x-1)*arccosh(c*x)-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+3/2*I
*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d+
3/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(3\,c^2d^{\frac{3}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2-3\sqrt{-c^2dx^2+d}\,c^2d-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^2}\right)a+b\int\frac{\left(-c^2d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 +
 d)^(5/2)/(d*x^2))*a + b*integrate((-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*
x + 1)*sqrt(c*x - 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**3,x)
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**3, x)
```

3.86 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=321

$$\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4}-\frac{3c^4d\sqrt{d-c^2dx^2}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\right.}{4\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^4+3/8*c^2*d*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)/x^2-1/12*b*c*d*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+5/8*b*c^3*d*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-3/4*c^4*d*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/8*I*b*c^4*d*polylog(2,-I*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-3/8*I*b*c^4*d*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2
*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 333, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5738, 30, 5761, 4180, 2279, 2391, 14}

$$\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{3ibc^4d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\right.}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*c
^3*d*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c^2*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - (d*(1 - c*x)*(1 + c*x)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) - (3*c^4*d*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*
x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*I)/8)*b*c^4*d*Sqrt[d - c^2*d*x^2
]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right)\ldots}{4\sqrt{-1+cx}\,\sqrt{\ldots}}$$

$$= \frac{3c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2} - \frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a\ldots\right.}{4x^4}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a\ldots\right.}{8x\ldots}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a\ldots\right.}{8x\ldots}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a\ldots\right.}{8x\ldots}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5bc^3d\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2d\sqrt{d-c^2dx^2}\left(a\ldots\right.}{8x\ldots}$$

Mathematica [A] time = 1.28, size = 574, normalized size = 1.79

$$\frac{-15ac^4d^2x^4\sqrt{\frac{cx-1}{cx+1}} + 21ac^2d^2x^2\sqrt{\frac{cx-1}{cx+1}} + 9ac^4d^{3/2}x^4\sqrt{\frac{cx-1}{cx+1}}\log(x)\sqrt{d-c^2dx^2} - 9ac^4d^{3/2}x^4\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\log\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^2*x + 2*b*c^2*d^2*x^2 + 15*b*c^3*d^2*x^3 - 15*b*c^4*d^2*x^4 - 6*a
*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 21*a*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)] - 15*a*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x] + 21*b*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCo
sh[c*x] - 15*b*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (9*I)*
b*c^4*d^2*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^5*d^2*x^5*
ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (9*I)*b*c^4*d^2*x^4*ArcCosh[c*x]*L
og[1 + I/E^ArcCosh[c*x]] + (9*I)*b*c^5*d^2*x^5*ArcCosh[c*x]*Log[1 + I/E^Arc
Cosh[c*x]] + 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
x^2]*Log[x] - 9*a*c^4*d^(3/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d
*x^2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (9*I)*b*c^4*d^2*x^4*(-1 + c*x)
*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (9*I)*b*c^4*d^2*x^4*(-1 + c*x)*PolyLog[2
, I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.73, size = 570, normalized size = 1.78

$$-\frac{a\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{4d\,x^4}+\frac{a\,c^2\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}}}{8d\,x^2}+\frac{a\,c^4\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{8}-\frac{3a\,c^4 d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)}{8}+\frac{3a\,c^4\sqrt{-c^2 d\,x^2+}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(5/2)+1/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(5/2)+1/8*
a*c^4*(-c^2*d*x^2+d)^(3/2)-3/8*a*c^4*d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+
d)^(1/2))/x)+3/8*a*c^4*(-c^2*d*x^2+d)^(1/2)*d+5/8*b*d*(-d*(c^2*x^2-1))^(1/2
)/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4+5/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(
1/2)/x/(c*x-1)^(1/2)*c^3-7/8*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^2/(c*x-1)
*arccosh(c*x)*c^2-1/12*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/x^3/(c*x-1)
^(1/2)*c+1/4*b*d*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/x^4/(c*x-1)*arccosh(c*x)+3/
8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d*c^4-3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))*d*c^4+3/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dil
og(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d*c^4-3/8*I*b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)*d*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(3\,c^4 d^{\frac{3}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4-3\sqrt{-c^2dx^2+d}\,c^4d-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{dx^2}+\frac{2\left(-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] -1/8*(3*c^4*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - (-c^2*d*x^2 + d)^(3/2)*c^4 - 3*sqrt(-c^2*d*x^2 + d)*c^4*d - (-c^2*d*x^2
+ d)^(5/2)*c^2/(d*x^2) + 2*(-c^2*d*x^2 + d)^(5/2)/(d*x^4))*a + b*integrate(
(-c^2*d*x^2 + d)^(3/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2))/x^5, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\,\mathrm{acosh}\,(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))/x**5, x)
```

3.87 $\int x^4 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=454

$$\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{d^2x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{128c^2}+\frac{1}{10}x^5\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}\right.$$

```
[Out] 1/16*d*x^5*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/10*x^5*(-c^2*d*x^2+d)^
(5/2)*(a+b*arccosh(c*x))-3/256*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2
)/c^4-1/128*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2+1/32*d^2*x^
5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+3/512*b*d^2*x^2*(-c^2*d*x^2+d)^(1
/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/512*b*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-31/960*b*c*d^2*x^6*(-c^2*d*x^2+d)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)+21/640*b*c^3*d^2*x^8*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-1/100*b*c^5*d^2*x^10*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-3/512*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c^5/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 485, normalized size of antiderivative = 1.07, number of steps used = 15, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14, 266, 43}

$$\frac{1}{32}d^2x^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{10}d^2x^5(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{16}d^2x^5(1-cx)($$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(512*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(b*d^2*x^4*Sqrt[d - c^2*d*x^2])/(512*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (31*
b*c*d^2*x^6*Sqrt[d - c^2*d*x^2])/(960*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (21*b
*c^3*d^2*x^8*Sqrt[d - c^2*d*x^2])/(640*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*x^10*Sqrt[d - c^2*d*x^2])/(100*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(256*c^4) - (d^2*x^3*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (d^2*x^5*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/32 + (d^2*x^5*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/16 + (d^2*x^5*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x]))/10 - (3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(512*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^4 \left(d - c^2dx^2\right)^{5/2} \left(a + b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d - c^2dx^2}\right)\int x^4(-1 + cx)^{5/2}(1 + cx)^{5/2}\left(a + b\cosh^{-1}(cx)\right) dx}{\sqrt{-1 + cx}\sqrt{1 + cx}}$$

$$= \frac{1}{10}d^2x^5(1 - cx)^2(1 + cx)^2\sqrt{d - c^2dx^2}\left(a + b\cosh^{-1}(cx)\right) - \frac{\left(d^2\sqrt{d\right.}$$

$$= \frac{1}{16}d^2x^5(1 - cx)(1 + cx)\sqrt{d - c^2dx^2}\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{10}d^2x^5(1$$

$$= \frac{1}{32}d^2x^5\sqrt{d - c^2dx^2}\left(a + b\cosh^{-1}(cx)\right) + \frac{1}{16}d^2x^5(1 - cx)(1 + cx)\sqrt{}$$

$$= -\frac{31bcd^2x^6\sqrt{d - c^2dx^2}}{960\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{21bc^3d^2x^8\sqrt{d - c^2dx^2}}{640\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{bc^5d^2x^{10}\sqrt{d}}{100\sqrt{-1 + cx}}$$

$$= \frac{bd^2x^4\sqrt{d - c^2dx^2}}{512c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{31bcd^2x^6\sqrt{d - c^2dx^2}}{960\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{21bc^3d^2x^8\sqrt{d}}{640\sqrt{-1 + cx}}$$

$$= \frac{3bd^2x^2\sqrt{d - c^2dx^2}}{512c^3\sqrt{-1 + cx}\sqrt{1 + cx}} + \frac{bd^2x^4\sqrt{d - c^2dx^2}}{512c\sqrt{-1 + cx}\sqrt{1 + cx}} - \frac{31bcd^2x^6\sqrt{}}{960\sqrt{-1 + }}$$

Mathematica [A] time = 6.59, size = 581, normalized size = 1.28

$$-\frac{3ad^{5/2}\tan^{-1}\left(\frac{cx\sqrt{-d(c^2x^2-1)}}{\sqrt{d}(c^2x^2-1)}\right)}{256c^5} + \sqrt{-d\left(c^2x^2 - 1\right)}\left(\frac{1}{10}ac^4d^2x^9 - \frac{3ad^2x}{256c^4} - \frac{21}{80}ac^2d^2x^7 - \frac{ad^2x^3}{128c^2} + \frac{31}{160}ad^2x^5\right) + \frac{bd^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-3*a*d^2*x)/(256*c^4) - (a*d^2*x^3)/(128*c^2) +
 (31*a*d^2*x^5)/160 - (21*a*c^2*d^2*x^7)/80 + (a*c^4*d^2*x^9)/10) - (3*a*d^
(5/2)*ArcTan[(c*x*Sqrt[-(d*(-1 + c^2*x^2))])/(Sqrt[d]*(-1 + c^2*x^2))])/(25
6*c^5) + (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(18*Cosh[2*ArcCosh[c*x]] -
9*Cosh[4*ArcCosh[c*x]] - 2*(36*ArcCosh[c*x]^2 + Cosh[6*ArcCosh[c*x]] + 18*A
rcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 18*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 6
*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]])))/(2304*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)) + (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(1440*ArcCosh[c*x]^2 -
576*Cosh[2*ArcCosh[c*x]] + 144*Cosh[4*ArcCosh[c*x]] + 64*Cosh[6*ArcCosh[c*x
]] + 9*Cosh[8*ArcCosh[c*x]] + 1152*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 576*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]] - 384*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] -
 72*ArcCosh[c*x]*Sinh[8*ArcCosh[c*x]]))/(36864*c^5*Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)) - (b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(50400*ArcCosh[c*x]^
2 - 25200*Cosh[2*ArcCosh[c*x]] + 3600*Cosh[4*ArcCosh[c*x]] + 2600*Cosh[6*Ar
cCosh[c*x]] + 675*Cosh[8*ArcCosh[c*x]] + 72*Cosh[10*ArcCosh[c*x]] + 50400*A
rcCosh[c*x]*Sinh[2*ArcCosh[c*x]] - 14400*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]
- 15600*ArcCosh[c*x]*Sinh[6*ArcCosh[c*x]] - 5400*ArcCosh[c*x]*Sinh[8*ArcCos
h[c*x]] - 720*ArcCosh[c*x]*Sinh[10*ArcCosh[c*x]]))/(3686400*c^5*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2x^8 - 2\,ac^2d^2x^6 + ad^2x^4 + \left(bc^4d^2x^8 - 2\,bc^2d^2x^6 + bd^2x^4\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^8 - 2*a*c^2*d^2*x^6 + a*d^2*x^4 + (b*c^4*d^2*x^8 - 2*
b*c^2*d^2*x^6 + b*d^2*x^4)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)x^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^4, x)
```

maple [A] time = 0.81, size = 690, normalized size = 1.52

$$-\frac{a\,x^3\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{10c^2d}-\frac{3ax\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{80c^4d}+\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{160c^4}+\frac{adx\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{128c^4}+\frac{3a\,d^2x\sqrt{-c^2d\,x^2+d}}{256c^4}+\frac{3a\,d^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/10*a*x^3*(-c^2*d*x^2+d)^(7/2)/c^2/d-3/80*a/c^4*x*(-c^2*d*x^2+d)^(7/2)/d+
1/160*a/c^4*x*(-c^2*d*x^2+d)^(5/2)+1/128*a/c^4*d*x*(-c^2*d*x^2+d)^(3/2)+3/2
56*a/c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+3/256*a/c^4*d^3/(c^2*d)^(1/2)*arctan((c
^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/100*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x
+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^10+21/640*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1
)^(1/2)*c^3/(c*x-1)^(1/2)*x^8-31/960*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(
1/2)*c/(c*x-1)^(1/2)*x^6+1/512*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c
/(c*x-1)^(1/2)*x^4+3/512*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c^3/(c*
x-1)^(1/2)*x^2+1/10*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*arccos
h(c*x)*x^11-29/80*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(
c*x)*x^9+73/160*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*
x)*x^7-129/640*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^
5-1/256*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x^3+3
/256*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^4/(c*x-1)*arccosh(c*x)*x-101/12
28800*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c^5/(c*x-1)^(1/2)-3/512*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*arccosh(c*x)^2*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{1280}\left(\frac{128\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^3}{c^2d}-\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^4}+\frac{48\left(-c^2dx^2+d\right)^{\frac{7}{2}}x}{c^4d}-\frac{10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}{c^4}-\frac{15\sqrt{-c^2dx^2+d}}{c^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/1280*(128*(-c^2*d*x^2 + d)^(7/2)*x^3/(c^2*d) - 8*(-c^2*d*x^2 + d)^(5/2)*
x/c^4 + 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^4*d) - 10*(-c^2*d*x^2 + d)^(3/2)*d*x
/c^4 - 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^4 - 15*d^(5/2)*arcsin(c*x)/c^5)*a +
b*integrate((-c^2*d*x^2 + d)^(5/2)*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4 \,(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^4*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.88 $\int x^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=371

$$-\frac{5d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{128c^2}+\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}x^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(c\right.$$

```
[Out] 5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/8*x^3*(-c^2*d*x^2+d)^(
5/2)*(a+b*arccosh(c*x))-5/128*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
/c^2+5/64*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+5/256*b*d^2*x^2*(
-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/768*b*c*d^2*x^4*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/288*b*c^3*d^2*x^6*(-c^2*d*x^2
+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/64*b*c^5*d^2*x^8*(-c^2*d*x^2+d)^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/256*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d
)^(1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 402, normalized size of antiderivative = 1.08, number of steps used = 13, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5759, 5676, 30, 14, 266, 43}

$$\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2x^3(1-cx)^2(cx+1)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}d^2x^3(1-cx)(cx+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2])/(256*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5
9*b*c*d^2*x^4*Sqrt[d - c^2*d*x^2])/(768*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17
*b*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2])/(288*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(128*c^2) + (5*d^2*x^3*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/64 + (5*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/48 + (d^2*x^3*(1 - c*x)^2*(1 + c*x)^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*d^2*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(256*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

```
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(5d^2\sqrt{d-c\right.}{}$$

$$=\frac{5}{48}d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}d^2x^3(1-c$$

$$=\frac{5}{64}d^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5}{48}d^2x^3(1-cx)(1+cx)\sqrt{d\,-}$$

$$=-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-c^2dx^2}}{288\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bc^5d^2x^8\sqrt{d-c^2}}{64\sqrt{-1+cx}\sqrt{1}}$$

$$=\frac{5bd^2x^2\sqrt{d-c^2dx^2}}{256c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{59bcd^2x^4\sqrt{d-c^2dx^2}}{768\sqrt{-1+cx}\sqrt{1+cx}}+\frac{17bc^3d^2x^6\sqrt{d-}}{288\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 4.55, size = 415, normalized size = 1.12

$$\frac{-2880ad^{5/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+192acd^2x\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(48c^6x^6-136c^4x^4+118c^2x^2-15\right)\sqrt{d-}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (192*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-1
5 + 118*c^2*x^2 - 136*c^4*x^4 + 48*c^6*x^6) - 2880*a*d^(5/2)*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*
x^2))] - 576*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c
*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) - 64*b*d^2*Sqrt[d - c^2*d*x^2]*
(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*
Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*
ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])) + b*d^2*Sqrt[d - c^2*d*x^2]*(-1440*A
rcCosh[c*x]^2 + 576*Cosh[2*ArcCosh[c*x]] - 144*Cosh[4*ArcCosh[c*x]] - 64*Co
sh[6*ArcCosh[c*x]] - 9*Cosh[8*ArcCosh[c*x]] + 24*ArcCosh[c*x]*(-48*Sinh[2*A
rcCosh[c*x]] + 24*Sinh[4*ArcCosh[c*x]] + 16*Sinh[6*ArcCosh[c*x]] + 3*Sinh[8
*ArcCosh[c*x]])))/(73728*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2x^6-2\,ac^2d^2x^4+ad^2x^2+\left(bc^4d^2x^6-2\,bc^2d^2x^4+bd^2x^2\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^6 - 2*a*c^2*d^2*x^4 + a*d^2*x^2 + (b*c^4*d^2*x^6 - 2*
b*c^2*d^2*x^4 + b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\text{arcosh}\,(cx)+a)x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*x^2, x)
```

maple [A] time = 0.49, size = 581, normalized size = 1.57

$$-\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{8c^2d}+\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{48c^2}+\frac{5adx\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{192c^2}+\frac{5a\,d^2x\sqrt{-c^2d\,x^2+d}}{128c^2}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{128c^2\sqrt{c^2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/8*a*x*(-c^2*d*x^2+d)^(7/2)/c^2/d+1/48*a/c^2*x*(-c^2*d*x^2+d)^(5/2)+5/192
*a/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/128*a/c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/12
8*a/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-5/25
6*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^2*d
^2-1/64*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8+17
/288*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-59/76
8*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+5/256*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+1/8*b*(-d*(c^2*x
^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-23/48*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+127/192*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-133/384*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+5/128*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+359/73728*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{384}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^2}-\frac{48\left(-c^2dx^2+d\right)^{\frac{7}{2}}x}{c^2d}+\frac{10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}{c^2}+\frac{15\sqrt{-c^2dx^2+d}\,d^2x}{c^2}+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right.}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] 1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d)
 + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 +
15*d^(5/2)*arcsin(c*x)/c^3)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*x^2*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.89 $\int \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=293

$$\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right) - \frac{5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right)^2}{32 b c \sqrt{cx-1} \sqrt{cx+1}} + \frac{1}{6} x \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx\right.$$

```
[Out] 5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/6*x*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))+5/16*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-25/9
6*b*c*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/96*b*c^3*d
^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*d^2*(-c^2*x^
2+1)^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/32*d^2*(a+b*arc
cosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5685, 5683, 5676, 30, 14, 261}

$$\frac{5}{16} d^2 x \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right) + \frac{1}{6} d^2 x (1-cx)^2 (cx+1)^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right) + \frac{5}{24} d^2 x (1-cx)(cx+1$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-25*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
5*b*c^3*d^2*x^4*Sqrt[d - c^2*d*x^2])/(96*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b
*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(36*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/16 + (5*d^2*x*(1 -
c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/24 + (d^2*x*(1 - c
*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (5*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{\left(5d^2\sqrt{d-c^2dx\right.}}{}$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{24}d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\right.$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5}{16}d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+$$

$$= -\frac{25bcd^2x^2\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5bc^3d^2x^4\sqrt{d-c^2dx^2}}{96\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-\right.}}{36c\sqrt{-1+cx}\sqrt{1+\right.}}$$

Mathematica [A] time = 2.47, size = 347, normalized size = 1.18

$$\frac{-720ad^{5/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+48acd^2x\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8c^4x^4-26c^2x^2+33\right)\sqrt{d-c^2dx^2}-288bd}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (48*a*c*d^2*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(33
- 26*c^2*x^2 + 8*c^4*x^4) - 720*a*d^(5/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 288*b*d^2*
Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] -
Sinh[2*ArcCosh[c*x]])) + 36*b*d^2*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + C
osh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b*d^2*Sqrt[d -
 c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cosh[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCos
h[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*ArcCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]]
 + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCosh[c*x]])))/(2304*c*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\mathrm{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.31, size = 462, normalized size = 1.58

$$\frac{ax\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{6}+\frac{5adx\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{24}+\frac{5a\,d^2x\sqrt{-c^2d\,x^2+d}}{16}+\frac{5a\,d^3\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{16\sqrt{c^2d}}-\frac{5b\sqrt{-d\left(c^2x^2-1\right)}}{32\sqrt{cx-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*x*(-c^2*d*x^2+d)^(5/2)+5/24*a*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a*d^2*x*(
-c^2*d*x^2+d)^(1/2)+5/16*a*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))-5/32*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*
arccosh(c*x)^2*d^2-1/36*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^5*x^6+13/96*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*c^3*x^4-11/32*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*
x^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^6*arccosh(c*x)*x^7-1
7/24*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+59/4
8*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-11/16*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+299/2304*b*(-d*(
c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x+10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx+15\sqrt{-c^2dx^2+d}\,d^2x+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right)}{c}\right)a+b\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a + b*integrate((-c^2*d*x
^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{acosh}(c x)) \left(d - c^2 d x^2\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.90 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=284

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16b\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{4}c^2dx\left(d-c^2dx^2\right)^{3/2}\left(a+b\,c\right.$$

```
[Out] -5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-(-c^2*d*x^2+d)^(5/2)*(
a+b*arccosh(c*x))/x-15/8*c^2*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)+
9/16*b*c^3*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*b*
c^5*d^2*x^4*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/16*c*d^2*(a
+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d
^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 315, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5740, 5685, 5683, 5676, 30, 14, 266, 43}

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5}{4}c^2d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{15cd^2\sqrt{d-c^2d}}{16b\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (9*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
b*c^5*d^2*x^4*Sqrt[d - c^2*d*x^2])/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*
c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 - (5*c^2*d^2*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/4 - (d^2*(1 - c*x)^
2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (15*c*d^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{\left(bcd^2\sqrt{d-c\right.}{\sqrt{-1}}$$

$$= -\frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-cx)}{}$$

$$= -\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{5}{4}c^2d^2x(1-cx)(1+cx)\sqrt{}$$

$$= \frac{9bc^3d^2x^2\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^4\sqrt{d-c^2dx^2}}{16\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}$$

Mathematica [A] time = 1.83, size = 305, normalized size = 1.07

$$\frac{1}{128}d^2\left(240ac\sqrt{d}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + \frac{16a\left(2c^4x^4-9c^2x^2-8\right)\sqrt{d-c^2dx^2}}{x} + 64bc\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)}{\sqrt{\frac{cx-}{cx+}}}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (d^2*((16*a*Sqrt[d - c^2*d*x^2]*(-8 - 9*c^2*x^2 + 2*c^4*x^4))/x + 240*a*c*S
qrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 64*b*c*
Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x]
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (32*b*c*Sqrt[d - c^2*d*x^2]*(2*
ArcCosh[c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b*c*Sqrt[d - c^2*d*x^2]*(8*Arc
Cosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/128
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.57, size = 550, normalized size = 1.94

$$-\frac{a\left(-c^{2}d\,x^{2}+d\right)^{\frac{7}{2}}}{dx}-a\,c^{2}x\left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}}-\frac{5a\,c^{2}dx\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}{4}-\frac{15a\,c^{2}d^{2}x\sqrt{-c^{2}d\,x^{2}+d}}{8}-\frac{15a\,c^{2}d^{3}\arctan\left(\frac{\sqrt{\ldots}}{\sqrt{-c^{2}\ldots}}\right.}{8\sqrt{c^{2}d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(7/2)-a*c^2*x*(-c^2*d*x^2+d)^(5/2)-5/4*a*c^2*d*x*(-c^
2*d*x^2+d)^(3/2)-15/8*a*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-15/8*a*c^2*d^3/(c^2*
d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+15/16*b*(-d*(c^2*x^2-
1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*d^2*c+b*(-d*(c^2*x^2-1
))^(1/2)*arccosh(c*x)*d^2/(c*x+1)/(c*x-1)/x-1/16*b*(-d*(c^2*x^2-1))^(1/2)*d
^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^4+9/16*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^
3/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*d^2*c/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c+1/4*b*(-d*(c^2*x^2
-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^5-11/8*b*(-d*(c^2*x^2-1))
^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/8*b*(-d*(c^2*x^2-1))^(1/2
)*d^2*c^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x-33/128*b*(-d*(c^2*x^2-1))^(1/2)*d^
2*c/(c*x+1)^(1/2)/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(10\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}dx+15\sqrt{-c^{2}dx^{2}+d}\,c^{2}d^{2}x+15\,cd^{\frac{5}{2}}\arcsin\left(cx\right)+\frac{8\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}}{x}\right)a+b\int\frac{\left(-c^{2}dx^{2}+d\right)^{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima"
)
```

```
[Out] -1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x
 + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a + b*integrate((
-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx$

Optimal. Leaf size=293

$$\frac{5c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.$$

```
[Out] 5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x-1/3*(-c^2*d*x^2+d)^(5/2
)*(a+b*arccosh(c*x))/x^3+5/2*c^4*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1
/2)-1/6*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*
c^5*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/4*c^3*d^2*(a
+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b
*c^3*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5798, 5740, 5683, 5676, 30, 14, 266, 43}

$$\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)-\frac{5c^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4b\sqrt{cx-1}\sqrt{cx+1}}+\frac{5c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^
5*d^2*x^2*Sqrt[d - c^2*d*x^2])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^
2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 + (5*c^2*d^2*(1 - c*x)*(1 +
 c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x) - (d^2*(1 - c*x)^2*(1
 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (5*c^3*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (7*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} + \frac{\left(bcd^2\sqrt{d-c\right.}{3\sqrt{-1}}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x} - \frac{d^2(1-cx)^2(1}{}$$

$$= \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) + \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-}{3x}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{6x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^2\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}$$

Mathematica [A] time = 1.43, size = 319, normalized size = 1.09

$$\frac{-4d^3\left(a\sqrt{\frac{cx-1}{cx+1}}\left(3c^6x^6+11c^4x^4-16c^2x^2+2\right)-14bc^3x^3(cx-1)\log(cx)+bcx(1-cx)\right)-60ac^3d^{5/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{c}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (30*b*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^2 - 60*a*c^3*d^(5/2)*x^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sq
rt[d]*(-1 + c^2*x^2))] + 3*b*c^3*d^3*x^3*(-1 + c*x)*Cosh[2*ArcCosh[c*x]] -
4*d^3*(b*c*x*(1 - c*x) + a*Sqrt[(-1 + c*x)/(1 + c*x)]*(2 - 16*c^2*x^2 + 11*
c^4*x^4 + 3*c^6*x^6) - 14*b*c^3*x^3*(-1 + c*x)*Log[c*x]) - 2*b*d^3*(-1 + c*
x)*ArcCosh[c*x]*(4*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 - c*x + 7*c^2*x^2 + 7*c^3
*x^3) + 3*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.79, size = 1407, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] 4/3*a*c^2/d/x*(-c^2*d*x^2+d)^(7/2)-1/3*a/d/x^3*(-c^2*d*x^2+d)^(7/2)+4/3*a*c
^4*x*(-c^2*d*x^2+d)^(5/2)-147*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+35*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*a
rccosh(c*x)*c^5+190/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1
)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-23/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*
c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2+147*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c
^8-203*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(
c*x-1)*arccosh(c*x)*c^6-49/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^
2*x^2+1)*x^3*c^6+7/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)
*x*c^4+1/8*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)+1/3
*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)
*arccosh(c*x)-21/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x
^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-1/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4
*x^4-15*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-7/3*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*
x)*c^3+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^
3-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)*x+49/6*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*
c^8-28/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)
/(c*x-1)*c^6+7/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(
c*x+1)/(c*x-1)*c^4+5/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+
1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*x^2-5/4*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)^2*d^2*c^3+14/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^3-7/3*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3+5/3*a
*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*a*c^
4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(10\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4dx+15\sqrt{-c^2dx^2+d}\,c^4d^2x+15\,c^3d^{\frac{5}{2}}\arcsin\left(cx\right)+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{x}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima"
)
```

```
[Out] 1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
+ 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x
^2 + d)^(7/2)/(d*x^3))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Timed out
```

3.92 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx$

Optimal. Leaf size=293

$$-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5}+\frac{c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+\frac{c^5d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\,\sqrt{cx+1}}-$$

```
[Out] 1/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^3-1/5*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))/x^5-c^4*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/
x-1/20*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/30*b
*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*c^5*d^2*(
a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/1
5*b*c^5*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 324, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5798, 5740, 5738, 29, 5676, 14, 266, 43}

$$\frac{c^5d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2b\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{c^4d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}\,\left(a+b\right.}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(20*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11*
b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(30*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c^4
*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/x + (c^2*d^2*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^3) - (d^2*(1 - c*x)^2*(
1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*x^5) + (c^5*d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + (23*b*c^5*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^6}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5x^5} + \frac{\left(bcd^2\sqrt{d-c^2dx\right.}}{5\sqrt{-1+}}$$

$$= \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{d^2(1-cx)^2(1+c}{}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x} + \frac{c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x^3}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{20x^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2\sqrt{d-c^2dx^2}}{30x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}}{}$$

Mathematica [A] time = 3.59, size = 400, normalized size = 1.37

$$\frac{d^2\left(120ac^5\sqrt{d}\,x^5\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+8ad\sqrt{\frac{cx-1}{cx+1}}\left(c^2x^2-1\right)\left(23c^4x^4-11c^2x^2+3\right)-60bc^4dx^4(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^6,x]
```

```
[Out] (d^2*(8*a*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + c^2*x^2)*(3 - 11*c^2*x^2 + 23*
c^4*x^4) + 120*a*c^5*Sqrt[d]*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 40*b*c^2*
d*x^2*(1 - c*x)*(c*x - 2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c
*x] + 2*c^3*x^3*Log[c*x]) - 60*b*c^4*d*x^4*(1 - c*x)*(2*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - b*d*(
1 - c*x)*(20*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + Cosh[5*Arc
Cosh[c*x]]*Log[c*x] + Cosh[3*ArcCosh[c*x]]*(-1 + 5*Log[c*x]) + c*x*(3 + 10*
Log[c*x]) - 5*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Sinh[5*ArcCo
sh[c*x]])))/(120*x^5*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^6}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^6, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.86, size = 2429, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x)
```

```
[Out] 23/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^5+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)*arccosh(c*x)^2*d^2*c^5-46/15*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d^2*c^5-175/4*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*c^5-5819/30*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6
+325*c^4*x^4-75*c^2*x^2+9)*x^9/(c*x+1)/(c*x-1)*c^14+18791/60*b*(-d*(c^2*x^2
-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^7/(c*x
+1)/(c*x-1)*c^12-943/6*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x
^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1)*c^10+207/5*b*(-d*(c^2*x^2-
1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c*x+
1)/(c*x-1)*c^8-69/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6
+325*c^4*x^4-75*c^2*x^2+9)*x/(c*x+1)/(c*x-1)*c^6+9/5*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^5/(c*x+1)/(c*x
-1)*arccosh(c*x)-1329/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*
x^6+325*c^4*x^4-75*c^2*x^2+9)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9+1889/12*b
*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^
2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+141/20*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x^2/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c^3-9/20*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+
325*c^4*x^4-75*c^2*x^2+9)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+69/5*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/(c*x
+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+759/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2
/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^6/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c^11+2/15*a*c^2/d/x^3*(-c^2*d*x^2+d)^(7/2)-1/5*a/d/x^5*(-c^2*d*x^
2+d)^(7/2)-8/15*a*c^6*x*(-c^2*d*x^2+d)^(5/2)+759/20*b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3*c^8-69/20*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+
9)*x*c^6+5819/30*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325
*c^4*x^4-75*c^2*x^2+9)*x^7*c^12-7153/60*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*
c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5*c^10-8/15*a*c^4/d/x*(-c^2
*d*x^2+d)^(7/2)-2/3*a*c^6*d*x*(-c^2*d*x^2+d)^(3/2)-a*c^6*d^2*x*(-c^2*d*x^2+
d)^(1/2)-a*c^6*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2
))+1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-
75*c^2*x^2+9)*x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-1173*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*
x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+1495/3*b*(-d*(c^2*x^2-1))
^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^4/(c*x+1)^
(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-115*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035
*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*arccosh(c*x)*c^7-1587*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^
6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+3519*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x
^2+9)*x^7/(c*x+1)/(c*x-1)*arccosh(c*x)*c^12-9595/3*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^5/(c*x+1)/(c*x-1
)*arccosh(c*x)*c^10+5318/3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c
^6*x^6+325*c^4*x^4-75*c^2*x^2+9)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-9602/
```

```
15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^
2*x^2+9)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+777/5*b*(-d*(c^2*x^2-1))^(1/2)*
d^2/(1035*c^8*x^8-765*c^6*x^6+325*c^4*x^4-75*c^2*x^2+9)/x/(c*x+1)/(c*x-1)*a
rccosh(c*x)*c^4-117/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(1035*c^8*x^8-765*c^6*x^
6+325*c^4*x^4-75*c^2*x^2+9)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(10\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^6dx+15\sqrt{-c^2dx^2+d}\,c^6d^2x+15\,c^5d^{\frac{5}{2}}\arcsin\left(cx\right)+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^4}{x}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{dx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^6,x, algorithm="maxima"
)
```

```
[Out] -1/15*(10*(-c^2*d*x^2 + d)^(3/2)*c^6*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^6*d^2*
x + 15*c^5*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^4/x - 2*(-c^2*d
*x^2 + d)^(7/2)*c^2/(d*x^3) + 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^5))*a + b*integ
rate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^6, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^6, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**6,x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7dx^7}-\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^7d^2\log(x)\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{cx-1}\sqrt{cx+1}}+$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^7-1/42*b*c*d^2*(-c^2*d*x^2
+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/28*b*c^3*d^2*(-c^2*d*x^2+d)^(1/
2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/14*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c^7*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 234, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$-\frac{d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7}-\frac{3bc^5d^2\sqrt{d-c^2dx^2}}{14x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2\sqrt{}}{42x^6\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(42*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(28*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*
c^5*d^2*Sqrt[d - c^2*d*x^2])/(14*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(
1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*x^7) -
(b*c^7*d^2*Sqrt[d - c^2*d*x^2]*Log[x])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^8}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx\right.}{7\sqrt{-1+}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx\right.}{14}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7x^7} - \frac{\left(bcd^2\sqrt{d-c^2dx\right.}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{42x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3bc^3d^2\sqrt{d-c^2dx^2}}{28x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3bc^5d^2\sqrt{d-c}}{14x^2\sqrt{-1+cx}\,\sqrt{}}$$

Mathematica [A] time = 0.10, size = 105, normalized size = 0.48

$$\frac{d^2\sqrt{d-c^2dx^2}\left(12(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\left(12c^6x^6\log(x)+18c^4x^4-9c^2x^2+2\right)\right)}{84x^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^8,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(12*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCos
h[c*x]) - b*c*x*(2 - 9*c^2*x^2 + 18*c^4*x^4 + 12*c^6*x^6*Log[x])))/(84*x^7*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.59, size = 703, normalized size = 3.21

$$\left[\frac{12\left(bc^8d^2x^8-4\,bc^6d^2x^6+6\,bc^4d^2x^4-4\,bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+6\left(bc^9d^2x^9-bc^7d^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="fricas"
)
```

```
[Out] [1/84*(12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 6*(b*c^9*d
^2*x^9 - b*c^7*d^2*x^7)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(
-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2))
- (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*c^3*d^2*x^
3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^8*d^2*x^8
 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d
*x^2 + d))/(c^2*x^9 - x^7), -1/84*(12*(b*c^9*d^2*x^9 - b*c^7*d^2*x^7)*sqrt(
```

```
d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x
^4 - (c^2 + 1)*d*x^2 + d)) - 12*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*
d^2*x^4 - 4*b*c^2*d^2*x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*
x^2 - 1)) + (18*b*c^5*d^2*x^5 - (18*b*c^5 - 9*b*c^3 + 2*b*c)*d^2*x^7 - 9*b*
c^3*d^2*x^3 + 2*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 12*(a*c
^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*s
qrt(-c^2*d*x^2 + d))/(c^2*x^9 - x^7)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.94, size = 3775, normalized size = 17.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x)
```

```
[Out] 17/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*
c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5*c^12+3/14*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^1
3/(c*x+1)/(c*x-1)*c^20-27/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c
^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11/(c*x+1)/(c*x-1)
*c^18+73/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x
^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9/(c*x+1)/(c*x-1)*c^16-67/42*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)*x^7/(c*x+1)/(c*x-1)*c^14+11/14*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1
)*x^5/(c*x+1)/(c*x-1)*c^12-17/84*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-
21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x
-1)*c^10+1/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8
*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^8-5/28*b*(-d*(c
^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4
*x^4-7*c^2*x^2+1)*x^3*c^10+1/42*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-2
1*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x*c^8-3/14*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c
^4*x^4-7*c^2*x^2+1)*x^11*c^18+3/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12
-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^9*c^16-83/84*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^
6+21*c^4*x^4-7*c^2*x^2+1)*x^7*c^14+1/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12
*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^7/(c*x+1
)/(c*x-1)*arccosh(c*x)+21/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^
10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^8/(c*x+1)^(1/2)/(c*
x-1)^(1/2)*c^15-119/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^
10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^13+47/4*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c
^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^11-109/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^
8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^9-1/
7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*
x^6+21*c^4*x^4-7*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7-41
```

```
/28*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^
6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+23/84*b*(
-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+2
1*c^4*x^4-7*c^2*x^2+1)/x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-1/42*b*(-d*(c^2*
x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^
4-7*c^2*x^2+1)/x^6/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-3/2*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2
+1)*x^10/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^17-165/7*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x
/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+55/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12
*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^3/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^4-11/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-2
1*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/x^5/(c*x+1)/(c*x-
1)*arccosh(c*x)*c^2-1/7*a/d/x^7*(-c^2*d*x^2+d)^(7/2)+b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2
+1)*x^13/(c*x+1)/(c*x-1)*arccosh(c*x)*c^20-7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^11
/(c*x+1)/(c*x-1)*arccosh(c*x)*c^18-3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x
^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^4/(c*x+1)^
(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^11+b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*
x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^2/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^9-b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*
x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^12/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^19+3*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c
^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^10/(c
*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^17+5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/
(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^6
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^13-5*b*(-d*(c^2*x^2-1))^(1/2)*d
^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*
x^8/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^15+23*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2
+1)*x^9/(c*x+1)/(c*x-1)*arccosh(c*x)*c^16-47*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(
7*c^12*x^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^7/
(c*x+1)/(c*x-1)*arccosh(c*x)*c^14+66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x
^12-21*c^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^5/(c*x+1)/
(c*x-1)*arccosh(c*x)*c^12-66*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c
^10*x^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*
arccosh(c*x)*c^10+330/7*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x
^10+35*c^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^8+55/12*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(7*c^12*x^12-21*c^10*x^10+35*c
^8*x^8-35*c^6*x^6+21*c^4*x^4-7*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7-1
/7*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2)*d^2*c^7+2/7*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)*arccosh(c*x)*d^2*c^7
```

maxima [C] time = 0.96, size = 224, normalized size = 1.02

$$\frac{\left(6\,c^8 d^4 \sqrt{-\frac{1}{c^4 d}}\,\log\left(x^2-\frac{1}{c^2}\right)+6i\,(-1)^{-2\,c^2 d x^2+2\,d}\,c^6 d^{\frac{7}{2}}\log\left(-2\,c^2 d+\frac{2\,d}{x^2}\right)+\frac{11\,\sqrt{-c^4 d x^4+2\,c^2 d x^2-d}\,c^4 d^3}{x^2}-\frac{7\,\sqrt{-c^4 d x^4+2\,c^2 d x^2}}{x^4}}{84\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^8,x, algorithm="maxima"
)
```

```
[Out] 1/84*(6*c^8*d^4*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + 6*I*(-1)^(-2*c^2*d*x^2
+ 2*d)*c^6*d^(7/2)*log(-2*c^2*d + 2*d/x^2) + 11*sqrt(-c^4*d*x^4 + 2*c^2*d*x
^2 - d)*c^4*d^3/x^2 - 7*sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*c^2*d^3/x^4 + 2*
sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*d^3/x^6)*b*c/d - 1/7*(-c^2*d*x^2 + d)^(7
/2)*b*arccosh(c*x)/(d*x^7) - 1/7*(-c^2*d*x^2 + d)^(7/2)*a/(d*x^7)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^{5/2}}{x^8}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8,x)
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^8, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**8,x)
[Out] Timed out
```

3.94 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx$

Optimal. Leaf size=314

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{9dx^9}-\frac{2c^2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{63dx^7}-\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bc^9d^2 1}{63\sqrt{}}$$

```
[Out] -1/9*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^9-2/63*c^2*(-c^2*d*x^2+d)^
(7/2)*(a+b*arccosh(c*x))/d/x^7-1/189*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+1/42*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-1/21*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/72*b*c*d^2*(-c^2*x^2+1)^4*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-2/63*b*c^9*d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 448, normalized size of antiderivative = 1.43, number of steps used = 7, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 97, 12, 103, 95, 5733, 446, 78, 43}

$$\frac{2c^8d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x}+\frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63x^3}-\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5}+$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] -(b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(189*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(42*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^7*d^2*Sqrt[d - c^2*d*x^2])/(21*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(72*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(21*x^5) + (c^6*d^2*
Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x^3) + (2*c^8*d^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x) + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*x^7) - (d^2*(1 - c*x)^2*(1 + c*x)
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*x^9) - (2*b*c^9*d^2*Sqrt[d
- c^2*d*x^2]*Log[x])/(63*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{10}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{63x^3}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{63x^3}$$

$$= -\frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(\right.}{63x^3}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} +$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^4\sqrt{d-c^2dx^2}}{72x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21x^5} +$$

$$= -\frac{bc^3d^2\sqrt{d-c^2dx^2}}{189x^6\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^5d^2\sqrt{d-c^2dx^2}}{42x^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^7d^2\sqrt{d-c}}{21x^2\sqrt{-1+cx}}$$

Mathematica [A] time = 0.17, size = 147, normalized size = 0.47

$$\frac{d^2\sqrt{d-c^2dx^2}\left(48c^2x^2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+168(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-bcx\right.}{1512x^9\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^10,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(168*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCo
sh[c*x]) + 48*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x])
 - b*c*x*(21 - 76*c^2*x^2 + 90*c^4*x^4 - 12*c^6*x^6 + 48*c^8*x^8*Log[x]))/
(1512*x^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.71, size = 795, normalized size = 2.53

$$\left[\frac{24\left(2\,bc^{10}d^2x^{10}-bc^8d^2x^8-16\,bc^6d^2x^6+34\,bc^4d^2x^4-26\,bc^2d^2x^2+7\,bd^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="fricas
")
```

```
[Out] [1/1512*(24*(2*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^
4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt
(c^2*x^2 - 1)) + 24*(b*c^11*d^2*x^11 - b*c^9*d^2*x^9)*sqrt(-d)*log((c^2*d*x
^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*s
qrt(-d) - d)/(c^2*x^4 - x^2)) + (12*b*c^7*d^2*x^7 - 90*b*c^5*d^2*x^5 - (12*
b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*b*c^3*d^2*x^3 - 21*b*c*d
```

```
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 24*(2*a*c^10*d^2*x^10 - a*c^
8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 26*a*c^2*d^2*x^2 + 7*a*d^
2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9), -1/1512*(48*(b*c^11*d^2*x^11 - b
*c^9*d^2*x^9)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 +
1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 24*(2*b*c^10*d^2*x^10 - b*c
^8*d^2*x^8 - 16*b*c^6*d^2*x^6 + 34*b*c^4*d^2*x^4 - 26*b*c^2*d^2*x^2 + 7*b*d
^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (12*b*c^7*d^2*x^7 -
 90*b*c^5*d^2*x^5 - (12*b*c^7 - 90*b*c^5 + 76*b*c^3 - 21*b*c)*d^2*x^9 + 76*
b*c^3*d^2*x^3 - 21*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 24*(
2*a*c^10*d^2*x^10 - a*c^8*d^2*x^8 - 16*a*c^6*d^2*x^6 + 34*a*c^4*d^2*x^4 - 2
6*a*c^2*d^2*x^2 + 7*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^11 - x^9)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.09, size = 5006, normalized size = 15.94

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x)
```

```
[Out] result too large to display
```

maxima [A] time = 0.52, size = 187, normalized size = 0.60

$$-\frac{1}{1512}\left(48\,c^{8}\sqrt{-d}\,d^{2}\log(x)-\frac{12\,c^{6}\sqrt{-d}\,d^{2}x^{6}-90\,c^{4}\sqrt{-d}\,d^{2}x^{4}+76\,c^{2}\sqrt{-d}\,d^{2}x^{2}-21\,\sqrt{-d}\,d^{2}}{x^{8}}\right)bc-\frac{1}{63}\,b\left(\frac{2\left(-c^{2}d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^10,x, algorithm="maxima
")
```

```
[Out] -1/1512*(48*c^8*sqrt(-d)*d^2*log(x) - (12*c^6*sqrt(-d)*d^2*x^6 - 90*c^4*sqr
t(-d)*d^2*x^4 + 76*c^2*sqrt(-d)*d^2*x^2 - 21*sqrt(-d)*d^2)/x^8)*b*c - 1/63*
b*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^2 + d)^(7/2)/(d*x^9))
*arccosh(c*x) - 1/63*a*(2*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^7) + 7*(-c^2*d*x^
2 + d)^(7/2)/(d*x^9))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^{10}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^10, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**10,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(c x)\right)}{x^{12}} \, dx$

Optimal. Leaf size=385

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{11dx^{11}}-\frac{4c^2\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{99dx^9}-\frac{8c^4\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx\right.}{693dx^7}$$

```
[Out] -1/11*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/d/x^11-4/99*c^2*(-c^2*d*x^2+d
)^(7/2)*(a+b*arccosh(c*x))/d/x^9-8/693*c^4*(-c^2*d*x^2+d)^(7/2)*(a+b*arccos
h(c*x))/d/x^7-1/110*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^10/(c*x-1)^(1/2)/(c*x+1)
^(1/2)+23/792*b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x^8/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-113/4158*b*c^5*d^2*(-c^2*d*x^2+d)^(1/2)/x^6/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1
/924*b*c^7*d^2*(-c^2*d*x^2+d)^(1/2)/x^4/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/693*b
*c^9*d^2*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-8/693*b*c^11*
d^2*ln(x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 519, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 97, 12, 103, 95, 5733, 1251, 893}

$$\frac{8c^{10}d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693x}+\frac{4c^8d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693x^3}+\frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{231x^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(110*x^10*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2
3*b*c^3*d^2*Sqrt[d - c^2*d*x^2])/(792*x^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
113*b*c^5*d^2*Sqrt[d - c^2*d*x^2])/(4158*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*c^7*d^2*Sqrt[d - c^2*d*x^2])/(924*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(2*b*c^9*d^2*Sqrt[d - c^2*d*x^2])/(693*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(5*c^4*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*x^7) + (c^6*d^2*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(231*x^5) + (4*c^8*d^2*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(693*x^3) + (8*c^10*d^2*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x]))/(693*x) + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x]))/(99*x^9) - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(11*x^11) - (8*b*c^11*d^2*Sqrt[d - c^2
*d*x^2]*Log[x])/(693*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
```

```
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(d-c^2dx^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^{12}}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))}{x^{12}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{231x^5}$$

$$= -\frac{5c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{231x^7} + \frac{c^6d^2\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{231x^5}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{110x^{10}\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{23bc^3d^2\sqrt{d-c^2dx^2}}{792x^8\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{113bc^5d}{4158x^6\sqrt{-}}$$

Mathematica [A] time = 0.21, size = 165, normalized size = 0.43

$$\frac{d^2\sqrt{d-c^2dx^2}\left(480c^2x^2(cx-1)^{7/2}\left(2c^2x^2+7\right)(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+7560(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\right.\right.}{83160x^{11}\sqrt{cx-1}\,\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^12,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(7560*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcC
osh[c*x]) + 480*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(7 + 2*c^2*x^2)*(a
 + b*ArcCosh[c*x]) - b*c*x*(756 - 2415*c^2*x^2 + 2260*c^4*x^4 - 90*c^6*x^6
- 240*c^8*x^8 + 960*c^10*x^10*Log[x])))/(83160*x^11*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

fricas [A] time = 0.62, size = 879, normalized size = 2.28

$$\left[\frac{120\left(8\,bc^{12}d^2x^{12}-4\,bc^{10}d^2x^{10}-bc^8d^2x^8-116\,bc^6d^2x^6+274\,bc^4d^2x^4-224\,bc^2d^2x^2+63\,bd^2\right)\sqrt{-c^2dx^2+d}\,\mathrm{l}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="fricas
")
```

```
[Out] [1/83160*(120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*
b*c^6*d^2*x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 480*(b*c^13*d^2*x^13 - b*c^11*d^
2*x^11)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + (240*b*c^9*d^2
*x^9 + 90*b*c^7*d^2*x^7 - (240*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 -
 756*b*c)*d^2*x^11 - 2260*b*c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*
x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 120*(8*a*c^12*d^2*x^12 - 4*a*c^
10*d^2*x^10 - a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a
```

```
*c^2*d^2*x^2 + 63*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^2*x^13 - x^11), -1/83160*
(960*(b*c^13*d^2*x^13 - b*c^11*d^2*x^11)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d
)*sqrt(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) -
120*(8*b*c^12*d^2*x^12 - 4*b*c^10*d^2*x^10 - b*c^8*d^2*x^8 - 116*b*c^6*d^2*
x^6 + 274*b*c^4*d^2*x^4 - 224*b*c^2*d^2*x^2 + 63*b*d^2)*sqrt(-c^2*d*x^2 + d
)*log(c*x + sqrt(c^2*x^2 - 1)) - (240*b*c^9*d^2*x^9 + 90*b*c^7*d^2*x^7 - (2
40*b*c^9 + 90*b*c^7 - 2260*b*c^5 + 2415*b*c^3 - 756*b*c)*d^2*x^11 - 2260*b*
c^5*d^2*x^5 + 2415*b*c^3*d^2*x^3 - 756*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 120*(8*a*c^12*d^2*x^12 - 4*a*c^10*d^2*x^10 - a*c^8*d^2*x^8
- 116*a*c^6*d^2*x^6 + 274*a*c^4*d^2*x^4 - 224*a*c^2*d^2*x^2 + 63*a*d^2)*sqr
t(-c^2*d*x^2 + d))/(c^2*x^13 - x^11)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**B**] time = 1.31, size = 6379, normalized size = 16.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x)
```

```
[Out] result too large to display
```

maxima [**A**] time = 0.80, size = 251, normalized size = 0.65

$$-\frac{1}{83160}\left(960\,c^{10}\sqrt{-d}\,d^2\log(x) - \frac{240\,c^8\sqrt{-d}\,d^2x^8 + 90\,c^6\sqrt{-d}\,d^2x^6 - 2260\,c^4\sqrt{-d}\,d^2x^4 + 2415\,c^2\sqrt{-d}\,d^2x^2 - 756}{x^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^12,x, algorithm="maxima
")
```

```
[Out] -1/83160*(960*c^10*sqrt(-d)*d^2*log(x) - (240*c^8*sqrt(-d)*d^2*x^8 + 90*c^6
*sqrt(-d)*d^2*x^6 - 2260*c^4*sqrt(-d)*d^2*x^4 + 2415*c^2*sqrt(-d)*d^2*x^2 -
 756*sqrt(-d)*d^2)/x^10)*b*c - 1/693*b*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7
) + 28*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^
11))*arccosh(c*x) - 1/693*a*(8*(-c^2*d*x^2 + d)^(7/2)*c^4/(d*x^7) + 28*(-c^
2*d*x^2 + d)^(7/2)*c^2/(d*x^9) + 63*(-c^2*d*x^2 + d)^(7/2)/(d*x^11))
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^{12}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^12, x)
```

<u>**sympy**</u> **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**12,x)
[Out] Timed out
```

3.96 $\int x^7 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=458

$$\frac{\left(d-c^2dx^2\right)^{13/2}\left(a+b\cosh^{-1}(cx)\right)}{13c^8d^4}-\frac{3\left(d-c^2dx^2\right)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^8d^3}+\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^8d^2}-\frac{(d-}{}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^8/d+1/3*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^8/d^2-3/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c
^8/d^3+1/13*(-c^2*d*x^2+d)^(13/2)*(a+b*arccosh(c*x))/c^8/d^4+16/3003*b*d^2*
x*(-c^2*d*x^2+d)^(1/2)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/9009*b*d^2*x^3*(-c
^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/5005*b*d^2*x^5*(-c^2*d*
x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/21021*b*d^2*x^7*(-c^2*d*x^2+
d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-53/3861*b*c*d^2*x^9*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+27/1573*b*c^3*d^2*x^11*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/169*b*c^5*d^2*x^13*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 527, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1810}

$$-\frac{d^2x^6(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{13c^2}-\frac{6d^2x^4(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{143c^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (16*b*d^2*x*Sqrt[d - c^2*d*x^2])/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(8*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(9009*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 (2*b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(5005*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (5*b*d^2*x^7*Sqrt[d - c^2*d*x^2])/(21021*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (53*b*c*d^2*x^9*Sqrt[d - c^2*d*x^2])/(3861*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (27*b*c^3*d^2*x^11*Sqrt[d - c^2*d*x^2])/(1573*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (b*c^5*d^2*x^13*Sqrt[d - c^2*d*x^2])/(169*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (16*d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(3003*c^8) - (8*d^2*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a +
 b*ArcCosh[c*x]))/(429*c^6) - (6*d^2*x^4*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(143*c^4) - (d^2*x^6*(1 - c*x)^3*(1 + c*x)^
3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(13*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^7 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^7(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1}{}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1}{}$$

$$= -\frac{16d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3003c^8} - \frac{8d^2x^2(1}{}$$

$$= \frac{16bd^2x\sqrt{d-c^2dx^2}}{3003c^7\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{8bd^2x^3\sqrt{d-c^2dx^2}}{9009c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bd}{5005c^3}$$

Mathematica [A] time = 0.26, size = 193, normalized size = 0.42

$$\frac{d^2\sqrt{d-c^2dx^2}\left(231c^5x^6(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{2(cx-1)^{7/2}(cx+1)^{7/2}\left(63c^4x^4+28c^2x^2+8\right)\left(a+b\cosh^{-1}(cx)\right)}{c}\right.}{3003c^7\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(16*x + (8*c^2*x^3)/3 + (6*c^4*x^5)/5 + (5*c^6*
x^7)/7 - (371*c^8*x^9)/9 + (567*c^10*x^11)/11 - (231*c^12*x^13)/13) + 231*c
^5*x^6*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (2*(-1 + c*x
)^(7/2)*(1 + c*x)^(7/2)*(8 + 28*c^2*x^2 + 63*c^4*x^4)*(a + b*ArcCosh[c*x]))
/c))/(3003*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.63, size = 353, normalized size = 0.77

$$\frac{45045 \left(231 \, bc^{14}d^2x^{14} - 798 \, bc^{12}d^2x^{12} + 938 \, bc^{10}d^2x^{10} - 376 \, bc^8d^2x^8 - bc^6d^2x^6 - 2 \, bc^4d^2x^4 - 8 \, bc^2d^2x^2 + 16 \, bd^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/135270135*(45045*(231*b*c^14*d^2*x^14 - 798*b*c^12*d^2*x^12 + 938*b*c^10*
d^2*x^10 - 376*b*c^8*d^2*x^8 - b*c^6*d^2*x^6 - 2*b*c^4*d^2*x^4 - 8*b*c^2*d^
2*x^2 + 16*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (8004
15*b*c^13*d^2*x^13 - 2321865*b*c^11*d^2*x^11 + 1856855*b*c^9*d^2*x^9 - 3217
5*b*c^7*d^2*x^7 - 54054*b*c^5*d^2*x^5 - 120120*b*c^3*d^2*x^3 - 720720*b*c*d
^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 45045*(231*a*c^14*d^2*x^14 -
 798*a*c^12*d^2*x^12 + 938*a*c^10*d^2*x^10 - 376*a*c^8*d^2*x^8 - a*c^6*d^2*
x^6 - 2*a*c^4*d^2*x^4 - 8*a*c^2*d^2*x^2 + 16*a*d^2)*sqrt(-c^2*d*x^2 + d))/(
c^10*x^2 - c^8)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.88, size = 2374, normalized size = 5.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/13*x^6*(-c^2*d*x^2+d)^(7/2)/c^2/d+6/13/c^2*(-1/11*x^4*(-c^2*d*x^2+d)^
(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*
d*x^2+d)^(7/2))))+b*(1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-1+22784*c^10*x^10-1
6896*c^8*x^8+6496*c^6*x^6-1204*c^4*x^4+85*c^2*x^2+4096*x^14*c^14+4096*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^13*c^13-15360*x^12*c^12-9984*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^7*c^7+2912*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c-364*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-13312*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^11*c^11+16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9)*(-1+13*
arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*(c^2*x^2-1))^(1/2)*(1-33
28*c^10*x^10+4096*c^8*x^8-2352*c^6*x^6+620*c^4*x^4-61*c^2*x^2+1024*x^12*c^1
2+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^5*c^5-11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+220*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^3*c^3+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^9*c^9)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-
d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(c*x-1)^(
```

```
1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-280*c^4*x^
4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccosh(c*x))*
d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6
*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1
)+3/40960*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)
+5/24576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*d^
2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-5/2048*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1)+5/24576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*
x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+3/40960*(-d*(c^2*x^2-1))^
(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x
^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)-3/200704*(-d*(c^2*x^2-1))^
(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+1
04*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x
))*d^2/(c*x+1)/c^8/(c*x-1)-1/110592*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^8/(c*x-1)+1/991232*(-d*
(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+1024*x^12*c
^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*c^10*x^10-2816*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5
*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x^4+11*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2/(c*x+1)
/c^8/(c*x-1)+1/1384448*(-d*(c^2*x^2-1))^(1/2)*(-4096*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^13*c^13+4096*x^14*c^14+13312*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-
15360*x^12*c^12-16640*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+22784*c^10*x^10+9
984*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-16896*c^8*x^8-2912*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^5*c^5+6496*c^6*x^6+364*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-12
04*c^4*x^4-13*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+85*c^2*x^2-1)*(1+13*arccosh(c
*x))*d^2/(c*x+1)/c^8/(c*x-1))
```

maxima [A] time = 0.95, size = 313, normalized size = 0.68

$$-\frac{1}{3003}\left(\frac{231\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^6}{c^2d}+\frac{126\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^4}{c^4d}+\frac{56\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^6d}+\frac{16\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^8d}\right)b\operatorname{arcosh}(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/(c^2*d) + 126*(-c^2*d*x^2 + d)^(7/2
)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^6*d) + 16*(-c^2*d*x^2 + d)
^(7/2)/(c^8*d))*b*arccosh(c*x) - 1/3003*(231*(-c^2*d*x^2 + d)^(7/2)*x^6/(c^
2*d) + 126*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^4*d) + 56*(-c^2*d*x^2 + d)^(7/2)*x
^2/(c^6*d) + 16*(-c^2*d*x^2 + d)^(7/2)/(c^8*d))*a - 1/135270135*(800415*c^1
2*sqrt(-d)*d^2*x^13 - 2321865*c^10*sqrt(-d)*d^2*x^11 + 1856855*c^8*sqrt(-d)
*d^2*x^9 - 32175*c^6*sqrt(-d)*d^2*x^7 - 54054*c^4*sqrt(-d)*d^2*x^5 - 120120
*c^2*sqrt(-d)*d^2*x^3 - 720720*sqrt(-d)*d^2*x)*b/c^7
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^7\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^7*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.97 $\int x^5 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=378

$$-\frac{\left(d-c^2dx^2\right)^{11/2}\left(a+b\cosh^{-1}(cx)\right)}{11c^6d^3}+\frac{2\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^6d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^6d}-\frac{1}{4}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^6/d+2/9*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^6/d^2-1/11*(-c^2*d*x^2+d)^(11/2)*(a+b*arccosh(c*x))/c
^6/d^3+8/693*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4
/2079*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1155
*b*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-113/4851*b*c*
d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+23/891*b*c^3*d^2*x
^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/121*b*c^5*d^2*x^11*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 429, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 1153}

$$-\frac{d^2x^4(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{11c^2}-\frac{4d^2x^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{99c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (8*b*d^2*x*Sqrt[d - c^2*d*x^2])/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4
*b*d^2*x^3*Sqrt[d - c^2*d*x^2])/(2079*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b*d^2*x^5*Sqrt[d - c^2*d*x^2])/(1155*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (113
*b*c*d^2*x^7*Sqrt[d - c^2*d*x^2])/(4851*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (23
*b*c^3*d^2*x^9*Sqrt[d - c^2*d*x^2])/(891*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*c^5*d^2*x^11*Sqrt[d - c^2*d*x^2])/(121*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (8*
d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(693*
c^6) - (4*d^2*x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(99*c^4) - (d^2*x^4*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(11*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
```

```
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^5 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right) \int x^5(-1+cx)^{5/2}(1+cx)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-c}{}$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-c}{}$$

$$= -\frac{8d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{693c^6} - \frac{4d^2x^2(1-c}{}$$

$$= \frac{8bd^2x\sqrt{d-c^2dx^2}}{693c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bd^2x^3\sqrt{d-c^2dx^2}}{2079c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bd^2x^5\sqrt{d}}{1155c\sqrt{-1+}}$$

Mathematica [A] time = 0.20, size = 175, normalized size = 0.46

$$\frac{d^2\sqrt{d-c^2dx^2}\left(63c^3x^4(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+\frac{4(cx-1)^{7/2}(cx+1)^{7/2}\left(7c^2x^2+2\right)\left(a+b\cosh^{-1}(cx)\right)}{c}+b\left(-\frac{63}{11}c^{10}\right.\right.}{693c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(b*(8*x + (4*c^2*x^3)/3 + (3*c^4*x^5)/5 - (113*c^6
*x^7)/7 + (161*c^8*x^9)/9 - (63*c^10*x^11)/11) + 63*c^3*x^4*(-1 + c*x)^(7/2
```

```
)*(1 + c*x)^(7/2)*(a + b*ArcCosh[c*x]) + (4*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2
)*(2 + 7*c^2*x^2)*(a + b*ArcCosh[c*x]))/c))/(693*c^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])
```

fricas [A] time = 0.50, size = 317, normalized size = 0.84

$$\frac{3465\left(63\,bc^{12}d^2x^{12}-224\,bc^{10}d^2x^{10}+274\,bc^8d^2x^8-116\,bc^6d^2x^6-bc^4d^2x^4-4\,bc^2d^2x^2+8\,bd^2\right)\sqrt{-c^2dx^2+d}\ 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/2401245*(3465*(63*b*c^12*d^2*x^12 - 224*b*c^10*d^2*x^10 + 274*b*c^8*d^2*x
^8 - 116*b*c^6*d^2*x^6 - b*c^4*d^2*x^4 - 4*b*c^2*d^2*x^2 + 8*b*d^2)*sqrt(-c
^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (19845*b*c^11*d^2*x^11 - 61985
*b*c^9*d^2*x^9 + 55935*b*c^7*d^2*x^7 - 2079*b*c^5*d^2*x^5 - 4620*b*c^3*d^2*
x^3 - 27720*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 3465*(63*a*
c^12*d^2*x^12 - 224*a*c^10*d^2*x^10 + 274*a*c^8*d^2*x^8 - 116*a*c^6*d^2*x^6
 - a*c^4*d^2*x^4 - 4*a*c^2*d^2*x^2 + 8*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^8*x^
2 - c^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 1840, normalized size = 4.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/11*x^4*(-c^2*d*x^2+d)^(7/2)/c^2/d+4/11/c^2*(-1/9*x^2*(-c^2*d*x^2+d)^(
7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2)))+b*(1/247808*(-d*(c^2*x^2-1))^(
1/2)*(1-3328*c^10*x^10+4096*c^8*x^8-2352*c^6*x^6+620*c^4*x^4-61*c^2*x^2+102
4*x^12*c^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-1232*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^5*c^5-11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+220*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*x^3*c^3+1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11-2816*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^9*c^9)*(-1+11*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1
/165888*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x
^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/
c^6/(c*x-1)+1/10240*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c
```

```
^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh
(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1024*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/1
024*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+
arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+5/9216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/10240*(-d*(c^2*
x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
13*c^2*x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-5/100352*(-d*(c^2*
x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arc
cosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)-1/165888*(-d*(c^2*x^2-1))^(1/2)*(-256*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^
6+120*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^6/(c*x-1)+1/247
808*(-d*(c^2*x^2-1))^(1/2)*(-1024*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^11*c^11+102
4*x^12*c^12+2816*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^9*c^9-3328*c^10*x^10-2816*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+4096*c^8*x^8+1232*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^5*c^5-2352*c^6*x^6-220*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+620*c^4*x
^4+11*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-61*c^2*x^2+1)*(1+11*arccosh(c*x))*d^2
/(c*x+1)/c^6/(c*x-1))
```

maxima [A] time = 0.88, size = 249, normalized size = 0.66

$$-\frac{1}{693}\left(\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^4}{c^2d}+\frac{28\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^4d}+\frac{8\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^6d}\right)b\operatorname{arcosh}(cx)-\frac{1}{693}\left(\frac{63\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^4}{c^2d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/693*(63*(-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x
^2/(c^4*d) + 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*b*arccosh(c*x) - 1/693*(63*(
-c^2*d*x^2 + d)^(7/2)*x^4/(c^2*d) + 28*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^4*d) +
 8*(-c^2*d*x^2 + d)^(7/2)/(c^6*d))*a - 1/2401245*(19845*c^10*sqrt(-d)*d^2*x
^11 - 61985*c^8*sqrt(-d)*d^2*x^9 + 55935*c^6*sqrt(-d)*d^2*x^7 - 2079*c^4*sq
rt(-d)*d^2*x^5 - 4620*c^2*sqrt(-d)*d^2*x^3 - 27720*sqrt(-d)*d^2*x)*b/c^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^5*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.98 $\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=298

$$\frac{\left(d-c^2dx^2\right)^{9/2}\left(a+b\cosh^{-1}(cx)\right)}{9c^4d^2}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^4d}-\frac{bcd^2x^5\sqrt{d-c^2dx^2}}{21\sqrt{cx-1}\sqrt{cx+1}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{cx-1}\sqrt{cx+}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^4/d+1/9*(-c^2*d*x^2+d)^(9/2)
*(a+b*arccosh(c*x))/c^4/d^2+2/63*b*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/189*b*d^2*x^3*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/21*b*c*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+19/441*b*c^3*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8
1*b*c^5*d^2*x^9*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 331, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 100, 12, 74, 5733, 373}

$$-\frac{d^2x^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9c^2}-\frac{2d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (2*b*d^2*x*Sqrt[d - c^2*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*
d^2*x^3*Sqrt[d - c^2*d*x^2])/(189*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^
2*x^5*Sqrt[d - c^2*d*x^2])/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (19*b*c^3*d^
2*x^7*Sqrt[d - c^2*d*x^2])/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^9*Sqrt[d - c^2*d*x^2])/(81*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*d^2*(1 - c*
x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(63*c^4) - (d^2*
x^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*c^
2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
```

```
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^3\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2(1-cx}{}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2(1-cx}{}$$

$$=-\frac{2d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{63c^4}-\frac{d^2x^2(1-cx}{}$$

$$=\frac{2bd^2x\sqrt{d-c^2dx^2}}{63c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bd^2x^3\sqrt{d-c^2dx^2}}{189c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2x^5\sqrt{d-c^2}}{21\sqrt{-1+cx}\sqrt{1}}$$

Mathematica [A] time = 0.15, size = 160, normalized size = 0.54

$$\frac{d^2\sqrt{d-c^2dx^2}\left(7c^2x^2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)+2(cx-1)^{7/2}(cx+1)^{7/2}\left(a+b\cosh^{-1}(cx)\right)-\frac{7}{9}bcx\left(\right.\right.}{63c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*((-7*b*c*x*(-1 + c^2*x^2)^4)/9 + (25*b*c*(x - c^2*
x^3 + (3*c^4*x^5)/5 - (c^6*x^7)/7))/9 + 2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*
(a + b*ArcCosh[c*x]) + 7*c^2*x^2*(-1 + c*x)^(7/2)*(1 + c*x)^(7/2)*(a + b*Ar
cCosh[c*x])))/(63*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.45, size = 281, normalized size = 0.94

$$63\left(7\,bc^{10}d^2x^{10}-26\,bc^8d^2x^8+34\,bc^6d^2x^6-16\,bc^4d^2x^4-bc^2d^2x^2+2\,bd^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] 1/3969*(63*(7*b*c^10*d^2*x^10 - 26*b*c^8*d^2*x^8 + 34*b*c^6*d^2*x^6 - 16*b*
c^4*d^2*x^4 - b*c^2*d^2*x^2 + 2*b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) - (49*b*c^9*d^2*x^9 - 171*b*c^7*d^2*x^7 + 189*b*c^5*d^2*x^5 -
 21*b*c^3*d^2*x^3 - 126*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) +
 63*(7*a*c^10*d^2*x^10 - 26*a*c^8*d^2*x^8 + 34*a*c^6*d^2*x^6 - 16*a*c^4*d^2
*x^4 - a*c^2*d^2*x^2 + 2*a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 1102, normalized size = 3.70

$$a\left(-\frac{x^2\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{9c^2d}-\frac{2\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{63d\,c^4}\right)+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(256c^{10}x^{10}-704c^8x^8+256\sqrt{cx+1}\,\sqrt{cx-1}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] a*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b*(
1/41472*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-
280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arcc
osh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^
8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3
*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c
^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(
c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(
c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arc
cosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*
c^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-
1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh
(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

maxima [A] time = 0.95, size = 185, normalized size = 0.62

$$-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)a-\frac{(49\,c^8\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4
*d))*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*
d*x^2 + d)^(7/2)/(c^4*d))*a - 1/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqr
t(-d)*d^2*x^7 + 189*c^4*sqrt(-d)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sq
rt(-d)*d^2*x)*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.99 $\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=218

$$-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2d}+\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))/c^2/d+1/7*b*d^2*x*(-c^2*d*x^2+
d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/7*b*c*d^2*x^3*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/35*b*c^3*d^2*x^5*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-1/49*b*c^5*d^2*x^7*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5798, 5718, 194}

$$-\frac{d^2(1-cx)^3(cx+1)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{bc^5d^2x^7\sqrt{d-c^2dx^2}}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*d^2*x*Sqrt[d - c^2*d*x^2])/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2
*x^3*Sqrt[d - c^2*d*x^2])/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d^2*x
^5*Sqrt[d - c^2*d*x^2])/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*x^7*
Sqrt[d - c^2*d*x^2])/(49*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(1 - c*x)^3*(
1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(7*c^2)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\sqrt{d-c^2d\right.}{7c\sqrt{-1}}$$
$$=-\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c^2}-\frac{\left(bd^2\sqrt{d-c^2d\right.}{}$$
$$=\frac{bd^2x\sqrt{d-c^2dx^2}}{7c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{bcd^2x^3\sqrt{d-c^2dx^2}}{7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3bc^3d^2x^5\sqrt{d-c^2dx^2}}{35\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.25, size = 117, normalized size = 0.54

$$\frac{d^2\sqrt{d-c^2dx^2}\left(35a\left(c^2x^2-1\right)^4+35b\left(c^2x^2-1\right)^4\cosh^{-1}(cx)+bcx\sqrt{cx-1}\sqrt{cx+1}\left(-5c^6x^6+21c^4x^4-35c^2x^2+\right.\right.}{245c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(35*a*(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(35 - 35*c^2*x^2 + 21*c^4*x^4 - 5*c^6*x^6) + 35*b*(-1 + c^2*x^2)^
4*ArcCosh[c*x]))/(245*c^2*(-1 + c^2*x^2))
```

fricas [A] time = 0.57, size = 241, normalized size = 1.11

$$\frac{35\left(bc^8d^2x^8-4bc^6d^2x^6+6bc^4d^2x^4-4bc^2d^2x^2+bd^2\right)\sqrt{-c^2dx^2+d}\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(5bc^7d^2x^7-21bc^5d\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/245*(35*(b*c^8*d^2*x^8 - 4*b*c^6*d^2*x^6 + 6*b*c^4*d^2*x^4 - 4*b*c^2*d^2*
x^2 + b*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (5*b*c^7*d
^2*x^7 - 21*b*c^5*d^2*x^5 + 35*b*c^3*d^2*x^3 - 35*b*c*d^2*x)*sqrt(-c^2*d*x^
2 + d)*sqrt(c^2*x^2 - 1) + 35*(a*c^8*d^2*x^8 - 4*a*c^6*d^2*x^6 + 6*a*c^4*d^
2*x^4 - 4*a*c^2*d^2*x^2 + a*d^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.35, size = 956, normalized size = 4.39

$$-\frac{a\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{7c^2d}+b\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\left(64c^8x^8-144c^6x^6+64\sqrt{cx+1}\sqrt{cx-1}x^7c^7+104c^4x^4-112\sqrt{cx+1}\sqrt{c}\right.}{6272(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/7*a/c^2/d*(-c^2*d*x^2+d)^(7/2)+b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*
x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)
/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^
2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*
(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcc
osh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c
^2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))
^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*
x^2-1)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(
1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+10
4*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x)
)*d^2/(c*x+1)/c^2/(c*x-1))
```

maxima [A] time = 0.68, size = 118, normalized size = 0.54

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b\operatorname{arcosh}\left(cx\right)}{7\,c^2d}-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}a}{7\,c^2d}-\frac{\left(5\,c^6\sqrt{-d}\,d^3x^7-21\,c^4\sqrt{-d}\,d^3x^5+35\,c^2\sqrt{-d}\,d^3x^3-35\,\sqrt{-d}\,d^3\right.}{245\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b*arccosh(c*x)/(c^2*d) - 1/7*(-c^2*d*x^2 + d)^(
7/2)*a/(c^2*d) - 1/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 +
35*c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.100 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx$

Optimal. Leaf size=379

$$d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)-\frac{2d^2\sqrt{d-c^2dx^2}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{5}\left(d-c^2dx^2\right)^{5/2}\left(a+b\,\mathrm{c}\right.$$

```
[Out] 1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))+1/5*(-c^2*d*x^2+d)^(5/2)*(a+b
*arccosh(c*x))+d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)-23/15*b*c*d^2*x*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/45*b*c^3*d^2*x^3*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/25*b*c^5*d^2*x^5*(-c^2*d*x^2+d
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d^2*(a+b*arccosh(c*x))*arctan(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
I*b*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*d^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 410, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5745, 5743, 5761, 4180, 2279, 2391, 8, 194}

$$\frac{ibd^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+d^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-23*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (11
*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2])/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*
c^5*d^2*x^5*Sqrt[d - c^2*d*x^2])/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]) + (d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/3 + (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/5 - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b*d^2
*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) - (I*b*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right) - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)}{}$$

$$= \frac{1}{3}d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right) + \frac{1}{5}d^2(1-cx)^2(1+$$

$$= -\frac{8bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{23bcd^2x\sqrt{d-c^2dx^2}}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{11bc^3d^2x^3\sqrt{d-c^2dx^2}}{45\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^5\sqrt{d-c^2dx^2}}{25\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 3.85, size = 471, normalized size = 1.24

$$-ad^{5/2}\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)+\frac{1}{15}ad^2\left(3c^4x^4-11c^2x^2+23\right)\sqrt{d-c^2dx^2}+ad^{5/2}\log(x)+\frac{bd^2\sqrt{d-c^2dx^2}\,\left(i\mathrm{Li}_2\left(-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*d^2*Sqrt[d - c^2*d*x^2]*(23 - 11*c^2*x^2 + 3*c^4*x^4))/15 - (b*d^2*Sqrt[
d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh
[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
a*d^(5/2)*Log[x] - a*d^(5/2)*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (b*d^2*
Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x
*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCo
sh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^A
rcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)) - (b*d^2*Sqrt[d - c^2*d*x^2]*(25*Cosh[3*ArcCosh[c*x]] + 9*(-50*
c*x + Cosh[5*ArcCosh[c*x]]) + 15*ArcCosh[c*x]*(30*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x) - 5*Sinh[3*ArcCosh[c*x]] - 3*Sinh[5*ArcCosh[c*x]])))/(3600*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(ac^4d^2x^4-2\,ac^2d^2x^2+ad^2+\left(bc^4d^2x^4-2\,bc^2d^2x^2+bd^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.53, size = 620, normalized size = 1.64

$$\frac{\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} a}{5}+\frac{ad\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}{3}-a\,d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2 d\,x^2+d}}{x}\right)+a\sqrt{-c^2 d\,x^2+d}\,d^2-\frac{ib\sqrt{-d\left(c^2x^2-1\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/5*(-c^2*d*x^2+d)^(5/2)*a+1/3*a*d*(-c^2*d*x^2+d)^(3/2)-a*d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+a*(-c^2*d*x^2+d)^(1/2)*d^2-I*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*d^2-I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2-1/25*b*(-d*(c^2*x^2-1
))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^5*c^5+11/45*b*(-d*(c^2*x^2-1))^(
1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^3*c^3-23/15*b*(-d*(c^2*x^2-1))^(1/2)
*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x*c-23/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*
x+1)/(c*x-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2+I*b*(-d*(c
^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2)))*d^2+1/5*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh
(c*x)*x^6*c^6-14/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*
x)*x^4*c^4+34/15*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)*
x^2*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(15\,d^{\frac{5}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-3\left(-c^2dx^2+d\right)^{\frac{5}{2}}-5\left(-c^2dx^2+d\right)^{\frac{3}{2}}d-15\sqrt{-c^2dx^2+d}\,d^2\right)a+b\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] -1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2
 + d)*d^2)*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=404

$$-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)+\frac{5c^2d^2\sqrt{d-c^2dx^2}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{6}c^2d\left(d-c^2d\right.$$

```
[Out] -5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))-1/2*(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))/x^2-5/2*c^2*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)
-1/2*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+7/3*b*c^3*d
^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*c^5*d^2*x^3*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*c^2*d^2*(a+b*arccosh(c*x))*
arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-5/2*I*b*c^2*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/2*I*b*c^2*d^2*polylog
(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 435, normalized size of antiderivative = 1.08, number of steps used = 14, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {5798, 5740, 5745, 5743, 5761, 4180, 2279, 2391, 8, 270}

$$-\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5ibc^2d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{5}{2}c^2d^2\sqrt{d-c^2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*b*c^
3*d^2*x*Sqrt[d - c^2*d*x^2])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^2*
x^3*Sqrt[d - c^2*d*x^2])/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/2 - (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/6 - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(2*x^2) + (5*c^2*d^2*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)*E^ArcCosh
[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (((5*I)/2)*b*c^2*d^2*Sqrt[d - c^2*
d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2x^2} + \frac{\left(bcd^2\sqrt{d-c\right.}{2\sqrt{-1}}$$

$$= -\frac{5}{6}c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right) - \frac{d^2(1-cx)^2}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}}{6\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{7bc^3d^2x\sqrt{d-c^2dx^2}}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x^3\sqrt{d-c^2dx^2}}{9\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 4.29, size = 596, normalized size = 1.48

$$\frac{1}{36}d^2\left(90ac^2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)-90ac^2\sqrt{d}\,\log(x)+\frac{6a\left(2c^4x^4-14c^2x^2-3\right)\sqrt{d-c^2dx^2}}{x^2}-\frac{72bc^2\sqrt{}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (d^2*((6*a*Sqrt[d - c^2*d*x^2]*(-3 - 14*c^2*x^2 + 2*c^4*x^4))/x^2 + (b*c^2*
Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*Ar
cCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
- 90*a*c^2*Sqrt[d]*Log[x] + 90*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d
*x^2]] - (72*b*c^2*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*x)/(1 + c*x)]
*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + I*ArcCosh[c*x
]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*
PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)) + (18*b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)/(1
+ c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(1 +
c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
```

```
 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)/(1
+ c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2])))/36
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.74, size = 667, normalized size = 1.65

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{2d\,x^2}-\frac{a\,c^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{2}-\frac{5a\,c^2d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{6}+\frac{5a\,c^2d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2}-\frac{5a\,c^2\sqrt{-c^2d\,x^2+}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(7/2)-1/2*a*c^2*(-c^2*d*x^2+d)^(5/2)-5/6*a*c^2*
d*(-c^2*d*x^2+d)^(3/2)+5/2*a*c^2*d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(
1/2))/x)-5/2*a*c^2*(-c^2*d*x^2+d)^(1/2)*d^2+5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*c^2*d^2-1/9*b*(-d*(c^2*x^2-1))^(1/2)*c^5*d^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)*c^3*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*
x-1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/x/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c+5/2*I*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*c^2*d^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2*d^2
+1/2*b*(-d*(c^2*x^2-1))^(1/2)*d^2/x^2/(c*x+1)/(c*x-1)*arccosh(c*x)+11/6*b*(
-d*(c^2*x^2-1))^(1/2)*c^2*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)-5/2*I*b*(-d*(c^2
*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*c^2*d^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)*c^6*d^2/(c*x+1)/(c*x-1)*a
rccosh(c*x)*x^4-8/3*b*(-d*(c^2*x^2-1))^(1/2)*c^4*d^2/(c*x+1)/(c*x-1)*arccos
h(c*x)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(15\,c^2d^{\frac{5}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-3\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2-5\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d-15\sqrt{-c^2dx^2+d}\,c^2d^2-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima"
)
```

```
[Out] 1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(
-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a + b*integrate
((-c^2*d*x^2 + d)^(5/2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx$

Optimal. Leaf size=407

$$\frac{5c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8x^2}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{4x^4}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)$$

```
[Out] 5/8*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/x^2-1/4*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))/x^4+15/8*c^4*d^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(
1/2)-1/12*b*c*d^2*(-c^2*d*x^2+d)^(1/2)/x^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+9/8*
b*c^3*d^2*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^2*x*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/4*c^4*d^2*(a+b*arccosh(c*
x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+15/8*I*b*c^4*d^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15/8*I*b*c^4*d^2*p
olylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.08, antiderivative size = 438, normalized size of antiderivative = 1.08, number of steps used = 15, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {5798, 5740, 5743, 5761, 4180, 2279, 2391, 8, 14, 270}

$$\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15ibc^4d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{15}{8}c^4d^2\sqrt{d-c^2dx^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] -(b*c*d^2*Sqrt[d - c^2*d*x^2])/(12*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9*b
*c^3*d^2*Sqrt[d - c^2*d*x^2])/(8*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d
^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*c^4*d^2*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/8 + (5*c^2*d^2*(1 - c*x)*(1 + c*x)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*x^2) - (d^2*(1 - c*x)^2*(1 + c*x
)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(4*x^4) - (15*c^4*d^2*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(4*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (((15*I)/8)*b*c^4*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, (-I)
*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((15*I)/8)*b*c^4*d^2*Sq
rt[d - c^2*d*x^2]*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{x^5}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{4x^4} + \frac{\left(bcd^2\sqrt{d-c^2dx\right.}{4\sqrt{-1+}$$

$$= \frac{5c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{8x^2} - \frac{d^2(1-cx)^2(1+}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^3d^2\sqrt{d-c^2dx^2}}{2x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{4\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}}{12x^3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9bc^3d^2\sqrt{d-c^2dx^2}}{8x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.50, size = 660, normalized size = 1.62

$$\frac{-24ac^6d^3x^6\sqrt{\frac{cx-1}{cx+1}} - 3ac^4d^3x^4\sqrt{\frac{cx-1}{cx+1}} + 33ac^2d^3x^2\sqrt{\frac{cx-1}{cx+1}} + 45ac^4d^{5/2}x^4\sqrt{\frac{cx-1}{cx+1}}\,\log(x)\sqrt{d-c^2dx^2} - 45ac^4d^{5/2}x^4}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]))/x^5,x]
```

```
[Out] (-2*b*c*d^3*x + 2*b*c^2*d^3*x^2 + 27*b*c^3*d^3*x^3 - 27*b*c^4*d^3*x^4 - 24*
b*c^5*d^3*x^5 + 24*b*c^6*d^3*x^6 - 6*a*d^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 33*
a*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*a*c^4*d^3*x^4*Sqrt[(-1 + c*x)/
(1 + c*x)] - 24*a*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)] - 6*b*d^3*Sqrt[(-1
 + c*x)/(1 + c*x)]*ArcCosh[c*x] + 33*b*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x
)]*ArcCosh[c*x] - 3*b*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] -
 24*b*c^6*d^3*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + (45*I)*b*c^4*d^
3*x^4*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^5*d^3*x^5*ArcCosh
[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (45*I)*b*c^4*d^3*x^4*ArcCosh[c*x]*Log[1 +
 I/E^ArcCosh[c*x]] + (45*I)*b*c^5*d^3*x^5*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[
c*x]] + 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]
*Log[x] - 45*a*c^4*d^(5/2)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^
2]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*P
olyLog[2, (-I)/E^ArcCosh[c*x]] + (45*I)*b*c^4*d^3*x^4*(-1 + c*x)*PolyLog[2,
 I/E^ArcCosh[c*x]])/(24*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^5, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.84, size = 691, normalized size = 1.70

$$-\frac{a\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{4d\,x^4}+\frac{3a\,c^2\left(-c^2d\,x^2+d\right)^{\frac{7}{2}}}{8d\,x^2}+\frac{3a\,c^4\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{8}+\frac{5a\,c^4d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{8}-\frac{15a\,c^4d^{\frac{5}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{}}{x}\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x)
```

```
[Out] -1/4*a/d/x^4*(-c^2*d*x^2+d)^(7/2)+3/8*a*c^2/d/x^2*(-c^2*d*x^2+d)^(7/2)+3/8*
a*c^4*(-c^2*d*x^2+d)^(5/2)+5/8*a*c^4*d*(-c^2*d*x^2+d)^(3/2)-15/8*a*c^4*d^(5
/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+15/8*a*c^4*(-c^2*d*x^2+d)^(1
/2)*d^2+b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^2-b
*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x+1/8*b*(-d*(c^
2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)+9/8*b*d^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x+1)^(1/2)/x/(c*x-1)^(1/2)*c^3-11/8*b*d^2*(-d*(c^2*x^2-1))^(1
/2)/(c*x+1)/x^2/(c*x-1)*arccosh(c*x)*c^2-1/12*b*d^2*(-d*(c^2*x^2-1))^(1/2)/
(c*x+1)^(1/2)/x^3/(c*x-1)^(1/2)*c+1/4*b*d^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/
x^4/(c*x-1)*arccosh(c*x)-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4+15/8*I*b*(-d
*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*dilog(1+I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))*d^2*c^4-15/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4+1
5/8*I*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(
1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*d^2*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(15\,c^4d^{\frac{5}{2}}\log\left(\frac{2\,\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-3\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^4-5\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d-15\,\sqrt{-c^2dx^2+d}\,c^4d^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/x^5,x, algorithm="maxima"
)
```

```
[Out] -1/8*(15*c^4*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)
) - 3*(-c^2*d*x^2 + d)^(5/2)*c^4 - 5*(-c^2*d*x^2 + d)^(3/2)*c^4*d - 15*sqrt
(-c^2*d*x^2 + d)*c^4*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)*c^2/(d*x^2) + 2*(-c^2*d
*x^2 + d)^(7/2)/(d*x^4))*a + b*integrate((-c^2*d*x^2 + d)^(5/2)*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1))/x^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2))/x^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acosh}\,(cx))}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))/x**5,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))/x**5, x)
```

3.103 $\int \sqrt{1-x^2} \cosh^{-1}(x)\,dx$

Optimal. Leaf size=66

$$-\frac{\sqrt{1-x}\,x^2}{4\sqrt{x-1}}+\frac{1}{2}\sqrt{1-x^2}\,x\cosh^{-1}(x)-\frac{\sqrt{1-x}\,\cosh^{-1}(x)^2}{4\sqrt{x-1}}$$

```
[Out] -1/4*x^2*(1-x)^(1/2)/(-1+x)^(1/2)-1/4*arccosh(x)^2*(1-x)^(1/2)/(-1+x)^(1/2)
+1/2*x*arccosh(x)*(-x^2+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5713, 5683, 5676, 30}

$$-\frac{\sqrt{1-x^2}\,x^2}{4\sqrt{x-1}\,\sqrt{x+1}}+\frac{1}{2}\sqrt{1-x^2}\,x\cosh^{-1}(x)-\frac{\sqrt{1-x^2}\,\cosh^{-1}(x)^2}{4\sqrt{x-1}\,\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -(x^2*Sqrt[1 - x^2])/(4*Sqrt[-1 + x]*Sqrt[1 + x]) + (x*Sqrt[1 - x^2]*ArcCos
h[x])/2 - (Sqrt[1 - x^2]*ArcCosh[x]^2)/(4*Sqrt[-1 + x]*Sqrt[1 + x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{1-x^2}\cosh^{-1}(x)\,dx = \frac{\sqrt{1-x^2}\int \sqrt{-1+x}\sqrt{1+x}\cosh^{-1}(x)\,dx}{\sqrt{-1+x}\sqrt{1+x}}$$
$$= \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\int x\,dx}{2\sqrt{-1+x}\sqrt{1+x}} - \frac{\sqrt{1-x^2}\int \frac{\cosh^{-1}(x)}{\sqrt{-1+x}\sqrt{1+x}}\,dx}{2\sqrt{-1+x}\sqrt{1+x}}$$
$$= -\frac{x^2\sqrt{1-x^2}}{4\sqrt{-1+x}\sqrt{1+x}} + \frac{1}{2}x\sqrt{1-x^2}\cosh^{-1}(x) - \frac{\sqrt{1-x^2}\cosh^{-1}(x)^2}{4\sqrt{-1+x}\sqrt{1+x}}$$

Mathematica [A] time = 0.12, size = 54, normalized size = 0.82

$$-\frac{\sqrt{-((x-1)(x+1))}\left(\cosh\left(2\cosh^{-1}(x)\right)+2\cosh^{-1}(x)\left(\cosh^{-1}(x)-\sinh\left(2\cosh^{-1}(x)\right)\right)\right)}{8\sqrt{\frac{x-1}{x+1}}(x+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - x^2]*ArcCosh[x],x]
```

```
[Out] -1/8*(Sqrt[-((-1 + x)*(1 + x))]*(Cosh[2*ArcCosh[x]] + 2*ArcCosh[x]*(ArcCosh
[x] - Sinh[2*ArcCosh[x]])))/(Sqrt[(-1 + x)/(1 + x)]*(1 + x))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-x^2+1}\,\mathrm{arcosh}(x), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-x^2 + 1)*arccosh(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2+1}\,\mathrm{arcosh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-x^2 + 1)*arccosh(x), x)
```

maple [B] time = 0.39, size = 152, normalized size = 2.30

$$-\frac{\sqrt{-x^2+1}\,\mathrm{arccosh}(x)^2}{4\sqrt{-1+x}\,\sqrt{1+x}}+\frac{\sqrt{-x^2+1}\,\left(2x^3-2x+2\sqrt{1+x}\,\sqrt{-1+x}\,x^2-\sqrt{-1+x}\,\sqrt{1+x}\right)(-1+2\,\mathrm{arccosh}(x))}{16\,(-1+x)\,(1+x)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x)*(-x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-x^2+1)^(1/2)/(-1+x)^(1/2)/(1+x)^(1/2)*arccosh(x)^2+1/16*(-x^2+1)^(1/
2)*(2*x^3-2*x+2*(1+x)^(1/2)*(-1+x)^(1/2)*x^2-(-1+x)^(1/2)*(1+x)^(1/2))*(-1+
2*arccosh(x))/(-1+x)/(1+x)+1/16*(-x^2+1)^(1/2)*(-2*(1+x)^(1/2)*(-1+x)^(1/2)
*x^2+2*x^3+(-1+x)^(1/2)*(1+x)^(1/2)-2*x)*(1+2*arccosh(x))/(-1+x)/(1+x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x)*(-x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \mathrm{acosh}(x)\sqrt{1-x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x)*(1 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x)*(1 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-(x-1)(x+1)}\,\mathrm{acosh}(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x)*(-x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))*acosh(x), x)
```

3.104 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=236

$$-\frac{x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c^2d}-\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^6d}-\frac{4x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15c^4d}-\frac{bx^5\sqrt{c}}{25c}$$

```
[Out] -8/15*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)-4/45*b*x^3*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/25*b*x^5*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-8/15*(a+b*arccosh(c*x))*(-c^2*d*x^2+
d)^(1/2)/c^6/d-4/15*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/5*x
^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.71, antiderivative size = 260, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^4(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{5c^2\sqrt{d-c^2dx^2}}-\frac{4x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{15c^4\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(c\right.}{15c^6\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-8*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (4*b*x
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(45*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^5*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(25*c*Sqrt[d - c^2*d*x^2]) - (8*(1 - c*x)*(1 + c*
x)*(a + b*ArcCosh[c*x]))/(15*c^6*Sqrt[d - c^2*d*x^2]) - (4*x^2*(1 - c*x)*(1
 + c*x)*(a + b*ArcCosh[c*x]))/(15*c^4*Sqrt[d - c^2*d*x^2]) - (x^4*(1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x]))/(5*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{5c^2\sqrt{d-c^2dx^2}}+\frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{5c^2\sqrt{d-c^2dx^2}}\\
&= -\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}}-\frac{4x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{15c^4\sqrt{d-c^2dx^2}}-\frac{x^4(1-cx)(1}{5}\\
&= -\frac{4bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}}-\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(1+cx)\left(a+b\cosh\right.}{15c^6\sqrt{d-c^2dx^2}}\\
&= -\frac{8bx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}}-\frac{4bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{45c^3\sqrt{d-c^2dx^2}}-\frac{bx^5\sqrt{-1+cx}\,\sqrt{1+cx}}{25c\sqrt{d-c^2dx^2}}-\frac{8}{}
\end{aligned}
$$

Mathematica [A] time = 0.30, size = 140, normalized size = 0.59

$$
\frac{\sqrt{d-c^2dx^2}\left(-15a\left(3c^6x^6+c^4x^4+4c^2x^2-8\right)+bcx\sqrt{cx-1}\,\sqrt{cx+1}\left(9c^4x^4+20c^2x^2+120\right)-15b\left(3c^6x^6+c^4\right.\right.}{225c^6d(cx-1)(cx+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2
+ 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 15*b*(-8 + 4*c
^2*x^2 + c^4*x^4 + 3*c^6*x^6)*ArcCosh[c*x]))/(225*c^6*d*(-1 + c*x)*(1 + c*x
))
```

fricas [A] time = 0.74, size = 176, normalized size = 0.75

$$
-\frac{15\left(3\,bc^6x^6+bc^4x^4+4\,bc^2x^2-8\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^5x^5+20\,bc^3x^3+120\,bcx\right)\sqrt{-c}}{225\left(c^8dx^2-c^6d\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/225*(15*(3*b*c^6*x^6 + b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 +
d)*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^5*x^5 + 20*b*c^3*x^3 + 120*b*c*x)*
```

```
sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 15*(3*a*c^6*x^6 + a*c^4*x^4 + 4*a*
c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 670, normalized size = 2.84

$$a\left(-\frac{x^4\sqrt{-c^2d\,x^2+d}}{5c^2d}+\frac{-\frac{4x^2\sqrt{-c^2d\,x^2+d}}{15c^2d}-\frac{8\sqrt{-c^2d\,x^2+d}}{15d\,c^4}}{c^2}\right)+b\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(16c^6x^6-28c^4x^4+16\sqrt{cx+1}\,\sqrt{cx-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+
d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*
(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(
-1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^
4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x
^2-1)*(1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1)
)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2
*x^2-1)*(1+5*arccosh(c*x))/c^6/d/(c^2*x^2-1))
```

maxima [A] time = 0.93, size = 195, normalized size = 0.83

$$-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b\,\mathrm{arcosh}\,(cx)-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx}}{c^4d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4
*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b*arccosh(c*x) - 1/15*(3*sqrt(-c^2*d*
x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x
^2 + d)/(c^6*d))*a + 1/225*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*
sqrt(-d)*x)*b/(c^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}{\sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.105 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=212

$$-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{4c^2d}+\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{16bc^5\sqrt{d-c^2dx^2}}-\frac{3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c^4d}-\frac{b}{}$$

```
[Out] -3/16*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/16*b*x^4
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+3/16*(a+b*arccosh(c*x))
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arccos
h(c*x))*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/4*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c^2/d
```

Rubi [A] time = 0.65, antiderivative size = 228, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{4c^2\sqrt{d-c^2dx^2}}-\frac{3x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(c\right.}{16bc^5\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-3*b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c*Sqrt[d - c^2*d*x^2]) - (3*x*(1 - c*x
)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(8*c^4*Sqrt[d - c^2*d*x^2]) - (x^3*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(4*c^2*Sqrt[d - c^2*d*x^2]) + (3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(16*b*c^5*Sqrt[d - c^2*d*x^2
])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{4c^2\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(c\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{4c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}} - \frac{x^3(1-cx)(1}{4c}$$

$$= -\frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{16c^3\sqrt{d-c^2dx^2}} - \frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}}{16c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\,c\right.}{8c^4\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.93, size = 171, normalized size = 0.81

$$\frac{-\frac{16acx\left(2c^2x^2+3\right)\sqrt{d-c^2dx^2}}{d} - \frac{48a\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(-16\cosh\left(2\cosh^{-1}(cx)\right)-\cosh\left(4\cosh^{-1}(cx)\right)+4\cosh^{-1}(cx)\left(6\cosh^{-1}\right.\right.}{\sqrt{d-c^2dx^2}}}{128c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-16*a*c*x*(3 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2])/d - (48*a*ArcTan[(c*x*Sqrt
[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(-16*Cosh[2*ArcCosh[c*x]] - Cosh[4*ArcCosh[c*x]] + 4*Arc
Cosh[c*x]*(6*ArcCosh[c*x] + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]]))
)/Sqrt[d - c^2*d*x^2])/(128*c^5)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4\,\text{arcosh}\,(cx)+ax^4\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^2*d*x^2 - d)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.82, size = 408, normalized size = 1.92

$$-\frac{a\,x^3\sqrt{-c^2 d\,x^2+d}}{4c^2 d}-\frac{3ax\sqrt{-c^2 d\,x^2+d}}{8c^4 d}+\frac{3a\arctan\left(\frac{\sqrt{c^2 d}\;x}{\sqrt{-c^2 d\,x^2+d}}\right)}{8c^4\sqrt{c^2 d}}-\frac{3b\sqrt{-d\left(c^2x^2-1\right)}\;\sqrt{cx-1}\;\sqrt{cx+1}\;\mathrm{arccosh}\left(c x\right)}{16d\,c^5\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a/c^4*x/d*(-c^2*d*x^2+d)^(1/2)+3/
8*a/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-3/16*b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*arccosh
(c*x)^2+1/16*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^4+3/16*b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2-1/4*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*x^5
-1/8*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3+3/8*b*(-d*
(c^2*x^2-1))^(1/2)/d/c^4/(c^2*x^2-1)*arccosh(c*x)*x-15/128*b*(-d*(c^2*x^2-1
))^(1/2)/d/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}a\left(\frac{2\sqrt{-c^2dx^2+d}\,x^3}{c^2d}+\frac{3\sqrt{-c^2dx^2+d}\,x}{c^4d}-\frac{3\arcsin\left(cx\right)}{c^5\sqrt{d}}\right)+b\int\frac{x^4\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*a*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^4*
d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.106 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=156

$$-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^2d}-\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c^4d}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

```
[Out] -2/3*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/9*b*x^3*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))*(-c^
2*d*x^2+d)^(1/2)/c^4/d-1/3*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/
d
```

Rubi [A] time = 0.50, antiderivative size = 172, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}}-\frac{2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}}-\frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{9c\sqrt{d-c^2dx^2}}-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c*Sqrt[d - c^2*d*x^2]) - (2*(1 - c*x)*(1 +
 c*x)*(a + b*ArcCosh[c*x]))/(3*c^4*Sqrt[d - c^2*d*x^2]) - (x^2*(1 - c*x)*(1
 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
```

```
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{3c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}} - \frac{x^2(1-cx)(1+cx)\cdots}{3c^2\sqrt{d\cdots}}$$

$$= -\frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}\cdots\right.}{3c^4\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.24, size = 113, normalized size = 0.72

$$\frac{\sqrt{d-c^2dx^2}\left(-3a\left(c^4x^4+c^2x^2-2\right)+bcx\sqrt{cx-1}\,\sqrt{cx+1}\left(c^2x^2+6\right)-3b\left(c^4x^4+c^2x^2-2\right)\cosh^{-1}(cx)\right)}{9c^4d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*
a*(-2 + c^2*x^2 + c^4*x^4) - 3*b*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]))/(9
*c^4*d*(-1 + c*x)*(1 + c*x))
```

fricas [A] time = 0.52, size = 146, normalized size = 0.94

$$-\frac{3\left(bc^4x^4+bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^3x^3+6\,bcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,+3\left(ac^4x^4\cdots\right.}{9\left(c^6dx^2-c^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/9*(3*(b*c^4*x^4 + b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c
^2*x^2 - 1)) - (b*c^3*x^3 + 6*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)
 + 3*(a*c^4*x^4 + a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.56, size = 382, normalized size = 2.45

$$a\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\sqrt{c}\right.}{72c^4d\left(c^2x^2-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b*(-
1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c^4/d/
(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^4/d/(c^2*x^2-1)
-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/c^4/d
/(c^2*x^2-1))
```

maxima [A] time = 0.84, size = 131, normalized size = 0.84

$$-\frac{1}{3}\,b\left(\frac{\sqrt{-c^2dx^2+d}\,x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right)\operatorname{arcosh}\left(cx\right)-\frac{1}{3}\,a\left(\frac{\sqrt{-c^2dx^2+d}\,x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right)+\frac{\left(c^2\sqrt{-d}\,x^3+\right.}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d))*
arccosh(c*x) - 1/3*a*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2
+ d)/(c^4*d)) + 1/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-d)*x)*b/(c^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.107 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=132

$$-\frac{x\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^2d}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

```
[Out] -1/4*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/4*(a+b*arcc
osh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*x*(a
+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.40, antiderivative size = 140, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^3\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}}{4c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -(b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*Sqrt[d - c^2*d*x^2]) - (x*(1 - c
*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(2*c^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^3*Sqrt[d - c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2 \left(a + b\cosh^{-1}(cx)\right)}{\sqrt{d - c^2dx^2}}\,dx = \frac{\left(\sqrt{-1 + cx}\,\sqrt{1 + cx}\right) \int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d - c^2dx^2}}$$

$$= -\frac{x(1-cx)(1+cx)\left(a + b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d - c^2dx^2}} + \frac{\left(\sqrt{-1 + cx}\,\sqrt{1 + cx}\right) \int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2c^2\sqrt{d - c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{-1 + cx}\,\sqrt{1 + cx}}{4c\sqrt{d - c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a + b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d - c^2dx^2}} + \frac{\sqrt{-1 + cx}\,\sqrt{1}}{4b}$$

Mathematica [A] time = 0.67, size = 141, normalized size = 1.07

$$\frac{-\frac{4acx\sqrt{d-c^2dx^2}}{d} - \frac{4a\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\,(c^2x^2-1)}\right)}{\sqrt{d}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)-\cosh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{d-c^2dx^2}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-4*a*c*x*Sqrt[d - c^2*d*x^2])/d - (4*a*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(
Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] + Sinh[2*ArcCosh[c*x
]])))/Sqrt[d - c^2*d*x^2])/(8*c^3)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2 + d}\left(bx^2\operatorname{arcosh}(cx) + ax^2\right)}{c^2dx^2 - d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^2*d*x^2 - d)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x^2}{\sqrt{-c^2dx^2 + d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.53, size = 291, normalized size = 2.20

$$-\frac{ax\sqrt{-c^2d\,x^2 + d}}{2c^2d} + \frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{4d\,c^3\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right.}}{2d\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a/c^2/(c^2*d)^(1/2)*arctan((c^2*d)^
(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/4*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d
/(c^2*x^2-1)*arccosh(c*x)*x^3+1/4*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2
-1)*arccosh(c*x)*x-1/8*b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2d}-\frac{\arcsin\left(cx\right)}{c^3\sqrt{d}}\right)+b\int\frac{x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + b*int
egrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.108 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=72

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2d}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.21, antiderivative size = 80, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {5798, 5718, 8}

$$-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2])) - ((1 - c*x)*
(1 + c*x)*(a + b*ArcCosh[c*x]))/(c^2*Sqrt[d - c^2*d*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\
&= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}-\frac{\left(b\sqrt{-1+cx}\sqrt{1+cx}\right)\int 1\,dx}{c\sqrt{d-c^2dx^2}}\\
&= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}}-\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 85, normalized size = 1.18

$$\frac{\sqrt{d-c^2dx^2}\left(-ac^2x^2+a+\left(b-bc^2x^2\right)\cosh^{-1}(cx)+bcx\sqrt{cx-1}\sqrt{cx+1}\right)}{c^2d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
(b - b*c^2*x^2)*ArcCosh[c*x]))/(c^2*d*(-1 + c*x)*(1 + c*x))
```

fricas [A] time = 0.52, size = 117, normalized size = 1.62

$$\frac{\sqrt{-c^2dx^2+d}\sqrt{c^2x^2-1}\,bcx-\left(bc^2x^2-b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(ac^2x^2-a\right)\sqrt{-c^2dx^2+d}}{c^4dx^2-c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^2*x^2 - b)*sqrt(-c^2*d
*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - (a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 +
d))/(c^4*d*x^2 - c^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\mathrm{arcosh}\,(cx)+a)x}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.25, size = 158, normalized size = 2.19

$$-\frac{a\sqrt{-c^2d\,x^2+d}}{c^2d}+b\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)(-1+\mathrm{arccosh}\,(cx))}{2c^2d\left(c^2x^2-1\right)}-\frac{\sqrt{-d\left(c^2x^2-1\right)}\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/c^2/d*(-c^2*d*x^2+d)^(1/2)+b*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))/c^2/d/(c^2*x^2-1))
```

maxima [A] time = 1.25, size = 63, normalized size = 0.88

$$\frac{b\sqrt{-d}\,x}{cd}-\frac{\sqrt{-c^2dx^2+d}\,b\,\mathrm{arcosh}\,(cx)}{c^2d}-\frac{\sqrt{-c^2dx^2+d}\,a}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*x)/(c^2*d) - sqrt(-c^
2*d*x^2 + d)*a/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.109 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/
2)
```

Rubi [A] time = 0.12, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 1.00

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c*Sqrt[d - c^2*d
*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

maple [A] time = 0.06, size = 89, normalized size = 1.68

$$\frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}-\frac{b\sqrt{-(cx-1)(cx+1)d}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{2cd\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-(c*x-1
)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(c*x)
^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}\,dx+\frac{a\arcsin(cx)}{c\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
 + a*arcsin(c*x)/(c*sqrt(d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.110 $\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=151

$$\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx}}{\sqrt{d}}$$

```
[Out] 2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+I*b*polylog(2,
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5761, 4180, 2279, 2391}

$$-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\,\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]]
)/Sqrt[d - c^2*d*x^2] - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E
^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Pol
yLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int (a+bx)\mathrm{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Sub}}{\sqrt{}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(ib\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Sub}}{\sqrt{d-}}$$

$$= \frac{2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{ib\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{Li}_2\left(-\right.}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.29, size = 153, normalized size = 1.01

$$-\frac{a\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right)}{\sqrt{d}}+\frac{a\log(x)}{\sqrt{d}}-\frac{ib\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right)-\mathrm{Li}_2\left(ie^{-\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(\right.\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a*Log[x])/Sqrt[d] - (a*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] - (I*
b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c
*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyL
og[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{c^2dx^3-dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^3 - d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x), x)
```

maple [A] time = 0.37, size = 327, normalized size = 2.17

$$-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2dx^2+d}}{x}\right)}{\sqrt{d}}+\frac{ib\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}\left(cx\right)\ln\left(1+i\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right)\right)}{d\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)+I*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilo
g(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}\,x}\,dx-\frac{a\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x)
, x) - a*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(cx\right)}{x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.111 $\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=71

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{dx}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}$$

```
[Out] -b*c*ln(x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*
x))*(-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.30, antiderivative size = 79, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {5798, 5724, 29}

$$-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -(((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x^2])) - (b*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/Sqrt[d - c^2*d*x^2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.07, size = 71, normalized size = 1.00

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{x} - bc\log(x)\right)}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x]))/x - b*c*Log[x]))/Sqrt[d - c^2*d*x^2]
```

fricas [A] time = 0.58, size = 265, normalized size = 3.73

$$\left[-\frac{bc\sqrt{-d}\,x\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\left(x^4-1\right)\sqrt{-d}-d}{c^2x^4-x^2}\right)+2\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+2\sqrt{-c}}{2\,dx}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/2*(b*c*sqrt(-d)*x*log((c^2*d*x^6 + c^2*d*x^2 - d*x^4 + sqrt(-c^2*d*x^2
+ d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)/(c^2*x^4 - x^2)) + 2*sqrt(-c
^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(d
*x), (b*c*sqrt(d)*x*arctan(sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^2 + 1)
*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - sqrt(-c^2*d*x^2 + d)*b*log(c*
x + sqrt(c^2*x^2 - 1)) - sqrt(-c^2*d*x^2 + d)*a)/(d*x)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\sqrt{-c^2dx^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

maple [B] time = 0.38, size = 219, normalized size = 3.08

$$-\frac{a\sqrt{-c^2d\,x^2+d}}{dx}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}\,(cx)\,c}{d\left(c^2x^2-1\right)}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}\,(cx)\,x\,c^2}{\left(c^2x^2-1\right)d}+\frac{b\sqrt{\cdot}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a/d/x*(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/
(c^2*x^2-1)/d*c^2+b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d+b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

maxima [C] time = 0.54, size = 116, normalized size = 1.63

$$-\frac{\left(c^2 d\sqrt{-\frac{1}{c^4 d}}\log\left(x^2-\frac{1}{c^2}\right)+i\,(-1)^{-2\,c^2dx^2+2\,d}\sqrt{d}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)\right)bc}{2\,d}-\frac{\sqrt{-c^2dx^2+d}\,b\,\mathrm{arcosh}\,(cx)}{dx}-\frac{\sqrt{-c^2dx^2+}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d)
*sqrt(d)*log(-2*c^2*d + 2*d/x^2))*b*c/d - sqrt(-c^2*d*x^2 + d)*b*arccosh(c*
x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{acosh}\,(cx)}{x^2\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.112 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=238

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2dx^2}+\frac{c^2\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}-\frac{ibc^2\sqrt{cx-1}\sqrt{cx-}}{2\sqrt{d}}$$

```
[Out] 1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)+c^2*(a+b*arccosh
(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
(-c^2*d*x^2+d)^(1/2)-1/2*I*b*c^2*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+1/2*I*b*c^2*polylog(
2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*
x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.54, antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)*(
1 + c*x)*(a + b*ArcCosh[c*x]))/(2*x^2*Sqrt[d - c^2*d*x^2]) + (c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/Sqrt[d - c
^2*d*x^2] - ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^Arc
Cosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((I/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*PolyLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+cx}\,\sqrt{\right.}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{\left(c^2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Su}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+\right.}{\sqrt{d}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+\right.}{\sqrt{d}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2x^2\sqrt{d-c^2dx^2}} + \frac{c^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+\right.}{\sqrt{d}}$$

Mathematica [A] time = 1.09, size = 309, normalized size = 1.30

$$\frac{1}{2}\left(-\frac{a\sqrt{d-c^2dx^2}}{dx^2} - \frac{ac^2\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)}{\sqrt{d}} + \frac{ac^2\log(x)}{\sqrt{d}} + \frac{b(cx+1)\left(-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\,\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right)+i\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-((a*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a*c^2*Log[x])/Sqrt[d] - (a*c^2*Log[d
 + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (b*(1 + c*x)*(c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 + c*x)/
(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 +
 c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))/2
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)}{c^2dx^5-dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^2*d*x^5 - d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\sqrt{-c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

maple [B] time = 0.65, size = 489, normalized size = 2.05

$$-\frac{a\sqrt{-c^2d\,x^2+d}}{2d\,x^2}-\frac{a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2\sqrt{d}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,c^2}{2d\left(c^2x^2-1\right)}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx+1}\,\sqrt{c\cdots}}{2d\left(c^2x^2-1\right)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a/d/x^2*(-c^2*d*x^2+d)^(1/2)-1/2*a*c^2/d^(1/2)*ln((2*d+2*d^(1/2)*(-c^2
*d*x^2+d)^(1/2))/x)-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)/x^2*arccosh(c*x)+1/2*I*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-1/2*I*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))*c^2+1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
-1/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*d
ilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{c^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{\sqrt{d}}+\frac{\sqrt{-c^2dx^2+d}}{dx^2}\right)a+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) +
 sqrt(-c^2*d*x^2 + d)/(d*x^2))*a + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\operatorname{acosh}(c\,x)}{x^3\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+b\operatorname{acosh}(cx)}{x^3\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.113 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2c^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3dx^3}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}-\frac{2bc^3\sqrt{cx-1}\,\sqrt{cx+}}{3\sqrt{d-c^2d}}$$

```
[Out] 1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*b*c^3*ln(x
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/d/x^3-2/3*c^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/d
/x
```

Rubi [A] time = 0.51, antiderivative size = 171, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5748, 5724, 29, 30}

$$-\frac{2c^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3x\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6x^2\sqrt{d-c^2dx^2}}-\frac{2bc^3\sqrt{cx-}}{3\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*x^3*Sqrt[d - c^2*d*x^2]) - (2*c^2*(1 -
c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*x*Sqrt[d - c^2*d*x^2]) - (2*b*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*Sqrt[d - c^2*d*x^2])
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
```

```
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^3}\,dx}{3\sqrt{d-c^2dx^2}} + \frac{\left(2c^2\sqrt{-1+cx}\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+cx)\left(a+b\right.}{3x\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3x^3\sqrt{d-c^2dx^2}} - \frac{2c^2(1-cx)(1+cx)\left(a+b\right.}{3x\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.33, size = 174, normalized size = 1.12

$$-\frac{\sqrt{d-c^2dx^2}\left(4ac^2x^2\sqrt{cx-1}\,\sqrt{cx+1}+2a\sqrt{cx-1}\,\sqrt{cx+1}+6bc^3x^3-4bc^3x^3\log(cx-1)-4bc^3x^3\log\left(\frac{1}{cx-1}+1\right.\right.}{6dx^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] -1/6*(Sqrt[d - c^2*d*x^2]*(b*c*x + 6*b*c^3*x^3 + 2*a*Sqrt[-1 + c*x]*Sqrt[1
+ c*x] + 4*a*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(1 + 2*c^2*x^2)*ArcCosh[c*x] - 4*b*c^3*x^3*Log[-1 + c*x] - 4*b*c^
3*x^3*Log[1 + (-1 + c*x)^(-1)]))/(d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.86, size = 479, normalized size = 3.09

$$\left[-\frac{2\left(2bc^4x^4-bc^2x^2-b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+2\left(bc^5x^5-bc^3x^3\right)\sqrt{-d}\,\log\left(\frac{c^2dx^6+c^2dx^2-dx^4+\sqrt{-c^2dx^2}}{c^2x^4}\right.}{6\left(c^2dx^5-dx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] [-1/6*(2*(2*b*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(
c^2*x^2 - 1)) + 2*(b*c^5*x^5 - b*c^3*x^3)*sqrt(-d)*log((c^2*d*x^6 + c^2*d*x
^2 - d*x^4 + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*(x^4 - 1)*sqrt(-d) - d)
/(c^2*x^4 - x^2)) - sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1
) + 2*(2*a*c^4*x^4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^
```

```
3), 1/6*(4*(b*c^5*x^5 - b*c^3*x^3)*sqrt(d)*arctan(sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1)*(x^2 + 1)*sqrt(d)/(c^2*d*x^4 - (c^2 + 1)*d*x^2 + d)) - 2*(2*b
*c^4*x^4 - b*c^2*x^2 - b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))
 + sqrt(-c^2*d*x^2 + d)*(b*c*x^3 - b*c*x)*sqrt(c^2*x^2 - 1) - 2*(2*a*c^4*x^
4 - a*c^2*x^2 - a)*sqrt(-c^2*d*x^2 + d))/(c^2*d*x^5 - d*x^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\sqrt{-c^2dx^2 + d}\, x^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

maple [B] time = 0.66, size = 854, normalized size = 5.51

$$-\frac{a\sqrt{-c^2d\,x^2+d}}{3d\,x^3}-\frac{2a\,c^2\sqrt{-c^2d\,x^2+d}}{3dx}-\frac{4b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\,c^3}{3d\left(c^2x^2-1\right)}-\frac{2b\sqrt{-d\left(c^2x^2-1\right)}}{3d\left(3c^4x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a/d/x^3*(-c^2*d*x^2+d)^(1/2)-2/3*a*c^2/d/x*(-c^2*d*x^2+d)^(1/2)-4/3*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*
x)*c^3-2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x-1)*(
c*x+1)*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8+2
*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)*c^5-2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2
-1)*x^3*arccosh(c*x)*c^6-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^
2-1)*x*(c*x-1)*(c*x+1)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*
x^2-1)*x^3*c^6+2/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c
^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^
4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-1/3*b*(-d*(c^2*x^2-1))^(
1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*
x^4-2*c^2*x^2-1)/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x
^4-2*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1
/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)+2/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)*c^3
```

maxima [A] time = 0.97, size = 134, normalized size = 0.86

$$\frac{1}{6}\left(\frac{4\,c^2\sqrt{-d}\,\log(x)}{d}-\frac{\sqrt{-d}}{dx^2}\right)bc-\frac{1}{3}\,b\left(\frac{2\,\sqrt{-c^2dx^2+d}\,c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)\operatorname{arcosh}(cx)-\frac{1}{3}\,a\left(\frac{2\,\sqrt{-c^2dx^2+d}\,c^2}{dx}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*b*c - 1/3*b*(2*sqrt(-c^2*d
*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/3*a*(2
*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x^4\,\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x^4\sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.114 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{3c^6d^3}+\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^6d^2}+\frac{a+b\cosh^{-1}(cx)}{c^6d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{d-c^2dx^2}\tanh^{-1}}{c^6d^2\sqrt{cx-1}\sqrt{cx+}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/c^6/d^3+(a+b*arccosh(c*x))/c^6
/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^6/d^2-5
/3*b*x*(-c^2*d*x^2+d)^(1/2)/c^5/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*x^3*(
-c^2*d*x^2+d)^(1/2)/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*arctanh(c*x)*(-c^
2*d*x^2+d)^(1/2)/c^6/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 262, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 98, 21, 100, 12, 74, 5733, 1153, 208}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{4x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^4d\sqrt{d-c^2dx^2}}+\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}}+\frac{bx^3\sqrt{cx}}{9c^3d\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (5*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^4*(a + b*
ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8*(1 - c*x)*(1 + c*x)*(a + b*
ArcCosh[c*x]))/(3*c^6*d*Sqrt[d - c^2*d*x^2]) + (4*x^2*(1 - c*x)*(1 + c*x)*(
a + b*ArcCosh[c*x]))/(3*c^4*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcTanh[c*x])/(c^6*d*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(1+}{3c^4d}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-cx)(1+}{3c^4d}$$

$$= \frac{5bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1}{}$$

$$= \frac{5bx\sqrt{-1+cx}\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{9c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{8(1}{}$$

Mathematica [A] time = 0.11, size = 145, normalized size = 0.62

$$\frac{-3ac^4x^4-12ac^2x^2+24a+bc^3x^3\sqrt{cx-1}\sqrt{cx+1}-3b\left(c^4x^4+4c^2x^2-8\right)\cosh^{-1}(cx)+15bcx\sqrt{cx-1}\sqrt{cx+}}{9c^6d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (24*a - 12*a*c^2*x^2 - 3*a*c^4*x^4 + 15*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
+ b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 3*b*(-8 + 4*c^2*x^2 + c^4*x^4)*A
rcCosh[c*x] + 9*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(9*c^6*d*Sqrt[
d - c^2*d*x^2])
```

fricas [A] time = 0.61, size = 489, normalized size = 2.10

$$\left[\frac{12\left(bc^4x^4+4\,bc^2x^2-8\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-9\left(bc^2x^2-b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\,(}{c^6x^6}\right.}{36\,(c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/36*(12*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sq
rt(c^2*x^2 - 1)) - 9*(b*c^2*x^2 - b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4
 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*s
qrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(b*c^3*x^3 + 15*b*c
*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 12*(a*c^4*x^4 + 4*a*c^2*x^2 -
8*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^2*x^2 - c^6*d^2), -1/18*(9*(b*c^2*x^2 - b
)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*
d*x^4 - d)) - 6*(b*c^4*x^4 + 4*b*c^2*x^2 - 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*
x + sqrt(c^2*x^2 - 1)) + 2*(b*c^3*x^3 + 15*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 6*(a*c^4*x^4 + 4*a*c^2*x^2 - 8*a)*sqrt(-c^2*d*x^2 + d))/(c^
8*d^2*x^2 - c^6*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.68, size = 431, normalized size = 1.85

$$-\frac{a\,x^4}{3c^2d\sqrt{-c^2d\,x^2+d}}-\frac{4a\,x^2}{3c^4d\sqrt{-c^2d\,x^2+d}}+\frac{8a}{3c^6d\sqrt{-c^2d\,x^2+d}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(cx+\sqrt{cx-\right.}{c^6d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)-4/3*a/c^4*x^2/d/(-c^2*d*x^2+d)^(1/2)+
8/3*a/c^6/d/(-c^2*d*x^2+d)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-8/3*b*
(-d*(c^2*x^2-1))^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)+1/3*b*(-d*(c^2*x^2-
1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^4+4/3*b*(-d*(c^2*x^2-1))^(1/2)
/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1
/9*b*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^3-5/3*b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{x^4}{\sqrt{-c^2dx^2+d}\,c^2d}+\frac{4\,x^2}{\sqrt{-c^2dx^2+d}\,c^4d}-\frac{8}{\sqrt{-c^2dx^2+d}\,c^6d}\right)+\frac{1}{9}b\left(\frac{\frac{\left(c^4\sqrt{d}\,x^4+16\,c^2\sqrt{d}\,x^2-8\,\sqrt{d}\right)\sqrt{cx+1}\,\sqrt{cx-1}}{\sqrt{-cx+1}}-\frac{3\left(c^5\sqrt{d}\right.}{}}{\sqrt{cx}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*a*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^4*
d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/9*b*(((c^4*sqrt(d)*x^4 + 16*c^2*sq
rt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(-c*x + 1) - 3*(c^5*
sqrt(d)*x^5 + 4*c^3*sqrt(d)*x^3 - 8*c*sqrt(d)*x + (c^4*sqrt(d)*x^4 + 4*c^2*
sqrt(d)*x^2 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/(sqrt(c*x + 1)*c^7*d^2*x + (c*x + 1)*sqrt
(c*x - 1)*c^6*d^2) + 9*integrate(1/9*(3*c^7*sqrt(d)*x^7 + 9*c^5*sqrt(d)*x^5
 - 36*c^3*sqrt(d)*x^3 + 24*c*sqrt(d)*x + (3*c^6*sqrt(d)*x^6 + 8*c^4*sqrt(d)
*x^4 - 52*c^2*sqrt(d)*x^2 + 32*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*log(c*x -
 1)))/(sqrt(-c*x + 1)*((c^7*d^2*x^2 - c^5*d^2)*e^(3/2*log(c*x + 1) + log(c*
x - 1)) + 2*(c^8*d^2*x^3 - c^6*d^2*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1)) +
 (c^9*d^2*x^4 - c^7*d^2*x^2)*sqrt(c*x + 1))), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \left(a + b \operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.115 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=226

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^5d\sqrt{d-c^2dx^2}}+\frac{3x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2c^4d^2}-\frac{b\sqrt{cx-1}\sqrt{cx}}{2c^5d\sqrt{}}$$

```
[Out] x^3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+1/4*b*x^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-3/4*(a+b*arccosh(c*x))^2*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/b/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] time = 0.68, antiderivative size = 237, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {5798, 5752, 5759, 5676, 30, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{3x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}}-\frac{3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{4bc^5d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-}}{4c^3d\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c^3*d*Sqrt[d - c^2*d*x^2]) + (x^3*(
a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (3*x*(1 - c*x)*(1 + c*x)
*(a + b*ArcCosh[c*x]))/(2*c^4*d*Sqrt[d - c^2*d*x^2]) - (3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(4*b*c^5*d*Sqrt[d - c^2*d*x^2]) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+\right.}{}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^4d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\right.}{2c^{\cdots}}$$

$$= \frac{3bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cos\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\sqrt{1+cx}}{4c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)(1+cx)\left(a+b\cosh\right.}{2c^4d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.48, size = 192, normalized size = 0.85

$$\frac{-4acdx\left(c^2x^2-3\right)+12a\sqrt{d}\sqrt{d-c^2dx^2}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+bd\left(8cx\cosh^{-1}(cx)-\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(8\log\left(\sqrt{\frac{cx}{cx}}\right.\right.\right.}{8c^5d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a*c*d*x*(-3 + c^2*x^2) + 12*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*S
qrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(8*c*x*ArcCosh[c*x] - S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*ArcCosh[c*x]
] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x]*Sinh[2*Arc
Cosh[c*x]])))/(8*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.40, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(bx^4 \operatorname{arcosh}(cx) + ax^4\right)\sqrt{-c^2dx^2 + d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.76, size = 445, normalized size = 1.97

$$-\frac{a\,x^3}{2c^2d\sqrt{-c^2d\,x^2+d}}+\frac{3ax}{2c^4d\sqrt{-c^2d\,x^2+d}}-\frac{3a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^4d\sqrt{c^2d}}+\frac{3b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(}{4d^2c^5\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a/c^4*x/d/(-c^2*d*x^2+d)^(1/2)-3/
2*a/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/4*b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arcc
osh(c*x)^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^
3-1/4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x
^2-1)*arccosh(c*x)-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh
(c*x)*x+1/8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2
*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{x^3}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{3\,x}{\sqrt{-c^2dx^2+d}\,c^4d}+\frac{3\,\arcsin(cx)}{c^5d^{\frac{3}{2}}}\right)+b\int\frac{x^4\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*a*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*d)
 + 3*arcsin(c*x)/(c^5*d^(3/2))) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)}{\left(- d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.116 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=150

$$\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2}+\frac{a+b\cosh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{c^4d^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{bx\sqrt{d-c^2dx^2}}{c^3d^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)/c^4/d^2-b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^2/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-b*arctanh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^4/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2
)
```

Rubi [A] time = 0.39, antiderivative size = 163, normalized size of antiderivative = 1.09, number of steps used = 4, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 98, 21, 74, 5733, 388, 208}

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx+1}}{c^3d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^2*(a +
b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*(1 - c*x)*(1 + c*x)*(a +
b*ArcCosh[c*x]))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{c^4d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1\right.}{d\sqrt{d}$$
$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}\right.}{c^4d\sqrt{d-c^2dx^2}}$$
$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}\right.}{c^4d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.07, size = 97, normalized size = 0.65

$$\frac{-ac^2x^2+2a+b\left(2-c^2x^2\right)\cosh^{-1}(cx)+bcx\sqrt{cx-1}\,\sqrt{cx+1}\,+b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{c^4d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(2 - c^2*x^2)*Arc
Cosh[c*x] + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(c^4*d*Sqrt[d - c^
2*d*x^2])
```

fricas [A] time = 0.46, size = 429, normalized size = 2.86

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx-4\left(bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^2x^2-b\right)\sqrt{-d}\,\log\left(-\right.}{4\left(c^6d^2x^2-c^4d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - 4*(b*c^2*x^2 - 2*b)
*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 - b)*sqrt(-
d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x
^2 - 1)) - 4*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2
), -1/2*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + (b*c^2*x^2 - b)*s
qrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x
^4 - d)) - 2*(b*c^2*x^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2
- 1)) - 2*(a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^6*d^2*x^2 - c^4*d^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 313, normalized size = 2.09

$$-\frac{a\,x^{2}}{c^{2} d \sqrt{-c^{2} d\,x^{2}+d}}+\frac{2 a}{d\,c^{4} \sqrt{-c^{2} d\,x^{2}+d}}+\frac{b \sqrt{-d \left(c^{2} x^{2}-1\right)}\,\mathrm{arccosh}\left(c x\right) x^{2}}{c^{2} d^{2} \left(c^{2} x^{2}-1\right)}-\frac{b \sqrt{-d \left(c^{2} x^{2}-1\right)}\,\sqrt{c x+1}\,\sqrt{c x-1}\,x}{c^{3} d^{2} \left(c^{2} x^{2}-1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a/d/c^4/(-c^2*d*x^2+d)^(1/2)+b*(-d*(c^2
*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-b*(-d*(c^2*x^2-1))^(1/2
)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2*b*(-d*(c^2*x^2-1))^(1
/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [A] time = 0.93, size = 157, normalized size = 1.05

$$-\frac{1}{2}\,bc\left(\frac{2\,\sqrt{-d}\,x}{c^{4} d^{2}}+\frac{\sqrt{-d}\,\log\left(cx+1\right)}{c^{5} d^{2}}-\frac{\sqrt{-d}\,\log\left(cx-1\right)}{c^{5} d^{2}}\right)-b\left(\frac{x^{2}}{\sqrt{-c^{2} dx^{2}+d}\,c^{2} d}-\frac{2}{\sqrt{-c^{2} dx^{2}+d}\,c^{4} d}\right)\mathrm{arcosh}\left(cx\right)-a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(-
d)*log(c*x - 1)/(c^5*d^2)) - b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqrt(
-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d)
- 2/(sqrt(-c^2*d*x^2 + d)*c^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^{3}\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^{2}\,d\,x^{2}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))}{(-d (cx - 1) (cx + 1))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.117 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=143

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^2*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5752, 5676, 260}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(c^2*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^3*d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2 \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^{3/2}} dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2(a+b\cosh^{-1}(cx))}{(-1+cx)^{3/2}(1+cx)^{3/2}} dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\sqrt{\right.}}{cd\sqrt{}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^3d\sqrt{d-c^2dx^2}} - \frac{b\sqrt{-1+cx}\sqrt{1}}{2c^3d}$$

Mathematica [A] time = 0.68, size = 159, normalized size = 1.11

$$\frac{2a\sqrt{d}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + 2acdx + bd\left(2cx\cosh^{-1}(cx) - \sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(2\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)+\right.\right.}{2c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (2*a*c*d*x + 2*a*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2
])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)])))/(2*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(bx^2\,\text{arcosh}\,(cx)+ax^2\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.47, size = 279, normalized size = 1.95

$$\frac{ax}{c^2d\sqrt{-c^2d\,x^2+d}} - \frac{a\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}} + \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccosh}\,(cx)^2}{2d^2c^3\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+1/2*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)
*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{\arcsin(cx)}{c^3d^{\frac{3}{2}}}\right)+b\int\frac{x^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] a*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + b*integrat
e(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\operatorname{acosh}(cx)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.118 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=76

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(1/2)+b*arctanh(c*x)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {5798, 5718, 207}

$$\frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*ArcTanh[c*x])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$
$$= \frac{a+b\cosh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.24, size = 90, normalized size = 1.18

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^2d^2\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\tanh^{-1}(cx)}{c^2d^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(c^2*d^2*(-1 + c^2*x^2))) - (b
*Sqrt[-(d*(-1 + c^2*x^2))]*ArcTanh[c*x])/(c^2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

fricas [A] time = 0.78, size = 327, normalized size = 4.30

$$\left[-\frac{4\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\left(bc^2x^2-b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4dx^4-5\,c^2dx^2-4\left(c^3x^3+cx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,\sqrt{-}}{c^6x^6-3\,c^4x^4+3\,c^2x^2-1}\right.}{4\left(c^4d^2x^2-c^2d^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)) + (b*c^2*x^2 -
 b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x
)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4
 + 3*c^2*x^2 - 1)) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2), -1/
2*((b*c^2*x^2 - b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*
c*sqrt(d)*x/(c^4*d*x^4 - d)) + 2*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*
x^2 - 1)) + 2*sqrt(-c^2*d*x^2 + d)*a)/(c^4*d^2*x^2 - c^2*d^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.24, size = 198, normalized size = 2.61

$$\frac{a}{c^2d\sqrt{-c^2d\,x^2+d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}\,(cx)}{c^2d^2\left(c^2x^2-1\right)} + \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(cx+\sqrt{cx-1}\,\sqrt{cx+1}\right.}{c^2d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)

[Out] a/c^2/d/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*a
rccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
x-1)^(1/2)(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\left(\frac{\frac{\left(c\sqrt{d}x+\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{-cx+1}}+\frac{\sqrt{cx+1}\sqrt{cx-1}\sqrt{d}}{\sqrt{-cx+1}}}{\sqrt{cx+1}\,c^3d^2x+(cx+1)\sqrt{cx-1}\,c^2d^2}-\int\frac{c^2x^3}{\sqrt{-cx+1}\left(\left(c^2d^{\frac{3}{2}}x^2-d^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}\log(cx+1)+\log(cx-1\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] b*(((c*sqrt(d)*x + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d))*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/sqrt(-c*x + 1) + sqrt(c*x + 1)*sqrt(c*x - 1)*sqrt(d)/sq
rt(-c*x + 1))/(sqrt(c*x + 1)*c^3*d^2*x + (c*x + 1)*sqrt(c*x - 1)*c^2*d^2) -
 integrate((c^2*x^3 + c*x^2*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)) - x)/(s
qrt(-c*x + 1)*((c^2*d^(3/2)*x^2 - d^(3/2))*e^(3/2*log(c*x + 1) + log(c*x -
1)) + 2*(c^3*d^(3/2)*x^3 - c*d^(3/2)*x)*e^(log(c*x + 1) + 1/2*log(c*x - 1))
 + (c^4*d^(3/2)*x^4 - c^2*d^(3/2)*x^2)*sqrt(c*x + 1))), x)) + a/(sqrt(-c^2*
d*x^2 + d)*c^2*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2),x)

[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)

[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)

3.119 $\int \frac{a+b\cosh^{-1}(cx)}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=84

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)-1/2*b*ln(-c^2*x^2+1)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5713, 5688, 260}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}\log\left(1-c^2x^2\right)}{2cd\sqrt{d-c^2dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.86

$$\frac{2acx - b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right) + 2bcx\cosh^{-1}(cx)}{2cd\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(3/2),x]

[Out] (2*a*c*x + 2*b*c*x*ArcCosh[c*x] - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*c*d*Sqrt[d - c^2*d*x^2])

fricas [F] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^4d^2x^4-2\,c^2d^2x^2+d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(3/2), x)

maple [B] time = 0.20, size = 180, normalized size = 2.14

$$\frac{ax}{d\sqrt{-c^2d\,x^2+d}} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccosh}\,(cx)}{d^2c\left(c^2x^2-1\right)} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\text{arccosh}\,(cx)\,x}{d^2\left(c^2x^2-1\right)} + \frac{b\sqrt{-d\left(c^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)

[Out] a/d*x/(-c^2*d*x^2+d)^(1/2)-b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)

maxima [A] time = 0.70, size = 70, normalized size = 0.83

$$-\frac{bc\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)}{2\,d} + \frac{bx\,\text{arcosh}\,(cx)}{\sqrt{-c^2dx^2+d}\,d} + \frac{ax}{\sqrt{-c^2dx^2+d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")

[Out] -1/2*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b*x*arccosh(c*x)/(sqrt(-c^2*d*x^2 + d)*d) + a*x/(sqrt(-c^2*d*x^2 + d)*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(c x \right)}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.120 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=229

$$\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\,\sqrt{cx+1}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*arctan(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1
/2)+b*arctanh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-I*b*p
olylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d/(-c^2*d*x^2+d)^(1/2)+I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 5756, 5761, 4180, 2279, 2391, 207}

$$-\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) +
(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (I*
b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d -
 c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}\,\tanh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a\right.}{d\sqrt{d-}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1}}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1}}{}$$

$$= \frac{a+b\cosh^{-1}(cx)}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1}}{}$$

Mathematica [A] time = 2.57, size = 301, normalized size = 1.31

$$-\frac{\frac{a\sqrt{d-c^2dx^2}}{c^2x^2-1} + a\sqrt{d}\,\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) + a\left(-\sqrt{d}\right)\log(x) + \frac{ibd\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\text{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right) - \sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\text{Li}_2\left(ie^{-\text{co}}\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a*Sqrt[d]*Log[x] + a*Sqrt[d]*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (I*b*d*(I*ArcCosh[c*x] + Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^5 - 2*c^2*d^2
*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\operatorname{arcosh}(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

maple [B] time = 0.42, size = 511, normalized size = 2.23

$$\frac{a}{d\sqrt{-c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{3}{2}}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)}{d^2\left(c^2x^2-1\right)}+\frac{ib\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a/d/(-c^2*d*x^2+d)^(1/2)-a/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/
x)-b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)+I*b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arcco
sh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*dilog(1+I*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)}{d^{\frac{3}{2}}}-\frac{1}{\sqrt{-c^2dx^2+d}\,d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(sq
rt(-c^2*d*x^2 + d)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))
/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{x\,{\left(d - c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.121 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=158

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}+\frac{bc\log(x)\sqrt{d-c^2dx^2}}{d^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\sqrt{d-c^2dx^2}\log\left(1-c^2x^2\right)}{2d^2\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] (-a-b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccosh(c*x))/d/(
-c^2*d*x^2+d)^(1/2)+b*c*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1
)^(1/2)+1/2*b*c*ln(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 159, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 103, 12, 39, 5733, 446, 72}

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log(x)}{d\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCosh
[c*x]))/(d*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])
/(d*Sqrt[d - c^2*d*x^2]) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^
2])/(2*d*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
```

```
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{-1+2c^2x^2}{x\left(1-c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{-1+2c^2}{x\left(1-c^2\right.}\right.}{2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int\left(-\frac{1}{x}-\right.\right.}{2d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\log(x)}{d\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+}}{}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.72

$$\frac{4ac^2x^2-2a-bcx\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-c^2x^2\right)+2b\left(2c^2x^2-1\right)\cosh^{-1}(cx)-2bcx\sqrt{cx-1}\,\sqrt{cx+1}\,\log(x)}{2dx\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a + 4*a*c^2*x^2 + 2*b*(-1 + 2*c^2*x^2)*ArcCosh[c*x] - 2*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*Log[x] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(2*d*x*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,\left(b\,\text{arcosh}\left(cx\right)+a\right)}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^6 - 2*c^2*d^2
*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}} x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

maple [A] time = 0.36, size = 242, normalized size = 1.53

$$-\frac{a}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2a\,c^2x}{d\sqrt{-c^2d\,x^2+d}}-\frac{2b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)\,c}{d^2\left(c^2x^2-1\right)}-\frac{2b\sqrt{-d\left(c^2x^2-1\right)}\,\text{ar}}{\left(c^2x^2-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a/d/x/(-c^2*d*x^2+d)^(1/2)+2*a*c^2/d*x/(-c^2*d*x^2+d)^(1/2)-2*b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c-2*b
*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+b*(-d*(c^2*x^2-1
))^(1/2)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)
*c
```

maxima [A] time = 0.74, size = 144, normalized size = 0.91

$$\frac{1}{2}\,bc\left(\frac{\sqrt{-d}\,\log(cx+1)}{d^2}+\frac{\sqrt{-d}\,\log(cx-1)}{d^2}+\frac{2\sqrt{-d}\,\log(x)}{d^2}\right)+\left(\frac{2\,c^2x}{\sqrt{-c^2dx^2+d}\,d}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx}\right)b\operatorname{arcosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/2*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d)
*log(x)/d^2) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*
d*x))*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x
^2 + d)*d*x))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x^2\left(d - c^2\,d\,x^2\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.122 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=329

$$\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}}+\frac{3c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}-\frac{3ibc^2\sqrt{c}}{}$$

```
[Out] 3/2*c^2*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(1/2)+1/2*(-a-b*arccosh(c*x))/d
/x^2/(-c^2*d*x^2+d)^(1/2)+1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x/(-c^2*d*x
^2+d)^(1/2)+3*c^2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+b*c^2*arctanh(c*x)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-3/2*I*b*c^2*polylog(2,-I*(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+
d)^(1/2)+3/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5748, 5756, 5761, 4180, 2279, 2391, 207, 325}

$$-\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d\sqrt{d-c^2dx^2}}+\frac{3c^2\left(a+b\right.}{2d\sqrt{d-}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x*Sqrt[d - c^2*d*x^2]) + (3*c^2*(a
+ b*ArcCosh[c*x]))/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])/(2*d*x^
2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (b*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(d*Sqrt[d - c^2*d*x^2]) - (((3*I)/2)*b*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c
^2*d*x^2]) + (((3*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^A
rcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\left(-1+c^2x^2\right)}\,dx}{2d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{-1+cx}\,\sqrt{1+c\right.}{2d\sqrt{d}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(3c^2\sqrt{-1+cx}\right.}{2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{bc^2\sqrt{-1+cx}}{d\sqrt{d}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)}{2d\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{2dx^2\sqrt{d-c^2dx^2}} + \frac{3c^2\sqrt{-1+cx}}{}$$

Mathematica [A] time = 4.45, size = 405, normalized size = 1.23

$$\frac{1}{2}\left(-\frac{3ac^2\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)}{d^{3/2}} + \frac{3ac^2\log(x)}{d^{3/2}} - \frac{a\left(3c^2x^2-1\right)\sqrt{d-c^2dx^2}}{d^2x^2\left(c^2x^2-1\right)} - \frac{bc^2\left(\left(\frac{1}{c^2x^2}-1\right)\cosh^{-1}(cx)+\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-((a*(-1 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(d^2*x^2*(-1 + c^2*x^2))) + (3*
a*c^2*Log[x])/d^(3/2) - (3*a*c^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/d^(3
/2) - (b*c^2*(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^
2*x^2))*ArcCosh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] -
(3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2
*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2))/(d*Sqrt[d - c^2*d*x^2]))/2
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^7 - 2*c^2*d^2
*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

maple [B] time = 0.64, size = 648, normalized size = 1.97

$$-\frac{a}{2d\,x^2\sqrt{-c^2d\,x^2+d}}+\frac{3a\,c^2}{2d\sqrt{-c^2d\,x^2+d}}-\frac{3a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{3}{2}}}-\frac{3b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,c^2}{2d^2\left(c^2x^2-1\right)}-\frac{b\sqrt{-d\,(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(1/2)+3/2*a*c^2/d/(-c^2*d*x^2+d)^(1/2)-3/2*a*c^
2/d^(3/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-3/2*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^2-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(c
^2*x^2-1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(c^2*x^2-1)/x^2*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)/d^2*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2-b*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln(1+c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*c^2+3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*c^2-3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x
^2-1)/d^2*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2-3/2*I*
b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(
1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+3/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{3\,c^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)}{d^{\frac{3}{2}}}-\frac{3\,c^2}{\sqrt{-c^2dx^2+d}\,d}+\frac{1}{\sqrt{-c^2dx^2+d}\,dx^2}\right)a+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2)
 - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a + b*i
ntegrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^3
), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)),x)
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^3 \left(-d (cx - 1) (cx + 1)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.123 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^2x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bc^3\log(x)\sqrt{d-c}}{3d^2\sqrt{cx-1}\sqrt{cx}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccosh(c*x
))/d/x/(-c^2*d*x^2+d)^(1/2)+8/3*c^4*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(
1/2)-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^2/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/3*b
*c^3*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c^3*l
n(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5798, 103, 12, 39, 5733, 1251, 893}

$$\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}}-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{6dx^2\sqrt{d-c^2dx^2}}-\frac{5bc^3\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{lo}}{3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2*Sqrt[d - c^2*d*x^2]) - (a + b*A
rcCosh[c*x])/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[c*x]))/(
3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x]))/(3*d*Sqrt[d - c
^2*d*x^2]) - (5*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*d*Sqrt[d - c^
2*d*x^2]) - (b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(2*d*Sqrt
[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
```

```
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+\right.}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6dx^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(cx)\right)}{3dx\sqrt{d-c^2dx^2}} + \frac{8c^4x\left(a+b\cos\right.}{3d\sqrt{d-c^2}}$$

Mathematica [A] time = 0.12, size = 161, normalized size = 0.64

$$\frac{16ac^4x^4-8ac^2x^2-2a-10bc^3x^3\sqrt{cx-1}\,\sqrt{cx+1}\,\log(x)+2b\left(8c^4x^4-4c^2x^2-1\right)\cosh^{-1}(cx)-3bc^3x^3\sqrt{cx-1}}{6dx^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (-2*a - 8*a*c^2*x^2 + 16*a*c^4*x^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2
*b*(-1 - 4*c^2*x^2 + 8*c^4*x^4)*ArcCosh[c*x] - 10*b*c^3*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Log[x] - 3*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2
*x^2])/(6*d*x^3*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^4d^2x^8-2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^4*d^2*x^8 - 2*c^2*d^2
*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

maple [B] time = 0.66, size = 1050, normalized size = 4.20

$$-\frac{a}{3d\,x^3\sqrt{-c^2d\,x^2+d}}-\frac{4a\,c^2}{3dx\sqrt{-c^2d\,x^2+d}}+\frac{8a\,c^4x}{3d\sqrt{-c^2d\,x^2+d}}-\frac{16b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccosh}\,(cx)\,c^3}{3d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*a*c^2/d/x/(-c^2*d*x^2+d)^(1/2)+8/3*a*
c^4/d*x/(-c^2*d*x^2+d)^(1/2)-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^3+32/3*b*(-d*(c^2*x^2-1))^(1/2)/
d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*x+1)*(c*x-1)*c^8-32/3*b*(-d*(c^2*x^2-1))
^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10-16/3*b*(-d*(c^2*x^2-1))^(1/2)/d
^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6+16*b*(-d*(c^2*x^2-1))^(1
/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(
8*c^4*x^4-7*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-64/
3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6
-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*(c*x+1)*(c*x-1)
*c^4-4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6+8/3*b*(
-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*c^3+8*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*
x*arccosh(c*x)*c^4-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^
4-7*c^2*x^2-1)*x*c^4+4*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
/x*arccosh(c*x)*c^2-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1
)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4
*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3+
```

```
5/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{8\,c^4x}{\sqrt{-c^2dx^2+d}\,d}-\frac{4\,c^2}{\sqrt{-c^2dx^2+d}\,dx}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx^3}\right)a+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a + b*integrate(log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(cx\right)}{x^4\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.124 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{c^6d^3}-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{c^6d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^6d\left(d-c^2dx^2\right)^{3/2}}+\frac{11b\sqrt{d-c^2dx^2}\tanh^{-1}(cx)}{6c^6d^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx}{c^5d^3\sqrt{}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^6/d/(-c^2*d*x^2+d)^(3/2)-2*(a+b*arccosh(c*x))/c^6/
d^2/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c^6/d^3+b*
x*(-c^2*d*x^2+d)^(1/2)/c^5/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*x*(-c^2*d*
x^2+d)^(1/2)/c^5/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+11/6*b*arctan
h(c*x)*(-c^2*d*x^2+d)^(1/2)/c^6/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 280, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 98, 21, 74, 5733, 12, 1157, 388, 206}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{8(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)}{3c^6d^2\sqrt{d-c^2dx^2}}-\frac{bx\sqrt{cx-1}\sqrt{cx}}{c^5d^2\sqrt{d-c^2dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -((b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^5*d^2*Sqrt[d - c^2*d*x^2])) + (b*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])
 - (4*x^2*(a + b*ArcCosh[c*x]))/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^4*(a +
 b*ArcCosh[c*x]))/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (8*
(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]))/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) -
 (11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^6*d^2*Sqrt[d - c^2*d
*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
```

```
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5 \left(a + b\cosh^{-1}(cx)\right)}{\left(d - c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1+cx)\left(a\right.}{3c^6d^2\sqrt{d-}}$$

$$= -\frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{8(1-cx)(1+cx)\left(a\right.}{3c^6d^2\sqrt{d-}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx\right.}{3c^2d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} +$$

$$= -\frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}} +$$

Mathematica [A] time = 0.17, size = 167, normalized size = 0.69

$$\frac{6ac^4x^4 - 24ac^2x^2 + 16a - 6bc^3x^3\sqrt{cx-1}\,\sqrt{cx+1} - 11b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)\tanh^{-1}(cx) + 2b\left(3c^4x^4 - 12c\right.}{6c^6d^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (16*a - 24*a*c^2*x^2 + 6*a*c^4*x^4 + 5*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] -
 6*b*c^3*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*(8 - 12*c^2*x^2 + 3*c^4*x^4
)*ArcCosh[c*x] - 11*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c
*x])/(6*c^6*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

fricas [A] time = 0.83, size = 529, normalized size = 2.18

$$\left[-\frac{8\left(3\,bc^4x^4 - 12\,bc^2x^2 + 8\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx + \sqrt{c^2x^2-1}\right) + 11\left(bc^4x^4 - 2\,bc^2x^2 + b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5\,c^4d}{}\right.}{24\left(c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [-1/24*(8*(3*b*c^4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1)) + 11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)*log(-(c^6*d
*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x^3 + c*x)*sqrt(-c^2*d*x^2 + d)*s
qrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)) - 4*(
6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) + 8*(3*a*c^4*
x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c^8*d^3*x
^2 + c^6*d^3), 1/12*(11*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt
(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) - 4*(3*b*c^
4*x^4 - 12*b*c^2*x^2 + 8*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1
)) + 2*(6*b*c^3*x^3 - 5*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 4*(
```

```
3*a*c^4*x^4 - 12*a*c^2*x^2 + 8*a)*sqrt(-c^2*d*x^2 + d))/(c^10*d^3*x^4 - 2*c
^8*d^3*x^2 + c^6*d^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.65, size = 466, normalized size = 1.92

$$-\frac{a\,x^4}{c^2 d\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}+\frac{4a\,x^2}{c^4 d\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{8a}{3c^6 d\left(-c^2 d\,x^2+d\right)^{\frac{3}{2}}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,x^2}{c^4d^3\left(c^2x^2-1\right)}+\frac{b\sqrt{-d\left(c^2x\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a*x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)+4*a/c^4*x^2/d/(-c^2*d*x^2+d)^(3/2)-8/3*a/
c^6/d/(-c^2*d*x^2+d)^(3/2)-b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arc
cosh(c*x)*x^2+b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x+b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)+2*b
*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)*x^2+1/6*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-5/3*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)+11/6*b*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))-11/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{9}\,b\left(\frac{\frac{\left(9\,c^4\sqrt{d}\,x^4-8\,\sqrt{d}\right)\sqrt{cx+1}\,\sqrt{cx-1}}{\sqrt{-cx+1}}-\frac{3\left(3\,c^5\sqrt{d}\,x^5-12\,c^3\sqrt{d}\,x^3+8\,c\sqrt{d}\,x+\left(3\,c^4\sqrt{d}\,x^4-12\,c^2\sqrt{d}\,x^2+8\,\sqrt{d}\right)\sqrt{cx+1}\,\sqrt{cx-1}\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{c\right.}{\sqrt{-cx+1}}}{\left(c^8d^3x^2-c^6d^3\right)(cx+1)\sqrt{cx-1}+\left(c^9d^3x^3-c^7d^3x\right)\sqrt{cx+1}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/9*b*(((9*c^4*sqrt(d)*x^4 - 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1)/sqrt(-
c*x + 1) - 3*(3*c^5*sqrt(d)*x^5 - 12*c^3*sqrt(d)*x^3 + 8*c*sqrt(d)*x + (3*c
^4*sqrt(d)*x^4 - 12*c^2*sqrt(d)*x^2 + 8*sqrt(d))*sqrt(c*x + 1)*sqrt(c*x - 1
))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c*x + 1))/((c^8*d^3*x^2 - c
^6*d^3)*(c*x + 1)*sqrt(c*x - 1) + (c^9*d^3*x^3 - c^7*d^3*x)*sqrt(c*x + 1))
+ 9*integrate(1/9*(9*c^7*sqrt(d)*x^7 - 45*c^5*sqrt(d)*x^5 + 60*c^3*sqrt(d)*
x^3 - 24*c*sqrt(d)*x + (9*c^6*sqrt(d)*x^6 - 54*c^4*sqrt(d)*x^4 + 60*c^2*sqr
t(d)*x^2 - 16*sqrt(d))*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1)))/(sqrt(-c*x
+ 1)*((c^9*d^3*x^4 - 2*c^7*d^3*x^2 + c^5*d^3)*e^(3/2*log(c*x + 1) + log(c*x
 - 1)) + 2*(c^10*d^3*x^5 - 2*c^8*d^3*x^3 + c^6*d^3*x)*e^(log(c*x + 1) + 1/2
*log(c*x - 1)) + (c^11*d^3*x^6 - 2*c^9*d^3*x^4 + c^7*d^3*x^2)*sqrt(c*x + 1)
)), x)) - 1/3*a*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2
 + d)^(3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
[Out] int((x^5*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{5}\left(a+b\operatorname{acosh}{\left(cx \right)}\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral(x**5*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.125 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-1}\sqrt{cx+1}\log\left(1\right.}{3c^5d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(3/2)-x*(a+b*arccosh(c*x))/c^4/d^2/(-c^2*
d*x^2+d)^(1/2)+1/2*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/d
^2/(-c^2*d*x^2+d)^(1/2)+2/3*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
5/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 251, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5752, 5676, 260, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{2bc^5d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}}{6c^5d^2\left(1-c^2x\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^5*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) - (x*(a + b*ArcCosh[c*x]))/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*
ArcCosh[c*x]))/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(2*b*c^5*d^2*Sqrt[d - c^2*d
*x^2]) + (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*c^5*d^2*Sqr
t[d - c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$=\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{c^2d^2\sqrt{d-c^2dx^2}}+\ldots$$

$$=-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\ldots}{c^4d^2\sqrt{d-\ldots}}$$

$$=-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}+\frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a\ldots}{2bc^5d^2\sqrt{d\ldots}}$$

$$=\frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^5d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{x\left(a+b\cosh^{-1}(cx)\right)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-\ldots}}$$

Mathematica [A] time = 0.76, size = 225, normalized size = 1.00

$$\frac{\frac{2acx\left(4c^2x^2-3\right)\sqrt{d-c^2dx^2}}{\left(c^2x^2-1\right)^2}-6a\sqrt{d}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+\frac{bd\left(-\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)+2cx\cosh^{-1}(cx)}{c^2x^2-1}-8cx\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+\ldots\right.\right.\right.}{\sqrt{d-c^2dx^2}}}{6c^5d^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((2*a*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 6*a*Sqrt
[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (b*d*(-8*c
*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x
```

```
])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x]^2
+ 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2])/(6*c^
5*d^3)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(bx^4 \operatorname{arcosh}(cx) + ax^4\right)\sqrt{-c^2dx^2 + d}}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-(b*x^4*arccosh(c*x) + a*x^4)*sqrt(-c^2*d*x^2 + d)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.84, size = 1519, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)-a/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+a/c^4
/d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/2*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c
*x)^2+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2
*x^2-1)*arccosh(c*x)-32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6+
32*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)*c^2/d^3*arccosh(c*x)*x^7-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6
*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x+1)*(c*x-1)*x^5+8/3*b*(-d*(c^2*x^2-
1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^7+84*
b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/
c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-76*b*(-d*(c^2*x^2-1))^(1
/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+
14/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/c^2/d^3*(c*x+1)*(c*x-1)*x^3+4*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*
c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-22
/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+1
6)/d^3*x^5-220/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^
4-71*c^2*x^2+16)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+181/3
*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)
/c^2/d^3*arccosh(c*x)*x^3-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x+1)*(c*x-1)*x-13/2*b*(-d*(c^2*x^2-1
))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2+20/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^
```

```
6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*x^3+64/3*b*(-d*(c^2*x^2-1))^(1/2)/(24*
c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)*(c*x+1)^
(1/2)*(c*x-1)^(1/2)-16*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*
c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)*x+8/3*b*(-d*(c^2*x^2-1))^(1/2)/
(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*
x+1)^(1/2)-2*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c^4/d^3*x-4/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(x\left(\frac{3\,x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)-\frac{x}{\sqrt{-c^2dx^2+d}\,c^4d^2}+\frac{3\,\arcsin\left(cx\right)}{c^5d^{\frac{5}{2}}}\right)a+b\int\frac{x^4\log\left(cx+\sqrt{cx+1}\right.}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^
4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a +
 b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5
/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.126 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=158

$$-\frac{a+b\cosh^{-1}(cx)}{c^4d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3c^4d\left(d-c^2dx^2\right)^{3/2}}+\frac{5b\sqrt{d-c^2dx^2}\,\tanh^{-1}(cx)}{6c^4d^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bx\sqrt{d-c^2dx^2}}{6c^3d^3(cx-1)^{3/2}(cx+1)^{3/2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^4/d/(-c^2*d*x^2+d)^(3/2)+(-a-b*arccosh(c*x))/c^4/d
^2/(-c^2*d*x^2+d)^(1/2)+1/6*b*x*(-c^2*d*x^2+d)^(1/2)/c^3/d^3/(c*x-1)^(3/2)/
(c*x+1)^(3/2)+5/6*b*arctanh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^4/d^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 243, normalized size of antiderivative = 1.54, number of steps used = 5, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 94, 89, 21, 37, 5733, 12, 385, 206}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{(1-cx)^2\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{c^4d^2(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]) - (a + b*ArcCosh[c*x])/(c^4*d^2*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (x^3
*(a + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) +
((1 - c*x)^2*(a + b*ArcCosh[c*x]))/(3*c^4*d^2*(1 + c*x)*Sqrt[d - c^2*d*x^2]
) - (5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^4*d^2*Sqrt[d - c^2
*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
```

```
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
1])))
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1
))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)),
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] &&  !(SumSimpl
erQ[p, 1] &&  !SumSimplerQ[m, 1])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2\left(a+b\right.}{3c^4d^2(1+cx)}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3cd^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(1-cx)^2\left(a+b\right.}{3c^4d^2(1+cx)}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-}\right.}{3cd^2(1-cx)(1+cx)}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{c^4d^2(1+cx)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-}\right.}{3cd^2(1-cx)(1+cx)}$$

Mathematica [A] time = 0.13, size = 122, normalized size = 0.77

$$\frac{-6ac^2x^2+4a+b\left(4-6c^2x^2\right)\cosh^{-1}(cx)-5b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2-1\right)\tanh^{-1}(cx)-bcx\sqrt{cx-1}\,\sqrt{cx+1}}{6c^4d^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a - 6*a*c^2*x^2 - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*(4 - 6*c^2*x^2)
*ArcCosh[c*x] - 5*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*ArcTanh[c*x
])/(6*c^4*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

fricas [A] time = 0.69, size = 469, normalized size = 2.97

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx+8\left(3\,bc^2x^2-2\,b\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)-5\left(bc^4x^4-2\,bc^2x^2+b\right)}{24\left(c^8d^3x^4-2\,c^6d^3x^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*(3*b*c^2*x^2 - 2*
b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) - 5*(b*c^4*x^4 - 2*b*c
^2*x^2 + b)*sqrt(-d)*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 + 4*(c^3*x
^3 + c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3
*c^4*x^4 + 3*c^2*x^2 - 1)) + 8*(3*a*c^2*x^2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c
^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3), 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1)*b*c*x + 5*(b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c
^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d)*x/(c^4*d*x^4 - d)) + 4*(3*b*c^2*x
^2 - 2*b)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 4*(3*a*c^2*x^
2 - 2*a)*sqrt(-c^2*d*x^2 + d))/(c^8*d^3*x^4 - 2*c^6*d^3*x^2 + c^4*d^3)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.54, size = 313, normalized size = 1.98

$$\frac{a\,x^{2}}{c^{2}d\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}-\frac{2a}{3d\,c^{4}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}+\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\mathrm{arccosh}\left(cx\right)x^{2}}{d^{3}\left(c^{2}x^{2}-1\right)^{2}c^{2}}+\frac{b\sqrt{-d\left(c^{2}x^{2}-1\right)}\,\sqrt{cx+1}\,\sqrt{cx-1}}{6d^{3}\left(c^{2}x^{2}-1\right)^{2}c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a*x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*a/d/c^4/(-c^2*d*x^2+d)^(3/2)+b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)*x^2+1/6*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-2/3*b*(-d*(c^2
*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)-5/6*b*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)-1)+5/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3
/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [A] time = 0.70, size = 175, normalized size = 1.11

$$\frac{1}{12}\,bc\left(\frac{2\sqrt{-d}\,x}{c^{6}d^{3}x^{2}-c^{4}d^{3}}+\frac{5\sqrt{-d}\,\log\left(cx+1\right)}{c^{5}d^{3}}-\frac{5\sqrt{-d}\,\log\left(cx-1\right)}{c^{5}d^{3}}\right)+\frac{1}{3}\,b\left(\frac{3\,x^{2}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}d}-\frac{2}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{4}d}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/12*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(c
^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 1/3*b*(3*x^2/((-c^2*d*x^2 +
d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a*(3
*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.127 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=133

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{6c^3d^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6c^3d\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 160, normalized size of antiderivative = 1.20, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5724, 266, 43}

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{6c^3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (x^3*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*
x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*c^3*d^2*Sqrt[d
 - c^2*d*x^2])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{\left(-1+c^2x^2\right)^2}dx}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{x}{\left(-1+c^2x\right)^2}dx, x, x\right.}{6d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{1}{c^2\left(-1+c^2x\right)^2}+\frac{}{c^2}\right.\right.}{6d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+}}{6c^3d^2\sqrt{d}}$$

Mathematica [A] time = 0.21, size = 101, normalized size = 0.76

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\frac{b\left(\frac{1}{1-c^2x^2}+\log\left(1-c^2x^2\right)\right)}{c^3}-\frac{2x^3\left(a+b\cosh^{-1}(cx)\right)}{(cx-1)^{3/2}(cx+1)^{3/2}}\right)}{6d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((-2*x^3*(a + b*ArcCosh[c*x]))/((-1 + c*x)^(3
/2)*(1 + c*x)^(3/2)) + (b*((1 - c^2*x^2)^(-1) + Log[1 - c^2*x^2]))/c^3))/(6
*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(bx^2\,\text{arcosh}\,(cx)+ax^2\right)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*x^2*arccosh(c*x) + a*x^2)/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)x^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.54, size = 1228, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/c^2/d*x/(-c^2*d*x^2+d)^(3/2)-1/3*a/c^2/d^2*x/(-c^2*d*x^2+d)^(1/2)+2/3
*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*a
rccosh(c*x)-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*
x^2+1)*c^3/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^6+b*(-d*(c^2*x^2-
1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)
*x^7-1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)*c^2/d^3*(c*x-1)*(c*x+1)*x^5+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^
8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)*x^4-b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*
c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8
-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(c*x-1)*(c*x+1)*x^3+1/2*b*(-d*(c^2*x
^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*(c*x-1)^(1/
2)*(c*x+1)^(1/2)*x^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)*c^2/d^3*x^5-4/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)*x^2+1/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x
^2+1)/d^3*arccosh(c*x)*x^3-1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^
6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/6*b*(-d*(
c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*x^3+1/3*
b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d
^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/(3
*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1
/2)-1/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x
^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [A] time = 0.68, size = 169, normalized size = 1.27

$$\frac{1}{6}bc\left(\frac{\sqrt{-d}}{c^6d^3x^2-c^4d^3}-\frac{\sqrt{-d}\log(cx+1)}{c^4d^3}-\frac{\sqrt{-d}\log(cx-1)}{c^4d^3}\right)-\frac{1}{3}b\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d^2}-\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}\right)\text{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^3)
 - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 1/3*b*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d^
2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a*(x/(sqrt(-c^2*d
*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + b \operatorname{acosh}\left(cx\right)\right)}{\left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.128 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=127

$$\frac{a+b\cosh^{-1}(cx)}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{6cd\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*x*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*arctanh(c*x)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 154, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {5798, 5718, 199, 207}

$$\frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}(cx)}{6c^2d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^
2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*c^2*d^2*Sqrt[d - c^2
*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{\left(-1+c^2x^2\right)^2}\,dx}{3cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(b\sqrt{-1+cx}\,\sqrt{1+\right.}{6cd^2\sqrt{d}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\,\sqrt{1+}{6c^2d^2\sqrt{d-}}$$

Mathematica [A] time = 0.26, size = 119, normalized size = 0.94

$$\frac{\sqrt{d-c^2dx^2}\left(2a+bcx\sqrt{cx-1}\,\sqrt{cx+1}\,+2b\cosh^{-1}(cx)\right)}{6c^2d^3\left(c^2x^2-1\right)^2} - \frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\tanh^{-1}(cx)}{6c^2d^3\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*ArcCos
h[c*x]))/(6*c^2*d^3*(-1 + c^2*x^2)^2) - (b*Sqrt[-(d*(-1 + c^2*x^2))]*ArcTan
h[c*x])/(6*c^2*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.60, size = 421, normalized size = 3.31

$$\left[\frac{4\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,bcx+8\sqrt{-c^2dx^2+d}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^4x^4-2\,bc^2x^2+b\right)\sqrt{-d}\,\log\left(-\frac{c^6dx^6+5}{}\right.}{24\left(c^6d^3x^4-2\,c^4d^3x^2+c^2d^3\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(4*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x + 8*sqrt(-c^2*d*x^2 +
 d)*b*log(c*x + sqrt(c^2*x^2 - 1)) - (b*c^4*x^4 - 2*b*c^2*x^2 + b)*sqrt(-d)
*log(-(c^6*d*x^6 + 5*c^4*d*x^4 - 5*c^2*d*x^2 - 4*(c^3*x^3 + c*x)*sqrt(-c^2*
d*x^2 + d)*sqrt(c^2*x^2 - 1)*sqrt(-d) - d)/(c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2
 - 1)) + 8*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3),
 1/12*(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*c*x - (b*c^4*x^4 - 2*b*c^
2*x^2 + b)*sqrt(d)*arctan(2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*c*sqrt(d
)*x/(c^4*d*x^4 - d)) + 4*sqrt(-c^2*d*x^2 + d)*b*log(c*x + sqrt(c^2*x^2 - 1)
) + 4*sqrt(-c^2*d*x^2 + d)*a)/(c^6*d^3*x^4 - 2*c^4*d^3*x^2 + c^2*d^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)x}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccosh(c*x) + a)*x/(-c^2*d*x^2 + d)^(5/2), x)

maple [B] time = 0.29, size = 249, normalized size = 1.96

$$\frac{a}{3c^2d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx+1}\,\sqrt{cx-1}\,x}{6d^3\left(c^2x^2-1\right)^2c}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\mathrm{arccosh}\left(cx\right)}{3d^3\left(c^2x^2-1\right)^2c^2}+\frac{b\sqrt{-d\left(c^2x^2-1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)

[Out] 1/3*a/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)+1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-1/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int\frac{x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx+\frac{a}{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")

[Out] b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2), x) + 1/3*a/((-c^2*d*x^2 + d)^(3/2)*c^2*d)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2),x)

[Out] int((x*(a + b*acosh(c*x)))/(d - c^2*d*x^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)

[Out] Integral(x*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)

3.129 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=162

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-c^2x^2\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd\left(d-c^2dx^2\right)^{3/2}}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/6*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/d/(-c^2*d*x^2+d)^(3/2)+2/3*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)
^(1/2)-1/3*b*ln(-c^2*x^2+1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2+d
)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 189, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5713, 5691, 5688, 260, 261}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{b\sqrt{cx-1}\sqrt{cx+1}\log\left(1-\right.}{3cd^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]
) + (2*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcC
osh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
```

```
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\right.}{}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx}}$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{6cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx}}$$

Mathematica [A] time = 0.09, size = 132, normalized size = 0.81

$$\frac{4ac^3x^3-6acx-2b\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-1\right)\log\left(1-c^2x^2\right)+2bcx\left(2c^2x^2-3\right)\cosh^{-1}(cx)-b\sqrt{cx-1}\sqrt{cx+}}{6cd^2\left(c^2x^2-1\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (-6*a*c*x + 4*a*c^3*x^3 - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*b*c*x*(-3 + 2*
c^2*x^2)*ArcCosh[c*x] - 2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-1 + c^2*x^2)*Log
[1 - c^2*x^2])/(6*c*d^2*(-1 + c^2*x^2)*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^6 - 3*c^4*d^
3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.30, size = 1073, normalized size = 6.62

$$\frac{ax}{3d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{2ax}{3d^2\sqrt{-c^2d\,x^2+d}}-\frac{4b\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)}{3d^3c\left(c^2x^2-1\right)}+\frac{2b\sqrt{-d\left(c^2x^2-1\right)}\,c^4\,(c}{3\left(3c^6x^6-10c^4x^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d*x/(-c^2*d*x^2+d)^(3/2)+2/3*a/d^2*x/(-c^2*d*x^2+d)^(1/2)-4/3*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x
)+2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*
(c*x-1)*(c*x+1)*x^5-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^6/d^3*x^7+2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c^3/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-2*b*(-d*(c^2
*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*x^5
-5/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*(
c*x-1)*(c*x+1)*x^3+7/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^
2*x^2-4)*c^4/d^3*x^5-14/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11
*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2+17/3*b*(-d*(
c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*
x^3+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*(c*x-1
)*(c*x+1)*x+1/2*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c
^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)-4*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arcc
osh(c*x)*x-2/3*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)
/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c
^4*x^4+11*c^2*x^2-4)/d^3*x+2/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^3/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [A] time = 0.71, size = 157, normalized size = 0.97

$$\frac{1}{6}bc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\sqrt{-d}\,\log(cx+1)}{c^2d^3}+\frac{2\sqrt{-d}\,\log(cx-1)}{c^2d^3}\right)+\frac{1}{3}b\left(\frac{2x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)\operatorname{arcos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*d^
3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 1/3*b*(2*x/(sqrt(-c^2*d*x^2 + d)*
d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a*(2*x/(sqrt(-c^2*d
*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))/(d - c^2*d*x^2)^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.130 $\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=317

$$\frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(cx)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{ib\sqrt{cx-1}\sqrt{cx+1}}{d^2\sqrt{d-}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+(a+b*arccosh(c*x))/d^2/(-c^2*
d*x^2+d)^(1/2)+1/6*b*c*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2
*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+7/6*b*arctanh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-I*b*polylog(2,-I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)
^(1/2)+I*b*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 332, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5756, 5761, 4180, 2279, 2391, 207, 199}

$$-\frac{ib\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{ib\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{a+b\cosh^{-1}(}{3d^2(1-cx)(cx+1)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^
2]) + (a + b*ArcCosh[c*x])/(d^2*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCosh[c*x])
/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^
2]) + (7*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTanh[c*x])/(6*d^2*Sqrt[d - c^2*d
*x^2]) - (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])
/(d^2*Sqrt[d - c^2*d*x^2]) + (I*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I
*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1\right.}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{\right.}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{7b\sqrt{}}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{}}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{}}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{d^2\sqrt{d-c^2dx^2}} + \frac{a+b\cosh^{-1}(cx)}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{2\sqrt{}}{}$$

Mathematica [A] time = 7.08, size = 377, normalized size = 1.19

$$-\frac{a\log\left(\sqrt{d}\sqrt{-d\left(c^2x^2-1\right)}+d\right)}{d^{5/2}}+\sqrt{-d\left(c^2x^2-1\right)}\left(\frac{a}{3d^3\left(c^2x^2-1\right)^2}-\frac{a}{d^3\left(c^2x^2-1\right)}\right)+\frac{a\log(x)}{d^{5/2}}+\frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\Big(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(a/(3*d^3*(-1 + c^2*x^2)^2) - a/(d^3*(-1 + c^2*x^
2))) + (a*Log[x])/d^(5/2) - (a*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/
d^(5/2) + (b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c*x]*Coth[Arc
Cosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]*Log[1 - I/E
^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - 28*Log[Tan
h[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (24*I)*PolyLo
g[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]*Sinh[ArcC
osh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*ArcCosh[c*x]
*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^7 - 3*c^4*d^
3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}(cx) + a}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}} x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

maple [A] time = 0.48, size = 619, normalized size = 1.95

$$\frac{a}{3d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{a}{d^2\sqrt{-c^2d\,x^2+d}}-\frac{a\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{d^{\frac{5}{2}}}-\frac{b\sqrt{-d\left(c^2x^2-1\right)}\,\text{arccosh}(cx)\,x^2c^2}{d^3\left(c^2x^2-1\right)^2}+\frac{b\sqrt{-d\left(c^2\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a/d/(-c^2*d*x^2+d)^(3/2)+a/d^2/(-c^2*d*x^2+d)^(1/2)-a/d^(5/2)*ln((2*d+2
*d^(1/2)*(-c^2*d*x^2+d)^(1/2))/x)-b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^
2*arccosh(c*x)*x^2*c^2+1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2*a
rccosh(c*x)+7/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c
^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)-7/6*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^
2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+I*b*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
)+I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*di
log(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a\left(\frac{3\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{-c^2dx^2+d}\,d^2}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*a*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2) -
 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + b*integrate
(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\text{acosh}(c\,x)}{x\left(d - c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x(-d(cx-1)(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*acosh(c*x))/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.131 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d-c^2dx^2)^{5/2}}\,dx$

Optimal. Leaf size=248

$$\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d-c^2dx^2}}{6d^3\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)}+\frac{bc\log(}{d^3\sqrt{cx}}$$

```
[Out] (-a-b*arccosh(c*x))/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccosh(c*x))/d
/(-c^2*d*x^2+d)^(3/2)+8/3*c^2*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)
-1/6*b*c*(-c^2*d*x^2+d)^(1/2)/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+
b*c*ln(x)*(-c^2*d*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/6*b*c*ln(-
c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 279, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 103, 12, 40, 39, 5733, 1251, 893}

$$\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{4c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]
) + (8*c^2*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*Arc
Cosh[c*x])/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (4*c^2*x*(a +
b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(d^2*Sqrt[d - c^2*d*x^2]) - (5*b*c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(6*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
```

```
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c^2x\left(a+b\cosh^{-1}(\right.}{3d^2(1-cx)(1+cx)\sqrt{d-}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{d^2x(1-cx)(1+cx)\sqrt{d-c^2d}}$$

Mathematica [A] time = 0.38, size = 147, normalized size = 0.59

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\frac{4c^2x\left(2c^2x^2-3\right)\left(a+b\cosh^{-1}(cx)\right)}{3(cx-1)^{3/2}(cx+1)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{x(cx-1)^{3/2}(cx+1)^{3/2}} - \frac{1}{6}bc\left(\frac{1}{c^2x^2-1} + 5\log\left(1-c^2x^2\right) + 6\log(x)\right)\right)}{d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*(1
+ c*x)^(3/2)) + (4*c^2*x*(-3 + 2*c^2*x^2)*(a + b*ArcCosh[c*x]))/(3*(-1 + c*
x)^(3/2)*(1 + c*x)^(3/2)) - (b*c*((-1 + c^2*x^2)^(-1) + 6*Log[x] + 5*Log[1
- c^2*x^2]))/6))/(d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^8 - 3*c^4*d^
3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

maple [B] time = 0.46, size = 1350, normalized size = 5.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -a/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*a*c^2/d*x/(-c^2*d*x^2+d)^(3/2)+8/3*a*c^2/d^
2*x/(-c^2*d*x^2+d)^(1/2)-16/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+32/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x+1)*(c*x-1)*c^8-32/3*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10-80/3*b*(-d*(c
^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*(c*x+1)*(c*x-1
)*c^6+112/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9
)*x^7*c^8+64/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^
2-9)*x^4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5-64/3*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6+20*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x+1)*
(c*x-1)*c^4-140/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^5*c^6-136/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+56*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-
4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x+1
)*(c*x-1)*c^2+4/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2
*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+24*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*c^4+24*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*c-44*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x
*arccosh(c*x)*c^2-3/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26
*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-4*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8
*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+9*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)+5/3*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2-1)*c+b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2
*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a\left(\frac{8c^2x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{4c^2x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}-\frac{3}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/2)
*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + b*integrate(log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x^2\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.132 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=479

$$\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\sqrt{cx-1}\,\sqrt{cx+1}\,\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d\left(d-c^2dx^2\right)^{3/2}}-\frac{a+}{2dx^2}$$

```
[Out] 5/6*c^2*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/2)+1/2*(-a-b*arccosh(c*x))/d
/x^2/(-c^2*d*x^2+d)^(3/2)+5/2*c^2*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/
2)+3/4*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*d*x^2+d)^(1/2)-1/4*b*c*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+5/12*b*c
^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+5*c^
2*(a+b*arccosh(c*x))*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+13/6*b*c^2*arctanh(c*x)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*I*b*c^2*polylog(2,-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1
/2)+5/2*I*b*c^2*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 509, normalized size of antiderivative = 1.06, number of steps used = 16, number of rules used = 11, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {5798, 5748, 5756, 5761, 4180, 2279, 2391, 207, 199, 290, 325}

$$-\frac{5ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5ibc^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)}{2d^2\sqrt{d-c^2dx^2}}+\frac{5c^2\left(a+b\right.}{6d^2(1-cx)(c}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (3*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*Sqrt[d - c^2*d*x^2]) - (b*c*S
qrt[-1 + c*x]*Sqrt[1 + c*x])/(4*d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(5*b*c^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(12*d^2*(1 - c^2*x^2)*Sqrt[d - c^2
*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x]))/(2*d^2*Sqrt[d - c^2*d*x^2]) + (5*c^
2*(a + b*ArcCosh[c*x]))/(6*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (
a + b*ArcCosh[c*x])/(2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (
5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTan[E^ArcCosh[c*
x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (13*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Arc
Tanh[c*x])/(6*d^2*Sqrt[d - c^2*d*x^2]) - (((5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + ((
(5*I)/2)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, I*E^ArcCosh[c*x]])/(
d^2*Sqrt[d - c^2*d*x^2])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 290

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[((
c*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*n*(p + 1)), x] + Dist[(m + n*(p + 1)
 + 1)/(a*n*(p + 1)), Int[(c*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b
, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x
]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
```

```
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\left(-1+c^2x^2\right)^2}\,dx}{2d^2\sqrt{d-c^2dx^2}} + \frac{\left(5c^2\sqrt{\right.}}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}}{2d^2x^2(1-cx)(1+cx)}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5}{}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5}{}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5}{}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5}{}$$

$$= \frac{3bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\sqrt{d-c^2dx^2}} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{4d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5bc^3x\sqrt{-1+cx}\,\sqrt{1+cx}}{12d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5}{}$$

Mathematica [A] time = 7.29, size = 500, normalized size = 1.04

$$-\frac{5ac^2\log\left(\sqrt{d}\sqrt{-d\left(c^2x^2-1\right)}+d\right)}{2d^{5/2}}+\frac{5ac^2\log(x)}{2d^{5/2}}+\sqrt{-d\left(c^2x^2-1\right)}\left(-\frac{2ac^2}{d^3\left(c^2x^2-1\right)}+\frac{ac^2}{3d^3\left(c^2x^2-1\right)^2}-\frac{a}{2d^3x^2}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a/(d^3*x^2) + (a*c^2)/(3*d^3*(-1 + c^2*x^2)
^2) - (2*a*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a*c^2*Log[x])/(2*d^(5/2)) - (5*a
*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (b*c^2*((6*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*(-1 + c*x)*(1 + c*x)*ArcCos
h[c*x])/(c^2*x^2) + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 - Coth[ArcCosh[c
*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - 26*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (30*I)*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (30*I)*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] - 26*ArcCosh[c
*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x]*Tanh[ArcCo
sh[c*x]/2]^2))/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^9 - 3*c^4*d^
3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

maple [A] time = 0.74, size = 801, normalized size = 1.67

$$-\frac{a}{2d\,x^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{5a\,c^2}{6d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{5a\,c^2}{2d^2\sqrt{-c^2d\,x^2+d}}-\frac{5a\,c^2\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{-c^2d\,x^2+d}}{x}\right)}{2d^{\frac{5}{2}}}-\frac{5b\sqrt{-d\left(c^2x^2-1\right)}}{2d^3\left(c^4x^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -1/2*a/d/x^2/(-c^2*d*x^2+d)^(3/2)+5/6*a*c^2/d/(-c^2*d*x^2+d)^(3/2)+5/2*a*c^
2/d^2/(-c^2*d*x^2+d)^(1/2)-5/2*a*c^2/d^(5/2)*ln((2*d+2*d^(1/2)*(-c^2*d*x^2+
```

```
d)^(1/2))/x)-5/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x^2*arc
cosh(c*x)*c^4-1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x^2+1)*x*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*c^3+10/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^
2*x^2+1)*arccosh(c*x)*c^2+1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4*x^4-2*c^2*x
^2+1)/x*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-1/2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^4
*x^4-2*c^2*x^2+1)/x^2*arccosh(c*x)+13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)*c^2
-13/6*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*
ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2)))*c^2-5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^3/(c^2*x^2-1)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+
5/2*I*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*
dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2+5/2*I*b*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}a\left(\frac{15c^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{d^{\frac{5}{2}}}-\frac{15c^2}{\sqrt{-c^2dx^2+d}\,d^2}-\frac{5c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{3}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^2}\right)+b\int\frac{\log\left(cx+\right.}{\left(-c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/6*a*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5
/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d)
+ 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + b*integrate(log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acosh}\,(cx)}{x^3\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.133 $\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=338

$$-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{a+b\cosh^{-1}(cx)}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{bc\sqrt{d}}{6d^3x^2\sqrt{cx}}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arccosh(c*x))
/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arccosh(c*x))/d/(-c^2*d*x^2+d)^(3/
2)+16/3*c^4*x*(a+b*arccosh(c*x))/d^2/(-c^2*d*x^2+d)^(1/2)-1/6*b*c*(-c^2*d*x
^2+d)^(1/2)/d^3/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c^3*(-c^2*d*x^2+d)^(1
/2)/d^3/(-c^2*x^2+1)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/3*b*c^3*ln(x)*(-c^2*d*x^
2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/3*b*c^3*ln(-c^2*x^2+1)*(-c^2*d
*x^2+d)^(1/2)/d^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 383, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {5798, 103, 12, 40, 39, 5733, 1799, 1620}

$$\frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)}{d^2x(1-cx)(cx+1)\sqrt{d-c^2dx^2}}-\frac{a+b\cos}{3d^2x^3(1-cx)(cx+}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*x^2*Sqrt[d - c^2*d*x^2]) + (b*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) +
(16*c^4*x*(a + b*ArcCosh[c*x]))/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCos
h[c*x])/(3*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (2*c^2*(a + b
*ArcCosh[c*x]))/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*
(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (8*
b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[x])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (4
*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[1 - c^2*x^2])/(3*d^2*Sqrt[d - c^2*d
*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5733

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)^(m_)*((d1_) + (e1_.)*(x_))^(p_
)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> With[{u = IntHide[x^m*(1 + c*x)^
p*(-1 + c*x)^p, x]}, Dist[(-(d1*d2))^p*(a + b*ArcCosh[c*x]), u, x] - Dist[b
*c*(-(d1*d2))^p, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x]
, x], x]] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p - 1/2] && (IGtQ[(m + 1)/2, 0] || ILtQ[(m + 2*
p + 3)/2, 0]) && NeQ[p, -2^(-1)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh\right.}{d^2x(1-cx)(1+cx)}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh\right.}{d^2x(1-cx)(1+cx)}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh\right.}{d^2x(1-cx)(1+cx)}$$

$$= \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{a+b\cosh^{-1}(cx)}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{2c^2\left(a+b\cosh\right.}{d^2x(1-cx)(1+cx)}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2x^2\sqrt{d-c^2dx^2}} + \frac{bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}{6d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{16c^4x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}} - \frac{}{3d}$$

Mathematica [A] time = 0.44, size = 169, normalized size = 0.50

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\frac{2c^2\left(8c^4x^4-12c^2x^2+3\right)\left(a+b\cosh^{-1}(cx)\right)}{x(cx-1)^{3/2}(cx+1)^{3/2}} + \frac{a+b\cosh^{-1}(cx)}{x^3(cx-1)^{3/2}(cx+1)^{3/2}} - bc\left(\frac{1}{2x^2\left(c^2x^2-1\right)} + 4c^2\log\left(1-c^2x^2\right) + 8c^2\right.\right.}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/(x^3*(-1 + c*x)^(3/2)*(
1 + c*x)^(3/2)) + (2*c^2*(3 - 12*c^2*x^2 + 8*c^4*x^4)*(a + b*ArcCosh[c*x]))
/(x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)) - b*c*(1/(2*x^2*(-1 + c^2*x^2)) + 8*c
^2*Log[x] + 4*c^2*Log[1 - c^2*x^2])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)/(c^6*d^3*x^10 - 3*c^4*d
^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx)+a}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

maple [B] time = 0.67, size = 1878, normalized size = 5.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x)
[Out] -560/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^
2*x^2-1)*x^7*c^10-2*a*c^2/d/x/(-c^2*d*x^2+d)^(3/2)-1/3*a/d/x^3/(-c^2*d*x^2+
d)^(3/2)-8/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4
-10*c^2*x^2-1)*x*(c*x+1)*(c*x-1)*c^4+2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8
*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^
5-1/6*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2
*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-32/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c^3+16/3*b*(-d*(c^2*x
^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*arccosh(c*
x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*
c^3+128/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10
*c^2*x^2-1)*x^9*(c*x+1)*(c*x-1)*c^12-320/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12
*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^7*(c*x+1)*(c*x-1)*c^10+80*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*
x^5*(c*x+1)*(c*x-1)*c^8-40/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^
6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-128*b*(-d*(c^2*x^2-1
))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^4*arccosh(c*
x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^7+176/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c
^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*c^5+64*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*
c^4*x^4-10*c^2*x^2-1)*x^6*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^9+8/3*
a*c^4/d*x/(-c^2*d*x^2+d)^(3/2)+16/3*a*c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)+280/3*
b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1
)*x^5*c^8-32/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x
^4-10*c^2*x^2-1)*x^3*c^6-8/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^
6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*c^4+1/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c
^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/x^3*arccosh(c*x)-128/3*b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^11*c^
14+448/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*
c^2*x^2-1)*x^9*c^12-64*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+
35*c^4*x^4-10*c^2*x^2-1)*x^7*arccosh(c*x)*c^10+160*b*(-d*(c^2*x^2-1))^(1/2)
/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x^5*arccosh(c*x)*c^8-3
44/3*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*
x^2-1)*x^3*arccosh(c*x)*c^6+12*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*
c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)*x*arccosh(c*x)*c^4+6*b*(-d*(c^2*x^2-1))^(1
/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2-1)/x*arccosh(c*x)*c^2-
2*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(12*c^8*x^8-36*c^6*x^6+35*c^4*x^4-10*c^2*x^2
-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3
```

maxima [A] time = 0.71, size = 276, normalized size = 0.82

$$\frac{1}{6}\,bc\left(\frac{8\,c^2\sqrt{-d}\,\log\left(cx+1\right)}{d^3}+\frac{8\,c^2\sqrt{-d}\,\log\left(cx-1\right)}{d^3}+\frac{16\,c^2\sqrt{-d}\,\log(x)}{d^3}+\frac{\sqrt{-d}}{c^2d^3x^4-d^3x^2}\right)+\frac{1}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/6*b*c*(8*c^2*sqrt(-d)*log(c*x + 1)/d^3 + 8*c^2*sqrt(-d)*log(c*x - 1)/d^3
+ 16*c^2*sqrt(-d)*log(x)/d^3 + sqrt(-d)/(c^2*d^3*x^4 - d^3*x^2)) + 1/3*(16*
c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6*c
^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*b*arcco
sh(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 +
 d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/
2)*d*x^3))*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.134 $\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=246

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}-\frac{4\sqrt{ax-1}\sqrt{ax+1}\log\left(1-a^2x^2\right)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}$$

```
[Out] 1/5*x*arccosh(a*x)/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)/c^2/(-a^2*c*x
^2+c)^(3/2)+8/15*x*arccosh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2)+1/20*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^(1/2)+2/15*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)-4/15*ln(-a^2*x^2+1)*
(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 276, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5713, 5691, 5688, 260, 261}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}-\frac{4\sqrt{ax-1}\sqrt{ax+1}\log\left(1-a^2x^2\right)}{15ac^3\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2
]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*
c*x^2]) + (8*x*ArcCosh[a*x])/(15*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]
)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x])/
(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) - (4*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Log[1 - a^2*x^2])/(15*a*c^3*Sqrt[c - a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
```

```
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(a\sqrt{\cdot\right.}}{}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}}{15c^3(1-ax)(1+a}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}} + \frac{}{5c^3(}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{8x\cosh^{-1}(ax)}{15c^3\sqrt{c-a^2cx^2}} + \frac{}{5c^3(}$$

Mathematica [A] time = 0.09, size = 116, normalized size = 0.47

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\left(-8a^2x^2-16\left(a^2x^2-1\right)^2\log\left(1-a^2x^2\right)+11\right)+4ax\left(8a^4x^4-20a^2x^2+15\right)\cosh^{-1}(ax)}{60ac^3\left(a^2x^2-1\right)^2\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] (4*a*x*(15 - 20*a^2*x^2 + 8*a^4*x^4)*ArcCosh[a*x] + Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*(11 - 8*a^2*x^2 - 16*(-1 + a^2*x^2)^2*Log[1 - a^2*x^2]))/(60*a*c^3*(-
1 + a^2*x^2)^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 + 6
*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

giac [A] time = 4.41, size = 142, normalized size = 0.58

$$\frac{1}{60}\sqrt{-c}\left(\frac{16\log\left(\left|a^2x^2-1\right|\right)}{ac^4}-\frac{24\,a^4x^4-56\,a^2x^2+35}{\left(a^2x^2-1\right)^2ac^4}\right)-\frac{\sqrt{-a^2cx^2+c}\left(4\left(\frac{2\,a^4x^2}{c}-\frac{5\,a^2}{c}\right)x^2+\frac{15}{c}\right)x\log\left(ax+\sqrt{a^2x^2\right.}}{15\left(a^2cx^2-c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] 1/60*sqrt(-c)*(16*log(abs(a^2*x^2 - 1))/(a*c^4) - (24*a^4*x^4 - 56*a^2*x^2
+ 35)/((a^2*x^2 - 1)^2*a*c^4)) - 1/15*sqrt(-a^2*c*x^2 + c)*(4*(2*a^4*x^2/c
- 5*a^2/c)*x^2 + 15/c)*x*log(a*x + sqrt(a^2*x^2 - 1))/(a^2*c*x^2 - c)^3
```

maple [A] time = 0.46, size = 419, normalized size = 1.70

$$-\frac{16\sqrt{-c\left(a^2x^2-1\right)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)}{15c^4a\left(a^2x^2-1\right)}-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(8x^5a^5-20x^3a^3-8\sqrt{ax+1}\,\sqrt{ax-1}\,x^4a^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -16/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)
*arccosh(a*x)-1/60*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(
1/2)*(a*x-1)^(1/2)*x^4*a^4+15*a*x+16*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-8*
(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-64*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-64*x^
8*a^8+248*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5+280*x^6*a^6+160*a^4*x^4*arcco
sh(a*x)-340*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-456*x^4*a^4-380*a^2*x^2*arc
cosh(a*x)+165*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+328*a^2*x^2+256*arccosh(a*x)-
88)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4
+8/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*
ln((a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2-1)
```

maxima [A] time = 0.44, size = 191, normalized size = 0.78

$$-\frac{1}{60}a\left(\frac{16\sqrt{-\frac{1}{a^4c}}\log\left(x^2-\frac{1}{a^2}\right)}{c^3}+\frac{3}{\left(a^6c^3x^4\sqrt{-\frac{1}{c}}-2\,a^4c^3x^2\sqrt{-\frac{1}{c}}+a^2c^3\sqrt{-\frac{1}{c}}\right)c}-\frac{8}{\left(a^4c^2x^2\sqrt{-\frac{1}{c}}-a^2c^2\sqrt{-\frac{1}{c}}\right)c^2}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -1/60*a*(16*sqrt(-1/(a^4*c))*log(x^2 - 1/a^2)/c^3 + 3/((a^6*c^3*x^4*sqrt(-1
/c) - 2*a^4*c^3*x^2*sqrt(-1/c) + a^2*c^3*sqrt(-1/c))*c) - 8/((a^4*c^2*x^2*s
qrt(-1/c) - a^2*c^2*sqrt(-1/c))*c^2)) + 1/15*(8*x/(sqrt(-a^2*c*x^2 + c)*c^3
) + 4*x/((-a^2*c*x^2 + c)^(3/2)*c^2) + 3*x/((-a^2*c*x^2 + c)^(5/2)*c))*arcc
osh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)}{\left(c-a^2\,c\,x^2\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{\left(-c(ax-1)(ax+1)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.135 $\int \frac{x^4 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=145

$$\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^2}{16a^5\sqrt{1-ax}}-\frac{3x^2\sqrt{ax-1}}{16a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{4a^2}-\frac{3x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{8a^4}-\frac{x^4\sqrt{ax-1}}{16a\sqrt{1-ax}}$$

```
[Out] -3/16*x^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/16*x^4*(a*x-1)^(1/2)/a/(-a*x+1
)^(1/2)+3/16*arccosh(a*x)^2*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-3/8*x*arccosh(
a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.50, antiderivative size = 206, normalized size of antiderivative = 1.42, number of steps used = 6, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}}{16a\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}}{16a^3\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}}-\frac{3x(1-ax)(ax+1)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}}+3\surd$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(16*a^3*Sqrt[1 - a^2*x^2]) - (x^4*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])/(16*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x)*(1 + a
*x)*ArcCosh[a*x])/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 + a*x)*ArcC
osh[a*x])/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x]^2)/(16*a^5*Sqrt[1 - a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+\right.}{4}$$

$$= -\frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)\cosh^{-1}(a}{4a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\sqrt{1+ax}}{16a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\sqrt{1+ax}}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3}{}$$

Mathematica [A] time = 0.26, size = 93, normalized size = 0.64

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-16\cosh\left(2\cosh^{-1}(ax)\right)-\cosh\left(4\cosh^{-1}(ax)\right)+4\cosh^{-1}(ax)\left(6\cosh^{-1}(ax)+8\sinh\left(2\cos\right.\right.\right.}{128a^5\sqrt{-((ax-1)(ax+1))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-16*Cosh[2*ArcCosh[a*x]] - Cosh[4*Ar
cCosh[a*x]] + 4*ArcCosh[a*x]*(6*ArcCosh[a*x] + 8*Sinh[2*ArcCosh[a*x]] + Sin
h[4*ArcCosh[a*x]])))/(128*a^5*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^4\,\text{arcosh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\text{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [B] time = 0.80, size = 456, normalized size = 3.14

$$-\frac{3\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,\text{arccosh}(ax)^2}{16a^5\left(a^2x^2-1\right)} - \frac{\sqrt{-a^2x^2+1}\left(8x^5a^5-12x^3a^3+8\sqrt{ax+1}\sqrt{ax-1}\,x^4a^4+4a\right.}{256}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^2-1/256*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*
x-1)^(1/2)*x^4*a^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2
)*(a*x+1)^(1/2))*(-1+4*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2
)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x
+1)^(1/2))*(-1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x
^3*a^3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1
/2))*(1+2*arccosh(a*x))/a^5/(a^2*x^2-1)-1/256*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5
-12*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*a^2*x^2*(a*x-1)^(
1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+4*arccosh(a*x))/a^5/(a^2
*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\mathrm{acosh}\left(a\,x\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \operatorname{acosh}{\left(ax \right)}}{\sqrt{-\left(ax - 1\right)\left(ax + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.136 $\int \frac{x^3 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=110

$$-\frac{2x\sqrt{ax-1}}{3a^3\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{3a^2}-\frac{2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{3a^4}-\frac{x^3\sqrt{ax-1}}{9a\sqrt{1-ax}}$$

```
[Out] -2/3*x*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/9*x^3*(a*x-1)^(1/2)/a/(-a*x+1)^(1
/2)-2/3*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arccosh(a*x)*(-a^2*x^2+
1)^(1/2)/a^2
```

Rubi [A] time = 0.39, antiderivative size = 158, normalized size of antiderivative = 1.44, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5798, 5759, 5718, 8, 30}

$$-\frac{x^3\sqrt{ax-1}\sqrt{ax+1}}{9a\sqrt{1-a^2x^2}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}}{3a^3\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}}-\frac{2(1-ax)(ax+1)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a^3*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[-
1 + a*x]*Sqrt[1 + a*x])/(9*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*Ar
cCosh[a*x])/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*
x])/(3*a^2*Sqrt[1 - a^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\right.}{3a\sqrt{}}$$
$$= -\frac{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^2\sqrt{1-a^2x^2}} -$$
$$= -\frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}}{3a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-a}{}$$

Mathematica [A] time = 0.13, size = 74, normalized size = 0.67

$$-\frac{ax\sqrt{ax-1}\,\sqrt{ax+1}\left(a^2x^2+6\right)-3\left(a^4x^4+a^2x^2-2\right)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/9*(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(6 + a^2*x^2) - 3*(-2 + a^2*x^2 + a^
4*x^4)*ArcCosh[a*x])/(a^4*Sqrt[1 - a^2*x^2])
```

fricas [A] time = 1.18, size = 101, normalized size = 0.92

$$-\frac{3\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)-\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}}{9\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 -
 1)) - (a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1))/(a^6*x^2 - a
^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.53, size = 311, normalized size = 2.83

$$-\frac{\sqrt{-a^2x^2+1}\,\left(4x^4a^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\,\sqrt{ax+1}\,-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+1\right)(-1+3\,\mathrm{arccosh}\,(ax))}{72a^4\left(a^2x^2-1\right)}-\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/72*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x+
1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(-1+3*arccosh(a*x))/a^4/(a^2*
x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(
-1+arccosh(a*x))/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^4/(a^2*x^2-1)-1/72*(-a^2*x^2+1)
^(1/2)*(4*x^4*a^4-5*a^2*x^2-4*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x+1)
^(1/2)*(a*x-1)^(1/2)*a*x+1)*(1+3*arccosh(a*x))/a^4/(a^2*x^2-1)
```

maxima [C] time = 0.90, size = 62, normalized size = 0.56

$$\frac{1}{9}\,a\left(\frac{i\,x^3}{a^2}+\frac{6i\,x}{a^4}\right)-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)\mathrm{arcosh}\,(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/9*a*(I*x^3/a^2 + 6*I*x/a^4) - 1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a
^2*x^2 + 1)/a^4)*arccosh(a*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{acosh}\,(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,\mathrm{acosh}\,(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.137 $\int \frac{x^2 \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{2a^2}-\frac{x^2\sqrt{ax-1}}{4a\sqrt{1-ax}}$$

```
[Out] -1/4*x^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+1/4*arccosh(a*x)^2*(a*x-1)^(1/2)/a^
3/(-a*x+1)^(1/2)-1/2*x*arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.32, antiderivative size = 125, normalized size of antiderivative = 1.42, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5759, 5676, 30}

$$-\frac{x^2\sqrt{ax-1}\,\sqrt{ax+1}}{4a\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{4a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(4*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*
(1 + a*x)*ArcCosh[a*x])/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^2)/(4*a^3*Sqrt[1 - a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\right.}{2a}$$

$$= -\frac{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(a}{4a^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.16, size = 75, normalized size = 0.85

$$-\frac{\sqrt{-((ax-1)(ax+1))}\left(2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)+\sinh\left(2\cosh^{-1}(ax)\right)\right)-\cosh\left(2\cosh^{-1}(ax)\right)\right)}{8a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/8*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-Cosh[2*ArcCosh[a*x]] + 2*ArcCosh[a*x]
*(ArcCosh[a*x] + Sinh[2*ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1
 + a*x))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\operatorname{arcosh}(ax)}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [B] time = 0.48, size = 223, normalized size = 2.53

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2}{4a^3\left(a^2x^2-1\right)} - \frac{\sqrt{-a^2x^2+1}\,\left(2x^3a^3-2ax+2a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-}\right.}{16a^3\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^2-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a
*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(-1+2*arccosh(a*x))/a^3/(a^2*x^2-1
```

```
)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^
(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(1+2*arccosh(a*x))/a^3/(a^2*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{acosh}\left(a\,x\right)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
[Out] int((x^2*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\operatorname{acosh}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(x**2*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.138 $\int \frac{x\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=49

$$-\frac{\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)}{a^2}-\frac{x\sqrt{ax-1}}{a\sqrt{1-ax}}$$

```
[Out] -x*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-arccosh(a*x)*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.49, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5798, 5718, 8}

$$-\frac{x\sqrt{ax-1}\,\sqrt{ax+1}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -((x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2])) - ((1 - a*x)*(1 +
 a*x)*ArcCosh[a*x])/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}-\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int 1\,dx}{a\sqrt{1-a^2x^2}}\\
&= -\frac{x\sqrt{-1+ax}\,\sqrt{1+ax}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 55, normalized size = 1.12

$$\frac{\left(a^2x^2-1\right)\cosh^{-1}(ax)-ax\sqrt{ax-1}\sqrt{ax+1}}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-(a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (-1 + a^2*x^2)*ArcCosh[a*x])/(a^2*Sq
rt[1 - a^2*x^2])
```

fricas [A] time = 0.61, size = 72, normalized size = 1.47

$$\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\,ax+\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x + (-a^2*x^2 + 1)^(3/2)*log(a*x +
sqrt(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

giac [C] time = 0.73, size = 40, normalized size = 0.82

$$-\frac{i\,x}{a}-\frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -I*x/a - sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))/a^2
```

maple [B] time = 0.21, size = 123, normalized size = 2.51

$$-\frac{\sqrt{-a^2x^2+1}\,\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)(-1+\mathrm{arccosh}\,(ax))}{2a^2\left(a^2x^2-1\right)}-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax\cdots\right.}{2a^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(-1+arc
cosh(a*x))/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a
*x-1)^(1/2)*a*x-1)*(1+arccosh(a*x))/a^2/(a^2*x^2-1)
```

maxima [C] time = 0.66, size = 28, normalized size = 0.57

$$\frac{i\,x}{a}-\frac{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] I*x/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\mathrm{acosh}\,(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*acosh(a*x))/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.139 $\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^2}{2a\sqrt{1-ax}}$$

```
[Out] 1/2*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*a*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.06, size = 51, normalized size = 1.59

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^2}{2a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] -1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(a\,x)}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(1 - a^2*x^2)^(1/2),x)
[Out] int(acosh(a*x)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.140 $\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=103

$$-\frac{i\sqrt{ax-1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{i\sqrt{ax-1}\,\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2\sqrt{ax-1}\,\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

```
[Out] 2*arccosh(a*x)*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+
1)^(1/2)-I*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)+I*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)
/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 142, normalized size of antiderivative = 1.38, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {5798, 5761, 4180, 2279, 2391}

$$-\frac{i\sqrt{ax-1}\,\sqrt{ax+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{i\sqrt{ax-1}\,\sqrt{ax+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\text{cos}}{\sqrt{1}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1
 - a^2*x^2] - (I*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x
]])/Sqrt[1 - a^2*x^2] + (I*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcC
osh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int x\,\text{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \log\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(i\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\log(1}{x}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{i\sqrt{-1+ax}\sqrt{1+ax}\,\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right.}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.14, size = 113, normalized size = 1.10

$$\frac{i\sqrt{-((ax-1)(ax+1))}\left(\text{Li}_2\left(-ie^{-\cosh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)\left(\log\left(1-ie^{-\cosh^{-1}(ax)}\right)-\log\left(1+\right.\right.\right.}{\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[-((-1 + a*x)*(1 + a*x))]*(ArcCosh[a*x]*(Log[1 - I/E^ArcCosh[a*x]] -
 Log[1 + I/E^ArcCosh[a*x]]) + PolyLog[2, (-I)/E^ArcCosh[a*x]] - PolyLog[2,
I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}(ax)}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}(ax)}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)

maple [B] time = 0.36, size = 270, normalized size = 2.62

$$\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)\ln\left(1+i\left(ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)\right)}{a^2x^2-1}-\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x)

[Out] I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*l
n(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)
*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*ln(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2)))+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*dilog(1
+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a
*x+1)^(1/2)/(a^2*x^2-1)*dilog(1-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(arccosh(a*x)/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)),x)

[Out] int(acosh(a*x)/(x*(1 - a^2*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(acosh(a*x)/x/(-a**2*x**2+1)**(1/2),x)

[Out] Integral(acosh(a*x)/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)

3.141 $\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=48

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{x}-\frac{a\sqrt{ax-1}\log(x)}{\sqrt{1-ax}}$$

[Out] -a*ln(x)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-arccosh(a*x)*(-a^2*x^2+1)^(1/2)/x

Rubi [A] time = 0.25, antiderivative size = 72, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 5724, 29}

$$-\frac{a\sqrt{ax-1}\sqrt{ax+1}\log(x)}{\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]

[Out] -(((1 - a*x)*(1 + a*x)*ArcCosh[a*x])/(x*Sqrt[1 - a^2*x^2])) - (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[x])/Sqrt[1 - a^2*x^2]

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}-\frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x}\,dx}{\sqrt{1-a^2x^2}}\\
&= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}}-\frac{a\sqrt{-1+ax}\sqrt{1+ax}\log(x)}{\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [**A**] time = 0.03, size = 57, normalized size = 1.19

$$\frac{\left(a^2x^2-1\right)\cosh^{-1}(ax)-ax\sqrt{ax-1}\sqrt{ax+1}\log(x)}{x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-1 + a^2*x^2)*ArcCosh[a*x] - a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[x])/(x*
Sqrt[1 - a^2*x^2])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: NotImplementedError >> the translation of the FriCAS obje
ct sage2 to sage is not yet implemented
```

giac [**C**] time = 0.52, size = 80, normalized size = 1.67

$$\frac{1}{2}\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}\,|a|+a\right)|a|}-\frac{\sqrt{-a^2x^2+1}\,|a|+a}{x|a|}\right)\log\left(ax+\sqrt{a^2x^2-1}\right)-i\,a\log\left(|x|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*a
bs(a) + a)/(x*abs(a)))*log(a*x + sqrt(a^2*x^2 - 1)) - I*a*log(abs(x))
```

maple [**B**] time = 0.36, size = 168, normalized size = 3.50

$$-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)\,a}{a^2x^2-1}-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-1\right)\operatorname{arccosh}(ax)}{x\left(a^2x^2-1\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*
a-(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*
x)/x/(a^2*x^2-1)+(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)
*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a
```

maxima [**C**] time = 0.86, size = 73, normalized size = 1.52

$$-\frac{1}{2}\left(a^2\sqrt{-\frac{1}{a^4}}\log\left(x^2-\frac{1}{a^2}\right)+i\,(-1)^{-2\,a^2x^2+2}\log\left(-2\,a^2+\frac{2}{x^2}\right)\right)a-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^2*sqrt(-1/a^4)*log(x^2 - 1/a^2) + I*(-1)^(-2*a^2*x^2 + 2)*log(-2*a^
2 + 2/x^2))*a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a\,x)}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.142 $\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=167

$$-\frac{ia^2\sqrt{ax-1}\,\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}{2x^2}+\frac{a^2\sqrt{ax-1}\cosh^{-1}(ax)\tan}{\sqrt{1-ax}}$$

```
[Out] 1/2*a*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)+a^2*arccosh(a*x)*arctan(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*I*a^2*polylog(2,-I*(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+1/2*I*a^2*polyl
og(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*
arccosh(a*x)*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.47, antiderivative size = 230, normalized size of antiderivative = 1.38, number of steps used = 9, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {5798, 5748, 5761, 4180, 2279, 2391, 30}

$$-\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{ia^2\sqrt{ax-1}\sqrt{ax+1}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}}{2x\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*x*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 +
a*x)*ArcCosh[a*x])/(2*x^2*Sqrt[1 - a^2*x^2]) + (a^2*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((I/2)*a^2*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x
^2] + ((I/2)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]])
/Sqrt[1 - a^2*x^2]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+ax}\right.}{2\sqrt{1-}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x\right.}{2\sqrt{1-a^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\,\mathrm{t}}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\,\mathrm{t}}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\,\mathrm{t}}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.29, size = 234, normalized size = 1.40

$$\frac{(ax+1)\left(-ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\,\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(ax)}\right)+ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\,\mathrm{Li}_2\left(ie^{-\cosh^{-1}(ax)}\right)-ia^2x^2\sqrt{\frac{ax-1}{ax+1}}\,\cosh^{-1}(ax)\log\left(1-\right.\right.}{2x^2\sqrt{1-}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((1 + a*x)*(a*x*Sqrt[(-1 + a*x)/(1 + a*x)] - ArcCosh[a*x] + a*x*ArcCosh[a*x
] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a
*x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCos
h[a*x]] - I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, (-I)/E^ArcCosh[a*
x]] + I*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/E^ArcCosh[a*x]]))/(
2*x^2*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 2.27, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [A] time = 0.61, size = 349, normalized size = 2.09

$$-\frac{\left(a^2x^2\operatorname{arccosh}(ax)+\sqrt{ax+1}\,\sqrt{ax-1}\,ax-\operatorname{arccosh}(ax)\right)\sqrt{-a^2x^2+1}}{2\left(a^2x^2-1\right)x^2}+\frac{i\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosl}}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(a^2*x^2*arccosh(a*x)+(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-arccosh(a*x))*(-
a^2*x^2+1)^(1/2)/(a^2*x^2-1)/x^2+I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)
^(1/2)*arccosh(a*x)*ln(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^
2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*ln(1-I*(
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)+I*(-a^2*x^2+1)^(1/2)*(a
*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1+I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/
(2*a^2*x^2-2)-I*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*dilog(1-I*(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*a^2/(2*a^2*x^2-2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(acosh(a*x)/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)/x**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.143 $\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=98

$$\frac{4bc\sqrt{cx-1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-cx}}+\frac{2(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f}$$

```
[Out] 2/5*(f*x)^(5/2)*(a+b*arccosh(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/f+4/
35*b*c*(f*x)^(7/2)*HypergeometricPFQ([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)*(c*
x-1)^(1/2)/f^2/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 111, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {5798, 5763}

$$\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{1-c^2x^2}}+\frac{2(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^
2])/(5*f) + (4*b*c*(f*x)^(7/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricP
FQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(35*f^2*Sqrt[1 - c^2*x^2])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

$$=\frac{2(fx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f}+\frac{4bc(fx)^{7/2}\sqrt{-1+cx}\,\sqrt{1+cx}}{35f^2\sqrt{1-}}$$

Mathematica [A] time = 0.11, size = 100, normalized size = 1.02

$$\frac{2}{35}x(fx)^{3/2}\left(\frac{2bcx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{\sqrt{1-c^2x^2}}+7\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c
^2*x^2] + (2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4,
7/4}, {9/4, 11/4}, c^2*x^2])/Sqrt[1 - c^2*x^2]))/35
```

fricas [F] time = 1.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\left(bfx\,\text{arcosh}\,(cx)+afx\right)\sqrt{fx}}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*x^
2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(fx\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int\frac{\left(fx\right)^{\frac{3}{2}}\left(a+b\,\text{arccosh}\,(cx)\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(fx\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(f\,x\right)^{3/2}}{\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(1 - c^2*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
[Out] Timed out
```

3.144 $\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=141

$$\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}\,(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f\sqrt{d-c^2dx^2}}$$

```
[Out] 4/35*b*c*(f*x)^(7/2)*HypergeometricPFQ([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(-c^2*d*x^2+d)^(1/2)+2/5*(f*x)^(5/2)*(a+b*ar
ccosh(c*x))*hypergeom([1/2, 5/4],[9/4],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f/(-c^2*
d*x^2+d)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {5798, 5763}

$$\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{35f^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{1-c^2x^2}\,(fx)^{5/2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{5f\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*(f*x)^(5/2)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, 5/4, 9/4, c^2*x^2])/(5*f*Sqrt[d - c^2*d*x^2]) + (4*b*c*(f*x)^(7/2)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^
2])/(35*f^2*Sqrt[d - c^2*d*x^2])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$=\frac{2(fx)^{5/2}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)}{5f\sqrt{d-c^2dx^2}}+\frac{4bc(fx)^{7/2}\sqrt{-1}}{}$$

Mathematica [A] time = 0.03, size = 115, normalized size = 0.82

$$\frac{2x(fx)^{3/2}\left(2bcx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)+7\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{5}{4};\frac{9}{4};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)\right)}{35\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2*x*(f*x)^(3/2)*(7*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, 5/4, 9/4, c^2*x^2] + 2*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeome
tricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2]))/(35*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(bfx\operatorname{arcosh}(cx)+afx\right)\sqrt{fx}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*f*x*arccosh(c*x) + a*f*x)*sqrt(f*x)/(c^2*
d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}(cx)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"giac")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}(cx)\right)}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}(cx)+a\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm=
"maxima")
```

```
[Out] integrate((f*x)^(3/2)*(b*arccosh(c*x) + a)/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(f\,x\right)^{3/2}}{\sqrt{d - c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^(3/2))/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
[Out] Timed out
```

3.145 $\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=429

$$-\frac{c^6d^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7)}+\frac{3c^4d^3(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{3c^2d^3(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m-}}{}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-3*c^2*d^3*(f*x)^(3+m)*(a+b*arcco
sh(c*x))/f^3/(3+m)+3*c^4*d^3*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)-c^6*d
^3*(f*x)^(7+m)*(a+b*arccosh(c*x))/f^7/(7+m)-b*c*d^3*(m^4+27*m^3+284*m^2+132
9*m+2271)*(f*x)^(2+m)*(-c^2*x^2+1)/f^2/(7+m)^2/(m^2+8*m+15)^2/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+b*c^3*d^3*(9+m)*(13+2*m)*(f*x)^(4+m)*(-c^2*x^2+1)/f^4/(5+m)^
2/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d^3*(f*x)^(6+m)*(-c^2*x^2+1)/f^
6/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d^3*(35*m^3+455*m^2+1813*m+2161
)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2
)/f^2/(7+m)^2/(m^2+3*m+2)/(m^2+8*m+15)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 2.76, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 5731, 12, 1610, 1809, 1267, 459, 365, 364}

$$-\frac{3c^2d^3(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{3c^4d^3(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{c^6d^3(fx)^{m+7}\left(a+b\cosh^{-1}(cx)\right)}{f^7(m+7)}+\frac{d^3(fx)^{m-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^3*(2271 + 1329*m + 284*m^2 + 27*m^3 + m^4)*(f*x)^(2 + m)*(1 - c^2*
x^2))/(f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (
b*c^3*d^3*(9 + m)*(13 + 2*m)*(f*x)^(4 + m)*(1 - c^2*x^2))/(f^4*(5 + m)^2*(7
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d^3*(f*x)^(6 + m)*(1 - c^2*x
^2))/(f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*(f*x)^(1 + m)*(a +
 b*ArcCosh[c*x]))/(f*(1 + m)) - (3*c^2*d^3*(f*x)^(3 + m)*(a + b*ArcCosh[c*x
]))/(f^3*(3 + m)) + (3*c^4*d^3*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5
+ m)) - (c^6*d^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)) - (3*b*c
*d^3*(2161 + 1813*m + 455*m^2 + 35*m^3)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{3c^2d^3(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= -\frac{bc^5d^3(fx)^{6+m}\left(1-c^2x^2\right)}{f^6(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} -$$

$$= \frac{bc^3d^3(9+m)(13+2m)(fx)^{4+m}\left(1-c^2x^2\right)}{f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^5d^3(fx)^{6+m}\left(1-\right.}{f^6(7+m)^2\sqrt{-1+cx}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^3\left(2271+1329m+284m^2+27m^3+m^4\right)(fx)^{2+m}\left(1-c^2x^2\right)}{f^2(3+m)^2(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.18, size = 387, normalized size = 0.90

$$d^3x(fx)^m\left(-\frac{c^6x^6\left(a+b\cosh^{-1}(cx)\right)}{m+7} + \frac{3c^4x^4\left(a+b\cosh^{-1}(cx)\right)}{m+5} - \frac{3c^2x^2\left(a+b\cosh^{-1}(cx)\right)}{m+3} + \frac{a+b\cosh^{-1}(cx)}{m+1} -\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^3*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (3*c^2*x^2*(a + b*ArcCosh[c*x
]))/(3 + m) + (3*c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (c^6*x^6*(a + b*Ar
cCosh[c*x]))/(7 + m) + (b*c^7*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2,
4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
- (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2
*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*x^3*Sqrt[1
 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7
*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hy
pergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]))
```

fricas [F] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^6d^3x^6 - 3\,ac^4d^3x^4 + 3\,ac^2d^3x^2 - ad^3 + \left(bc^6d^3x^6 - 3\,bc^4d^3x^4 + 3\,bc^2d^3x^2 - bd^3\right)\text{arcosh}\left(cx\right)\right)\left(fx\right)^m\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(-(a*c^6*d^3*x^6 - 3*a*c^4*d^3*x^4 + 3*a*c^2*d^3*x^2 - a*d^3 + (b*c
^6*d^3*x^6 - 3*b*c^4*d^3*x^4 + 3*b*c^2*d^3*x^2 - b*d^3)*arccosh(c*x))*(f*x)
^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(-c^{2} d \,x^{2}+d\right)^{3} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a c^{6} d^{3} f^{m} x^{7} x^{m}}{m+7}+\frac{3 \, a c^{4} d^{3} f^{m} x^{5} x^{m}}{m+5}-\frac{3 \, a c^{2} d^{3} f^{m} x^{3} x^{m}}{m+3}+\frac{\left(f x\right)^{m+1} a d^{3}}{f(m+1)}-\frac{\left(\left(m^{3}+9 \, m^{2}+23 \, m+15\right) b c^{6} d^{3} f^{m} x^{7}-3 \, \left(m\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] -a*c^6*d^3*f^m*x^7*x^m/(m + 7) + 3*a*c^4*d^3*f^m*x^5*x^m/(m + 5) - 3*a*c^2*
d^3*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) - ((m^3 + 9*m^2 +
 23*m + 15)*b*c^6*d^3*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^4*d^3*f^m*
x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^3*f^m*x^3 - (m^3 + 15*m^2 + 71*m
 + 105)*b*d^3*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^4 + 16*m
^3 + 86*m^2 + 176*m + 105) - integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c^7*d^3
*f^m*x^7 - 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^5*d^3*f^m*x^5 + 3*(m^3 + 13*m^2
 + 47*m + 35)*b*c^3*d^3*f^m*x^3 - (m^3 + 15*m^2 + 71*m + 105)*b*c*d^3*f^m*x
)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*m^3 + 86*m
^2 + 176*m + 105)*c*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^
4 - 16*m^3 - 86*m^2 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + integ
rate(((m^3 + 9*m^2 + 23*m + 15)*b*c^8*d^3*f^m*x^8 - 3*(m^3 + 11*m^2 + 31*m
+ 21)*b*c^6*d^3*f^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^4*d^3*f^m*x^4 -
(m^3 + 15*m^2 + 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2
+ 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^3\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^3*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d^3\left(\int\left(-a\left(fx\right)^m\right)dx+\int\left(-b\left(fx\right)^m\operatorname{acosh}\left(cx\right)\right)dx+\int 3ac^2x^2\left(fx\right)^m\,dx+\int\left(-3ac^4x^4\left(fx\right)^m\right)dx+\int ac^6x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] -d**3*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Int
egral(3*a*c**2*x**2*(f*x)**m, x) + Integral(-3*a*c**4*x**4*(f*x)**m, x) + I
ntegral(a*c**6*x**6*(f*x)**m, x) + Integral(3*b*c**2*x**2*(f*x)**m*acosh(c*
x), x) + Integral(-3*b*c**4*x**4*(f*x)**m*acosh(c*x), x) + Integral(b*c**6*
x**6*(f*x)**m*acosh(c*x), x))
```

3.146 $\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=307

$$\frac{c^4d^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{2c^2d^2(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd^2\left(15m^2\right.}{f^2}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-2*c^2*d^2*(f*x)^(3+m)*(a+b*arcco
sh(c*x))/f^3/(3+m)+c^4*d^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)-b*c*d^2
*(m^2+13*m+38)*(f*x)^(2+m)*(-c^2*x^2+1)/f^2/(3+m)^2/(5+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)+b*c^3*d^2*(f*x)^(4+m)*(-c^2*x^2+1)/f^4/(5+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-b*c*d^2*(15*m^2+100*m+149)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m
],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f^2/(m^2+3*m+2)/(m^2+8*m+15)^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {270, 5731, 12, 520, 1267, 459, 365, 364}

$$-\frac{2c^2d^2(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{c^4d^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd^2\left(15m\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^2*(38 + 13*m + m^2)*(f*x)^(2 + m)*(1 - c^2*x^2))/(f^2*(3 + m)^2*(5
 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (b*c^3*d^2*(f*x)^(4 + m)*(1 - c^2*
x^2))/(f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*(f*x)^(1 + m)*(a
+ b*ArcCosh[c*x]))/(f*(1 + m)) - (2*c^2*d^2*(f*x)^(3 + m)*(a + b*ArcCosh[c*
x]))/(f^3*(3 + m)) + (c^4*d^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))/(f^5*(5 +
 m)) - (b*c*d^2*(149 + 100*m + 15*m^2)*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2 + m)*(3
+ m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] && !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d - c^2dx^2\right)^2 \left(a + b\cosh^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a + b\cosh^{-1}(cx\right.}{f^3(3+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a + b\cosh^{-1}(cx\right.}{f^3(3+m)}$$

$$= \frac{d^2(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{2c^2d^2(fx)^{3+m}\left(a + b\cosh^{-1}(cx\right.}{f^3(3+m)}$$

$$= \frac{bc^3d^2(fx)^{4+m}\left(1 - c^2x^2\right)}{f^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^2(fx)^{1+m}\left(a + b\cosh^{-1}(cx)\right)}{f(1+m)}$$

$$= -\frac{bcd^2\left(38 + 13m + m^2\right)(fx)^{2+m}\left(1 - c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(\right.}{f^4(5+m)^2\sqrt{-1+}}$$

$$= -\frac{bcd^2\left(38 + 13m + m^2\right)(fx)^{2+m}\left(1 - c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(\right.}{f^4(5+m)^2\sqrt{-1+}}$$

$$= -\frac{bcd^2\left(38 + 13m + m^2\right)(fx)^{2+m}\left(1 - c^2x^2\right)}{f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2(fx)^{4+m}\left(\right.}{f^4(5+m)^2\sqrt{-1+}}$$

Mathematica [A] time = 0.45, size = 290, normalized size = 0.94

$$d^2x(fx)^m\left(\frac{c^4x^4\left(a + b\cosh^{-1}(cx)\right)}{m+5} - \frac{2c^2x^2\left(a + b\cosh^{-1}(cx)\right)}{m+3} + \frac{a + b\cosh^{-1}(cx)}{m+1} - \frac{bcx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}\right.}{\left(m^2+3m+2\right)\sqrt{cx-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d^2*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (2*c^2*x^2*(a + b*ArcCosh[c*x
]))/(3 + m) + (c^4*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*x*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + 3*m + m^2
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*x^3*Sqrt[1 - c^2*x^2]*Hypergeome
tric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (b*c^5*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (6
+ m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
)
```

fricas [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\left(cx\right)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2} d\, x^{2}+d\right)^{2} \left(a +b \,\mathrm{arccosh}\left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ac^{4}d^{2}f^{m}x^{5}x^{m}}{m+5}-\frac{2\,ac^{2}d^{2}f^{m}x^{3}x^{m}}{m+3}+\frac{\left(fx\right)^{m+1}ad^{2}}{f(m+1)}+\frac{\left(\left(m^{2}+4\,m+3\right)bc^{4}d^{2}f^{m}x^{5}-2\left(m^{2}+6\,m+5\right)bc^{2}d^{2}f^{m}x^{3}+\left(m^{2}+\right.}{m^{3}+9\,m^{2}+23\,m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] a*c^4*d^2*f^m*x^5*x^m/(m + 5) - 2*a*c^2*d^2*f^m*x^3*x^m/(m + 3) + (f*x)^(m
+ 1)*a*d^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*c^4*d^2*f^m*x^5 - 2*(m^2 + 6*m
+ 5)*b*c^2*d^2*f^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*m +
3)*b*c^5*d^2*f^m*x^5 - 2*(m^2 + 6*m + 5)*b*c^3*d^2*f^m*x^3 + (m^2 + 8*m + 1
5)*b*c*d^2*f^m*x)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m^2 + 2
3*m + 15)*c*x + ((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 1
5)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^6*d^2*
f^m*x^6 - 2*(m^2 + 6*m + 5)*b*c^4*d^2*f^m*x^4 + (m^2 + 8*m + 15)*b*c^2*d^2*
f^m*x^2)*x^m/((m^3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^2\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int a \left(fx\right)^{m}\, dx + \int b \left(fx\right)^{m} \operatorname{acosh}{\left(cx \right)}\, dx + \int \left(- 2 a c^{2} x^{2} \left(fx\right)^{m}\right)\, dx + \int a c^{4} x^{4} \left(fx\right)^{m}\, dx + \int \left(- 2 b c^{2} x^{2} \left(fx\right)^{m}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] d**2*(Integral(a*(f*x)**m, x) + Integral(b*(f*x)**m*acosh(c*x), x) + Integr
al(-2*a*c**2*x**2*(f*x)**m, x) + Integral(a*c**4*x**4*(f*x)**m, x) + Integr
al(-2*b*c**2*x**2*(f*x)**m*acosh(c*x), x) + Integral(b*c**4*x**4*(f*x)**m*a
cosh(c*x), x))
```

3.147 $\int (fx)^m \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=184

$$-\frac{c^2d(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd(3m+7)\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\right.}{f^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)-c^2*d*(f*x)^(3+m)*(a+b*arccosh(c*x
))/f^3/(3+m)+b*c*d*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(3+m)^2-b*c*
d*(7+3*m)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2
+1)^(1/2)/f^2/(3+m)^2/(m^2+3*m+2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {14, 5731, 12, 460, 126, 365, 364}

$$-\frac{c^2d(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}-\frac{bcd(3m+7)\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};\right.}{f^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*c*d*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(f^2*(3 + m)^2) + (d*(f*
x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) - (c^2*d*(f*x)^(3 + m)*(a + b*
ArcCosh[c*x]))/(f^3*(3 + m)) - (b*c*d*(7 + 3*m)*(f*x)^(2 + m)*Sqrt[1 - c^2*
x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*(1 + m)*(2
 + m)*(3 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
```

```
&& !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} -$$

$$= \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} - \frac{c^2d(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} -$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} -$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} -$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} -$$

$$= \frac{bcd(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}}{f^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} -$$

Mathematica [A] time = 0.24, size = 191, normalized size = 1.04

$$dx(fx)^m\left(-\frac{c^2x^2\left(a+b\cosh^{-1}(cx)\right)}{m+3} + \frac{a+b\cosh^{-1}(cx)}{m+1} - \frac{bcx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{\left(m^2+3m+2\right)\sqrt{cx-1}\,\sqrt{cx+1}} + \frac{bc^3x^3\sqrt{1-c}}{\left(m^2+7m\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] d*x*(f*x)^m*((a + b*ArcCosh[c*x])/(1 + m) - (c^2*x^2*(a + b*ArcCosh[c*x]))/
(3 + m) - (b*c*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m
)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*x^3*
Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((
12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

fricas [F] time = 1.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dx^2 - ad + \left(bc^2dx^2 - bd\right)\operatorname{arcosh}\left(cx\right)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*(f*x)^m, x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2} d\, x^{2}+d\right) \left(a+b \,\operatorname{arccosh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ac^2df^mx^3x^m}{m+3} - \frac{\left(bc^2df^m(m+1)x^3 - bdf^m(m+3)x\right)x^m \log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{m^2+4m+3} + \frac{\left(fx\right)^{m+1} ad}{f(m+1)} - \int \frac{}{\left(m^2+4m\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] -a*c^2*d*f^m*x^3*x^m/(m + 3) - (b*c^2*d*f^m*(m + 1)*x^3 - b*d*f^m*(m + 3)*x
)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m + 1
)*a*d/(f*(m + 1)) - integrate((b*c^3*d*f^m*(m + 1)*x^3 - b*c*d*f^m*(m + 3)*
x)*x^m/((m^2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^
2*x^2 - m^2 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) + integrate((b*c^4*
d*f^m*(m + 1)*x^4 - b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 -
 m^2 - 4*m - 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(d-c^2\,d\,x^2\right)\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-a\left(fx\right)^{m}\right)dx+\int\left(-b\left(fx\right)^{m}\operatorname{acosh}\left(cx\right)\right)dx+\int ac^{2}x^{2}\left(fx\right)^{m}dx+\int bc^{2}x^{2}\left(fx\right)^{m}\operatorname{acosh}\left(cx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] -d*(Integral(-a*(f*x)**m, x) + Integral(-b*(f*x)**m*acosh(c*x), x) + Integr
al(a*c**2*x**2*(f*x)**m, x) + Integral(b*c**2*x**2*(f*x)**m*acosh(c*x), x))
```

3.148 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Mathematica [A] time = 4.10, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x]
```

fricas [A] time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(b\operatorname{arcosh}(cx)+a)\left(fx\right)^m}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="fricas")
[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\operatorname{arcosh}(cx)+a)\left(fx\right)^m}{c^2dx^2-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="giac")
[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

maple [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}{-c^{2}d\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)\left(fx\right)^{m}}{c^{2}dx^{2}-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x, algorithm="maxima")
[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\left(f\,x\right)^{m}}{d-c^{2}\,d\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{a\left(fx\right)^{m}}{c^{2}x^{2}-1}\,dx+\int \frac{b\left(fx\right)^{m}\,\mathrm{acosh}\left(cx\right)}{c^{2}x^{2}-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d),x)
[Out] -(Integral(a*(f*x)**m/(c**2*x**2 - 1), x) + Integral(b*(f*x)**m*acosh(c*x)/
(c**2*x**2 - 1), x))/d
```

3.149 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{(1-m)\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)}{2d}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}-\frac{bc\sqrt{1-c^2x^2}\,(fx)^{m+2}\,{}_2F_1\left(\frac{3}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{2d^2f^2(m+2)\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] 1/2*(f*x)^(1+m)*(a+b*arccosh(c*x))/d^2/f/(-c^2*x^2+1)-1/2*b*c*(f*x)^(2+m)*h
ypergeom([3/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^2/f^2/(2+m)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*(1-m)*Unintegrable((f*x)^m*(a+b*arccosh(c*
x))/(-c^2*d*x^2+d),x)/d
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(2*d^2*f*(1 - c^2*x^2)) - (b*c*(f*x)^(
2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2, c^2*x
^2])/(2*d^2*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*Defer[Int]
[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(2*d)
```

Rubi steps

$$\begin{aligned}
\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx &= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{2d^2f}+\frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2d\ldots}}{2d}\\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2d}+\frac{\left(bc\sqrt{-1+c^2x^2}\ldots\right.}{2d^2f\sqrt{-1\ldots}}\\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}+\frac{(1-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2}\,dx}{2d}-\frac{\left(bc\sqrt{1-c^2x^2}\right)\ldots}{2d^2f\sqrt{-1+\ldots}}\\
&= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{2d^2f\left(1-c^2x^2\right)}-\frac{bc(fx)^{2+m}\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{3}{2},\frac{2+m}{2};\frac{4+m}{2};c^2x^2\right)}{2d^2f^2(2+m)\sqrt{-1+cx}\,\sqrt{1+cx}}+\ldots
\end{aligned}$$

Mathematica [A] time = 11.21, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^2, x]
```

fricas [A] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^{m}}{c^{4}d^{2}x^{4}-2\,c^{2}d^{2}x^{2}+d^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="fricas"
)
```

```
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2),
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^{m}}{\left(c^{2}dx^{2}-d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

maple [A] time = 1.25, size = 0, normalized size = 0.00

$$\int\frac{\left(fx\right)^{m}\left(a+b\,\text{arccosh}\,(cx)\right)}{\left(-c^{2}d\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^{m}}{\left(c^{2}dx^{2}-d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^2,x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(a+b\,\text{acosh}\,(c\,x)\right)\left(f\,x\right)^{m}}{\left(d-c^{2}\,d\,x^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{a(fx)^m}{c^4x^4-2c^2x^2+1}\,dx + \int \frac{b(fx)^m \operatorname{acosh}(cx)}{c^4x^4-2c^2x^2+1}\,dx}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**2,x)
```

```
[Out] (Integral(a*(f*x)**m/(c**4*x**4 - 2*c**2*x**2 + 1), x) + Integral(b*(f*x)**
m*acosh(c*x)/(c**4*x**4 - 2*c**2*x**2 + 1), x))/d**2
```

3.150 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$

Optimal. Leaf size=294

$$\frac{(1-m)(3-m)\mathrm{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d-c^2dx^2},x\right)}{8d^2}+\frac{(3-m)(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}+\frac{(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}-\frac{b}{}$$

```
[Out] 1/4*(f*x)^(1+m)*(a+b*arccosh(c*x))/d^3/f/(-c^2*x^2+1)^2+1/8*(3-m)*(f*x)^(1+
m)*(a+b*arccosh(c*x))/d^3/f/(-c^2*x^2+1)-1/8*b*c*(3-m)*(f*x)^(2+m)*hypergeo
m([3/2, 1+1/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^3/f^2/(2+m)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-1/4*b*c*(f*x)^(2+m)*hypergeom([5/2, 1+1/2*m],[2+1/2*m
],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^3/f^2/(2+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8
*(1-m)*(3-m)*Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d),x)/d^2
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(4*d^3*f*(1 - c^2*x^2)^2) + ((3 - m)*(
f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(8*d^3*f*(1 - c^2*x^2)) - (b*c*(3 - m)*(
f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[3/2, (2 + m)/2, (4 + m)/2,
 c^2*x^2])/(8*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(f*x)^(2
 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[5/2, (2 + m)/2, (4 + m)/2, c^2*x^
2])/(4*d^3*f^2*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((1 - m)*(3 - m)*Def
er[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2), x])/(8*d^2)
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx=\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}-\frac{(bc)\int \frac{(fx)^{1+m}}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{4d^3f}+\frac{(3-m)\int \frac{(fx)^m(a+}{(d-}}{4d}$$

$$=\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}+\frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}+\frac{(bc(3-}{}$$

$$=\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}+\frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}+\frac{((1-m}{}$$

$$=\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}+\frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}-\frac{bc(fx)^{2}}{4}$$

$$=\frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{4d^3f\left(1-c^2x^2\right)^2}+\frac{(3-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{8d^3f\left(1-c^2x^2\right)}-\frac{bc(3-}{}$$

Mathematica [A] time = 15.02, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^3}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3,x]

[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^3, x]

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="fricas")

[Out] integral(-(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="giac")

[Out] integrate(-(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)

maple [A] time = 1.37, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m(a+b\,\text{arccosh}\,(cx))}{\left(-c^2d\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)

[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\left(c^2dx^2-d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^3,x, algorithm="maxima")

[Out] -integrate((b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)^3, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,\left(f\,x\right)^m}{\left(d-c^2\,d\,x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3,x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**3,x)
[Out] Timed out
```

3.151 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=723

$$-\frac{15bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{15d^2\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\right.}{f(m+4)(m+6)\left(m^2+3m+2\right)}$$

```
[Out] 5*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)/(6+m)+(f*x)
^(1+m)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))/f/(6+m)+15*d^2*(f*x)^(1+m)*(
a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(6+m)/(m^2+6*m+8)+15*d^2*(f*x)^(1+
m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2
*d*x^2+d)^(1/2)/f/(6+m)/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-b*c
*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-15*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*b*c*d^2*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/
(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b*c^3*d^2*(f*x)^(4+m)*(-c^2
*d*x^2+d)^(1/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c^3*d^2*(
f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-b*c^5*d^2*(f*x)^(6+m)*(-c^2*d*x^2+d)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-15*b*c*d^2*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/
2+1/2*m, 2+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(6+m)/(m^2+5*m+
4)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.39, antiderivative size = 764, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {5798, 5745, 5743, 5763, 32, 14, 270}

$$-\frac{15bcd^2\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)(m+6)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{15d^2\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\right.}{f(m+4)(m+6)\left(m^2+3m\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(6 + m)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])) - (15*b*c*d^2*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^
2*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*(f*x
)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (5*b*c^3*d^2*(f*x)^(4 + m)*Sqrt[d - c^2*d*x^2])/(f^4*(4 +
m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d^2*(f*x)^(4 + m)*Sqr
t[d - c^2*d*x^2])/(f^4*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c
^5*d^2*(f*x)^(6 + m)*Sqrt[d - c^2*d*x^2])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (15*d^2*(f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d^2*(f*x)^(1 + m)*(1 - c*x)*(1 + c*x)*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + (d^2*(f*x)^(
1 + m)*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f
*(6 + m)) + (15*d^2*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f
*(4 + m)*(6 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (15*b*c*d^2*(f*x)^(
2 + m)*Sqrt[d - c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 +
m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right) dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{d^2(fx)^{1+m}(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(6+m)} - \frac{(5a}{}$$

$$= \frac{5d^2(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)(6+m)} + \frac{d^2(}{}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} + \frac{2bc^3d^2(fx)^{4+m}\sqrt{d-}}{f^4(4+m)(6+m)\sqrt{-1+}}$$

$$= -\frac{bcd^2(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{15bcd^2(fx)^{2+m}}{f^2(2+m)^2(4+m)(6+}$$

Mathematica [A] time = 1.31, size = 350, normalized size = 0.48

$$\frac{d^2x\sqrt{d-c^2dx^2}\,(fx)^m\left(\frac{15\left(-\frac{(m+2)\left(\frac{bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{m+2}+\frac{\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}\right)}{m+1}+(m+2)\sqrt{cx-1}\sqrt{cx+1}\,(a+}{(m+2)^2(m+4)}\right.}{(m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (d^2*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((5*b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4
+ m)))/(4 + m) - b*c*x*((2 + m)^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 +
 m)) - (5*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]))/(4 + m) +
(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x]) + (15*(-(b*c*x) + (2
+ m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]) - ((2 + m)*((Sqrt[1
- c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2
, c^2*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*HypergeometricPFQ[{1, 1
 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(2 + m)))/(1 + m)))/((2 +
 m)^2*(4 + m))))/((6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d^2x^4 - 2\,ac^2d^2x^2 + ad^2 + \left(bc^4d^2x^4 - 2\,bc^2d^2x^2 + bd^2\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2+d}\,\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral((a*c^4*d^2*x^4 - 2*a*c^2*d^2*x^2 + a*d^2 + (b*c^4*d^2*x^4 - 2*b*c^
2*d^2*x^2 + b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 2.22, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2} d\, x^{2}+d\right)^{\frac{5}{2}} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^{2} d x^{2} + d\right)^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)} \left(f x\right)^{m}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.152 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=455

$$-\frac{3bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3d\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right.}{f(m+4)\left(m^2+3m+2\right)\sqrt{1-cx}$$

```
[Out] (f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)+3*d*(f*x)^(1+m)
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(m^2+6*m+8)+3*d*(f*x)^(1+m)*(a+b
*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2+
d)^(1/2)/f/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d*(f*x)^(2
+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*
d*(f*x)^(2+m)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+b*c^3*d*(f*x)^(4+m)*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-3*b*c*d*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+
1/2*m, 2+1/2*m],c^2*x^2)*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(m^2+5*m+4)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.90, antiderivative size = 477, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5798, 5745, 5743, 5763, 32, 14}

$$-\frac{3bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3d\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2}\right.}{f(m+4)\left(m^2+3m+2\right)(1}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*(4 + m)*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2
+ m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c^3*d*(f*x)^(4 + m)*Sqrt[d
- c^2*d*x^2])/(f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*(f*x)^(1
+ m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (d*(f*
x)^(1 + m)*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f
*(4 + m)) + (3*d*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(4
 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (3*b*c*d*(f*x)^(2 + m)*Sqrt[d
- c^2*d*x^2]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2},
 c^2*x^2])/(f^2*(1 + m)*(2 + m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
```

```
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right) dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{d(fx)^{1+m}(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(4+m)} + \frac{(3c}{}$$

$$= \frac{3d(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f\left(8+6m+m^2\right)} + \frac{d(fx)^{1+m}(1-cx)(}{}$$

$$= -\frac{3bcd(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd(fx)^{2+m}\sqrt{}}{f^2(2+m)(4+m)\sqrt{}}$$

Mathematica [A] time = 0.79, size = 274, normalized size = 0.60

$$-\frac{dx\sqrt{d-c^2dx^2}\,(fx)^m\left(\frac{3bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{(m+1)(m+2)^2}+\frac{3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{(m+1)(m+2)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3\sqrt{cx-1}\,\sqrt{cx+1}\,(a+}{m+2}\right.}{(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((d*x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((3*b*c*x)/(2 + m)^2 + b*c*x*((2 + m)^(-
1) - (c^2*x^2)/(4 + m)) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*
x]))/(2 + m) + (-1 + c*x)^(3/2)*(1 + c*x)^(3/2)*(a + b*ArcCosh[c*x]) + (3*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/((1 + m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c*x
*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(
(1 + m)*(2 + m)^2)))/((4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

fricas [F] time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2dx^2-ad+\left(bc^2dx^2-bd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}\,\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(-(a*c^2*d*x^2 - a*d + (b*c^2*d*x^2 - b*d)*arccosh(c*x))*sqrt(-c^2*
d*x^2 + d)*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.95, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccosh}\left(cx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}\,(c\,x))\,\left(d - c^2\,d\,x^2\right)^{3/2}\,\left(f\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.153 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=278

$$-\frac{bc\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)(a\ldots}{f\left(m^2+3m+2\right)\sqrt{1-cx}\,\sqrt{cx+\ldots}}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f/(2+m)+(f*x)^(1+m)*(a+
b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*d*x^2
+d)^(1/2)/f/(m^2+3*m+2)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*(-c^2*
d*x^2+d)^(1/2)/f^2/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*Hype
rgeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(-c^2*d*x
^2+d)^(1/2)/f^2/(1+m)/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 288, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5743, 5763, 32}

$$-\frac{bc\sqrt{d-c^2dx^2}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\sqrt{d-c^2dx^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2}\ldots\right.}{f\left(m^2+3m+2\right)(1-cx)(c\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2])/(f^2*(2 + m)^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(f*
(2 + m)) + ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*
m + m^2)*(1 - c*x)*(1 + c*x)) - (b*c*(f*x)^(2 + m)*Sqrt[d - c^2*d*x^2]*Hype
rgeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(
1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
```

```
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\sqrt{d-c^2dx^2}\int (fx)^m\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}-\frac{\sqrt{d-c^2dx^2}\int\frac{(fx)^m(a+b}{\sqrt{-1+c}}}{(2+m)\sqrt{-1+cx}}$$

$$=-\frac{bc(fx)^{2+m}\sqrt{d-c^2dx^2}}{f^2(2+m)^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{(fx)^{1+m}\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{f(2+m)}$$

Mathematica [A] time = 0.29, size = 223, normalized size = 0.80

$$\frac{x\sqrt{d-c^2dx^2}(fx)^m\left(-bcx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)-(m+2)\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2}\right.\right.}{(m+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (x*(f*x)^m*Sqrt[d - c^2*d*x^2]*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (
2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometri
cPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 +
m)^2*(-1 + c*x)*(1 + c*x))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)\,(fx)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-c^2 d\,x^2 + d}\ (a + b \,\mathrm{arccosh}\,(cx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^2dx^2 + d}\,(b\,\mathrm{arcosh}\,(cx) + a)\,(fx)^m\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x)),x, algorithm="max
ima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}\,(c\,x))\ \sqrt{d - c^2\,d\,x^2}\ (f\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-d\,(cx - 1)\,(cx + 1)}\ (a + b\,\mathrm{acosh}\,(cx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x)), x)
```

3.154 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=176

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\dots}{f(m+1)\sqrt{d-c^2dx^2}}$$

```
[Out] b*c*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m
],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(1+m)/(2+m)/(-c^2*d*x^2+d)^(1/2)
+(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*
x^2)*(-c^2*x^2+1)^(1/2)/f/(1+m)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {5798, 5763}

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\dots}{f(m+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f
*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
 m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*
x^2])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{(fx)^{1+m}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{d-c^2dx^2}}+\frac{bc(fx)^{2+m}\dots}{\dots}\end{aligned}$$

Mathematica [A] time = 0.08, size = 147, normalized size = 0.84

$$\frac{x(fx)^m\left(bcx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)+(m+2)\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right.\right.}{(m+1)(m+2)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (x*(f*x)^m*((2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F
1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(
(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^2*d*x^2 - d)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m(a+b\,\text{arccosh}\,(cx))}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c\, x)) \left(f\, x\right)^m}{\sqrt{d - c^2\, d\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m (a + b \operatorname{acosh}(cx))}{\sqrt{-d\,(cx - 1)\,(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.155 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=300

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{df^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2 \dots\right.}{df(m+1)\sqrt{d-c^2d \dots}}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(1/2)+b*c*(f*x)^(2+m)*hyp
ergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/f^2/(2
+m)/(-c^2*d*x^2+d)^(1/2)-b*c*m*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1
+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/f^2/(1+
m)/(2+m)/(-c^2*d*x^2+d)^(1/2)-m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1
/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d/f/(1+m)/(-c^2*d*x^
2+d)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5756, 5763, 364}

$$-\frac{bcm\sqrt{cx-1}\sqrt{cx+1}(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{df^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{m\sqrt{1-c^2x^2}(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2 \dots\right.}{df(m+1)\sqrt{d-c^2d \dots}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d*f*Sqrt[d - c^2*d*x^2]) - (m*(f*x)^(
1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m
)/2, (3 + m)/2, c^2*x^2])/(d*f*(1 + m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2
 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/
2, c^2*x^2])/(d*f^2*(2 + m)*Sqrt[d - c^2*d*x^2]) - (b*c*m*(f*x)^(2 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m
/2, 2 + m/2}, c^2*x^2])/(d*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{1+m}}{-1+c^2x^2}\,dx}{df\sqrt{d-c^2dx^2}} - \frac{\left(m\sqrt{-1\right.}{}}{}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{df\sqrt{d-c^2dx^2}} - \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)\,{}_2F_1\left(\right.}{df(1+m)\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.25, size = 216, normalized size = 0.72

$$\frac{x(fx)^m\left(-bcmx\sqrt{cx-1}\,\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)-m(m+2)\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\right.\right.}{d(m+1)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometr
ic2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2]) + (1 + m)*((2 + m)*(a + b*ArcCos
h[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, 1 + m/2,
2 + m/2, c^2*x^2]) - b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/(d*(1 + m)*(2 + m)
*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^4*d^2*x^4 - 2
*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^{m}}{\left(-c^{2}dx^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} (a + b \operatorname{arccosh}(cx))}{\left(-c^{2}d\,x^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^{m}}{\left(-c^{2}dx^{2} + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(3/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,\left(f\,x\right)^{m}}{\left(d - c^{2}\,d\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} (a + b \operatorname{acosh}(cx))}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.156 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=450

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3d^2f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2}{3d^2f(}$$

```
[Out] 1/3*(f*x)^(1+m)*(a+b*arccosh(c*x))/d/f/(-c^2*d*x^2+d)^(3/2)+1/3*(2-m)*(f*x)
^(1+m)*(a+b*arccosh(c*x))/d^2/f/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(2-m)*(f*x)^(2
+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^2/f^2/(2+m)/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(f*x)^(2+m)*hypergeom([2, 1+1/2*m
],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/f^2/(2+m)/(-c^2*d*x^2+
d)^(1/2)-1/3*b*c*(2-m)*m*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m
],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/f^2/(1+m)/(
2+m)/(-c^2*d*x^2+d)^(1/2)-1/3*(2-m)*m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hyperg
eom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d^2/f/(1+m)/(-
c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {5798, 5756, 5763, 364}

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3d^2f^2(m+1)(m+2)\sqrt{d-c^2dx^2}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2}{3d^2f(}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*Sqrt[d - c^2*d*x^2])
+ ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d^2*f*(1 - c*x)*(1 + c*x)*Sqrt[d
- c^2*d*x^2]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c
*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(3*d^2*f*(1 + m
)*Sqrt[d - c^2*d*x^2]) + (b*c*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 +
m)*Sqrt[d - c^2*d*x^2]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*H
ypergeometric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d^2*f^2*(2 + m)*Sqrt
[d - c^2*d*x^2]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/
(3*d^2*f^2*(1 + m)*(2 + m)*Sqrt[d - c^2*d*x^2])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
```

```
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^{1+m}}{\left(-1+c^2x^2\right)^2}\,dx}{3d^2f\sqrt{d-c^2dx^2}} + \frac{((\cdots}{\cdots}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{bc(fx\cdots}{\cdots}$$

$$= \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f\sqrt{d-c^2dx^2}} + \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3d^2f(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{(2-\cdots}{\cdots}$$

Mathematica [A] time = 0.71, size = 319, normalized size = 0.71

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}\,(fx)^m\left(\frac{(m-2)\left(bcmx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)+m(m+2)\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}\cdots\right.\right.}{(m+1)(m+2)\sqrt{cx-1}\sqrt{cx+1}}\cdots\right.}{3d^2\sqrt{d-c^2\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/((-1 + c*x)
^(3/2)*(1 + c*x)^(3/2))) + (b*c*x*Hypergeometric2F1[2, 1 + m/2, 2 + m/2, c^
2*x^2])/(2 + m) + ((-2 + m)*(m*(2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x
])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - (1 + m)*((2 + m)
```

```
*(a + b*ArcCosh[c*x]) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F
1[1, 1 + m/2, 2 + m/2, c^2*x^2]) + b*c*m*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hyp
ergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1
+ m)*(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/(3*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m/(c^6*d^3*x^6 -
3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))}{(-c^2d\,x^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{(-c^2dx^2+d)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(-c^2*d*x^2+d)^(5/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{acosh}\,(c\,x))\,(f\,x)^m}{(d-c^2\,d\,x^2)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.157 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)$

Optimal. Leaf size=817

$$\frac{(cx\text{d1}+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{m+1}}{f(m+6)}+\frac{5\text{d1}\text{d2}(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}\left(a+b\cosh^{-1}(\right.}{f(m+4)(m+6)}$$

```
[Out] 5*d1*d2*(f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))
/f/(4+m)/(6+m)+(f*x)^(1+m)*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccos
h(c*x))/f/(6+m)+15*d1^2*d2^2*(f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/
2)*(-c*d2*x+d2)^(1/2)/f/(6+m)/(m^2+6*m+8)+15*d1^2*d2^2*(f*x)^(1+m)*(a+b*arc
cosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2
)*(-c*d2*x+d2)^(1/2)/f/(6+m)/(m^3+7*m^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2
)-b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)/
(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^
(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-5*b*c*d1^2*d2^2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(6+m
)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b*c^3*d1^2*d2^2*(f*x)^(4+m)*(c*
d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^4/(4+m)^2/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+2*b*c^3*d1^2*d2^2*(f*x)^(4+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^4
/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^5*d1^2*d2^2*(f*x)^(6+m)*(c*d1*
x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*b
*c*d1^2*d2^2*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m
, 2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(6+m)/
(m^2+5*m+4)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.58, antiderivative size = 827, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.171, Rules used = {5745, 5743, 5763, 32, 14, 270}

$$\frac{(cx\text{d1}+\text{d1})^{5/2}(\text{d2}-c\text{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)(fx)^{m+1}}{f(m+6)}+\frac{5\text{d1}\text{d2}(cx\text{d1}+\text{d1})^{3/2}(\text{d2}-c\text{d2}x)^{3/2}\left(a+b\cosh^{-1}(\right.}{f(m+4)(m+6)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2
 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - (15*b*c*d1^2*d2^2*(f*x)^(2 +
 m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*(4 + m)*(6 + m)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*
x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (5*b*c^3*d1^2*d2^2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x
])/(f^4*(4 + m)^2*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d1^2*d2^
2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^4*(4 + m)*(6 + m)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^5*d1^2*d2^2*(f*x)^(6 + m)*Sqrt[d1 + c*d
1*x]*Sqrt[d2 - c*d2*x])/(f^6*(6 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*
d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[
c*x]))/(f*(6 + m)*(8 + 6*m + m^2)) + (5*d1*d2*(f*x)^(1 + m)*(d1 + c*d1*x)^(
3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))/(f*(4 + m)*(6 + m)) + ((f*x)
^(1 + m)*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*x]))/(f*(
6 + m)) + (15*d1^2*d2^2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/(f*(4 + m)*(6 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) -
 (15*b*c*d1^2*d2^2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*Hyperg
eometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1
+ m)*(2 + m)^2*(4 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(fx)^{1+m}(\text{d1}+c\text{d1}x)^{5/2}(\text{d2}-c\text{d2}x)^{5/2}\left(a+b\,\text{co}\right.}{f(6+m)}$$

$$= \frac{5\text{d1}\text{d2}(fx)^{1+m}(\text{d1}+c\text{d1}x)^{3/2}(\text{d2}-c\text{d2}x)^{3/2}\left(a\right.}{f(4+m)(6+m)}$$

$$= -\frac{bc\text{d1}^2\text{d2}^2(fx)^{2+m}\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= -\frac{bc\text{d1}^2\text{d2}^2(fx)^{2+m}\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}{f^2(2+m)(6+m)\sqrt{-1+cx}\sqrt{1+cx}} -$$

Mathematica [A] time = 2.35, size = 387, normalized size = 0.47

$$\text{d1}^2\text{d2}^2x\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\,(fx)^m\left(\frac{5\left(\frac{3\left(-bcx\sqrt{cx-1}\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)-(m+2)\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(\right.\right.}{(m+1)(m+2)^2(cx-1}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1^2*d2^2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(-((b*c*x*((2 + m)
^(-1) - (2*c^2*x^2)/(4 + m) + (c^4*x^4)/(6 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x])) + (-1 + c^2*x^2)^2*(a + b*ArcCosh[c*x]) + (5*((b*c*x*(-(2 + m)^(-1)
+ (c^2*x^2)/(4 + m)))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (-1 + c*x)*(1 + c*x)
*(a + b*ArcCosh[c*x]) + (3*((1 + m)*(-(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*ArcCosh[c*x]) - (2 +
m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ
[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2]))/((1 + m)*(2 + m)^2
*(-1 + c*x)*(1 + c*x))))/(4 + m)))/(6 + m)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ac^4d_1^2d_2^2x^4 - 2\,ac^2d_1^2d_2^2x^2 + ad_1^2d_2^2 + \left(bc^4d_1^2d_2^2x^4 - 2\,bc^2d_1^2d_2^2x^2 + bd_1^2d_2^2\right)\text{arcosh}\left(cx\right)\right)\sqrt{cd_1x+d_1}\,\sqrt{-c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral((a*c^4*d1^2*d2^2*x^4 - 2*a*c^2*d1^2*d2^2*x^2 + a*d1^2*d2^2 + (b*c^
4*d1^2*d2^2*x^4 - 2*b*c^2*d1^2*d2^2*x^2 + b*d1^2*d2^2)*arccosh(c*x))*sqrt(c
*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cd_1x+d_1)^{\frac{5}{2}}(-cd_2x+d_2)^{\frac{5}{2}}(b\,\text{arcosh}\left(cx\right)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

maple [F] time = 2.97, size = 0, normalized size = 0.00

$$\int (fx)^m (cd1x + d1)^{\frac{5}{2}} (-cd2x + d2)^{\frac{5}{2}} (a + b \operatorname{arccosh}(cx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cd_1x + d_1)^{\frac{5}{2}}(-cd_2x + d_2)^{\frac{5}{2}}(b \operatorname{arcosh}(cx) + a) (fx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(5/2)*(-c*d2*x+d2)^(5/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^(5/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b \operatorname{acosh}(c\,x)) \left(f\,x\right)^m (d_1 + c\,d_1\,x)^{5/2} (d_2 - c\,d_2\,x)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(5/2)*(-c*d2*x+d2)**(5/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

3.158 $\int (fx)^m(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)$

Optimal. Leaf size=503

$$-\frac{3bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d}}}{f(}$$

```
[Out] (f*x)^(1+m)*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x))/f/(4+m)
+3*d1*d2*(f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2
)/f/(m^2+6*m+8)+3*d1*d2*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+
1/2*m],[3/2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(m^3+7*m
^2+14*m+8)/(-c*x+1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d1*d2*(f*x)^(2+m)*(c*d1*x+d1)
^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c
*d1*d2*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)/(4+m)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+b*c^3*d1*d2*(f*x)^(4+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x
+d2)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*c*d1*d2*(f*x)^(2+m)*
HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(c*d1
*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/(m^2+5*m+4)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 513, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5745, 5743, 5763, 32, 14}

$$-\frac{3bc\text{d1}\text{d2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2(m+4)\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\text{d1}\text{d2}\sqrt{1-c^2x^2}\sqrt{c\text{d1}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*b*c*d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 +
m)^2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d1*d2*(f*x)^(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (b*c^3*d1*d2*(f*x)^(4 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(
f^4*(4 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d
1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]))/(f*(8 + 6*m + m^2)) + (
(f*x)^(1 + m)*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*x]))
/(f*(4 + m)) + (3*d1*d2*(f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*S
qrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3
+ m)/2, c^2*x^2])/(f*(4 + m)*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x)) - (3*b*c*
d1*d2*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricPFQ[{
1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)^
2*(4 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh^{-1}(cx)\right)dx = \frac{(fx)^{1+m}(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}\left(a+b\cosh\right.}{f(4+m)}$$

$$= \frac{3\mathrm{d1}\mathrm{d2}(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}\left(a+b\,\mathrm{co}\right.}{f\left(8+6m+m^2\right)}$$

$$= -\frac{3bc\mathrm{d1}\mathrm{d2}(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2(4+m)\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc}{}$$

Mathematica [A] time = 1.05, size = 288, normalized size = 0.57

$$\frac{\mathrm{d1}\mathrm{d2}x\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}(fx)^m\left(-\frac{3bcx\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}} - \frac{3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{(m+1)(m+2)(cx-1)(cx+1)}\right.}{m+4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)*(a + b*ArcCosh[c*
x]),x]
```

```
[Out] (d1*d2*x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((-3*b*c*x)/((2 + m)^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*x*(-(2 + m)^(-1) + (c^2*x^2)/(4 + m))
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*(a + b*ArcCosh[c*x]))/(2 + m) - (-1 +
 c*x)*(1 + c*x)*(a + b*ArcCosh[c*x]) - (3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/((1 + m)*(2 +
m)*(-1 + c*x)*(1 + c*x)) - (3*b*c*x*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}
, {3/2 + m/2, 2 + m/2}, c^2*x^2])/((1 + m)*(2 + m)^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])))/(4 + m)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ac^2d_1d_2x^2 - ad_1d_2 + \left(bc^2d_1d_2x^2 - bd_1d_2\right)\operatorname{arcosh}(cx)\right)\sqrt{cd_1x + d_1}\,\sqrt{-cd_2x + d_2}\,\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(-(a*c^2*d1*d2*x^2 - a*d1*d2 + (b*c^2*d1*d2*x^2 - b*d1*d2)*arccosh(
c*x))*sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cd_1x + d_1)^{\frac{3}{2}}(-cd_2x + d_2)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

maple [F] time = 2.60, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m (cd1x + d1)^{\frac{3}{2}}\,(-cd2x + d2)^{\frac{3}{2}}\,(a + b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (cd_1x + d_1)^{\frac{3}{2}}(-cd_2x + d_2)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(3/2)*(-c*d2*x+d2)^(3/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^(3/2)*(b*arccosh(c*x) + a)*(f*
x)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{acosh}(c\,x))\,\left(f\,x\right)^m\,(d_1 + c\,d_1\,x)^{3/2}\,(d_2 - c\,d_2\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(3/2)*(-c*d2*x+d2)**(3/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Timed out
```

3.159 $\int (fx)^m \sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}\left(a+b\cosh^{-1}(cx)\right)$

Optimal. Leaf size=302

$$-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)}{f(m^2+}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(2+m)
+(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*
x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f/(m^2+3*m+2)/(-c*x+1)^(1/2)/(c*x
+1)^(1/2)-b*c*(f*x)^(2+m)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)/f^2/(2+m)^2/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m,
1+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)
/f^2/(1+m)/(2+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 312, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {5743, 5763, 32}

$$-\frac{bc\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-c\text{d2}x}(fx)^{m+2}{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sqrt{c\text{d1}x+\text{d1}}\sqrt{\text{d2}-}}{f(}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])/(f^2*(2 + m)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]*(a + b*ArcCosh[c*x]))/(f*(2 + m)) + ((f*x)^(1 + m)*Sqrt[d1 + c*d1*x]
*Sqrt[d2 - c*d2*x]*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1
[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(2 + 3*m + m^2)*(1 - c*x)*(1 + c*x
)) - (b*c*(f*x)^(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*HypergeometricP
FQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 +
 m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
```

```
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] && !IntegerQ[m]
```

Rubi steps

$$\int (fx)^m\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{(fx)^{1+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,\left(a+b\cosh^{-1}(cx)\right)}{f(2+m)}$$

$$= -\frac{bc(fx)^{2+m}\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}{f^2(2+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{(fx)^{1+m}\sqrt{\mathrm{d1}}}{}$$

Mathematica [A] time = 0.22, size = 229, normalized size = 0.76

$$\frac{x\sqrt{\mathrm{cd1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}\,(fx)^m\left(-bcx\sqrt{cx-1}\,\sqrt{cx+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)-(m+2)\sqrt{1-}\right.}{(m-}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*(a + b*ArcCosh[c*x]),
x]
```

```
[Out] (x*(f*x)^m*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]*((1 + m)*(-(b*c*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + a*(2 + m)*(-1 + c^2*x^2) + b*(2 + m)*(-1 + c^2*x^2)*A
rcCosh[c*x]) - (2 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometri
c2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2] - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])
)/((1 + m)*(2 + m)^2*(-1 + c*x)*(1 + c*x))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{cd_1x+d_1}\,\sqrt{-cd_2x+d_2}\,(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m,
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cd_1x+d_1}\,\sqrt{-cd_2x+d_2}\,(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="giac")
```

```
[Out] integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \sqrt{cd1x + d1}\, \sqrt{-cd2x + d2}\, \left(a + b\,\mathrm{arccosh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{cd_{1}x + d_{1}}\, \sqrt{-cd_{2}x + d_{2}}\, (b\,\mathrm{arcosh}\left(cx\right) + a) \left(fx\right)^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(c*d1*x+d1)^(1/2)*(-c*d2*x+d2)^(1/2)*(a+b*arccosh(c*x)),x
, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}\left(c\,x\right)) \left(f\,x\right)^{m}\, \sqrt{d_{1} + c\,d_{1}\,x}\, \sqrt{d_{2} - c\,d_{2}\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d_{1}\left(cx + 1\right)} \left(fx\right)^{m} \sqrt{-d_{2}\left(cx - 1\right)}\, \left(a + b\,\mathrm{acosh}\left(cx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(c*d1*x+d1)**(1/2)*(-c*d2*x+d2)**(1/2)*(a+b*acosh(c*x)),
x)
```

```
[Out] Integral(sqrt(d1*(c*x + 1))*(f*x)**m*sqrt(-d2*(c*x - 1))*(a + b*acosh(c*x))
, x)
```

3.160 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathbf{d1}+c\mathbf{d1}x}\,\sqrt{\mathbf{d2}-c\mathbf{d2}x}}\,dx$

Optimal. Leaf size=188

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{cd1x+d1}\,\sqrt{d2-cd2x}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)(}{f(m+1)\sqrt{cd1x+d1}\,\sqrt{d2-c}}$$

```
[Out] b*c*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m
],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-
c*d2*x+d2)^(1/2)+(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],
[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+
d2)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {5765, 5763}

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{f^2(m+1)(m+2)\sqrt{cd1x+d1}\,\sqrt{d2-cd2x}}+\frac{\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};c^2x^2\right)(}{f(m+1)\sqrt{cd1x+d1}\,\sqrt{d2-c}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]),x]
```

```
[Out] ((f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2
, (1 + m)/2, (3 + m)/2, c^2*x^2])/(f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*
d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*HypergeometricPFQ[
{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2])/(f^2*(1 + m)*(2 + m)
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}}$$

$$= \frac{(fx)^{1+m}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right){}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};c^2x^2\right)}{f(1+m)\sqrt{\text{d1}+c\text{d1}x}\sqrt{\text{d2}-c\text{d2}x}} + \frac{bc(fx)^{2+m}}{}$$

Mathematica [C] time = 6.27, size = 264, normalized size = 1.40

$$\frac{2^{-m-3}\sqrt{c\text{d1}x+\text{d1}}\left(\frac{cx}{cx+1}\right)^{1-m}(fx)^m\left(bm\left(\frac{cx}{cx+1}\right)^m\sinh\left(2\cosh^{-1}(cx)\right)\left(\sqrt{\pi}c(m+1)x\sqrt{\frac{cx-1}{cx+1}}\Gamma(m+1)\,{}_3\tilde{F}_2\left(1,\frac{m+}{2}\right.\right.\right.}{c^2\text{d1}m}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*
x]),x]
```

```
[Out] (2^(-3 - m)*(f*x)^m*((c*x)/(1 + c*x))^(1 - m)*Sqrt[d1 + c*d1*x]*(2^(3 + m)*
a*(1 + m)*(-1 + c*x)*AppellF1[-m, -m, 1/2, 1 - m, (1 + c*x)^(-1), 2/(1 + c*
x)] + b*m*((c*x)/(1 + c*x))^m*(-(2^(2 + m)*(-1 + c*x)*ArcCosh[c*x]*Hypergeo
metric2F1[1, (2 + m)/2, (3 + m)/2, c^2*x^2]) + c*(1 + m)*Sqrt[Pi]*x*Sqrt[(-
1 + c*x)/(1 + c*x)]*Gamma[1 + m]*HypergeometricPFQRegularized[{1, (2 + m)/2
, (2 + m)/2}, {(3 + m)/2, (4 + m)/2}, c^2*x^2])*Sinh[2*ArcCosh[c*x]]))/(c^2
*d1*m*(1 + m)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d2 - c*d2*x])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}\,(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{c^2d_1d_2x^2-d_1d_2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^2*d1*d2*x^2 - d1*d2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\,(fx)^m}{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(a+b\,\text{arccosh}\,(cx))}{\sqrt{cd1x+d1}\,\sqrt{-cd2x+d2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^m}{\sqrt{cd_1x + d_1}\,\sqrt{-cd_2x + d_2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,\left(f\,x\right)^m}{\sqrt{d_1 + c\,d_1\,x}\,\sqrt{d_2 - c\,d_2\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(1/2)*(d2 - c*d2*x)^(1/2)),
 x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m (a + b \operatorname{acosh}(cx))}{\sqrt{d_1 (cx + 1)}\,\sqrt{-d_2 (cx - 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(1/2)/(-c*d2*x+d2)**(1/2),
x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(sqrt(d1*(c*x + 1))*sqrt(-d2*(c*x - 1)
)), x)
```

3.161 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\mathbf{d1}+c\mathbf{d1}x)^{3/2}(\mathbf{d2}-c\mathbf{d2}x)^{3/2}}\,dx$

Optimal. Leaf size=336

$$-\frac{bcm\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{\mathrm{d1}\mathrm{d2}f^2(m+1)(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2}\right.}{\mathrm{d1}\mathrm{d2}f(m+1)\sqrt{c\mathrm{d1}x+\mathrm{d}}}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))/d1/d2/f/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
+b*c*(f*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/d1/d2/f^2/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-b*c*m*(f*x
)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1/d2/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-c*
d2*x+d2)^(1/2)-m*(f*x)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],
[3/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/d1/d2/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c
*d2*x+d2)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {5756, 5765, 5763, 364}

$$-\frac{bcm\sqrt{cx-1}\,\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{\mathrm{d1}\mathrm{d2}f^2(m+1)(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2}\right.}{\mathrm{d1}\mathrm{d2}f(m+1)\sqrt{c\mathrm{d1}x+\mathrm{d}}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(d1*d2*f*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c
*d2*x]) - (m*(f*x)^(1 + m)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeom
etric2F1[1/2, (1 + m)/2, (3 + m)/2, c^2*x^2])/(d1*d2*f*(1 + m)*Sqrt[d1 + c*
d1*x]*Sqrt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
Hypergeometric2F1[1, (2 + m)/2, (4 + m)/2, c^2*x^2])/(d1*d2*f^2*(2 + m)*Sqr
t[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*m*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, c
^2*x^2])/(d1*d2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] && !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} - \frac{m\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}}\,dx}{\mathrm{d1}\mathrm{d2}} - \frac{\left(bc\sqrt{-1+\right.}{\mathrm{d1}\mathrm{d2}f\sqrt{}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} + \frac{bc(fx)^{2+m}\sqrt{-1+cx}\,\sqrt{1+cx}\,{}_2F_1\left(1,\frac{2+}{2}\right.}{\mathrm{d1}\mathrm{d2}f^2(2+m)\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-c\mathrm{d2}x}} - \frac{m(fx)^{1+m}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}{\mathrm{d1}\mathrm{d2}f(1+m)\sqrt{\mathrm{d1}+c\mathrm{d1}x}}$$

Mathematica [F] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)
^(3/2)), x]
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{cd_1x+d_1}\,\sqrt{-cd_2x+d_2}\,(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^m}{c^4d_1^2d_2^2x^4-2\,c^2d_1^2d_2^2x^2+d_1^2d_2^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="fricas")
```

```
[Out] integral(sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m/
(c^4*d1^2*d2^2*x^4 - 2*c^2*d1^2*d2^2*x^2 + d1^2*d2^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^{m}}{(cd_1x + d_1)^{\frac{3}{2}}(-cd_2x + d_2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} (a + b \operatorname{arccosh}(cx))}{(cd1x + d1)^{\frac{3}{2}} (-cd2x + d2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)\left(fx\right)^{m}}{(cd_1x + d_1)^{\frac{3}{2}}(-cd_2x + d_2)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(3/2)*(-c*d2*x + d2)^
(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\, x))\, \left(f\, x\right)^{m}}{(d_1 + c\, d_1\, x)^{3/2}\, (d_2 - c\, d_2\, x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(3/2)*(d2 - c*d2*x)^(3/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(3/2)/(-c*d2*x+d2)**(3/2),
x)
```

```
[Out] Timed out
```

3.162 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\mathbf{d1}+c\mathbf{d1}x)^{5/2}(\mathbf{d2}-c\mathbf{d2}x)^{5/2}}\,dx$

Optimal. Leaf size=504

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3\mathrm{d1}^2\mathrm{d2}^2f^2(m+1)(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1}{3\mathrm{d1}^2\mathrm{d2}^2f(m+1)\sqrt{}}$$

```
[Out] 1/3*(f*x)^(1+m)*(a+b*arccosh(c*x))/d1/d2/f/(c*d1*x+d1)^(3/2)/(-c*d2*x+d2)^(
3/2)+1/3*(2-m)*(f*x)^(1+m)*(a+b*arccosh(c*x))/d1^2/d2^2/f/(c*d1*x+d1)^(1/2)
/(-c*d2*x+d2)^(1/2)+1/3*b*c*(2-m)*(f*x)^(2+m)*hypergeom([1, 1+1/2*m],[2+1/2
*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2/d2^2/f^2/(2+m)/(c*d1*x+d1)^(1
/2)/(-c*d2*x+d2)^(1/2)+1/3*b*c*(f*x)^(2+m)*hypergeom([2, 1+1/2*m],[2+1/2*m]
,c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2/d2^2/f^2/(2+m)/(c*d1*x+d1)^(1/2)
/(-c*d2*x+d2)^(1/2)-1/3*b*c*(2-m)*m*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2
*m, 1+1/2*m],[3/2+1/2*m, 2+1/2*m],c^2*x^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d1^2
/d2^2/f^2/(1+m)/(2+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)-1/3*(2-m)*m*(f*x
)^(1+m)*(a+b*arccosh(c*x))*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],c^2*x^2)*
(-c^2*x^2+1)^(1/2)/d1^2/d2^2/f/(1+m)/(c*d1*x+d1)^(1/2)/(-c*d2*x+d2)^(1/2)
```

Rubi [A] time = 1.45, antiderivative size = 504, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {5756, 5765, 5763, 364}

$$-\frac{bc(2-m)m\sqrt{cx-1}\sqrt{cx+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;c^2x^2\right)}{3\mathrm{d1}^2\mathrm{d2}^2f^2(m+1)(m+2)\sqrt{c\mathrm{d1}x+\mathrm{d1}}\sqrt{\mathrm{d2}-c\mathrm{d2}x}}-\frac{(2-m)m\sqrt{1-c^2x^2}\,(fx)^{m+1}\,{}_2F_1}{3\mathrm{d1}^2\mathrm{d2}^2f(m+1)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)
),x]
```

```
[Out] ((f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1*d2*f*(d1 + c*d1*x)^(3/2)*(d2 - c
*d2*x)^(3/2)) + ((2 - m)*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(3*d1^2*d2^2*f
*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - ((2 - m)*m*(f*x)^(1 + m)*Sqrt[1 - c
^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, c
^2*x^2])/(3*d1^2*d2^2*f*(1 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) + (b*c
*(2 - m)*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeometric2F1[1, (2
 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[d1 + c*d1*x]*Sq
rt[d2 - c*d2*x]) + (b*c*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Hypergeo
metric2F1[2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(3*d1^2*d2^2*f^2*(2 + m)*Sqrt[
d1 + c*d1*x]*Sqrt[d2 - c*d2*x]) - (b*c*(2 - m)*m*(f*x)^(2 + m)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m
/2}, c^2*x^2])/(3*d1^2*d2^2*f^2*(1 + m)*(2 + m)*Sqrt[d1 + c*d1*x]*Sqrt[d2 -
 c*d2*x])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
```

```
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5765

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[(Sqrt[1 + c
*x]*Sqrt[-1 + c*x])/(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[((f*x)^m*(a + b*
ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] /; FreeQ[{a, b, c,
d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[
n, 0] &&  !(GtQ[d1, 0] && LtQ[d2, 0]) && (IntegerQ[m] || EqQ[n, 1])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx = \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}}}{3\mathrm{d1}\mathrm{d2}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}\right.}{3\mathrm{d1}^2\mathrm{d2}^2f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}\right.}{3\mathrm{d1}^2\mathrm{d2}^2f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

$$= \frac{(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{3\mathrm{d1}\mathrm{d2}f(\mathrm{d1}+c\mathrm{d1}x)^{3/2}(\mathrm{d2}-c\mathrm{d2}x)^{3/2}} + \frac{(2-m)(fx)^{1+m}\left(a+b\cosh^{-1}\right.}{3\mathrm{d1}^2\mathrm{d2}^2f\sqrt{\mathrm{d1}+c\mathrm{d1}x}\,\sqrt{\mathrm{d2}-}}$$

Mathematica [F] time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{(\mathrm{d1}+c\mathrm{d1}x)^{5/2}(\mathrm{d2}-c\mathrm{d2}x)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)),x]

[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)
^(5/2)), x]
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{cd_1x+d_1}\sqrt{-cd_2x+d_2}\left(b\,\text{arcosh}\,(cx)+a\right)\left(fx\right)^m}{c^6d_1^3d_2^3x^6-3\,c^4d_1^3d_2^3x^4+3\,c^2d_1^3d_2^3x^2-d_1^3d_2^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="fricas")
```

```
[Out] integral(-sqrt(c*d1*x + d1)*sqrt(-c*d2*x + d2)*(b*arccosh(c*x) + a)*(f*x)^m
/(c^6*d1^3*d2^3*x^6 - 3*c^4*d1^3*d2^3*x^4 + 3*c^2*d1^3*d2^3*x^2 - d1^3*d2^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\,(cx)+a\right)\left(fx\right)^m}{\left(cd_1x+d_1\right)^{\frac{5}{2}}\left(-cd_2x+d_2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(a+b\,\text{arccosh}\,(cx)\right)}{\left(cd1x+d1\right)^{\frac{5}{2}}\left(-cd2x+d2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\,(cx)+a\right)\left(fx\right)^m}{\left(cd_1x+d_1\right)^{\frac{5}{2}}\left(-cd_2x+d_2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(c*d1*x+d1)^(5/2)/(-c*d2*x+d2)^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/((c*d1*x + d1)^(5/2)*(-c*d2*x + d2)^
(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\text{acosh}\,(c\,x)\right)\left(f\,x\right)^m}{\left(d_1+c\,d_1\,x\right)^{5/2}\left(d_2-c\,d_2\,x\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)),
x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/((d1 + c*d1*x)^(5/2)*(d2 - c*d2*x)^(5/2)),
 x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(c*d1*x+d1)**(5/2)/(-c*d2*x+d2)**(5/2),
x)
```

```
[Out] Timed out
```

3.163 $\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx$

Optimal. Leaf size=128

$$\frac{a\sqrt{ax-1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-ax}}+\frac{\cosh^{-1}(ax)(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{f(m+1)}$$

```
[Out] (f*x)^(1+m)*arccosh(a*x)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],a^2*x^2)/f/
(1+m)+a*(f*x)^(2+m)*HypergeometricPFQ([1, 1+1/2*m, 1+1/2*m],[3/2+1/2*m, 2+1
/2*m],a^2*x^2)*(a*x-1)^(1/2)/f^2/(1+m)/(2+m)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 141, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5798, 5763}

$$\frac{a\sqrt{ax-1}\,\sqrt{ax+1}\,(fx)^{m+2}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{f^2(m+1)(m+2)\sqrt{1-a^2x^2}}+\frac{\cosh^{-1}(ax)(fx)^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
```

```
[Out] ((f*x)^(1 + m)*ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^
2*x^2])/(f*(1 + m)) + (a*(f*x)^(2 + m)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Hyperge
ometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/(f^2*(1 +
 m)*(2 + m)*Sqrt[1 - a^2*x^2])
```

Rule 5763

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/(Sqrt[(d1_) + (
e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^(m + 1)*Sq
rt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])*Hypergeometric2F1[1/2, (1 + m)/2, (3 +
 m)/2, c^2*x^2])/(f*(m + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Simp[(b*
c*(f*x)^(m + 2)*HypergeometricPFQ[{1, 1 + m/2, 1 + m/2}, {3/2 + m/2, 2 + m/
2}, c^2*x^2])/(Sqrt[-(d1*d2)]*f^2*(m + 1)*(m + 2)), x] /; FreeQ[{a, b, c, d
1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[d
1, 0] && LtQ[d2, 0] &&  !IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{1-a^2x^2}} \, dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{(fx)^m \cosh^{-1}(ax)}{\sqrt{-1+ax}\,\sqrt{1+ax}} \, dx}{\sqrt{1-a^2x^2}}$$

$$= \frac{(fx)^{1+m}\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};a^2x^2\right)}{f(1+m)}+\frac{a(fx)^{2+m}\sqrt{-1+ax}\,\sqrt{1+ax}\,{}_3F_2\left(1,1+\right.}{f^2(1+m)(2+m)\sqrt{}}$$

Mathematica [A] time = 0.08, size = 124, normalized size = 0.97

$$\frac{x(fx)^m\left(\frac{ax\sqrt{ax-1}\sqrt{ax+1}\,{}_3F_2\left(1,\frac{m}{2}+1,\frac{m}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;a^2x^2\right)}{(m+2)\sqrt{1-a^2x^2}}+\cosh^{-1}(ax)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};a^2x^2\right)\right)}{m+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*ArcCosh[a*x])/Sqrt[1 - a^2*x^2],x]
[Out] (x*(f*x)^m*(ArcCosh[a*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, a^2*x
^2] + (a*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*HypergeometricPFQ[{1, 1 + m/2, 1 +
m/2}, {3/2 + m/2, 2 + m/2}, a^2*x^2])/((2 + m)*Sqrt[1 - a^2*x^2])))/(1 + m)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,(fx)^m\,\text{arcosh}\,(ax)}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*(f*x)^m*arccosh(a*x)/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(fx)^m\,\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

maple [F] time = 0.57, size = 0, normalized size = 0.00

$$\int\frac{(fx)^m\,\text{arccosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] int((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(fx)^m\,\text{arcosh}\,(ax)}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x)^m*arccosh(a*x)/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\text{acosh}\,(a\,x)\,(f\,x)^m}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((acosh(a*x)*(f*x)^m)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{acosh}(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*acosh(a*x)/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.164 $\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=266

$$-\frac{2}{343}a^6c^3x^7+\frac{234a^4c^3x^5}{6125}-\frac{1514a^2c^3x^3}{11025}+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{8}{35}c^3x\,($$

```
[Out] 4322/3675*c^3*x-1514/11025*a^2*c^3*x^3+234/6125*a^4*c^3*x^5-2/343*a^6*c^3*x
^7+16/105*c^3*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a-12/175*c^3*(a*x-1)
^(5/2)*(a*x+1)^(5/2)*arccosh(a*x)/a+2/49*c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)*ar
ccosh(a*x)/a+16/35*c^3*x*arccosh(a*x)^2+8/35*c^3*x*(-a^2*x^2+1)*arccosh(a*x
)^2+6/35*c^3*x*(-a^2*x^2+1)^2*arccosh(a*x)^2+1/7*c^3*x*(-a^2*x^2+1)^3*arcco
sh(a*x)^2-32/35*c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 0.68, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5681, 5718, 194, 5654, 8}

$$-\frac{2}{343}a^6c^3x^7+\frac{234a^4c^3x^5}{6125}-\frac{1514a^2c^3x^3}{11025}+\frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{8}{35}c^3x\,($$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```

```
[Out] (4322*c^3*x)/3675 - (1514*a^2*c^3*x^3)/11025 + (234*a^4*c^3*x^5)/6125 - (2*
a^6*c^3*x^7)/343 - (32*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(35*a
) + (16*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(105*a) - (12*c^
3*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])/(175*a) + (2*c^3*(-1 + a*x
)^(7/2)*(1 + a*x)^(7/2)*ArcCosh[a*x])/(49*a) + (16*c^3*x*ArcCosh[a*x]^2)/35
 + (8*c^3*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/35 + (6*c^3*x*(1 - a^2*x^2)^2*Arc
Cosh[a*x]^2)/35 + (c^3*x*(1 - a^2*x^2)^3*ArcCosh[a*x]^2)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
```

```
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^3\cosh^{-1}(ax)^2\,dx = \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^2+\frac{1}{7}(6c)\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2\,dx+\frac{1}{7}\left(2ac^3\right.$$
$$=\frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)}{49a}+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{1}{7}c^3x$$
$$=-\frac{12c^3(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{175a}+\frac{2c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax}{49a}$$
$$=\frac{2c^3x}{49}-\frac{2}{49}a^2c^3x^3+\frac{6}{245}a^4c^3x^5-\frac{2}{343}a^6c^3x^7+\frac{16c^3(-1+ax)^{3/2}(1+ax)^{3/2}\cosh}{105a}$$
$$=\frac{962c^3x}{3675}-\frac{1514a^2c^3x^3}{11025}+\frac{234a^4c^3x^5}{6125}-\frac{2}{343}a^6c^3x^7-\frac{32c^3\sqrt{-1+ax}\sqrt{1+ax}\cosh}{35a}$$
$$=\frac{4322c^3x}{3675}-\frac{1514a^2c^3x^3}{11025}+\frac{234a^4c^3x^5}{6125}-\frac{2}{343}a^6c^3x^7-\frac{32c^3\sqrt{-1+ax}\sqrt{1+ax}\cos}{35a}$$

Mathematica [A] time = 0.28, size = 125, normalized size = 0.47

$$\frac{c^3\left(-2250a^7x^7+14742a^5x^5-52990a^3x^3-11025ax\left(5a^6x^6-21a^4x^4+35a^2x^2-35\right)\cosh^{-1}(ax)^2+210\sqrt{ax-1}\right.}{385875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(453810*a*x - 52990*a^3*x^3 + 14742*a^5*x^5 - 2250*a^7*x^7 + 210*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 757*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*Ar
cCosh[a*x] - 11025*a*x*(-35 + 35*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[
a*x]^2))/(385875*a)
```

fricas [A] time = 0.64, size = 175, normalized size = 0.66

$$-\frac{2250\,a^7c^3x^7-14742\,a^5c^3x^5+52990\,a^3c^3x^3-453810\,ac^3x+11025\left(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x\right)}{3858}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/385875*(2250*a^7*c^3*x^7 - 14742*a^5*c^3*x^5 + 52990*a^3*c^3*x^3 - 45381
0*a*c^3*x + 11025*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a*c
^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 210*(75*a^6*c^3*x^6 - 351*a^4*c^3*x^
4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
1)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.21, size = 188, normalized size = 0.71

$$-\frac{c^3\left(55125\mathrm{arccosh}\left(ax\right)^2a^7x^7-15750\,\mathrm{arccosh}\left(ax\right)\sqrt{ax-1}\,\sqrt{ax+1}\,a^6x^6-231525\mathrm{arccosh}\left(ax\right)^2a^5x^5+737\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x)
```

```
[Out] -1/385875/a*c^3*(55125*arccosh(a*x)^2*a^7*x^7-15750*arccosh(a*x)*(a*x-1)^(1
/2)*(a*x+1)^(1/2)*a^6*x^6-231525*arccosh(a*x)^2*a^5*x^5+73710*arccosh(a*x)*
a^4*x^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)+2250*x^7*a^7+385875*a^3*x^3*arccosh(a*x
)^2-158970*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^2*x^2-14742*x^5*a^5-3
85875*a*x*arccosh(a*x)^2+453810*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)+52
990*x^3*a^3-453810*a*x)
```

maxima [A] time = 0.85, size = 178, normalized size = 0.67

$$-\frac{2}{343}\,a^6c^3x^7+\frac{234}{6125}\,a^4c^3x^5-\frac{1514}{11025}\,a^2c^3x^3+\frac{4322}{3675}\,c^3x+\frac{2}{3675}\left(75\,\sqrt{a^2x^2-1}\,a^4c^3x^6-351\,\sqrt{a^2x^2-1}\,a^2c^3x^4+75\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/343*a^6*c^3*x^7 + 234/6125*a^4*c^3*x^5 - 1514/11025*a^2*c^3*x^3 + 4322/3
675*c^3*x + 2/3675*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1
)*a^2*c^3*x^4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/
a^2)*a*arccosh(a*x) - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3
 - 35*c^3*x)*arccosh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)^2\,{\left(c-a^2\,c\,x^2\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2)^3, x)
```

sympy [A] time = 11.25, size = 243, normalized size = 0.91

$$\begin{cases} -\frac{a^6c^3x^7\,\mathrm{acosh}^2\left(ax\right)}{7}-\frac{2a^6c^3x^7}{343}+\frac{2a^5c^3x^6\sqrt{a^2x^2-1}\,\mathrm{acosh}\left(ax\right)}{49}+\frac{3a^4c^3x^5\,\mathrm{acosh}^2\left(ax\right)}{5}+\frac{234a^4c^3x^5}{6125}-\frac{234a^3c^3x^4\sqrt{a^2x^2-1}\,\mathrm{acosh}\left(ax\right)}{1225}-a\cdots \\ -\frac{\pi^2c^3x}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**2/7 - 2*a**6*c**3*x**7/343 + 2*a**5*
c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)/49 + 3*a**4*c**3*x**5*acosh(a*x)**
2/5 + 234*a**4*c**3*x**5/6125 - 234*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acos
h(a*x)/1225 - a**2*c**3*x**3*acosh(a*x)**2 - 1514*a**2*c**3*x**3/11025 + 15
14*a*c**3*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/3675 + c**3*x*acosh(a*x)**2 +
 4322*c**3*x/3675 - 4322*c**3*sqrt(a**2*x**2 - 1)*acosh(a*x)/(3675*a), Ne(a
, 0)), (-pi**2*c**3*x/4, True))
```

3.165 $\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=195

$$\frac{2}{125}a^4c^2x^5-\frac{76}{675}a^2c^2x^3+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{8}{15}c^2x\cosh^{-1}(ax)^2-\frac{2c^2(a}{}$$

```
[Out] 298/225*c^2*x-76/675*a^2*c^2*x^3+2/125*a^4*c^2*x^5+8/45*c^2*(a*x-1)^(3/2)*(
a*x+1)^(3/2)*arccosh(a*x)/a-2/25*c^2*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arccosh(a*
x)/a+8/15*c^2*x*arccosh(a*x)^2+4/15*c^2*x*(-a^2*x^2+1)*arccosh(a*x)^2+1/5*c
^2*x*(-a^2*x^2+1)^2*arccosh(a*x)^2-16/15*c^2*arccosh(a*x)*(a*x-1)^(1/2)*(a*
x+1)^(1/2)/a
```

Rubi [A] time = 0.46, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5681, 5718, 194, 5654, 8}

$$\frac{2}{125}a^4c^2x^5-\frac{76}{675}a^2c^2x^3+\frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{8}{15}c^2x\cosh^{-1}(ax)^2-\frac{2c^2(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```

```
[Out] (298*c^2*x)/225 - (76*a^2*c^2*x^3)/675 + (2*a^4*c^2*x^5)/125 - (16*c^2*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(15*a) + (8*c^2*(-1 + a*x)^(3/2)*(1
+ a*x)^(3/2)*ArcCosh[a*x])/(45*a) - (2*c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2)
*ArcCosh[a*x])/(25*a) + (8*c^2*x*ArcCosh[a*x]^2)/15 + (4*c^2*x*(1 - a^2*x^2
)*ArcCosh[a*x]^2)/15 + (c^2*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
```

```
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^2\,dx = \frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^2+\frac{1}{5}(4c)\int\left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx-\frac{1}{5}\left(2ac\right.$$

$$= -\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)}{25a}+\frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{1}{5}$$

$$= \frac{8c^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{45a}-\frac{2c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(a}{25a}$$

$$= \frac{58c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{15a}+\frac{8c}{}$$

$$= \frac{298c^2x}{225}-\frac{76}{675}a^2c^2x^3+\frac{2}{125}a^4c^2x^5-\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{15a}+\frac{8}{}$$

Mathematica [A] time = 0.23, size = 101, normalized size = 0.52

$$\frac{c^2\left(54a^5x^5-380a^3x^3+225ax\left(3a^4x^4-10a^2x^2+15\right)\cosh^{-1}(ax)^2-30\sqrt{ax-1}\sqrt{ax+1}\left(9a^4x^4-38a^2x^2+14\right.\right.}{3375a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^2,x]
```

```
[Out] (c^2*(4470*a*x - 380*a^3*x^3 + 54*a^5*x^5 - 30*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x] + 225*a*x*(15 - 10*a^2*x^2 + 3
*a^4*x^4)*ArcCosh[a*x]^2))/(3375*a)
```

fricas [A] time = 0.55, size = 142, normalized size = 0.73

$$\frac{54\,a^5c^2x^5-380\,a^3c^2x^3+4470\,ac^2x+225\left(3\,a^5c^2x^5-10\,a^3c^2x^3+15\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-30\left(9\,a^4c^2\right.}{3375\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(54*a^5*c^2*x^5 - 380*a^3*c^2*x^3 + 4470*a*c^2*x + 225*(3*a^5*c^2*x^
5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 30*(9*a^4
*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x
^2 - 1)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 140, normalized size = 0.72

$$\frac{c^2\left(675\operatorname{arccosh}(ax)^2 a^5x^5 - 270\operatorname{arccosh}(ax)a^4x^4\sqrt{ax-1}\,\sqrt{ax+1} - 2250a^3x^3\operatorname{arccosh}(ax)^2 + 1140\operatorname{arccosh}(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x)
```

```
[Out] 1/3375/a*c^2*(675*arccosh(a*x)^2*a^5*x^5-270*arccosh(a*x)*a^4*x^4*(a*x-1)^(
1/2)*(a*x+1)^(1/2)-2250*a^3*x^3*arccosh(a*x)^2+1140*arccosh(a*x)*(a*x-1)^(1
/2)*(a*x+1)^(1/2)*a^2*x^2+54*x^5*a^5+3375*a*x*arccosh(a*x)^2-4470*(a*x-1)^(
1/2)*(a*x+1)^(1/2)*arccosh(a*x)-380*x^3*a^3+4470*a*x)
```

maxima [A] time = 0.82, size = 134, normalized size = 0.69

$$\frac{2}{125}a^4c^2x^5 - \frac{76}{675}a^2c^2x^3 + \frac{298}{225}c^2x - \frac{2}{225}\left(9\sqrt{a^2x^2-1}\,a^2c^2x^4 - 38\sqrt{a^2x^2-1}\,c^2x^2 + \frac{149\sqrt{a^2x^2-1}\,c^2}{a^2}\right)a\operatorname{arcosh}(ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] 2/125*a^4*c^2*x^5 - 76/675*a^2*c^2*x^3 + 298/225*c^2*x - 2/225*(9*sqrt(a^2*
x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149*sqrt(a^2*x^2 - 1)
*c^2/a^2)*a*arccosh(a*x) + 1/15*(3*a^4*c^2*x^5 - 10*a^2*c^2*x^3 + 15*c^2*x)
*arccosh(a*x)^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)^2\,\left(c-a^2\,c\,x^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2)^2, x)
```

sympy [A] time = 3.90, size = 182, normalized size = 0.93

$$\begin{cases} \frac{a^4c^2x^5\operatorname{acosh}^2(ax)}{5} + \frac{2a^4c^2x^5}{125} - \frac{2a^3c^2x^4\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{25} - \frac{2a^2c^2x^3\operatorname{acosh}^2(ax)}{3} - \frac{76a^2c^2x^3}{675} + \frac{76ac^2x^2\sqrt{a^2x^2-1}\operatorname{acosh}(ax)}{225} + c^2x\operatorname{acos} \\ -\frac{\pi^2c^2x}{4} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**2,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**2/5 + 2*a**4*c**2*x**5/125 - 2*a**3*c
**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)/25 - 2*a**2*c**2*x**3*acosh(a*x)**2
/3 - 76*a**2*c**2*x**3/675 + 76*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)/
225 + c**2*x*acosh(a*x)**2 + 298*c**2*x/225 - 298*c**2*sqrt(a**2*x**2 - 1)*
acosh(a*x)/(225*a), Ne(a, 0)), (-pi**2*c**2*x/4, True))
```

3.166 $\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx$

Optimal. Leaf size=112

$$-\frac{2}{27}a^2cx^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{2c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{9a}-\frac{4c\sqrt{ax-1}\sqrt{a}}{}$$

```
[Out] 14/9*c*x-2/27*a^2*c*x^3+2/9*c*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)/a+2/
3*c*x*arccosh(a*x)^2+1/3*c*x*(-a^2*x^2+1)*arccosh(a*x)^2-4/3*c*arccosh(a*x)
*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 0.26, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5681, 5718, 5654, 8}

$$-\frac{2}{27}a^2cx^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{2c(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}{9a}-\frac{4c\sqrt{ax-1}\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```

```
[Out] (14*c*x)/9 - (2*a^2*c*x^3)/27 - (4*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x])/(3*a) + (2*c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])/(9*a) + (2
*c*x*ArcCosh[a*x]^2)/3 + (c*x*(1 - a^2*x^2)*ArcCosh[a*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^2\,dx = \frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^2+\frac{1}{3}(2c)\int \cosh^{-1}(ax)^2\,dx+\frac{1}{3}(2ac)\int x\sqrt{-1+ax}\,\sqrt{}$$
$$= \frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}{9a}+\frac{2}{3}cx\cosh^{-1}(ax)^2+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-}$$
$$= \frac{2cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}\,c}{9a}$$
$$= \frac{14cx}{9}-\frac{2}{27}a^2cx^3-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a}+\frac{2c(-1+ax)^{3/2}(1+ax)^{3/2}}{9a}$$

Mathematica [A] time = 0.12, size = 73, normalized size = 0.65

$$\frac{c\left(-2a^3x^3-9ax\left(a^2x^2-3\right)\cosh^{-1}(ax)^2+6\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2x^2-7\right)\cosh^{-1}(ax)+42ax\right)}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^2,x]
```

```
[Out] (c*(42*a*x - 2*a^3*x^3 + 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcC
osh[a*x] - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^2))/(27*a)
```

fricas [A] time = 0.62, size = 95, normalized size = 0.85

$$-\frac{2\,a^3cx^3-42\,acx+9\left(a^3cx^3-3\,acx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^2cx^2-7\,c\right)\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(2*a^3*c*x^3 - 42*a*c*x + 9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*
x^2 - 1))^2 - 6*(a^2*c*x^2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2
 - 1)))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.07, size = 90, normalized size = 0.80

$$-\frac{c\left(9a^3x^3\operatorname{arccosh}(ax)^2-6\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,a^2x^2-27ax\operatorname{arccosh}(ax)^2+42\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arcc}\right.}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^2,x)
```

```
[Out] -1/27/a*c*(9*a^3*x^3*arccosh(a*x)^2-6*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)*a^2*x^2-27*a*x*arccosh(a*x)^2+42*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*
x)+2*x^3*a^3-42*a*x)
```

maxima [A] time = 1.08, size = 76, normalized size = 0.68

$$-\frac{2}{27}\,a^2cx^3+\frac{2}{9}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{7\,\sqrt{a^2x^2-1}\,c}{a^2}\right)a\,\mathrm{arcosh}\left(ax\right)-\frac{1}{3}\left(a^2cx^3-3\,cx\right)\mathrm{arcosh}\left(ax\right)^2+\frac{14}{9}\,cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -2/27*a^2*c*x^3 + 2/9*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)
*a*arccosh(a*x) - 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^2 + 14/9*c*x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \mathrm{acosh}\left(a\,x\right)^2\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2*(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^2*(c - a^2*c*x^2), x)
```

sympy [A] time = 1.06, size = 105, normalized size = 0.94

$$\begin{cases} -\frac{a^2cx^3\,\mathrm{acosh}^2\,(ax)}{3}-\frac{2a^2cx^3}{27}+\frac{2acx^2\sqrt{a^2x^2-1}\,\mathrm{acosh}\,(ax)}{9}+cx\,\mathrm{acosh}^2\,(ax)+\frac{14cx}{9}-\frac{14c\sqrt{a^2x^2-1}\,\mathrm{acosh}\,(ax)}{9a} & \text{for } a\neq 0\\ -\frac{\pi^2cx}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**2,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**2/3 - 2*a**2*c*x**3/27 + 2*a*c*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)/9 + c*x*acosh(a*x)**2 + 14*c*x/9 - 14*c*sqrt(a*
*2*x**2 - 1)*acosh(a*x)/(9*a), Ne(a, 0)), (-pi**2*c*x/4, True))
```

3.167 $\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx$

Optimal. Leaf size=98

$$\frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\cosh^{-1}(ax}{}$$

```
[Out] 2*arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+2*arccosh(a*x
)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*arccosh(a*x)*polylog(2,
a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)
^(1/2))/a/c+2*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5694, 4182, 2531, 2282, 6589}

$$\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{2\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{2\text{PolyI}}{}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (2*ArcCosh[a*x]*PolyLog[
2, -E^ArcCosh[a*x]])/(a*c) - (2*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a
*c) - (2*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (2*PolyLog[3, E^ArcCosh[a*x]]
)/(a*c)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx = -\frac{\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\,\text{Subst}\left(\int x\log\left(1-e^x\right)\,dx, x, \cosh^{-1}(ax)\right)}{ac} - \frac{2\,\text{Sub}}{}$$
$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}_2\left(e\right.}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}_2\left(e\right.}{ac}$$
$$= \frac{2\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac} + \frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac} - \frac{2\cosh^{-1}(ax)\text{Li}_2\left(e\right.}{ac}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 0.97

$$\frac{2\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right) - 2\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right) - 2\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right) + 2\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right) + \cosh^{-1}}{ac}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^2*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^2*Log[1 + E^ArcCo
sh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 2*ArcCosh[a*x]*Poly
Log[2, E^ArcCosh[a*x]] - 2*PolyLog[3, -E^ArcCosh[a*x]] + 2*PolyLog[3, E^Arc
Cosh[a*x]])/(a*c)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{arcosh}\,(ax)^2}{a^2cx^2-c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\text{arcosh}\,(ax)^2}{a^2cx^2-c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c), x)
```

maple [A] time = 0.08, size = 201, normalized size = 2.05

$$-\frac{\operatorname{arccosh}(ax)^2 \ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{ac}-\frac{2\operatorname{arccosh}(ax)\operatorname{polylog}\left(2, ax+\sqrt{ax-1}\,\sqrt{ax+1}\right)}{ac}+\frac{2\operatorname{polylog}(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-2*arccosh(a*x)*
polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+2*polylog(3,a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))/a/c+1/a/c*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))+2*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-2*polyl
og(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\log(ax+1)-\log(ax-1)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^2}{2\,ac}-\int\frac{\left(\left(ax\log(ax+1)-ax\log(ax-1)\right)\sqrt{ax+1}\,\sqrt{ax-}\right.}{a^3cx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/
(a*c) - integrate(((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*sqrt
(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*log(a*
x + sqrt(a*x + 1)*sqrt(a*x - 1))/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)*sqrt(
a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)**2/(a**2*x**2 - 1), x)/c
```

3.168 $\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^2} dx$

Optimal. Leaf size=163

$$\frac{x\cosh^{-1}(ax)^2}{2c^2(1-a^2x^2)}+\frac{\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\frac{\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)}{ac^2}$$

```
[Out] 1/2*x*arccosh(a*x)^2/c^2/(-a^2*x^2+1)+arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))/a/c^2-arctanh(a*x)/a/c^2+arccosh(a*x)*polylog(2,-a*x-(a*
x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))/a/c^2-polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+polylo
g(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-arccosh(a*x)/a/c^2/(a*x-1)^(1/2)
/(a*x+1)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5689, 5718, 207, 5694, 4182, 2531, 2282, 6589}

$$\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac^2}-\frac{\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```

```
[Out] -(ArcCosh[a*x]/(a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])) + (x*ArcCosh[a*x]^2)/(
2*c^2*(1 - a^2*x^2)) + (ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - A
rcTanh[a*x]/(a*c^2) + (ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(a*c^2) -
(ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - PolyLog[3, -E^ArcCosh[a
*x]]/(a*c^2) + PolyLog[3, E^ArcCosh[a*x]]/(a*c^2)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c^2} + \frac{\int \frac{\cosh^{-1}(ax)^2}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\int \frac{1}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^2\text{csch}(x)\,dx, x, \cos\right.}{2ac^2}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(a}{ac^2}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(a}{ac^2}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(a}{ac^2}$$

$$= -\frac{\cosh^{-1}(ax)}{ac^2\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^2}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} - \frac{\tanh^{-1}(a}{ac^2}$$

Mathematica [A] time = 0.94, size = 191, normalized size = 1.17

$$\frac{-8\cosh^{-1}(ax)\text{Li}_2\left(-e^{-\cosh^{-1}(ax)}\right) + 8\cosh^{-1}(ax)\text{Li}_2\left(e^{-\cosh^{-1}(ax)}\right) - 8\text{Li}_3\left(-e^{-\cosh^{-1}(ax)}\right) + 8\text{Li}_3\left(e^{-\cosh^{-1}(ax)}\right) -}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-4*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] - ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]
^2 - 4*ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] + 4*ArcCosh[a*x]^2*Log[1 +
 E^(-ArcCosh[a*x])] + 8*Log[Tanh[ArcCosh[a*x]/2]] - 8*ArcCosh[a*x]*PolyLog[
2, -E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] - 8*P
olyLog[3, -E^(-ArcCosh[a*x])] + 8*PolyLog[3, E^(-ArcCosh[a*x])] - ArcCosh[a
*x]^2*Sech[ArcCosh[a*x]/2]^2 + 4*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(8*a*c^
2)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}\,(ax)^2}{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^2/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\,(ax)^2}{\left(a^2cx^2 - c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.25, size = 288, normalized size = 1.77

$$-\frac{\operatorname{arccosh}(ax)^2 x}{2\left(a^2x^2-1\right)c^2}-\frac{\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}}{a\left(a^2x^2-1\right)c^2}-\frac{\operatorname{arccosh}(ax)^2\ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2a\,c^2}-\operatorname{arccosh}(ax)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)*arccosh(a*x)^2/c^2*x-1/a/(a^2*x^2-1)*arccosh(a*x)/c^2*(a*x
-1)^(1/2)*(a*x+1)^(1/2)-1/2/a/c^2*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*
x+1)^(1/2))-arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+p
olylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+1/2/a/c^2*arccosh(a*x)^2*ln
(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))/a/c^2-polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-2
/a/c^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,ax-\left(a^2x^2-1\right)\log\left(ax+1\right)+\left(a^2x^2-1\right)\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^2}{4\left(a^3c^2x^2-ac^2\right)}-\int-\frac{\left(2\,a^3x^3+\left(2\,a^2x^2-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c^2*x^2 - a*c^2) - integrate(-1/2
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^4
- 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(a\,x\right)^2}{\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^4x^4-2a^2x^2+1}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**2/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.169 $\int \frac{\cosh^{-1}(ax)^2}{(c-a^2cx^2)^3}\,dx$

Optimal. Leaf size=258

$$-\frac{x}{12c^3\left(1-a^2x^2\right)}+\frac{3x\cosh^{-1}(ax)^2}{8c^3\left(1-a^2x^2\right)}+\frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2}+\frac{3\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(a}\right.}{4ac^3}$$

```
[Out] -1/12*x/c^3/(-a^2*x^2+1)+1/6*arccosh(a*x)/a/c^3/(a*x-1)^(3/2)/(a*x+1)^(3/2)
+1/4*x*arccosh(a*x)^2/c^3/(-a^2*x^2+1)^2+3/8*x*arccosh(a*x)^2/c^3/(-a^2*x^2
+1)+3/4*arccosh(a*x)^2*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/6*a
rctanh(a*x)/a/c^3+3/4*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/
2))/a/c^3-3/4*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3
-3/4*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/4*polylog(3,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/4*arccosh(a*x)/a/c^3/(a*x-1)^(1/2)/(a*x+
1)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5689, 5718, 199, 207, 5694, 4182, 2531, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{4ac^3}-\frac{3\text{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}+\text{3Pc}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```

```
[Out] -x/(12*c^3*(1 - a^2*x^2)) + ArcCosh[a*x]/(6*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x
)^(3/2)) - (3*ArcCosh[a*x])/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Arc
Cosh[a*x]^2)/(4*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^2)/(8*c^3*(1 - a^2
*x^2)) + (3*ArcCosh[a*x]^2*ArcTanh[E^ArcCosh[a*x]])/(4*a*c^3) - (5*ArcTanh[
a*x])/(6*a*c^3) + (3*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]])/(4*a*c^3) -
(3*ArcCosh[a*x]*PolyLog[2, E^ArcCosh[a*x]])/(4*a*c^3) - (3*PolyLog[3, -E^Ar
cCosh[a*x]])/(4*a*c^3) + (3*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} - \frac{a\int \frac{x\cosh^{-1}(ax)}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{2c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^2}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)^2}{8c^3\left(1-a^2x^2\right)} - \frac{\int \frac{1}{\left(-1+a^2x^2\right)^2}\,dx}{6c^3} + \frac{(3a}{}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(a}{4c^3\left(1-a^2x\right.}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(a}{4c^3\left(1-a^2x\right.}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(a}{4c^3\left(1-a^2x\right.}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(a}{4c^3\left(1-a^2x\right.}$$

$$= -\frac{x}{12c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)}{6ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{3\cosh^{-1}(ax)}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(a}{4c^3\left(1-a^2x\right.}$$

Mathematica [A] time = 4.81, size = 319, normalized size = 1.24

$$-\frac{72\left(2\cosh^{-1}(ax)\mathrm{Li}_2\left(-e^{-\cosh^{-1}(ax)}\right) - 2\cosh^{-1}(ax)\mathrm{Li}_2\left(e^{-\cosh^{-1}(ax)}\right) + 2\mathrm{Li}_3\left(-e^{-\cosh^{-1}(ax)}\right) - 2\mathrm{Li}_3\left(e^{-\cosh^{-1}(ax}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/192*(80*ArcCosh[a*x]*Coth[ArcCosh[a*x]/2] + 2*(-2 + 9*ArcCosh[a*x]^2)*Cs
ch[ArcCosh[a*x]/2]^2 - 2*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]*
Csch[ArcCosh[a*x]/2]^4 - 3*ArcCosh[a*x]^2*Csch[ArcCosh[a*x]/2]^4 - 160*Log[
Tanh[ArcCosh[a*x]/2]] + 72*(ArcCosh[a*x]^2*Log[1 - E^(-ArcCosh[a*x])] - Arc
Cosh[a*x]^2*Log[1 + E^(-ArcCosh[a*x])] + 2*ArcCosh[a*x]*PolyLog[2, -E^(-Arc
Cosh[a*x])] - 2*ArcCosh[a*x]*PolyLog[2, E^(-ArcCosh[a*x])] + 2*PolyLog[3, -
E^(-ArcCosh[a*x])] - 2*PolyLog[3, E^(-ArcCosh[a*x])]) + 2*(-2 + 9*ArcCosh[a
*x]^2)*Sech[ArcCosh[a*x]/2]^2 + 3*ArcCosh[a*x]^2*Sech[ArcCosh[a*x]/2]^4 - (
32*ArcCosh[a*x]*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 + a*x))^(3/2)*(1 +
a*x)^3) - 80*ArcCosh[a*x]*Tanh[ArcCosh[a*x]/2])/(a*c^3)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\operatorname{arcosh}(ax)^2}{a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^2/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arcosh}(ax)^2}{\left(a^2cx^2 - c\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^2/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.38, size = 443, normalized size = 1.72

$$-\frac{3a^2\operatorname{arccosh}(ax)^2 x^3}{8\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} - \frac{3a\operatorname{arccosh}(ax)\sqrt{ax-1}\,\sqrt{ax+1}\,x^2}{4\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} + \frac{a^2x^3}{12\left(x^4a^4 - 2a^2x^2 + 1\right)c^3} + \frac{5\operatorname{arccosh}(ax)^2 x}{8\left(x^4a^4 - 2a^2x^2 + 1\right)c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*x^3-3/4*a/(a^4*x^4-2*a^2*
x^2+1)/c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*x^2+1/12*a^2/(a^4*x^4-2
*a^2*x^2+1)/c^3*x^3+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*x+11/12/a/
(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/12/(a^
4*x^4-2*a^2*x^2+1)/c^3*x-5/3/a/c^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))
-3/8/a/c^3*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3/4*arccosh
(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/4*polylog(3,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/8/a/c^3*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))+3/4*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^
(1/2))/a/c^3-3/4*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(6\,a^3x^3 - 10\,ax - 3\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log(ax+1) + 3\left(a^4x^4 - 2\,a^2x^2 + 1\right)\log(ax-1)\right)\log\left(ax + \sqrt{ax+1}\,\sqrt{ax}\right.}{16\left(a^5c^3x^4 - 2\,a^3c^3x^2 + ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
)^2/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-1/8*(6*a^5*x^5 - 16*
a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x +
 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^6
*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqr
t(a*x - 1))/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^6*c
^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1)),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^2}{\left(c - a^2\,c\,x^2\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^2(ax)}{a^6x^6-3a^4x^4+3a^2x^2-1}\,dx}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**2/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

3.170 $\int x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=371

$$-\frac{x^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{15c^2}-\frac{2bcx^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2+$$

```
[Out] -856/3375*b^2*(-c^2*d*x^2+d)^(1/2)/c^4+22/3375*b^2*x^2*(-c^2*d*x^2+d)^(1/2)
/c^2+2/125*b^2*x^4*(-c^2*d*x^2+d)^(1/2)-2/15*(a+b*arccosh(c*x))^2*(-c^2*d*x
^2+d)^(1/2)/c^4-1/15*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+1/5*
x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+4/15*a*b*x*(-c^2*d*x^2+d)^(1/
2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/15*b^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(
1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/45*b*x^3*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c*x^5*(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.06, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {5798, 5743, 5759, 5718, 5654, 74, 5662, 100, 12}

$$\frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{2bcx^5\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{1}{5}x^4\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2+\frac{2bx^3\sqrt{d-c^2d}}{45c\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-856*b^2*Sqrt[d - c^2*d*x^2])/(3375*c^4) + (22*b^2*x^2*Sqrt[d - c^2*d*x^2]
)/(3375*c^2) + (2*b^2*x^4*Sqrt[d - c^2*d*x^2])/125 + (4*a*b*x*Sqrt[d - c^2*
d*x^2])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*x*Sqrt[d - c^2*d*x^2
]*ArcCosh[c*x])/(15*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*x^3*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(45*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*
c*x^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^4) - (x^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(15*c^2) + (x^4*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^2)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x^3\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{5}x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}}{5\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{2bcx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{15c^2}$$

$$= \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{2bx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2bcx^5\sqrt{}}{}$$

$$= -\frac{2b^2x^2\sqrt{d-c^2dx^2}}{135c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4abx\sqrt{d-c^2dx^2}}{15c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{8b^2\sqrt{d-c^2dx^2}}{27c^4} + \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{4}{15c^3}$$

$$= -\frac{856b^2\sqrt{d-c^2dx^2}}{3375c^4} + \frac{22b^2x^2\sqrt{d-c^2dx^2}}{3375c^2} + \frac{2}{125}b^2x^4\sqrt{d-c^2dx^2} + \frac{}{15}$$

Mathematica [A] time = 0.44, size = 237, normalized size = 0.64

$$\frac{\sqrt{d-c^2dx^2}\left(225a^2\left(c^2x^2-1\right)^2\left(3c^2x^2+2\right)-30abcx\sqrt{cx-1}\sqrt{cx+1}\left(9c^4x^4-5c^2x^2-30\right)+30b\cosh^{-1}(cx)\left(15a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2) - 30*a*b*c*x
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-30 - 5*c^2*x^2 + 9*c^4*x^4) + 2*b^2*(428 -
439*c^2*x^2 - 16*c^4*x^4 + 27*c^6*x^6) + 30*b*(15*a*(-1 + c^2*x^2)^2*(2 + 3
*c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(30 + 5*c^2*x^2 - 9*c^4*x^4)
)*ArcCosh[c*x] + 225*b^2*(-1 + c^2*x^2)^2*(2 + 3*c^2*x^2)*ArcCosh[c*x]^2))/
(3375*c^4*(-1 + c^2*x^2))
```

fricas [A] time = 0.52, size = 349, normalized size = 0.94

$$\frac{225\left(3\,b^2c^6x^6-4\,b^2c^4x^4-b^2c^2x^2+2\,b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(9\,abc^5x^5-5\,abc^3x^3-30\,abcx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/3375*(225*(3*b^2*c^6*x^6 - 4*b^2*c^4*x^4 - b^2*c^2*x^2 + 2*b^2)*sqrt(-c^2
*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 - 5*a*b*c^3*
x^3 - 30*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^5*x
```

```
^5 - 5*b^2*c^3*x^3 - 30*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 1
5*(3*a*b*c^6*x^6 - 4*a*b*c^4*x^4 - a*b*c^2*x^2 + 2*a*b)*sqrt(-c^2*d*x^2 + d
))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 - 4*(225*a^2
 + 8*b^2)*c^4*x^4 - (225*a^2 + 878*b^2)*c^2*x^2 + 450*a^2 + 856*b^2)*sqrt(-
c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.73, size = 1284, normalized size = 3.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/5*x^2*(-c^2*d*x^2+d)^(3/2)/c^2/d-2/15/d/c^4*(-c^2*d*x^2+d)^(3/2))+b
^2*(1/4000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/(c*x
+1)/c^4/(c*x-1)+1/864*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccos
h(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+
2)/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+
1/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*
x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arcc
osh(c*x)+2)/(c*x+1)/c^4/(c*x-1)+1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c
*x)^2+10*arccosh(c*x)+2)/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/800*(-d*(c^2*x^2-1))
^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2
*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c-1)*(-1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*
x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))
/(c*x+1)/c^4/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^4/(c*x-1)+1/80
0*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/(c*x+1)/c^4/(c*x-1))
```

maxima [A] time = 1.00, size = 326, normalized size = 0.88

$$-\frac{1}{15}b^2\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\operatorname{arcosh}(cx)^2-\frac{2}{15}ab\left(\frac{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{c^4d}\right)\text{ar}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/15*b^2*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/
(c^4*d))*arccosh(c*x)^2 - 2/15*a*b*(3*(-c^2*d*x^2 + d)^(3/2)*x^2/(c^2*d) +
2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d))*arccosh(c*x) - 1/15*a^2*(3*(-c^2*d*x^2 +
d)^(3/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(3/2)/(c^4*d)) + 2/3375*b^2*((27*
sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 11*sqrt(c^2*x^2 - 1)*sqrt(-d)*x^2 - 42
8*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/c^2 - 15*(9*c^4*sqrt(-d)*x^5 - 5*c^2*sqrt
(-d)*x^3 - 30*sqrt(-d)*x)*arccosh(c*x)/c^3) - 2/225*(9*c^4*sqrt(-d)*x^5 - 5
*c^2*sqrt(-d)*x^3 - 30*sqrt(-d)*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\sqrt{-d\,(cx-1)\,(cx+1)}\;(a+b\,\mathrm{acosh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

3.171 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=319

$$\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{8c^2}-\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}$$

```
[Out] -1/64*b^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+1/32*b^2*x^3*(-c^2*d*x^2+d)^(1/2)-1/8*
x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+1/4*x^3*(a+b*arccosh(c*x))^
2*(-c^2*d*x^2+d)^(1/2)-1/64*b^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-
1)^(1/2)/(c*x+1)^(1/2)+1/8*b*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/
(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/24*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(
1/2)/b/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.93, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {5798, 5743, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -(b^2*x*Sqrt[d - c^2*d*x^2])/(64*c^2) + (b^2*x^3*Sqrt[d - c^2*d*x^2])/32 -
(b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) + (b*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*c*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(8
*c^2) + (x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (Sqrt[d - c^2*
d*x^2]*(a + b*ArcCosh[c*x])^3)/(24*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\sqrt{d-c^2dx^2}\int x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{4}x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int\frac{x^2\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\sqrt{1+cx}}}{4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{-1+cx}\sqrt{1+cx}} - \frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{8c^2}$$

$$= \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^4\sqrt{d}}{}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{16c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} + \frac{bx^2\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{co}\right.}{8c\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx}{16c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{b^2x\sqrt{d-c^2dx^2}}{64c^2} + \frac{1}{32}b^2x^3\sqrt{d-c^2dx^2} - \frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx}{64c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 2.11, size = 241, normalized size = 0.76

$$-\frac{-96a^2cx\left(2c^2x^2-1\right)\sqrt{d-c^2dx^2} + 96a^2\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + \frac{12ab\sqrt{d-c^2dx^2}\left(8\cosh^{-1}(cx)^2+\cosh\left(4\cosh^{-1}(cx)\right)-4\cos\right.}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}}{768c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/768*(-96*a^2*c*x*(-1 + 2*c^2*x^2)*Sqrt[d - c^2*d*x^2] + 96*a^2*Sqrt[d]*A
rcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (12*a*b*Sqrt[d
- c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh
[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c
^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1
+ 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x)))/c^3
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2x^2\,\mathrm{arcosh}\,(cx)^2 + 2\,abx^2\,\mathrm{arcosh}\,(cx) + a^2x^2\right)\sqrt{-c^2dx^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.64, size = 767, normalized size = 2.40

$$-\frac{a^2x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{4c^2d}+\frac{a^2x\sqrt{-c^2d\,x^2+d}}{8c^2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8c^2\sqrt{c^2d}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,x^2}{8\sqrt{cx+1}\,c\sqrt{cx-1}}-\frac{b^2\sqrt{-d\left(c\right.}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a^2*x*(-c^2*d*x^2+d)^(3/2)/c^2/d+1/8*a^2/c^2*x*(-c^2*d*x^2+d)^(1/2)+1/
8*a^2/c^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/8*
b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2-1
/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^
4-1/24*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c
*x)^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)^2*x^5
-3/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3+1/8*b^2*
(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)^2*x-1/64*b^2*(-d*(c
^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*arccosh(c*x)+1/32*b^2*(-d*
(c^2*x^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*x^5-3/64*b^2*(-d*(c^2*x^2-1))^(1/2)/
(c*x+1)/(c*x-1)*x^3+1/64*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/c^2/(c*x-1)*x-1
/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^
2-1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/8*a*b*
(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2+1/2*a*b*(-d*(c^2*x
^2-1))^(1/2)/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5-3/4*a*b*(-d*(c^2*x^2-1))^
(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+
1)/c^2/(c*x-1)*arccosh(c*x)*x-1/64*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)
/c^3/(c*x-1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}a^2\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2}-\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2d}+\frac{\sqrt{d}\,\arcsin(cx)}{c^3}\right)+\int\sqrt{-c^2dx^2+d}\,b^2x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/8*a^2*(sqrt(-c^2*d*x^2 + d)*x/c^2 - 2*(-c^2*d*x^2 + d)^(3/2)*x/(c^2*d) +
sqrt(d)*arcsin(c*x)/c^3) + integrate(sqrt(-c^2*d*x^2 + d)*b^2*x^2*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2*d*x^2 + d)*a*b*x^2*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{-d\,(cx-1)\,(cx+1)}\,\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

3.172 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=186

$$\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{2}{27}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/c^2/d-14/27*b^2*(-c^2*d*x^2+
d)^(1/2)/c^2+2/27*b^2*x^2*(-c^2*d*x^2+d)^(1/2)+2/3*b*x*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/9*b*c*x^3*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 194, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5798, 5718, 5680, 12, 460, 74}

$$-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{9\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{cx-1}\sqrt{cx+1}}-\frac{(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3c^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-14*b^2*Sqrt[d - c^2*d*x^2])/(27*c^2) + (2*b^2*x^2*Sqrt[d - c^2*d*x^2])/27
 + (2*b*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*c*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (2*b*c*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^2)/(3*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2dx=\frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2}-\frac{\left(2b\sqrt{d-c^2dx^2}\right.}{}$$

$$=\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{9\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcx^3\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{9\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{2}{27}b^2x^2\sqrt{d-c^2dx^2}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2bcx^3\sqrt{d}}{}$$

$$=-\frac{14b^2\sqrt{d-c^2dx^2}}{27c^2}+\frac{2}{27}b^2x^2\sqrt{d-c^2dx^2}+\frac{2bx\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3c\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 0.37, size = 181, normalized size = 0.97

$$\frac{\sqrt{d-c^2dx^2}\left(9a^2\left(c^2x^2-1\right)^2-6abcx\sqrt{cx-1}\sqrt{cx+1}\left(c^2x^2-3\right)+6b\cosh^{-1}(cx)\left(3a\left(c^2x^2-1\right)^2+bcx\sqrt{cx-1}\right.\right.}{27c^2\left(c^2x^2-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(-6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-3 + c^2*x^2
) + 9*a^2*(-1 + c^2*x^2)^2 + 2*b^2*(7 - 8*c^2*x^2 + c^4*x^4) + 6*b*(b*c*x*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(3 - c^2*x^2) + 3*a*(-1 + c^2*x^2)^2)*ArcCosh[c
*x] + 9*b^2*(-1 + c^2*x^2)^2*ArcCosh[c*x]^2))/(27*c^2*(-1 + c^2*x^2))
```

fricas [A] time = 0.57, size = 280, normalized size = 1.51

$$\frac{9\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(abc^3x^3-3\,abcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] 1/27*(9*(b^2*c^4*x^4 - 2*b^2*c^2*x^2 + b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 - 3*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqr
t(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 - 3*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2
*x^2 - 1) - 3*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(-c^2*d*x^2 + d))*log
(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 - 2*(9*a^2 + 8*b^2)*c^
2*x^2 + 9*a^2 + 14*b^2)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.36, size = 726, normalized size = 3.90

$$-\frac{a^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{3c^2d}+b^2\left(\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\sqrt{cx+1}\,\sqrt{cx-1}\,xc+1\right)(9}{216\,(cx+1)\,c^2\,(cx-1)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/3*a^2/c^2/d*(-c^2*d*x^2+d)^(3/2)+b^2*(1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^
4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8
*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccos
h(c*x)^2-2*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+
2)/(c*x+1)/c^2/(c*x-1)+1/216*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(
9*arccosh(c*x)^2+6*arccosh(c*x)+2)/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/72*(-d*(c^
2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-
3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/(c*x+1)/c^2/(c*x-1
)-1/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-
1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)-1/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/(c*x+1)/c^2/(c*x-1)+1/72*
(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))/(c*x+1)/c^2
/(c*x-1))
```

maxima [A] time = 1.24, size = 204, normalized size = 1.10

$$\frac{2}{27}b^2\left(\frac{\sqrt{c^2x^2-1}\,\sqrt{-d}\,dx^2-\frac{7\sqrt{c^2x^2-1}\,\sqrt{-d}\,d}{c^2}}{d}-\frac{3\left(c^2\sqrt{-d}\,dx^3-3\sqrt{-d}\,dx\right)\operatorname{arcosh}(cx)}{cd}\right)-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\operatorname{arcosh}(c}{3\,c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 - 7*sqrt(c^2*x^2 - 1)*sqrt(-d)*
d/c^2)/d - 3*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*arccosh(c*x)/(c*d)) - 1/
3*(-c^2*d*x^2 + d)^(3/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/3*(-c^2*d*x^2 + d)^
(3/2)*a*b*arccosh(c*x)/(c^2*d) - 2/9*(c^2*sqrt(-d)*d*x^3 - 3*sqrt(-d)*d*x)*
a*b/(c*d) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\;(a+b\,\mathrm{acosh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

3.173 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=204

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}b^2$$

```
[Out] 1/4*b^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1
/2)+1/4*b^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-1/2*b*c*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-1/6*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5713, 5683, 5676, 5662, 90, 52}

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2-\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}b^2$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (b^2*x*Sqrt[d - c^2*d*x^2])/4 + (b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(4*c
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (x*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/2 - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.$$

$$= \frac{1}{4}b^2x\sqrt{d-c^2dx^2} - \frac{bcx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{1}{2}x\sqrt{d-c^2}$$

$$= \frac{1}{4}b^2x\sqrt{d-c^2dx^2} + \frac{b^2\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcx^2\sqrt{d-c^2dx^2}\left(a+\right.}{2\sqrt{-1+cx}\sqrt{}}$$

Mathematica [A] time = 1.09, size = 235, normalized size = 1.15

$$\frac{1}{24}\left(12a^2x\sqrt{d-c^2dx^2} - \frac{12a^2\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{c} - \frac{6ab\sqrt{d-c^2dx^2}\left(2\cosh^{-1}(cx)^2+\cosh\left(2\cosh^{-1}(cx)\right)-\right.}{c\sqrt{\frac{cx-1}{cx+1}}(cx+1)}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (12*a^2*x*Sqrt[d - c^2*d*x^2] - (12*a^2*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*
x^2])/(Sqrt[d]*(-1 + c^2*x^2))])/c - (6*a*b*Sqrt[d - c^2*d*x^2]*(2*ArcCosh[
c*x]^2 + Cosh[2*ArcCosh[c*x]] - 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]))/(c*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (b^2*Sqrt[d - c^2*d*x^2]*(-4*ArcCosh[
c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*ArcCosh[c*x]^2)*Sinh[
2*ArcCosh[c*x]]))/(c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))/24
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.30, size = 528, normalized size = 2.59

$$\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2}+\frac{a^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,c\operatorname{arccosh}(cx)\,x^2}{2\sqrt{cx+1}\,\sqrt{cx-1}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,c^2x^3}{4\,(cx+1)\,(cx-1)}-\frac{b^2\sqrt{-}}{4\,(cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 1/2*a^2*x*(-c^2*d*x^2+d)^(1/2)+1/2*a^2*d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)
*x/(-c^2*d*x^2+d)^(1/2))-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*c*arccosh(c*x)*x^2+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*
c^2*x^3-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*x-1/6*b^2*(-d*(c^2*x
^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^3+1/2*b^2*(-d*(c^2*
x^2-1))^(1/2)/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1
))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/(c
*x+1)^(1/2)/(c*x-1)^(1/2)/c*arccosh(c*x)-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1
)/(c*x-1)*c^2*arccosh(c*x)*x^3-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)
/(c*x-1)^(1/2)*c*x^2-a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)/(c*x-1)*arccosh(c*x
)*x+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{-c^2dx^2+d}\,x+\frac{\sqrt{d}\,\arcsin(cx)}{c}\right)a^2+\int\sqrt{-c^2dx^2+d}\,b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2+2\sqrt{-c^2dx^2+d}\,ab\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(-c^2*d*x^2 + d)*x + sqrt(d)*arcsin(c*x)/c)*a^2 + integrate(sqrt(-
c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(-c^2*d
*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))^2\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-d(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

3.174 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=402

$$\frac{2ib\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib\sqrt{d-c^2dx^2}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2abcx\sqrt{}}{\sqrt{cx-1}}$$

```
[Out] 2*b^2*(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*
c*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c*x*arccosh(c*x)
*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*(a+b*arccosh(c*x))^2*ar
ctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*(a+b*arcco
sh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*polylog
(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {5798, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74}

$$\frac{2ib\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ib\sqrt{d-c^2dx^2}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] 2*b^2*Sqrt[d - c^2*d*x^2] - (2*a*b*c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (2*b^2*c*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 - (2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + ((2*I)*b*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLo
g[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^Arc
Cosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*Sqrt[d - c^2*d*x^2]
*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\sqrt{1+cx}}dx}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(2i}{}$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\sqrt{d-c^2dx^2}\ \mathrm{Su}}{}$$

$$= -\frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} + \sqrt{d-c^2dx^2}\left(a+\right.$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= 2b^2\sqrt{d-c^2dx^2} - \frac{2abcx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{2b^2cx\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 1.34, size = 449, normalized size = 1.12

$$a^2\sqrt{d-c^2dx^2} - a^2\sqrt{d}\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) + a^2\sqrt{d}\log(cx) + \frac{2ab\sqrt{d-c^2dx^2}\left(i\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right) - i\mathrm{Li}_2\left(ie^{-\cosh^{-1}(cx)}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] a^2*Sqrt[d - c^2*d*x^2] + a^2*Sqrt[d]*Log[c*x] - a^2*Sqrt[d]*Log[d + Sqrt[d
]*Sqrt[d - c^2*d*x^2]] + (2*a*b*Sqrt[d - c^2*d*x^2]*(-(c*x) + Sqrt[(-1 + c*
x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] +
I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log[1 + I/E^ArcCo
sh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I/E^ArcCosh[c*x
]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*Sqrt[d - c^2*d*x^2]*(2 +
(2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) + ArcCosh[c*x]^2
+ (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 + I/E
^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*ArcCosh
[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] - 2*
PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)))
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\,\mathrm{arcosh}\,(cx)^2 + 2\,ab\,\mathrm{arcosh}\,(cx) + a^2\right)}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2\sqrt{-c^2d\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(\sqrt{d}\,\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\sqrt{-c^2dx^2+d}\right)a^2+\int\frac{\sqrt{-c^2dx^2+d}\,b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{x}+\frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] -(sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - sqrt(-c
^2*d*x^2 + d))*a^2 + integrate(sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/x + 2*sqrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x, x)
```

3.175 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=234

$$\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}+\frac{2bc\sqrt{d-}}{}$$

```
[Out] -(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x+c*(a+b*arccosh(c*x))^2*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/3*c*(a+b*arccosh(c*x))^3*(-c^2*
d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c*(a+b*arccosh(c*x))*ln(1+
1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-b^2*c*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*
d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5738, 5660, 3718, 2190, 2279, 2391, 5676}

$$\frac{b^2c\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}-\frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{cx-1}\sqrt{cx+1}}-$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x) - (c*Sqrt[d - c^2*d*x^2]*
(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (b^2*c*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(2*ArcCo
sh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{\left(2bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-1}(cx)}{x}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 1.71, size = 270, normalized size = 1.15

$$-\frac{a^2\sqrt{d-c^2dx^2}}{x}+a^2c\sqrt{d}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+abc\sqrt{d-c^2dx^2}\left(\frac{2\log(cx)+\cosh^{-1}(cx)^2}{\sqrt{\frac{cx-1}{cx+1}}(cx+1)}-\frac{2\cosh^{-1}(cx)}{cx}\right)+\frac{1}{3}b^2c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -((a^2*Sqrt[d - c^2*d*x^2])/x) + a^2*c*Sqrt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))] + a*b*c*Sqrt[d - c^2*d*x^2]*((-2*ArcCosh[c*x
])/(c*x) + (ArcCosh[c*x]^2 + 2*Log[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x))) + (b^2*c*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*((-3*ArcCosh[c*x])/(c*x) +
 (ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])/(Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[2,
 -E^(-2*ArcCosh[c*x])])/(1 - c*x)))/3
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2\right)}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.62, size = 582, normalized size = 2.49

$$-\frac{a^2\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}{dx}-a^2c^2x\sqrt{-c^2d\,x^2+d}-\frac{a^2c^2d\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)^3c}{3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{b^2\sqrt{-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(3/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(1/2)-a^2*c^2*d/(c^2
*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*c-b^2*(-d*(c^2*x^2-1)
)^(1/2)*arccosh(c*x)^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c-b^2*(-d*(c^2*x^2-1))^(
1/2)*arccosh(c*x)^2/(c*x+1)/(c*x-1)*x*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/(c*x+1)/(c*x-1)/x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c+a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)^2*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)*x*
c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/(c*x+1)/(c*x-1)/x+2*a*b*(-d*(
c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\left(c\sqrt{d}\,\arcsin(cx)+\frac{\sqrt{-c^2dx^2+d}}{x}\right)a^2+\int\frac{\sqrt{-c^2dx^2+d}\,b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{x^2}+\frac{2\sqrt{-c^2dx^2+d}\,ab\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] -(c*sqrt(d)*arcsin(c*x) + sqrt(-c^2*d*x^2 + d)/x)*a^2 + integrate(sqrt(-c^2
*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*sqrt(-c^2*
d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{d-c^2\,d\,x^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**2, x)
```

3.176 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=427

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bc}{}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^2-b*c*(a+b*arccosh(c*x))*(
-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+c^2*(a+b*arccosh(c*x))^2*
arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b^2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+I*b*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+I*b^2*c^2*polylog(3,-
I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5738, 5662, 92, 205, 5761, 4180, 2531, 2282, 6589}

$$-\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{ibc^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\right.}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -((b*c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (c^2*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b^2*c^2*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt
[1 + c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*c^2*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (I*b*c^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E
^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (I*b^2*c^2*Sqrt[d - c^2*d*
x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b
^2*c^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bc\sqrt{d-c^2dx^2}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2} d}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2}$$

$$= -\frac{bc\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2}$$

Mathematica [A] time = 85.47, size = 547, normalized size = 1.28

$$\frac{1}{2}a\left(-\frac{a\sqrt{d-c^2dx^2}}{x^2} + ac^2\sqrt{d}\log\left(\sqrt{d}\sqrt{d-c^2dx^2}+d\right) - ac^2\sqrt{d}\log(x) + \frac{2bd(cx+1)\left(ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\operatorname{Li}_2\left(-ie^{-\cosh}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (a*(-((a*Sqrt[d - c^2*d*x^2])/x^2) - a*c^2*Sqrt[d]*Log[x] + a*c^2*Sqrt[d]*L
og[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*b*d*(1 + c*x)*(c*x*Sqrt[(-1 + c*x)
/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] + I*c^2*x^2*Sqrt[(-1 + c*x)/(
1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[(-1 + c*x)
/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2])))/2 +
(b^2*c^2*Sqrt[d - c^2*d*x^2]*((2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(
c*x - c^2*x^2) - ArcCosh[c*x]^2/(c^2*x^2) - (I*((4*I)*ArcTan[Tanh[ArcCosh[c
*x]/2]] + ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]^2*Log[1 +
 I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - 2*Arc
Cosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcCosh[c*x]]
- 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))
/2
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^2\sqrt{-c^2d\,x^2+d}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(c^2\sqrt{d}\,\log\left(\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-\sqrt{-c^2dx^2+d}\,c^2-\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{dx^2}\right)a^2+\int \frac{\sqrt{-c^2dx^2+d}\,b^2\log\left(cx+\sqrt{c\right.}}{x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/2*(c^2*sqrt(d)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
sqrt(-c^2*d*x^2 + d)*c^2 - (-c^2*d*x^2 + d)^(3/2)/(d*x^2))*a^2 + integrate(
sqrt(-c^2*d*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 + 2*s
qrt(-c^2*d*x^2 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{acosh}\,(c\,x))^2\sqrt{d-c^2\,d\,x^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**3, x)
```

3.177 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=336

$$-\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3}-\frac{c^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/d/x^3+1/3*b^2*c^2*(-c^2*d*x^
2+d)^(1/2)/x-1/3*b^2*c^3*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-1/3*b*c*(-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^
2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*c^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/3*b*c^3*(a+b*arccosh(c*x))*ln(1+1/(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/3*b^2*c^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.58, antiderivative size = 344, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5724, 5729, 97, 12, 52, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b^2c^3\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}+\frac{c^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\right.}{3x^2\sqrt{cx-1}\sqrt{cx+1}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*ArcCosh[
c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*(1 - c^2*x^2)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^3*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
((1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) -
(2*b*c^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x]
)])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b^2*c^3*Sqrt[d - c^2*d*x^2]*PolyLog
[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bc\sqrt{d-c^2dx^2}\right)\int}{3\sqrt{-1+}}$$

$$= -\frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-cx)(1+cx)\sqrt{d-c^2}}{3}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{(1-c}{}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{c^3\sqrt{d}}{}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2d}}{3x^2\sqrt{-1+}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2d}}{3x^2\sqrt{-1+}}$$

$$= \frac{b^2c^2\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc\left(1-c^2x^2\right)\sqrt{d-c^2d}}{3x^2\sqrt{-1+}}$$

Mathematica [A] time = 1.01, size = 304, normalized size = 0.90

$$-\frac{d(cx+1)\left(a^2c^3x^3-a^2c^2x^2-a^2cx+a^2-2abc^3x^3\sqrt{\frac{cx-1}{cx+1}}\log(cx)-b\cosh^{-1}(cx)\left(-2a(cx-1)^2(cx+1)+2bc^3x^3\sqrt{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] -1/3*(d*(1 + c*x)*(a^2 - a^2*c*x - a^2*c^2*x^2 - b^2*c^2*x^2 + a^2*c^3*x^3
+ b^2*c^3*x^3 - a*b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*(-1 + c*x + c^2*x^
2 + c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2 - b*ArcCosh[c
*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*a*(-1 + c*x)^2*(1 + c*x) + 2*b*c^
3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x])]) - 2*a*b*c^3*
x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[c*x] + b^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 +
c*x)]*PolyLog[2, -E^(-2*ArcCosh[c*x])]))/(x^3*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.90, size = 2633, normalized size = 7.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x)
```

```
[Out] -1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(3/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x
^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-2/3*b^2*(-d*(c^2*x^2-1))
^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+b^2*(-d
*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x
)^2*c^8+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1)/
(c*x-1)*arccosh(c*x)*c^8+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^
2+1)*x^5/(c*x+1)/(c*x-1)*c^8-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^
2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-
3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^
4-3*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-3*b^2*(-d*(c^2*x^2-1))^(1/2
)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^6-5/3*b^2*(-
d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)
^2*c^2+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c
*x-1)*arccosh(c*x)^2*c^4-a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)
*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*
x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-1/3*a*b*(-d*(
c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-
1)^(1/2)*arccosh(c*x)^2*c^5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2
+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-1/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*
x)*c-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(
c*x-1)^(1/2)*arccosh(c*x)^2*c^7+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3
*c^2*x^2+1)*x^3*c^6-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*
x^3*c^6+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*c^4+2/3*b^
2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c^3-1/3
*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*
c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+2*a*b*(-d*(c^2*
x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*c^5+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+1
)/(c*x-1)*arccosh(c*x)*c^8-6*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^
2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+20/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(
3*c^4*x^4-3*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-10/3*a*b*(-d*(c^2
*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^2-1
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3*arccosh(c*x)*c^6+
1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x*arccosh(c*x)*c^4-1
```

```
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*c^3+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^5/(c*x+
1)/(c*x-1)*c^8-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^3/(
c*x+1)/(c*x-1)*c^6+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x
/(c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)
/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+
1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4
*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3+b^2*(-d*(c
^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccos
h(c*x)*c^3-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x^4/(c*x+1)^(
1/2)/(c*x-1)^(1/2)*c^7+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*c^2*x^2+1)*x
^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^4*x^4-3*
c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*c^3-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-2
/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(c^4d^2\sqrt{-\frac{1}{c^4d}}\log\left(x^2-\frac{1}{c^2}\right)+i\,(-1)^{-2\,c^2dx^2+2\,d}\,c^2d^{\frac{3}{2}}\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)+\frac{\sqrt{-c^4dx^4+2\,c^2dx^2-d}\,d}{x^2}\right)abc}{3\,d}+\frac{1}{3}\,b^2\left(\left(c^2\sqrt{d}\,x^2-\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3*(c^4*d^2*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d
)*c^2*d^(3/2)*log(-2*c^2*d + 2*d/x^2) + sqrt(-c^4*d*x^4 + 2*c^2*d*x^2 - d)*
d/x^2)*a*b*c/d + 1/3*b^2*((c^2*sqrt(d)*x^2 - sqrt(d))*sqrt(c*x + 1)*sqrt(-c
*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 - 3*integrate(2/3*((c*
x + 1)*sqrt(c*x - 1)*c^2*sqrt(d)*x + (c^3*sqrt(d)*x^2 - c*sqrt(d))*sqrt(c*x
 + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c*x^4 + sqrt(
c*x + 1)*sqrt(c*x - 1)*x^3), x)) - 2/3*(-c^2*d*x^2 + d)^(3/2)*a*b*arccosh(c
*x)/(d*x^3) - 1/3*(-c^2*d*x^2 + d)^(3/2)*a^2/(d*x^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{d-c^2\,d\,x^2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(-c**2*d*x**2+d)**(1/2)/x**4,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2/x**4, x)
```

3.178 $\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=495

$$-\frac{dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{35c^2}-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{7}x^4\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh\right.$$

```
[Out] 1/7*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-37384/385875*b^2*d*(-c^2*
d*x^2+d)^(1/2)/c^4+3358/385875*b^2*d*x^2*(-c^2*d*x^2+d)^(1/2)/c^2+484/42875
*b^2*d*x^4*(-c^2*d*x^2+d)^(1/2)-2/343*b^2*c^2*d*x^6*(-c^2*d*x^2+d)^(1/2)-2/
35*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^4-1/35*d*x^2*(a+b*arccosh(
c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+3/35*d*x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^
2+d)^(1/2)+4/35*a*b*d*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+4/35*b^2*d*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+2/105*b*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-16/175*b*c*d*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+2/49*b*c^3*d*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.67, antiderivative size = 507, normalized size of antiderivative = 1.02, number of steps used = 26, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5759, 5718, 5654, 74, 5662, 100, 12, 14, 5731, 460}

$$\frac{4abdx\sqrt{d-c^2dx^2}}{35c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}-\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{35}dx$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-37384*b^2*d*Sqrt[d - c^2*d*x^2])/(385875*c^4) + (3358*b^2*d*x^2*Sqrt[d -
c^2*d*x^2])/(385875*c^2) + (484*b^2*d*x^4*Sqrt[d - c^2*d*x^2])/42875 - (2*b
^2*c^2*d*x^6*Sqrt[d - c^2*d*x^2])/343 + (4*a*b*d*x*Sqrt[d - c^2*d*x^2])/(35
*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*b^2*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh
[c*x])/(35*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*x^3*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(105*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (16*b*c*d*x
^5*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(175*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (2*b*c^3*d*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(
35*c^4) - (d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(35*c^2) + (3*
d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/35 + (d*x^4*(1 - c*x)*(1
+ c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
```

```
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{7}dx^4(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(3d\sqrt{d-c^2\ldots\right.}{}$$

$$= -\frac{2bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a+\ldots\right.}{49\sqrt{-1+cx}\,\sqrt{1\ldots}}$$

$$= -\frac{16bcdx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{175\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3dx^7\sqrt{d-c^2dx^2}\left(a\ldots\right.}{49\sqrt{-1+cx}\ldots}$$

$$= \frac{6}{875}b^2dx^4\sqrt{d-c^2dx^2} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2dx^2} + \frac{2bdx^3\sqrt{d-c^2dx\ldots}}{105c\sqrt{-1\ldots}}$$

$$= -\frac{2b^2dx^2\sqrt{d-c^2dx^2}}{315c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2\ldots}$$

$$= \frac{22b^2dx^2\sqrt{d-c^2dx^2}}{7875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx^2}}{42875} - \frac{2}{343}b^2c^2dx^6\sqrt{d-c^2\ldots}$$

$$= -\frac{8b^2d\sqrt{d-c^2dx^2}}{63c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-c^2dx\ldots}}{42875}$$

$$= -\frac{856b^2d\sqrt{d-c^2dx^2}}{7875c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-c^2\ldots}}{42875}$$

$$= -\frac{37384b^2d\sqrt{d-c^2dx^2}}{385875c^4} + \frac{3358b^2dx^2\sqrt{d-c^2dx^2}}{385875c^2} + \frac{484b^2dx^4\sqrt{d-\ldots}}{42875}$$

Mathematica [A] time = 0.58, size = 262, normalized size = 0.53

$$-\frac{d\sqrt{d-c^2dx^2}\left(11025a^2\left(5c^2x^2+2\right)\left(c^2x^2-1\right)^3 - 210abcx\sqrt{cx-1}\,\sqrt{cx+1}\left(75c^6x^6-168c^4x^4+35c^2x^2+210\right) - \ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/385875*(d*Sqrt[d - c^2*d*x^2]*(11025*a^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2
) - 210*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^
4 + 75*c^6*x^6) + 2*b^2*(-18692 + 20371*c^2*x^2 + 499*c^4*x^4 - 3303*c^6*x^
6 + 1125*c^8*x^8) - 210*b*(-105*a*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2) + b*c*x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(210 + 35*c^2*x^2 - 168*c^4*x^4 + 75*c^6*x^6))
*ArcCosh[c*x] + 11025*b^2*(-1 + c^2*x^2)^3*(2 + 5*c^2*x^2)*ArcCosh[c*x]^2))
/(c^4*(-1 + c^2*x^2))
```

fricas [A] time = 0.68, size = 432, normalized size = 0.87

$$-\frac{11025\left(5\,b^2c^8dx^8 - 13\,b^2c^6dx^6 + 9\,b^2c^4dx^4 + b^2c^2dx^2 - 2\,b^2d\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2 - 210\left(75\,abc\ldots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] -1/385875*(11025*(5*b^2*c^8*d*x^8 - 13*b^2*c^6*d*x^6 + 9*b^2*c^4*d*x^4 + b^
2*c^2*d*x^2 - 2*b^2*d)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2
- 210*(75*a*b*c^7*d*x^7 - 168*a*b*c^5*d*x^5 + 35*a*b*c^3*d*x^3 + 210*a*b*c*
d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((75*b^2*c^7*d*x^7 - 168*
b^2*c^5*d*x^5 + 35*b^2*c^3*d*x^3 + 210*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt
(c^2*x^2 - 1) - 105*(5*a*b*c^8*d*x^8 - 13*a*b*c^6*d*x^6 + 9*a*b*c^4*d*x^4 +
 a*b*c^2*d*x^2 - 2*a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)
) + (1125*(49*a^2 + 2*b^2)*c^8*d*x^8 - 9*(15925*a^2 + 734*b^2)*c^6*d*x^6 +
(99225*a^2 + 998*b^2)*c^4*d*x^4 + (11025*a^2 + 40742*b^2)*c^2*d*x^2 - 2*(11
025*a^2 + 18692*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^6*x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.77, size = 1952, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] a^2*(-1/7*x^2*(-c^2*d*x^2+d)^(5/2)/c^2/d-2/35/d/c^4*(-c^2*d*x^2+d)^(5/2))+b
^2*(-1/43904*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/
16000*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)
/c^4/(c*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(
c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)
+2)*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c
*x-1)+1/1152*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2
+6*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)+1/16000*(-d*(c^2*x^2-1))^(1/2)*(-1
6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25
*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1)-1/43904*(-d*(c^2*x
^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arcco
sh(c*x)^2+14*arccosh(c*x)+2)*d/(c*x+1)/c^4/(c*x-1))+2*a*b*(-1/6272*(-d*(c^2
*x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c
^7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*ar
```

```
ccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6
-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccos
h(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2
*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c+1)*(-1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)
*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^
4/(c*x-1)-3/128*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^
2*x^2-1)*(1+arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1/2
)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)+1/3200*(-d
*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^4/(c*x-1)-1/6272*(-d*(c^
2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)
*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*a
rccosh(c*x))*d/(c*x+1)/c^4/(c*x-1))
```

maxima [A] time = 1.20, size = 388, normalized size = 0.78

$$-\frac{1}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)b^2\operatorname{arcosh}\left(cx\right)^2-\frac{2}{35}\left(\frac{5\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{c^4d}\right)ab\operatorname{arco}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -1/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4
*d))*b^2*arccosh(c*x)^2 - 2/35*(5*(-c^2*d*x^2 + d)^(5/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(5/2)/(c^4*d))*a*b*arccosh(c*x) - 1/35*(5*(-c^2*d*x^2 + d)^(
5/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(5/2)/(c^4*d))*a^2 - 2/385875*b^2*((1
125*sqrt(c^2*x^2 - 1)*c^4*sqrt(-d)*d*x^6 - 2178*sqrt(c^2*x^2 - 1)*c^2*sqrt(
-d)*d*x^4 - 1679*sqrt(c^2*x^2 - 1)*sqrt(-d)*d*x^2 + 18692*sqrt(c^2*x^2 - 1)
*sqrt(-d)*d/c^2)/c^2 - 105*(75*c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)*d*x^5
+ 35*c^2*sqrt(-d)*d*x^3 + 210*sqrt(-d)*d*x)*arccosh(c*x)/c^3) + 2/3675*(75*
c^6*sqrt(-d)*d*x^7 - 168*c^4*sqrt(-d)*d*x^5 + 35*c^2*sqrt(-d)*d*x^3 + 210*s
qrt(-d)*d*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

3.179 $\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=441

$$\frac{bdx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16c\sqrt{cx-1}\sqrt{cx+1}}-\frac{dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{16c^2}-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{48\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] 1/6*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+7/1152*b^2*d*x*(-c^2*d*x^
2+d)^(1/2)/c^2+43/1728*b^2*d*x^3*(-c^2*d*x^2+d)^(1/2)-1/108*b^2*c^2*d*x^5*(
-c^2*d*x^2+d)^(1/2)-1/16*d*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+
1/8*d*x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+7/1152*b^2*d*arccosh(c*
x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*d*x^2*(a+b*a
rccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/48*b*c*d*
x^4*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1
8*b*c^3*d*x^6*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)-1/48*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c^3/(c*x-1)^(1/2)
/(c*x+1)^(1/2)
```

Rubi [A] time = 1.48, antiderivative size = 453, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5759, 5676, 5662, 90, 52, 100, 12, 14, 5731, 460}

$$\frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18\sqrt{cx-1}\sqrt{cx+1}}-\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{8}dx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (7*b^2*d*x*Sqrt[d - c^2*d*x^2])/(1152*c^2) + (43*b^2*d*x^3*Sqrt[d - c^2*d*x
^2])/1728 - (b^2*c^2*d*x^5*Sqrt[d - c^2*d*x^2])/108 + (7*b^2*d*Sqrt[d - c^2
*d*x^2]*ArcCosh[c*x])/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*x^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - (7*b*c*d*x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (b*c^3*d*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
18*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^2)/(16*c^2) + (d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 +
(d*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/6 -
(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(48*b*c^3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
```

```
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2 dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{6}dx^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(d\sqrt{d-c^2d\right.}}{}$$

$$= -\frac{bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{12\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\right.}{18\sqrt{-1+cx}\sqrt{1+}}$$

$$= -\frac{7bcdx^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3dx^6\sqrt{d-c^2dx^2}\left(a+b\right.}{18\sqrt{-1+cx}\sqrt{1}}$$

$$= \frac{1}{64}b^2dx^3\sqrt{d-c^2dx^2} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2} + \frac{bdx^2\sqrt{d-c^2dx^2}\left(\right.}{16c\sqrt{-1+}}$$

$$= -\frac{b^2dx\sqrt{d-c^2dx^2}}{32c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}$$

$$= -\frac{b^2dx\sqrt{d-c^2dx^2}}{128c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}$$

$$= \frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}$$

$$= \frac{7b^2dx\sqrt{d-c^2dx^2}}{1152c^2} + \frac{43b^2dx^3\sqrt{d-c^2dx^2}}{1728} - \frac{1}{108}b^2c^2dx^5\sqrt{d-c^2dx^2}$$

Mathematica [A] time = 4.37, size = 485, normalized size = 1.10

$$\frac{-864a^2d^{3/2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 288a^2cdx\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8c^4x^4-14c^2x^2+3\right)\sqrt{d-c^2dx^2} - 216ab}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-288*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(3
 - 14*c^2*x^2 + 8*c^4*x^4) - 864*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 216*a*b
*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh
[c*x]*Sinh[4*ArcCosh[c*x]]) - 18*b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]
^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4
*ArcCosh[c*x]]) - 12*a*b*d*Sqrt[d - c^2*d*x^2]*(-72*ArcCosh[c*x]^2 + 18*Cos
h[2*ArcCosh[c*x]] - 9*Cosh[4*ArcCosh[c*x]] - 2*Cosh[6*ArcCosh[c*x]] + 12*Ar
cCosh[c*x]*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]] + Sinh[6*ArcCo
sh[c*x]])) + b^2*d*Sqrt[d - c^2*d*x^2]*(288*ArcCosh[c*x]^3 + 12*ArcCosh[c*x
]*(-18*Cosh[2*ArcCosh[c*x]] + 9*Cosh[4*ArcCosh[c*x]] + 2*Cosh[6*ArcCosh[c*x
]]) + 108*Sinh[2*ArcCosh[c*x]] - 27*Sinh[4*ArcCosh[c*x]] - 4*Sinh[6*ArcCosh
[c*x]] - 72*ArcCosh[c*x]^2*(-3*Sinh[2*ArcCosh[c*x]] + 3*Sinh[4*ArcCosh[c*x]
] + Sinh[6*ArcCosh[c*x]])))/(13824*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2c^2dx^4 - a^2dx^2 + \left(b^2c^2dx^4 - b^2dx^2\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^2dx^4 - abdx^2\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2 +}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^4 - a^2*d*x^2 + (b^2*c^2*d*x^4 - b^2*d*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^2*d*x^4 - a*b*d*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^2dx^2 + d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

maple [B] time = 0.96, size = 1021, normalized size = 2.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x-1/3*a*b
*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+11/12*a*b*(-
d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+1/18*a*b*(-d*(c
^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-7/48*a*b*(-d*(c^2*x^
2-1))^(1/2)*d/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+1/16*a*b*(-d*(c^2*x^2-1))^(
1/2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-1/6*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(
c*x+1)*c^4/(c*x-1)*arccosh(c*x)^2*x^7+11/24*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c
*x+1)*c^2/(c*x-1)*arccosh(c*x)^2*x^5+1/16*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x
+1)/c^2/(c*x-1)*arccosh(c*x)^2*x-17/24*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)
/(c*x-1)*arccosh(c*x)*x^3-1/16*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/c^3*arccosh(c*x)^2*d+1/18*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(
1/2)*c^3/(c*x-1)^(1/2)*arccosh(c*x)*x^6-7/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(
c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^4+1/16*b^2*(-d*(c^2*x^2-1))^(1/
2)*d/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2-1/6*a^2*x*(-c^2*d*x^2+d
)^(5/2)/c^2/d+1/16*a^2/c^2*d*x*(-c^2*d*x^2+d)^(1/2)+1/16*a^2/c^2*d^2/(c^2*d
)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/24*a^2/c^2*x*(-c^2*d
*x^2+d)^(3/2)+7/1152*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-1)
^(1/2)+7/1152*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*
arccosh(c*x)-1/48*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^
3*arccosh(c*x)^3*d-17/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arcco
sh(c*x)^2*x^3-1/108*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^4/(c*x-1)*x^7+59
/1728*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)*c^2/(c*x-1)*x^5-7/1152*b^2*(-d*(
c^2*x^2-1))^(1/2)*d/(c*x+1)/c^2/(c*x-1)*x-65/3456*b^2*(-d*(c^2*x^2-1))^(1/2
)*d/(c*x+1)/(c*x-1)*x^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}a^2\left(\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}{c^2} - \frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^2d} + \frac{3\sqrt{-c^2dx^2+d}\,dx}{c^2} + \frac{3\,d^{\frac{3}{2}}\arcsin(cx)}{c^3}\right) + \int \left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] 1/48*a^2*(2*(-c^2*d*x^2 + d)^(3/2)*x/c^2 - 8*(-c^2*d*x^2 + d)^(5/2)*x/(c^2*
d) + 3*sqrt(-c^2*d*x^2 + d)*d*x/c^2 + 3*d^(3/2)*arcsin(c*x)/c^3) + integrat
e((-c^2*d*x^2 + d)^(3/2)*b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 +
 2*(-c^2*d*x^2 + d)^(3/2)*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,(a+b\,\mathrm{acosh}\,(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

3.180 $\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=348

$$\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2d}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx\right.}{15\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/5*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/c^2/d-16/75*b^2*d*(-c^2*x^2+
1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-8/225*b^2*d*(-c^2*x^2+1)^2*(-c
^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-2/125*b^2*d*(-c^2*x^2+1)^3*(-c^2*d*x
^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)+2/5*b*d*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d
)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/15*b*c*d*x^3*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/25*b*c^3*d*x^5*(a+b*arccosh
(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.55, antiderivative size = 361, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 194, 5680, 12, 520, 1247, 698}

$$\frac{2bc^3dx^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{25\sqrt{cx-1}\sqrt{cx+1}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{cx-1}\sqrt{cx+1}}+\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{5c\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-16*b^2*d*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*c^2*(1 - c*x)*(1 + c*x))
- (8*b^2*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*c^2*(1 - c*x)*(1 + c*x
)) - (2*b^2*d*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*c^2*(1 - c*x)*(1 +
c*x)) + (2*b*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(5*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (4*b*c*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c^3*d*x^5*Sqrt[d - c^2*d*x^2]*(a
+ b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*x)^2*(1 +
c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(5*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
```

```
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{d(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2}+\frac{\left(2bd\sqrt{d-\right.}{}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{15\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{15\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{15\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{15\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=\frac{2bdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5c\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4bcdx^3\sqrt{d-c^2dx^2}\left(a+b\,c\right.}{15\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=-\frac{16b^2d\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{75c^2(1-cx)(1+cx)}-\frac{8b^2d\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}}{225c^2(1-cx)(1+cx)}-$$

Mathematica [A] time = 0.53, size = 208, normalized size = 0.60

$$-\frac{d\sqrt{d-c^2dx^2}\left(225a^2\left(c^2x^2-1\right)^3-30abcx\sqrt{cx-1}\,\sqrt{cx+1}\left(3c^4x^4-10c^2x^2+15\right)-30b\cosh^{-1}(cx)\left(bcx\sqrt{cx}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/1125*(d*Sqrt[d - c^2*d*x^2]*(225*a^2*(-1 + c^2*x^2)^3 - 30*a*b*c*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4) + 2*b^2*(-149 + 187*c
^2*x^2 - 47*c^4*x^4 + 9*c^6*x^6) - 30*b*(-15*a*(-1 + c^2*x^2)^3 + b*c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(15 - 10*c^2*x^2 + 3*c^4*x^4))*ArcCosh[c*x] + 225
*b^2*(-1 + c^2*x^2)^3*ArcCosh[c*x]^2))/(c^2*(-1 + c^2*x^2))
```

fricas [A] time = 0.57, size = 367, normalized size = 1.05

$$-\frac{225\left(b^2c^6dx^6-3\,b^2c^4dx^4+3\,b^2c^2dx^2-b^2d\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(3\,abc^5dx^5-10\,abc^3dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] -1/1125*(225*(b^2*c^6*d*x^6 - 3*b^2*c^4*d*x^4 + 3*b^2*c^2*d*x^2 - b^2*d)*sq
rt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(3*a*b*c^5*d*x^5 - 1
0*a*b*c^3*d*x^3 + 15*a*b*c*d*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30
*((3*b^2*c^5*d*x^5 - 10*b^2*c^3*d*x^3 + 15*b^2*c*d*x)*sqrt(-c^2*d*x^2 + d)*
```

```
sqrt(c^2*x^2 - 1) - 15*(a*b*c^6*d*x^6 - 3*a*b*c^4*d*x^4 + 3*a*b*c^2*d*x^2 -
 a*b*d)*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (9*(25*a^2 + 2
*b^2)*c^6*d*x^6 - (675*a^2 + 94*b^2)*c^4*d*x^4 + (675*a^2 + 374*b^2)*c^2*d*
x^2 - (225*a^2 + 298*b^2)*d)*sqrt(-c^2*d*x^2 + d))/(c^4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.45, size = 1270, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/5*a^2/c^2/d*(-c^2*d*x^2+d)^(5/2)+b^2*(-1/4000*(-d*(c^2*x^2-1))^(1/2)*(16
*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*
arccosh(c*x)^2-10*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-
1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d/(c*x+1
)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d
*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c
*x)^2+2*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*x-1)+1/288*(-d*(c^2*x^2-1))^(1/2)*
(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d/(c*x+1)/c^2/(c*
x-1)-1/4000*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)*d/
(c*x+1)/c^2/(c*x-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^
4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x)
)*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-
1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1
)-1/16*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*
(1+arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)+1/96*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(1+3*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1)-1/800*(-d*(c^2*x^2-1
))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^
2*x^2-1)*(1+5*arccosh(c*x))*d/(c*x+1)/c^2/(c*x-1))
```

maxima [A] time = 0.75, size = 278, normalized size = 0.80

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}b^2\operatorname{arcosh}\left(cx\right)^2}{5\,c^2d}-\frac{2}{1125}b^2\left(\frac{9\,\sqrt{c^2x^2-1}\,c^2\sqrt{-d}\,d^2x^4-38\,\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2x^2+\frac{149\,\sqrt{c^2x^2-1}\,\sqrt{-d}\,d^2}{c^2}}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/5*(-c^2*d*x^2 + d)^(5/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/1125*b^2*((9*sqr
t(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 - 38*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2*x^2
 + 149*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/d - 15*(3*c^4*sqrt(-d)*d^2*x^5 -
 10*c^2*sqrt(-d)*d^2*x^3 + 15*sqrt(-d)*d^2*x)*arccosh(c*x)/(c*d)) - 2/5*(-c
^2*d*x^2 + d)^(5/2)*a*b*arccosh(c*x)/(c^2*d) - 1/5*(-c^2*d*x^2 + d)^(5/2)*a
^2/(c^2*d) + 2/75*(3*c^4*sqrt(-d)*d^2*x^5 - 10*c^2*sqrt(-d)*d^2*x^3 + 15*sq
rt(-d)*d^2*x)*a*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\mathrm{acosh}\,(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

3.181 $\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=336

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{1}{4}x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+$$

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+15/64*b^2*d*x*(-c^2*d*x^2+d
)^(1/2)+1/32*b^2*d*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)+3/8*d*x*(a+b*arc
cosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+9/64*b^2*d*arccosh(c*x)*(-c^2*d*x^2+d)^(1
/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/8*b*c*d*x^2*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*d*(-c^2*x^2+1)^2*(a+b*arccos
h(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*d*(a+b*arcco
sh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 348, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5713, 5685, 5683, 5676, 5662, 90, 52, 5716, 38}

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{3}{8}dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{4}dx(1-cx)(cx+1)\sqrt{d-c^2dx^2}\left(a+b\text{co}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (15*b^2*d*x*Sqrt[d - c^2*d*x^2])/64 + (b^2*d*x*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2])/32 + (9*b^2*d*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(64*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - (3*b*c*d*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*d*x*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/8 + (d*x*(1 - c*x)*(1 + c*x)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (d*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x])^3)/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{4}dx(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(3d\sqrt{d-c^2dx^2}\right.}{}$$

$$= \frac{bd\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3}{8}dx\sqrt{d-c^2dx^2}\,(a+$$

$$= \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3bcdx^2\sqrt{d-c^2dx^2}\,(a+b\cosh^{-1}}{8\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} - \frac{3bcdx^2\sqrt{d}}{8}$$

$$= \frac{15}{64}b^2dx\sqrt{d-c^2dx^2} + \frac{1}{32}b^2dx(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{9b^2d\sqrt{d-}}{64c\sqrt{-}}$$

Mathematica [A] time = 2.95, size = 374, normalized size = 1.11

$$\frac{-288a^2d^{3/2}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 96a^2cdx\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2c^2x^2-5\right)\sqrt{d-c^2dx^2} - 192abd\sqrt{d-c^2d}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-96*a^2*c*d*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-5 + 2*c^2*x^2)*Sqrt[d
 - c^2*d*x^2] - 288*a^2*d^(3/2)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan
[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 192*a*b*d*Sqrt[d - c
^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*Arc
Cosh[c*x]])) - 32*b^2*d*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c
*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) +
 12*a*b*d*Sqrt[d - c^2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*
ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]) + b^2*d*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[
c*x]^3 + 12*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Si
nh[4*ArcCosh[c*x]]))/(768*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2c^2dx^2-a^2d+\left(b^2c^2dx^2-b^2d\right)\text{arcosh}\,(cx)^2+2\left(abc^2dx^2-abd\right)\text{arcosh}\,(cx)\right)\sqrt{-c^2dx^2+d}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.38, size = 775, normalized size = 2.31

$$\frac{x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}a^2}{4}+\frac{3a^2dx\sqrt{-c^2d\,x^2+d}}{8}+\frac{3a^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8\sqrt{c^2d}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,d\,c^3\mathrm{arccosh}\left(cx\right)x^4}{8\sqrt{cx+1}\,\sqrt{cx-1}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/4*x*(-c^2*d*x^2+d)^(3/2)*a^2+3/8*a^2*d*x*(-c^2*d*x^2+d)^(1/2)+3/8*a^2*d^2
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/8*b^2*(-d*(c^
2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*arccosh(c*x)*x^4-5/8*b^2*
(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*arccosh(c*x)*x^2-1/8
*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^3*d+
17/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c*arccosh(c*
x)-17/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*x-1/4*b^2*(-d*(c^2*x^
2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)^2*x^5+7/8*b^2*(-d*(c^2*x^2-1
))^(1/2)*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)^2*x^3-5/8*b^2*(-d*(c^2*x^2-1))^
(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x-1/32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/
(c*x+1)/(c*x-1)*c^4*x^5+19/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*
c^2*x^3-3/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*arccos
h(c*x)^2*d+1/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3
*x^4-5/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c*x^2-1/2
*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^4*arccosh(c*x)*x^5+7/4*a*b*
(-d*(c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*c^2*arccosh(c*x)*x^3-5/4*a*b*(-d*(
c^2*x^2-1))^(1/2)*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x+17/64*a*b*(-d*(c^2*x^2-1
))^(1/2)*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\left(2\left(-c^2dx^2+d\right)^{\frac{3}{2}}x+3\sqrt{-c^2dx^2+d}\,dx+\frac{3\,d^{\frac{3}{2}}\arcsin\left(cx\right)}{c}\right)a^2+\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2*(-c^2*d*x^2 + d)^(3/2)*x + 3*sqrt(-c^2*d*x^2 + d)*d*x + 3*d^(3/2)*ar
csin(c*x)/c)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2, x)
```

3.182 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=573

$$\frac{2ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ibd\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2a}{\sqrt{}}$$

```
[Out] 1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+68/27*b^2*d*(-c^2*d*x^2+d)^(1
/2)-2/27*b^2*c^2*d*x^2*(-c^2*d*x^2+d)^(1/2)+d*(a+b*arccosh(c*x))^2*(-c^2*d*
x^2+d)^(1/2)-2*a*b*c*d*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2
*b^2*c*d*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/
3*b*c*d*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+2/9*b*c^3*d*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-2*d*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b*d*(a+b*arccosh(c*x
))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b*d*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*
I*b^2*d*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 585, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460}

$$\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{2ibd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)\left(a+b\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d*x^2*Sqrt[d - c^2*d*x^2])/2
7 - (2*a*b*c*d*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b
^2*c*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
 (2*b*c*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (2*b*c^3*d*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2
 + (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/3 - (
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2
*I)*b*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b^2*d*Sqrt[d - c^2*d*x^2]*PolyL
og[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d*S
qrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
```

```
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{3}d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{\left(d\sqrt{d-c^2dx^2}\right)\int}{\sqrt{}}$$

$$= -\frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3dx^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{9\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcdx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{2bc^3d}{}$$

$$= -\frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b^2cdx\sqrt{d-c^2dx^2}\,\mathrm{c}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b}{}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b}{}$$

$$= \frac{68}{27}b^2d\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2dx^2\sqrt{d-c^2dx^2} - \frac{2abcdx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b}{}$$

Mathematica [A] time = 2.91, size = 650, normalized size = 1.13

$$-a^2d^{3/2}\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)-\frac{1}{3}a^2d\left(c^2x^2-4\right)\sqrt{d-c^2dx^2}+a^2d^{3/2}\log(cx)+\frac{2abd\sqrt{d-c^2dx^2}\left(i\mathrm{Li}_2\left(-ie^{-\cosh^{-1}}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] -1/3*(a^2*d*(-4 + c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (b^2*d*Sqrt[d - c^2*d*x^2
]*(2*(-13 + Cosh[2*ArcCosh[c*x]]) + 9*ArcCosh[c*x]^2*(-1 + Cosh[2*ArcCosh[c
*x]]) + (3*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*(9*c*x - Cosh[3*ArcCosh[
c*x]]))/(-1 + c*x)))/54 - (a*b*d*Sqrt[d - c^2*d*x^2]*(9*c*x + 12*((-1 + c*x
)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] - Cosh[3*ArcCosh[c*x]]))/(18*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + a^2*d^(3/2)*Log[c*x] - a^2*d^(3/2)*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + (2*a*b*d*Sqrt[d - c^2*d*x^2]*(-(c*x) +
 Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x] + c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*A
rcCosh[c*x] + I*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]*Log
[1 + I/E^ArcCosh[c*x]] + I*PolyLog[2, (-I)/E^ArcCosh[c*x]] - I*PolyLog[2, I
/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + b^2*d*Sqrt[d -
c^2*d*x^2]*(2 + (2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x])/(1 - c*x) +
 ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*
x]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c
*x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^Ar
cCosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^2 - abd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2 + d\right)^{\frac{3}{2}} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}\left(3\,d^{\frac{3}{2}}\log\left(\frac{2\sqrt{-c^2dx^2 + d}\,\sqrt{d}}{|x|} + \frac{2\,d}{|x|}\right) - \left(-c^2dx^2 + d\right)^{\frac{3}{2}} - 3\sqrt{-c^2dx^2 + d}\,d\right)a^2 + \int \frac{\left(-c^2dx^2 + d\right)^{\frac{3}{2}}b^2\log\left(cx\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] -1/3*(3*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) - (
-c^2*d*x^2 + d)^(3/2) - 3*sqrt(-c^2*d*x^2 + d)*d)*a^2 + integrate((-c^2*d*x
^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d*x^
2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + b\operatorname{acosh}\left(c\,x\right)\right)^2 \left(d - c^2\,d\,x^2\right)^{3/2}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d(cx-1)(cx+1))^{\frac{3}{2}}(a+b\operatorname{acosh}(cx))^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x,x)
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x, x)
```

3.183 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=453

$$-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{2b\sqrt{cx-1}\sqrt{cx+1}}+\frac{cd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x-1/4*b^2*c^2*d*x*(-c^2*d*x^2+d)
^(1/2)-3/2*c^2*d*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-5/4*b^2*c*d*ar
ccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/2*b*c^3*d*x^2
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d*
(-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+c*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+1/2*c*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+2*b*c*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b^2*c*d*polylog(2,-1
/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 465, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 14, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {5798, 5740, 5683, 5676, 5662, 90, 52, 5727, 5660, 3718, 2190, 2279, 2391, 38}

$$\frac{b^2cd\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a+b\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] -(b^2*c^2*d*x*Sqrt[d - c^2*d*x^2])/4 - (5*b^2*c*d*Sqrt[d - c^2*d*x^2]*ArcCo
sh[c*x])/(4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*c^3*d*x^2*Sqrt[d - c^2*d*x
^2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d*(1 - c^
2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) - (3*c^2*d*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (c*d*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d
*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (c*d*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (2*b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh
[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b^2*c*d*Sqrt[d - c^2*d*x^2]*Poly
Log[2, -E^(2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
```

```
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} - \frac{\left(2bcd\sqrt{d-c^2dx^2}\right.}{\sqrt{-}}$$

$$= \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3}{2}c^2dx\sqrt{d-c^2dx^2}\left(a\right.$$

$$= \frac{1}{2}b^2c^2dx\sqrt{d-c^2dx^2} + \frac{3bc^3dx^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bcd\left(\right.}{}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-c}}{2\sqrt{-}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-}}{2\sqrt{}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-}}{2\sqrt{}}$$

$$= -\frac{1}{4}b^2c^2dx\sqrt{d-c^2dx^2} - \frac{5b^2cd\sqrt{d-c^2dx^2}\cosh^{-1}(cx)}{4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3bc^3dx^2\sqrt{d-}}{2\sqrt{}}$$

Mathematica [A] time = 4.11, size = 433, normalized size = 0.96

$$\frac{36a^2cd^{3/2}x\sqrt{\frac{cx-1}{cx+1}}(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 12a^2d\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(c^2x^2+2\right)\sqrt{d-c^2dx^2} - 24abd\sqrt{d-c^2dx^2}\left(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (-12*a^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(2 + c^2*x^2)*Sqrt[d - c^2*
d*x^2] + 36*a^2*c*d^(3/2)*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcTan[(c*
x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 24*a*b*d*Sqrt[d - c^2*d*
x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*
x]^2 + 2*Log[c*x])) - 8*b^2*d*Sqrt[d - c^2*d*x^2]*(ArcCosh[c*x]*(3*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[c*x]*(3 + ArcCosh[
c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[2, -E^(-2*ArcCosh[
c*x])]) + 6*a*b*c*d*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh[c*x]] + 2*ArcCosh
[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + b^2*c*d*x*Sqrt[d - c^2*d*x^2
]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] - 3*(1 + 2*ArcCos
h[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/(24*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
)
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^2dx^2 - abd\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.68, size = 942, normalized size = 2.08

$$-\frac{a^2\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}{dx}-a^2c^2x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}-\frac{3a^2c^2dx\sqrt{-c^2d\,x^2+d}}{2}-\frac{3a^2c^2d^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2\sqrt{c^2d}}+\frac{b^2\sqrt{-d\left(c^2x^2\right.}}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(5/2)-a^2*c^2*x*(-c^2*d*x^2+d)^(3/2)-3/2*a^2*c^2*d*
x*(-c^2*d*x^2+d)^(1/2)-3/2*a^2*c^2*d^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x
/(-c^2*d*x^2+d)^(1/2))+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)*arccosh(c*x)^3*c*d+b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d-1/4*b^2*(-d*(c^2*
x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*x^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*
d/(c*x+1)/(c*x-1)*x-b^2*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*arccosh(c*x)^2-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*arccosh(c*x)-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^4*d/(c*x+1)/(c*x-1)*ar
ccosh(c*x)^2*x^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/(c*x-1)*arcco
sh(c*x)^2*x+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*c^3*d/(c*x+1)^(1/2)/(c*x-1)^(1/2
)*arccosh(c*x)*x^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c*d+b^2*(-d*(c^2*x^
2-1))^(1/2)*arccosh(c*x)^2*d/(c*x+1)/(c*x-1)/x+3/2*a*b*(-d*(c^2*x^2-1))^(1/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*c*d-a*b*(-d*(c^2*x^2-1))^(1/2
)*c^4*d/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*c^3
*d/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)
^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)-a*b*(-d*(c^2*x^2-1))^(1/2)*c^2*d/(c*x+1)/
(c*x-1)*arccosh(c*x)*x-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*c*d/(c*x+1)^(1/2)/(c*
x-1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d/(c*x+1)/(c*x-1)/x+2*
a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)*c*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(3\sqrt{-c^2dx^2+d}\,c^2dx+3\,cd^{\frac{3}{2}}\arcsin\left(cx\right)+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}}{x}\right)a^2+\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{c}\right.}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] -1/2*(3*sqrt(-c^2*d*x^2 + d)*c^2*d*x + 3*c*d^(3/2)*arcsin(c*x) + 2*(-c^2*d*
x^2 + d)^(3/2)/x)*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))^2\,\left(d - c^2\,d\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}\,(a + b\,\mathrm{acosh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**2, x)
```

3.184 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^2}{x^3}\,dx$

Optimal. Leaf size=630

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] -1/2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^2-2*b^2*c^2*d*(-c^2*d*x^2+
d)^(1/2)-3/2*c^2*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+3*a*b*c^3*d*x*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*b^2*c^3*d*x*arccosh(c*x)
*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d*(a+b*arccosh(c*x))*
(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c^3*d*x*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*c^2*d*(a+b*arccosh(
c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+b^2*c^2*d*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d
*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b*c^2*d*(a+b*arccosh(c*x))*po
lylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)+3*I*b*c^2*d*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*I
*b^2*c^2*d*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*I*b^2*c^2*d*polylog(3,I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.37, antiderivative size = 642, normalized size of antiderivative = 1.02, number of steps used = 18, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5740, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 14, 5731, 460, 92, 205}

$$-\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3ibc^2d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,ie^{\cosh^{-1}(cx)}\right)(}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] -2*b^2*c^2*d*Sqrt[d - c^2*d*x^2] + (3*a*b*c^3*d*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*b^2*c^3*d*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x
])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x]))/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d*x*Sqrt[d - c^2*d*x^2]
*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*c^2*d*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (d*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (3*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
b^2*c^2*d*Sqrt[d - c^2*d*x^2]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - ((3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((
3*I)*b*c^2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCos
h[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((3*I)*b^2*c^2*d*Sqrt[d - c^2*d*x
^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((3*I
)*b^2*c^2*d*Sqrt[d - c^2*d*x^2]*PolyLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} - \frac{\left(bcd\sqrt{d-c^2dx^2}\right.}{\sqrt{-}}$$

$$= -\frac{bcd\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bc^3dx\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{x\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cosh^{-}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cos}{\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cos}{\sqrt{-1+cx}\sqrt{1+c}}$$

$$= -2b^2c^2d\sqrt{d-c^2dx^2} + \frac{3abc^3dx\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3b^2c^3dx\sqrt{d-c^2dx^2}\cos}{\sqrt{-1+cx}\sqrt{1+c}}$$

Mathematica [A] time = 176.12, size = 1129, normalized size = 1.79

$$\frac{1}{2}d\sqrt{d-c^2dx^2}\left(\frac{4x^2\cosh^{-1}(cx)c^4}{(cx-1)^{3/2}\sqrt{cx+1}} - \frac{2x\cosh^{-1}(cx)^2c^3}{cx-1} - \frac{4x\cosh^{-1}(cx)c^3}{(cx-1)^{3/2}\sqrt{cx+1}} - \frac{2x\tan^{-1}\left(\frac{1}{\sqrt{c^2x^2-1}}\right)c^3}{(cx-1)\sqrt{c^2x^2-1}} - \frac{4xc^3}{cx-1} + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-(a^2*c^2*d) - (a^2*d)/(2*x^2))*Sqrt[-(d*(-1 + c^2*x^2))] - (3*a^2*c^2*d^(
3/2)*Log[x])/2 + (3*a^2*c^2*d^(3/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2)
)]])/2 - 2*a*b*c^2*d*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^Ar
cCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)) + (I*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]])
)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^2*(((-I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/
(c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^A
rcCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/
E^ArcCosh[c*x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^Ar
cCosh[c*x]] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c
*x]]))/Sqrt[-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d*Sqrt[d - c^2*d*x^2]*((4*c^2
)/(-1 + c*x) - (4*c^3*x)/(-1 + c*x) - (2*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2
)*Sqrt[1 + c*x]) + (2*c*ArcCosh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) -
(4*c^3*x*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (4*c^4*x^2*ArcCos
h[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (2*c^2*ArcCosh[c*x]^2)/(-1 + c*x
) + ArcCosh[c*x]^2/(x^2*(-1 + c*x)) - (2*c^3*x*ArcCosh[c*x]^2)/(-1 + c*x) +
```

```
(c*ArcCosh[c*x]^2)/(x - c*x^2) + (2*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1
 + c*x)*Sqrt[-1 + c^2*x^2]) - (2*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 +
 c*x)*Sqrt[-1 + c^2*x^2]) - ((3*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c
*x]^2*Log[1 - I/E^ArcCosh[c*x]])/(-1 + c*x) + ((3*I)*c^2*Sqrt[(-1 + c*x)/(1
 + c*x)]*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*
Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-
1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/
E^ArcCosh[c*x]])/(-1 + c*x) - ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog
[3, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((6*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)
]*PolyLog[3, I/E^ArcCosh[c*x]])/(-1 + c*x)))/2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2 - a^2d + \left(b^2c^2dx^2 - b^2d\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abc^2dx^2 - abd\right)\operatorname{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2 + d}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2 d\,x^2 + d\right)^{\frac{3}{2}} \left(a + b\operatorname{arccosh}\left(cx\right)\right)^2}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(3\,c^2d^{\frac{3}{2}}\log\left(\frac{2\sqrt{-c^2dx^2 + d}\,\sqrt{d}}{|x|} + \frac{2\,d}{|x|}\right) - \left(-c^2dx^2 + d\right)^{\frac{3}{2}}c^2 - 3\sqrt{-c^2dx^2 + d}\,c^2d - \frac{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}{dx^2}\right)a^2 + \int \frac{(-c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")
```

```
[Out] 1/2*(3*c^2*d^(3/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
- (-c^2*d*x^2 + d)^(3/2)*c^2 - 3*sqrt(-c^2*d*x^2 + d)*c^2*d - (-c^2*d*x^2 +
```

```
d)^(5/2)/(d*x^2))*a^2 + integrate((-c^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1))^2/x^3 + 2*(-c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))/x^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}\,(a+b\operatorname{acosh}(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**3,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**3, x)
```

3.185 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=426

$$\frac{c^2d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}-\frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh\right.}{3x^3}$$

```
[Out] -1/3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^3+1/3*b^2*c^2*d*(-c^2*d*x^
2+d)^(1/2)/x+c^2*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/x-1/3*b^2*c^3*
d*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*d*(
-c^2*x^2+1)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2)/(c*x+
1)^(1/2)-4/3*c^3*d*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-1/3*c^3*d*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)
^(1/2)/(c*x+1)^(1/2)-8/3*b*c^3*d*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/3*b
^2*c^3*d*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(
1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.17, antiderivative size = 438, normalized size of antiderivative = 1.03, number of steps used = 18, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5740, 5738, 5660, 3718, 2190, 2279, 2391, 5676, 5729, 97, 12, 52}

$$-\frac{4b^2c^3d\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{c^3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{3b\sqrt{cx-1}\sqrt{cx+1}}+\frac{4c^3d\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{3\sqrt{cx-1}\sqrt{cx+}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (b^2*c^2*d*Sqrt[d - c^2*d*x^2])/(3*x) - (b^2*c^3*d*Sqrt[d - c^2*d*x^2]*ArcC
osh[c*x])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d*(1 - c^2*x^2)*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c^
2*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (4*c^3*d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(1 - c*
x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (c^3*d*S
qrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (8*b*c^3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCo
sh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b^2*c^3*d*Sqrt[d - c^2*d*x
^2]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
```

```
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 5738

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (
-Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt
```

```
[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x] - Dist[
(c^2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f^2*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), Int[((f*x)^(m + 2)*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 +
c*x]), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1,
0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && LtQ[m, -1]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d(1-cx)(1+cx)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} - \frac{\left(2bcd\sqrt{d-c^2\cdots\right.}{3\cdots}$$

$$= -\frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c^2d\sqrt{d-c^2dx^2}\,\left(\cdots\right.}{\cdots}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\cdots$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{bcd\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\cdots$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\cdots}{3x^2\cdots}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\cdots}{3x^2\cdots}$$

$$= \frac{b^2c^2d\sqrt{d-c^2dx^2}}{3x} - \frac{b^2c^3d\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)}{3\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd\left(1-c^2x^2\right)\cdots}{3x^2\cdots}$$

Mathematica [A] time = 2.12, size = 583, normalized size = 1.37

$$-4a^2c^4d^2x^4\sqrt{\frac{cx-1}{cx+1}}+5a^2c^2d^2x^2\sqrt{\frac{cx-1}{cx+1}}-3a^2c^3d^{3/2}x^3\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)-a^2d^2\sqrt{\frac{cx-1}{cx+1}}+8abc^4d$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-(a*b*c*d^2*x) + a*b*c^2*d^2*x^2 - a^2*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 5*
a^2*c^2*d^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + b^2*c^2*d^2*x^2*Sqrt[(-1 + c*x
)/(1 + c*x)] - 4*a^2*c^4*d^2*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b^2*c^4*d^2*x
^4*Sqrt[(-1 + c*x)/(1 + c*x)] - b*d^2*(-1 + c*x)*(-3*a*c^3*x^3 + b*(-Sqrt[(
-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*c^2*x^2*Sqrt[(-1
+ c*x)/(1 + c*x)] + 4*c^3*x^3*(-1 + Sqrt[(-1 + c*x)/(1 + c*x)])))*ArcCosh[c
*x]^2 + b^2*c^3*d^2*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 3*a^2*c^3*d^(3/2)*x^3*S
qrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
2])/(Sqrt[d]*(-1 + c^2*x^2))] + b*d^2*(-1 + c*x)*ArcCosh[c*x]*(b*c*x + 2*a*
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x - 4*c^2*x^2 - 4*c^3*x^3) + 8*b*c^3*x^3*
Log[1 + E^(-2*ArcCosh[c*x])]) - 8*a*b*c^3*d^2*x^3*Log[c*x] + 8*a*b*c^4*d^2*
x^4*Log[c*x] - 4*b^2*c^3*d^2*x^3*(-1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/(3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2c^2dx^2-a^2d+\left(b^2c^2dx^2-b^2d\right)\operatorname{arcosh}(cx)^2+2\left(abc^2dx^2-abd\right)\operatorname{arcosh}(cx)\right)\sqrt{-c^2dx^2+d}}{x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.94, size = 2879, normalized size = 6.76

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x)
```

```
[Out] 16/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x
-1)*c^8-20/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x
+1)/(c*x-1)*c^6+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x
```

```
/(c*x+1)/(c*x-1)*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2
+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-16/3*b^2*(-d*(c^2*x^2-1
))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*arccosh(c*x)*c^6+4/3*b^2*(-d*(c^2*x
^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*arccosh(c*x)*c^4-16/3*a*b*(-d*(c^
2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3*c^6+4/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*d*c^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)*d*c^3+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(
c*x)^2*d*c^3+a^2*c^4*d*x*(-c^2*d*x^2+d)^(1/2)+a^2*c^4*d^2/(c^2*d)^(1/2)*arc
tan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/
(24*c^4*x^4-9*c^2*x^2+1)*x^3*c^6+3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4
-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3-a*b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*d*c^3+16/3*a*b*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*d*c^3-8/3*a*b*(-d*(c^2*x^2-1))
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)*d*c^3+20/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x
+1)/(c*x-1)*c^8-29/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*
x^3/(c*x+1)/(c*x-1)*c^6+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-
9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4
*x^4-9*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2-8/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2)*d*c^3+3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2
+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-8*b^2*(-d*(c^2*x^2-1))^(1/
2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+3*b^2*(-d
*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1
/2)*c^5-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(
1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^3-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(5/2)+2/
3*a^2*c^4*x*(-c^2*d*x^2+d)^(3/2)+2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)^(5/2)-28/3*
a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arc
cosh(c*x)*c^2+64*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/
(c*x+1)/(c*x-1)*arccosh(c*x)*c^8+146/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4
*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-64*a*b*(-d*(c^2*x^2-1)
)^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(
c*x)*c^7+24*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-104*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(2
4*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6-52*b^2*(-d*(c^2
*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^
2*c^6-32*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^4/(c*x+1)^
(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7-8*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c
^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5+12*b^2
*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1
)^(1/2)*arccosh(c*x)^2*c^5-8*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2
*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^5-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*
d/(24*c^4*x^4-9*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-1/3
*a*b*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)/x^2
/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c-20/3*b^2*(-d*(c^2*x^2-1))^(1/2)
*d/(24*c^4*x^4-9*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+73/3*b^2*(
-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(
c*x)^2*c^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x/(c*x
+1)/(c*x-1)*arccosh(c*x)*c^4-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-
9*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^2+32*b^2*(-d*(c^2*x^2-1))^(
1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^8+16/3
*b^2*(-d*(c^2*x^2-1))^(1/2)*d/(24*c^4*x^4-9*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*
arccosh(c*x)*c^8
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(3\sqrt{-c^2dx^2+d}\,c^4dx+3\,c^3d^{\frac{3}{2}}\arcsin\left(cx\right)+\frac{2\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2}{x}-\frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{dx^3}\right)a^2+\int\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}b^2\log\left(c\right.}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/3*(3*sqrt(-c^2*d*x^2 + d)*c^4*d*x + 3*c^3*d^(3/2)*arcsin(c*x) + 2*(-c^2*d
*x^2 + d)^(3/2)*c^2/x - (-c^2*d*x^2 + d)^(5/2)/(d*x^3))*a^2 + integrate((-c
^2*d*x^2 + d)^(3/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2*(-
c^2*d*x^2 + d)^(3/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2/x**4, x)
```

3.186 $\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=880

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^9}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{38bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^7}{441\sqrt{cx-1}\sqrt{cx+1}}-\frac{10b^2c^2d^2\sqrt{d-c^2dx^2}\,x^6}{3087}$$

```
[Out] 5/63*d*x^4*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/9*x^4*(-c^2*d*x^2+d)
^(5/2)*(a+b*arccosh(c*x))^2-37384/694575*b^2*d^2*(-c^2*d*x^2+d)^(1/2)/c^4+3
358/694575*b^2*d^2*x^2*(-c^2*d*x^2+d)^(1/2)/c^2+484/77175*b^2*d^2*x^4*(-c^2
*d*x^2+d)^(1/2)-10/3087*b^2*c^2*d^2*x^6*(-c^2*d*x^2+d)^(1/2)+16/2835*b^2*d^
2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+8/8505*b^2*d^2*(-c
^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+2/4725*b^2*d^2*(-c^2*
x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)-20/3969*b^2*d^2*(-c^2*x^
2+1)^4*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)+2/729*b^2*d^2*(-c^2*x^2+1)
^5*(-c^2*d*x^2+d)^(1/2)/c^4/(-c*x+1)/(c*x+1)-2/63*d^2*(a+b*arccosh(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/c^4-1/63*d^2*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(
1/2)/c^2+1/21*d^2*x^4*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+4/63*a*b*d^
2*x*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+4/63*b^2*d^2*x*arc
cosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2/189*b*d^2*
x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2
/21*b*c*d^2*x^5*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+38/441*b*c^3*d^2*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)-2/81*b*c^5*d^2*x^9*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 2.34, antiderivative size = 911, normalized size of antiderivative = 1.04, number of steps used = 34, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.621$, Rules used = {5798, 5745, 5743, 5759, 5718, 5654, 74, 5662, 100, 12, 14, 5731, 460, 270, 520, 1251, 897, 1153}

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^9}{81\sqrt{cx-1}\sqrt{cx+1}}+\frac{38bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^7}{441\sqrt{cx-1}\sqrt{cx+1}}-\frac{10b^2c^2d^2\sqrt{d-c^2dx^2}\,x^6}{3087}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-37384*b^2*d^2*Sqrt[d - c^2*d*x^2])/(694575*c^4) + (3358*b^2*d^2*x^2*Sqrt[
d - c^2*d*x^2])/(694575*c^2) + (484*b^2*d^2*x^4*Sqrt[d - c^2*d*x^2])/77175
- (10*b^2*c^2*d^2*x^6*Sqrt[d - c^2*d*x^2])/3087 + (4*a*b*d^2*x*Sqrt[d - c^2
*d*x^2])/(63*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (16*b^2*d^2*(1 - c^2*x^2)*
Sqrt[d - c^2*d*x^2])/(2835*c^4*(1 - c*x)*(1 + c*x)) + (8*b^2*d^2*(1 - c^2*x
^2)^2*Sqrt[d - c^2*d*x^2])/(8505*c^4*(1 - c*x)*(1 + c*x)) + (2*b^2*d^2*(1 -
 c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(4725*c^4*(1 - c*x)*(1 + c*x)) - (20*b^2*d
^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(3969*c^4*(1 - c*x)*(1 + c*x)) + (2
*b^2*d^2*(1 - c^2*x^2)^5*Sqrt[d - c^2*d*x^2])/(729*c^4*(1 - c*x)*(1 + c*x))
 + (4*b^2*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(63*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (2*b*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(189*
c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^5*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x]))/(21*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (38*b*c^3*d^2*x^7*Sqrt[
d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(441*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
2*b*c^5*d^2*x^9*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(81*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c
^4) - (d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(63*c^2) + (d^2*
x^4*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/21 + (5*d^2*x^4*(1 - c*x)*(
1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/63 + (d^2*x^4*(1 - c*x
)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
```

```
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
```

```
/; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] && !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int x^3 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^3(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{9}d^2x^4(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\right.}{}$$

$$= -\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{45\sqrt{-1+cx}\sqrt{1+cx}} + \frac{4bc^3d^2x^7\sqrt{d-c^2dx^2}}{63\sqrt{-1+c}}$$

$$= -\frac{8bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{105\sqrt{-1+cx}\sqrt{1+cx}} + \frac{38bc^3d^2x^7\sqrt{d-c^2dx}}{441\sqrt{-1+}}$$

$$= -\frac{2bcd^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{21\sqrt{-1+cx}\sqrt{1+cx}} + \frac{38bc^3d^2x^7\sqrt{d-c^2dx}}{441\sqrt{-1+}}$$

$$= \frac{2}{525}b^2d^2x^4\sqrt{d-c^2dx^2} - \frac{10b^2c^2d^2x^6\sqrt{d-c^2dx^2}}{3087} + \frac{2bd^2x^3\sqrt{d-}}{189c}$$

$$= -\frac{2b^2d^2x^2\sqrt{d-c^2dx^2}}{567c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175} - \frac{10b^2c^2d^2x^6\sqrt{d}}{3087}$$

$$= \frac{22b^2d^2x^2\sqrt{d-c^2dx^2}}{14175c^2} + \frac{484b^2d^2x^4\sqrt{d-c^2dx^2}}{77175} - \frac{10b^2c^2d^2x^6\sqrt{d}}{3087}$$

$$= -\frac{40b^2d^2\sqrt{d-c^2dx^2}}{567c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4\sqrt{d}}{77175}$$

$$= -\frac{856b^2d^2\sqrt{d-c^2dx^2}}{14175c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4\sqrt{}}{7717}$$

$$= -\frac{37384b^2d^2\sqrt{d-c^2dx^2}}{694575c^4} + \frac{3358b^2d^2x^2\sqrt{d-c^2dx^2}}{694575c^2} + \frac{484b^2d^2x^4}{77}$$

Mathematica [A] time = 0.75, size = 288, normalized size = 0.33

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3969a^2\left(7c^2x^2+2\right)\left(c^2x^2-1\right)^4 - 126abcx\sqrt{cx-1}\sqrt{cx+1}\left(49c^8x^8-171c^6x^6+189c^4x^4-21c^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3969*a^2*(-1 + c^2*x^2)^4*(2 + 7*c^2*x^2) - 126*a
*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-126 - 21*c^2*x^2 + 189*c^4*x^4 - 171*
c^6*x^6 + 49*c^8*x^8) + 2*b^2*(6140 - 7039*c^2*x^2 - 106*c^4*x^4 + 2152*c^6
*x^6 - 1490*c^8*x^8 + 343*c^10*x^10) + 126*b*(63*a*(-1 + c^2*x^2)^4*(2 + 7*
c^2*x^2) + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(126 + 21*c^2*x^2 - 189*c^4*x
^4 + 171*c^6*x^6 - 49*c^8*x^8))*ArcCosh[c*x] + 3969*b^2*(-1 + c^2*x^2)^4*(2
 + 7*c^2*x^2)*ArcCosh[c*x]^2))/(250047*c^4*(-1 + c^2*x^2))
```

fricas [A] time = 0.93, size = 558, normalized size = 0.63

$$\frac{3969\left(7\,b^2c^{10}d^2x^{10}-26\,b^2c^8d^2x^8+34\,b^2c^6d^2x^6-16\,b^2c^4d^2x^4-b^2c^2d^2x^2+2\,b^2d^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] 1/250047*(3969*(7*b^2*c^10*d^2*x^10 - 26*b^2*c^8*d^2*x^8 + 34*b^2*c^6*d^2*x
^6 - 16*b^2*c^4*d^2*x^4 - b^2*c^2*d^2*x^2 + 2*b^2*d^2)*sqrt(-c^2*d*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1))^2 - 126*(49*a*b*c^9*d^2*x^9 - 171*a*b*c^7*d^2
*x^7 + 189*a*b*c^5*d^2*x^5 - 21*a*b*c^3*d^2*x^3 - 126*a*b*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 126*((49*b^2*c^9*d^2*x^9 - 171*b^2*c^7*d^2
*x^7 + 189*b^2*c^5*d^2*x^5 - 21*b^2*c^3*d^2*x^3 - 126*b^2*c*d^2*x)*sqrt(-c^
2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 63*(7*a*b*c^10*d^2*x^10 - 26*a*b*c^8*d^2*x
^8 + 34*a*b*c^6*d^2*x^6 - 16*a*b*c^4*d^2*x^4 - a*b*c^2*d^2*x^2 + 2*a*b*d^2)
*sqrt(-c^2*d*x^2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (343*(81*a^2 + 2*b^2)
*c^10*d^2*x^10 - 2*(51597*a^2 + 1490*b^2)*c^8*d^2*x^8 + 2*(67473*a^2 + 2152
*b^2)*c^6*d^2*x^6 - 4*(15876*a^2 + 53*b^2)*c^4*d^2*x^4 - (3969*a^2 + 14078*
b^2)*c^2*d^2*x^2 + 2*(3969*a^2 + 6140*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^6*
x^2 - c^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.86, size = 2224, normalized size = 2.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] a^2*(-1/9*x^2*(-c^2*d*x^2+d)^(7/2)/c^2/d-2/63/d/c^4*(-c^2*d*x^2+d)^(7/2))+b
^2*(1/373248*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7
*c^7-280*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(81*a
rccosh(c*x)^2-18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/175616*(-d*(c^2*
x^2-1))^(1/2)*(64*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^
7+104*c^4*x^4-112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arcco
sh(c*x)^2-14*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1)
)^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)-3/256
*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arcco
```

```
sh(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1)+1/1728*(-d*(c^2*x^2-1))
^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1
)/c^4/(c*x-1)-3/175616*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-
56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*d^2/(c*x+1)/c
^4/(c*x-1)+1/373248*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-704*c^8*x^8
-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+41*c^2*x^2-
1)*(81*arccosh(c*x)^2+18*arccosh(c*x)+2)*d^2/(c*x+1)/c^4/(c*x-1))+2*a*b*(1/
41472*(-d*(c^2*x^2-1))^(1/2)*(256*c^10*x^10-704*c^8*x^8+256*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^9*c^9+688*c^6*x^6-576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7-28
0*c^4*x^4+432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+41*c^2*x^2-120*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3*c^3+9*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+9*arccos
h(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-
144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c
^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^4
/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*
x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*
x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/256*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arcco
sh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/576*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^
2*x^2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1)-3/25088*(-d*(c^2*x^2-1)
)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3
+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c
*x))*d^2/(c*x+1)/c^4/(c*x-1)+1/41472*(-d*(c^2*x^2-1))^(1/2)*(-256*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^9*c^9+256*c^10*x^10+576*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
7*c^7-704*c^8*x^8-432*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+688*c^6*x^6+120*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-280*c^4*x^4-9*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x*c+41*c^2*x^2-1)*(1+9*arccosh(c*x))*d^2/(c*x+1)/c^4/(c*x-1))
```

maxima [A] time = 0.81, size = 471, normalized size = 0.54

$$-\frac{1}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)b^2\operatorname{arcosh}(cx)^2-\frac{2}{63}\left(\frac{7\left(-c^2dx^2+d\right)^{\frac{7}{2}}x^2}{c^2d}+\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}}{c^4d}\right)ab\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -1/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4
*d))*b^2*arccosh(c*x)^2 - 2/63*(7*(-c^2*d*x^2 + d)^(7/2)*x^2/(c^2*d) + 2*(-
c^2*d*x^2 + d)^(7/2)/(c^4*d))*a*b*arccosh(c*x) - 1/63*(7*(-c^2*d*x^2 + d)^(
7/2)*x^2/(c^2*d) + 2*(-c^2*d*x^2 + d)^(7/2)/(c^4*d))*a^2 + 2/250047*b^2*((3
43*sqrt(c^2*x^2 - 1)*c^6*sqrt(-d)*d^2*x^8 - 1147*sqrt(c^2*x^2 - 1)*c^4*sqrt
(-d)*d^2*x^6 + 1005*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^2*x^4 + 899*sqrt(c^2*x
^2 - 1)*sqrt(-d)*d^2*x^2 - 6140*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^2/c^2)/c^2 - 6
3*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c^4*sqrt(-d)*d^
2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*arccosh(c*x)/c^3) - 2
/3969*(49*c^8*sqrt(-d)*d^2*x^9 - 171*c^6*sqrt(-d)*d^2*x^7 + 189*c^4*sqrt(-d
)*d^2*x^5 - 21*c^2*sqrt(-d)*d^2*x^3 - 126*sqrt(-d)*d^2*x)*a*b/c^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 (a + b \operatorname{acosh}(c x))^2 \left(d - c^2 d x^2\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
[Out] int(x^3*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
[Out] Timed out
```

3.187 $\int x^2 \left(d-c^2 d x^2\right)^{5/2} \left(a+b \cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=841

$$-\frac{b c^5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right) x^8}{32 \sqrt{cx-1} \sqrt{cx+1}}+\frac{b^2 c^4 d^2 \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\, x^7}{256 (1-cx)(cx+1)}+\frac{17 b c^3 d^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}\right.}{144 \sqrt{cx-1} \sqrt{cx+1}}$$

```
[Out] 5/48*d*x^3*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/8*x^3*(-c^2*d*x^2+d)
^(5/2)*(a+b*arccosh(c*x))^2+35/9216*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)/c^2+215/
13824*b^2*d^2*x^3*(-c^2*d*x^2+d)^(1/2)-5/864*b^2*c^2*d^2*x^5*(-c^2*d*x^2+d)
^(1/2)+73/12288*b^2*d^2*x*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c
*x+1)+73/18432*b^2*d^2*x^3*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)-43/4608*b^2*c^2*d^2*x^5*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)+1/256*b^2*c^4*d^2*x^7*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)
-5/128*d^2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2+5/64*d^2*x^3*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+35/9216*b^2*d^2*arccosh(c*x)*(-c^2*d
*x^2+d)^(1/2)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/128*b*d^2*x^2*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-59/384*b*c*d^2*x^4
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+17/144
*b*c^3*d^2*x^6*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-1/32*b*c^5*d^2*x^8*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-5/384*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c
^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-73/12288*b^2*d^2*arctanh(c*x/(c^2*x^2-1)^(1/
2))*(c^2*x^2-1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/c^3/(-c*x+1)/(c*x+1)
```

Rubi [A] time = 2.12, antiderivative size = 872, normalized size of antiderivative = 1.04, number of steps used = 30, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {5798, 5745, 5743, 5759, 5676, 5662, 90, 52, 100, 12, 14, 5731, 460, 266, 43, 520, 1267, 459, 321, 217, 206}

$$-\frac{b c^5 d^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}(cx)\right) x^8}{32 \sqrt{cx-1} \sqrt{cx+1}}+\frac{b^2 c^4 d^2 \left(1-c^2 x^2\right) \sqrt{d-c^2 d x^2}\, x^7}{256 (1-cx)(cx+1)}+\frac{17 b c^3 d^2 \sqrt{d-c^2 d x^2} \left(a+b \cosh^{-1}\right.}{144 \sqrt{cx-1} \sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (35*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/(9216*c^2) + (215*b^2*d^2*x^3*Sqrt[d - c
^2*d*x^2])/13824 - (5*b^2*c^2*d^2*x^5*Sqrt[d - c^2*d*x^2])/864 + (73*b^2*d^
2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(12288*c^2*(1 - c*x)*(1 + c*x)) + (7
3*b^2*d^2*x^3*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(18432*(1 - c*x)*(1 + c*x)
) - (43*b^2*c^2*d^2*x^5*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(4608*(1 - c*x)*
(1 + c*x)) + (b^2*c^4*d^2*x^7*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(256*(1 -
c*x)*(1 + c*x)) + (35*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(9216*c^3*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(128*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (59*b*c*d^2*x^4*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(384*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (17*b
*c^3*d^2*x^6*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(144*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*c^5*d^2*x^8*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(
32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x])^2)/(128*c^2) + (5*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^2)/64 + (5*d^2*x^3*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^2)/48 + (d^2*x^3*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^2)/8 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(38
4*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (73*b^2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt
[d - c^2*d*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(12288*c^3*(1 - c*x)*(1
+ c*x))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{1}{8}d^2x^3(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\right.}{}$$

$$= -\frac{bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{16\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bc^3d^2x^6\sqrt{d-c^2dx^2}\left(a\right.}{12\sqrt{-1+cx}}$$

$$= -\frac{11bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{96\sqrt{-1+cx}\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-c^2d}}{144\sqrt{-1+}}$$

$$= -\frac{59bcd^2x^4\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{384\sqrt{-1+cx}\sqrt{1+cx}} + \frac{17bc^3d^2x^6\sqrt{d-c^2d}}{144\sqrt{-1+}}$$

$$= \frac{5}{512}b^2d^2x^3\sqrt{d-c^2dx^2} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d-c^2dx^2} + \frac{b^2c^4d^2x^7\left(1\right.}{256(1}$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{256c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d}$$

$$= -\frac{5b^2d^2x\sqrt{d-c^2dx^2}}{1024c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d}$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d}$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d}$$

$$= \frac{35b^2d^2x\sqrt{d-c^2dx^2}}{9216c^2} + \frac{215b^2d^2x^3\sqrt{d-c^2dx^2}}{13824} - \frac{5}{864}b^2c^2d^2x^5\sqrt{d}$$

Mathematica [A] time = 5.91, size = 910, normalized size = 1.08

$$-\frac{d^2\left(-110592a^2c^8\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\,x^8 - 110592a^2c^7\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\,x^7 + 313344a^2c^6\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2}\,x^6\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/884736*(d^2*(34560*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2
] + 34560*a^2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 2718
72*a^2*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 271872*a^2*
c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^5*x^5
*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 313344*a^2*c^6*x^6*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^7*x^7*Sqrt[(-1 + c*
x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 110592*a^2*c^8*x^8*Sqrt[(-1 + c*x)/(1 +
 c*x)]*Sqrt[d - c^2*d*x^2] + 11520*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 +
 34560*a^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^
```

```
2])/(Sqrt[d]*(-1 + c^2*x^2))] + 34560*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 +
c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 13824*a*
b*Sqrt[d - c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 3456*a*b*Sqrt[d - c^2*d*x^2]*C
osh[4*ArcCosh[c*x]] - 1536*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 2
16*a*b*Sqrt[d - c^2*d*x^2]*Cosh[8*ArcCosh[c*x]] - 6912*b^2*Sqrt[d - c^2*d*x
^2]*Sinh[2*ArcCosh[c*x]] - 864*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*ArcCosh[c*x]]
 + 256*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 27*b^2*Sqrt[d - c^2*d
*x^2]*Sinh[8*ArcCosh[c*x]] + 24*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]*(576*b*C
osh[2*ArcCosh[c*x]] + 144*b*Cosh[4*ArcCosh[c*x]] - 64*b*Cosh[6*ArcCosh[c*x]
] + 9*b*Cosh[8*ArcCosh[c*x]] - 1152*a*Sinh[2*ArcCosh[c*x]] - 576*a*Sinh[4*A
rcCosh[c*x]] + 384*a*Sinh[6*ArcCosh[c*x]] - 72*a*Sinh[8*ArcCosh[c*x]]) - 28
8*b*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^2*(-120*a + 48*b*Sinh[2*ArcCosh[c*x]]
+ 24*b*Sinh[4*ArcCosh[c*x]] - 16*b*Sinh[6*ArcCosh[c*x]] + 3*b*Sinh[8*ArcCos
h[c*x]])))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^4d^2x^6 - 2\,a^2c^2d^2x^4 + a^2d^2x^2 + \left(b^2c^4d^2x^6 - 2\,b^2c^2d^2x^4 + b^2d^2x^2\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^4d^2x^6 - 2\,abc\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^6 - 2*a^2*c^2*d^2*x^4 + a^2*d^2*x^2 + (b^2*c^4*d^2*
x^6 - 2*b^2*c^2*d^2*x^4 + b^2*d^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^6
- 2*a*b*c^2*d^2*x^4 + a*b*d^2*x^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2 + d\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx) + a)^2x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*x^2, x)
```

maple [A] time = 0.91, size = 1312, normalized size = 1.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 359/36864*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)+35
9/36864*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c^3/(c*x-1)^(1/2)*arcc
osh(c*x)+1/256*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*x^9-263/1
3824*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*x^7+1915/55296*b^2*
(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*x^5-359/36864*b^2*(-d*(c^2*x
^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*x-5/384*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)/c^3*arccosh(c*x)^3*d^2-133/384*b^2*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3+5/128*a^2/c^2*d^2*x*(-c^2*d*
x^2+d)^(1/2)+5/128*a^2/c^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d
*x^2+d)^(1/2))-5/128*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
/c^3*arccosh(c*x)^2*d^2-133/192*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x
-1)*arccosh(c*x)*x^3+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)
*arccosh(c*x)^2*x^9-23/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1
)*arccosh(c*x)^2*x^7+127/192*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*
```

```
x-1)*arccosh(c*x)^2*x^5+5/128*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c
*x-1)*arccosh(c*x)^2*x-1/32*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^
5/(c*x-1)^(1/2)*arccosh(c*x)*x^8+17/144*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x
+1)^(1/2)*c^3/(c*x-1)^(1/2)*arccosh(c*x)*x^6-59/384*b^2*(-d*(c^2*x^2-1))^(1
/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*arccosh(c*x)*x^4+5/128*b^2*(-d*(c^2*x
^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*arccosh(c*x)*x^2-1/32*a*b*(-
d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^5/(c*x-1)^(1/2)*x^8+17/144*a*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c^3/(c*x-1)^(1/2)*x^6-59/384*a*b*(-d*
(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)*c/(c*x-1)^(1/2)*x^4+5/128*a*b*(-d*(c^2
*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/c/(c*x-1)^(1/2)*x^2-1081/110592*b^2*(-d*(c
^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*x^3-1/8*a^2*x*(-c^2*d*x^2+d)^(7/2)/c^2
/d+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^6/(c*x-1)*arccosh(c*x)*x^9-
23/24*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^4/(c*x-1)*arccosh(c*x)*x^7+1
27/96*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)*c^2/(c*x-1)*arccosh(c*x)*x^5+5
/64*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/c^2/(c*x-1)*arccosh(c*x)*x+5/192
*a^2/c^2*d*x*(-c^2*d*x^2+d)^(3/2)+1/48*a^2/c^2*x*(-c^2*d*x^2+d)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{384}\left(\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}{c^2}-\frac{48\left(-c^2dx^2+d\right)^{\frac{7}{2}}x}{c^2d}+\frac{10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx}{c^2}+\frac{15\sqrt{-c^2dx^2+d}\,d^2x}{c^2}+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right.}{c^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] 1/384*(8*(-c^2*d*x^2 + d)^(5/2)*x/c^2 - 48*(-c^2*d*x^2 + d)^(7/2)*x/(c^2*d)
 + 10*(-c^2*d*x^2 + d)^(3/2)*d*x/c^2 + 15*sqrt(-c^2*d*x^2 + d)*d^2*x/c^2 +
15*d^(5/2)*arcsin(c*x)/c^3)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*x^2*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*x^2
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.188 $\int x \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=470

$$\frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7\sqrt{cx-1}\sqrt{cx+1}}-\frac{\left(d-c^2dx^2\right)^{7/2}\left(a+b\cosh^{-1}(cx)\right)^2}{7c^2d}$$

```
[Out] -1/7*(-c^2*d*x^2+d)^(7/2)*(a+b*arccosh(c*x))^2/c^2/d-32/245*b^2*d^2*(-c^2*x
^2+1)*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-16/735*b^2*d^2*(-c^2*x^2+1)
^2*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-12/1225*b^2*d^2*(-c^2*x^2+1)^3
*(-c^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)-2/343*b^2*d^2*(-c^2*x^2+1)^4*(-c
^2*d*x^2+d)^(1/2)/c^2/(-c*x+1)/(c*x+1)+2/7*b*d^2*x*(a+b*arccosh(c*x))*(-c^2
*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/7*b*c*d^2*x^3*(a+b*arccosh(
c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+6/35*b*c^3*d^2*x^5*(
a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/49*b*c
^5*d^2*x^7*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)
```

Rubi [A] time = 0.68, antiderivative size = 485, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 194, 5680, 12, 1610, 1799, 1850}

$$-\frac{2bc^5d^2x^7\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{49\sqrt{cx-1}\sqrt{cx+1}}+\frac{6bc^3d^2x^5\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{35\sqrt{cx-1}\sqrt{cx+1}}-\frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (-32*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(245*c^2*(1 - c*x)*(1 + c*x
)) - (16*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(735*c^2*(1 - c*x)*(1
 + c*x)) - (12*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(1225*c^2*(1 -
c*x)*(1 + c*x)) - (2*b^2*d^2*(1 - c^2*x^2)^4*Sqrt[d - c^2*d*x^2])/(343*c^2*
(1 - c*x)*(1 + c*x)) + (2*b*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))
/(7*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x]))/(7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (6*b*c^3*d^2*x^5*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(35*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(2*b*c^5*d^2*x^7*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(49*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) - (d^2*(1 - c*x)^3*(1 + c*x)^3*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2)/(7*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^3(1+cx)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{7c^2} - \frac{\left(2bd^2\sqrt{d-\right.}{}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{7\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{7\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{7\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{7\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= \frac{2bd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{7c\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bcd^2x^3\sqrt{d-c^2dx^2}\left(a+b\,\mathrm{c}\right.}{7\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{32b^2d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{245c^2(1-cx)(1+cx)} - \frac{16b^2d^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}}{735c^2(1-cx)(1+cx)} -$$

Mathematica [A] time = 0.61, size = 234, normalized size = 0.50

$$\frac{d^2\sqrt{d-c^2dx^2}\left(3675a^2\left(c^2x^2-1\right)^4-210abcx\sqrt{cx-1}\,\sqrt{cx+1}\left(5c^6x^6-21c^4x^4+35c^2x^2-35\right)+210b\cosh^{-1}(cx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*(3675*a^2*(-1 + c^2*x^2)^4 - 210*a*b*c*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(-35 + 35*c^2*x^2 - 21*c^4*x^4 + 5*c^6*x^6) + 2*b^2*(21
61 - 2918*c^2*x^2 + 1108*c^4*x^4 - 426*c^6*x^6 + 75*c^8*x^8) + 210*b*(35*a*
(-1 + c^2*x^2)^4 + b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(35 - 35*c^2*x^2 + 21
*c^4*x^4 - 5*c^6*x^6))*ArcCosh[c*x] + 3675*b^2*(-1 + c^2*x^2)^4*ArcCosh[c*x
]^2))/(25725*c^2*(-1 + c^2*x^2))
```

fricas [A] time = 0.47, size = 477, normalized size = 1.01

$$\frac{3675\left(b^2c^8d^2x^8-4\,b^2c^6d^2x^6+6\,b^2c^4d^2x^4-4\,b^2c^2d^2x^2+b^2d^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-210\left(5\,abc^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] 1/25725*(3675*(b^2*c^8*d^2*x^8 - 4*b^2*c^6*d^2*x^6 + 6*b^2*c^4*d^2*x^4 - 4*
b^2*c^2*d^2*x^2 + b^2*d^2)*sqrt(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)
)^2 - 210*(5*a*b*c^7*d^2*x^7 - 21*a*b*c^5*d^2*x^5 + 35*a*b*c^3*d^2*x^3 - 35
```

```
*a*b*c*d^2*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 210*((5*b^2*c^7*d^2*
x^7 - 21*b^2*c^5*d^2*x^5 + 35*b^2*c^3*d^2*x^3 - 35*b^2*c*d^2*x)*sqrt(-c^2*d
*x^2 + d)*sqrt(c^2*x^2 - 1) - 35*(a*b*c^8*d^2*x^8 - 4*a*b*c^6*d^2*x^6 + 6*a
*b*c^4*d^2*x^4 - 4*a*b*c^2*d^2*x^2 + a*b*d^2)*sqrt(-c^2*d*x^2 + d))*log(c*x
 + sqrt(c^2*x^2 - 1)) + (75*(49*a^2 + 2*b^2)*c^8*d^2*x^8 - 12*(1225*a^2 + 7
1*b^2)*c^6*d^2*x^6 + 2*(11025*a^2 + 1108*b^2)*c^4*d^2*x^4 - 4*(3675*a^2 + 1
459*b^2)*c^2*d^2*x^2 + (3675*a^2 + 4322*b^2)*d^2)*sqrt(-c^2*d*x^2 + d))/(c^
4*x^2 - c^2)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.57, size = 1958, normalized size = 4.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/7*a^2/c^2/d*(-c^2*d*x^2+d)^(7/2)+b^2*(1/43904*(-d*(c^2*x^2-1))^(1/2)*(64
*c^8*x^8-144*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^3*c^3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(49*arccosh(c*x)^2-14*arcc
osh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x
^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(25*arccos
h(c*x)^2-10*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^
(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)*d^2/(c*x+1)/
c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c
^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(
-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh
(c*x)^2+2*arccosh(c*x)+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/384*(-d*(c^2*x^2-1))^(1
/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)*d^2/(c*x+1)/c
^2/(c*x-1)-1/3200*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)
+2)*d^2/(c*x+1)/c^2/(c*x-1)+1/43904*(-d*(c^2*x^2-1))^(1/2)*(-64*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5
-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(49*arccosh(c*x)^2+14*arccosh(c*x)+2)*
d^2/(c*x+1)/c^2/(c*x-1))+2*a*b*(1/6272*(-d*(c^2*x^2-1))^(1/2)*(64*c^8*x^8-1
44*c^6*x^6+64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+104*c^4*x^4-112*(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x^5*c^5-25*c^2*x^2+56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3-7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+7*arccosh(c*x))*d^2/(c*x+1)/c^2/
(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*
x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))*
```

```
d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-5/128*(-d*(
c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c
*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/128*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^
2+1)*(1+3*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)-1/640*(-d*(c^2*x^2-1))^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)*(1+5*arccosh(c*x))*d^2/(c*x+1)/c^2/(c*x-1)+1/6272*(-d*(c^2*x^2-1))^(1/2)*
(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^8+112*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+104*c^4
*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+1)*(1+7*arccosh(c*x))*d^2
/(c*x+1)/c^2/(c*x-1))
```

maxima [A] time = 0.67, size = 337, normalized size = 0.72

$$-\frac{\left(-c^2dx^2+d\right)^{\frac{7}{2}}b^2\operatorname{arcosh}(cx)^2}{7\,c^2d}-\frac{2\left(-c^2dx^2+d\right)^{\frac{7}{2}}ab\operatorname{arcosh}(cx)}{7\,c^2d}+\frac{2}{25725}\,b^2\left(\frac{75\,\sqrt{c^2x^2-1}\,c^4\sqrt{-d}\,d^3x^6-351\,\sqrt{c^2x}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -1/7*(-c^2*d*x^2 + d)^(7/2)*b^2*arccosh(c*x)^2/(c^2*d) - 2/7*(-c^2*d*x^2 +
d)^(7/2)*a*b*arccosh(c*x)/(c^2*d) + 2/25725*b^2*((75*sqrt(c^2*x^2 - 1)*c^4*
sqrt(-d)*d^3*x^6 - 351*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*d^3*x^4 + 757*sqrt(c^
2*x^2 - 1)*sqrt(-d)*d^3*x^2 - 2161*sqrt(c^2*x^2 - 1)*sqrt(-d)*d^3/c^2)/d -
105*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*c^2*sqrt(-d)*d^3
*x^3 - 35*sqrt(-d)*d^3*x)*arccosh(c*x)/(c*d)) - 1/7*(-c^2*d*x^2 + d)^(7/2)*
a^2/(c^2*d) - 2/245*(5*c^6*sqrt(-d)*d^3*x^7 - 21*c^4*sqrt(-d)*d^3*x^5 + 35*
c^2*sqrt(-d)*d^3*x^3 - 35*sqrt(-d)*d^3*x)*a*b/(c*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2, x)
```

3.189 $\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=486

$$\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2d}}{48bc}$$

```
[Out] 5/24*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/6*x*(-c^2*d*x^2+d)^(5/
2)*(a+b*arccosh(c*x))^2+245/1152*b^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+65/1728*b^2
*d^2*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)+1/108*b^2*d^2*x*(-c*x+1)^2*(c*
x+1)^2*(-c^2*d*x^2+d)^(1/2)+5/16*d^2*x*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^
(1/2)+115/1152*b^2*d^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-5/16*b*c*d^2*x^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+5/48*b*d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/18*b*d^2*(-c^2*x^2+1)^3*(a+b*a
rccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/48*d^2*(a
+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 517, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {5713, 5685, 5683, 5676, 5662, 90, 52, 5716, 38}

$$\frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{48c\sqrt{cx-1}\sqrt{cx+1}}-\frac{5d^2\sqrt{d-c^2d}}{48bc}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (245*b^2*d^2*x*Sqrt[d - c^2*d*x^2])/1152 + (65*b^2*d^2*x*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2])/1728 + (b^2*d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c
^2*d*x^2])/108 + (115*b^2*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(1152*c*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c*d^2*x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcC
osh[c*x]))/(16*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*b*d^2*(1 - c^2*x^2)^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(48*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (b*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(18*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (5*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^2)/16 + (5*d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/24 + (d^2*x*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*A
rcCosh[c*x])^2)/6 - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(48*
b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{6}d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(5d^2\sqrt{d-\right.}{}$$

$$= \frac{bd^2\left(1-c^2x^2\right)^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{18c\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5}{24}d^2x(1-cx)(1+$$

$$= \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2} + \frac{5bd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2}}{48c\sqrt{-1+c}}$$

$$= \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}b^2d^2x(1-cx)^2(1+cx)^2$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}$$

$$= \frac{245b^2d^2x\sqrt{d-c^2dx^2}}{1152} + \frac{65b^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{1728} + \frac{1}{108}$$

Mathematica [A] time = 3.49, size = 740, normalized size = 1.52

$$d^2\left(9504a^2c^2x^2\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2} + 9504a^2cx\sqrt{\frac{cx-1}{cx+1}}\sqrt{d-c^2dx^2} - 4320a^2c\sqrt{d}\,x\sqrt{\frac{cx-1}{cx+1}}\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 4\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*(9504*a^2*c*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 9504*a^
2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^3*x^3
*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] - 7488*a^2*c^4*x^4*Sqrt[(-1
 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^5*x^5*Sqrt[(-1 + c*x)/(
1 + c*x)]*Sqrt[d - c^2*d*x^2] + 2304*a^2*c^6*x^6*Sqrt[(-1 + c*x)/(1 + c*x)]
*Sqrt[d - c^2*d*x^2] - 1440*b^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x]^3 - 4320*a
^2*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqr
t[d]*(-1 + c^2*x^2))] - 4320*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*Arc
Tan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 3240*a*b*Sqrt[d -
 c^2*d*x^2]*Cosh[2*ArcCosh[c*x]] + 324*a*b*Sqrt[d - c^2*d*x^2]*Cosh[4*ArcCo
sh[c*x]] - 24*a*b*Sqrt[d - c^2*d*x^2]*Cosh[6*ArcCosh[c*x]] + 1620*b^2*Sqrt[
d - c^2*d*x^2]*Sinh[2*ArcCosh[c*x]] - 81*b^2*Sqrt[d - c^2*d*x^2]*Sinh[4*Arc
Cosh[c*x]] + 4*b^2*Sqrt[d - c^2*d*x^2]*Sinh[6*ArcCosh[c*x]] - 12*b*Sqrt[d -
 c^2*d*x^2]*ArcCosh[c*x]*(270*b*Cosh[2*ArcCosh[c*x]] - 27*b*Cosh[4*ArcCosh[
c*x]] + 2*b*Cosh[6*ArcCosh[c*x]] - 540*a*Sinh[2*ArcCosh[c*x]] + 108*a*Sinh[
4*ArcCosh[c*x]] - 12*a*Sinh[6*ArcCosh[c*x]]) + 72*b*Sqrt[d - c^2*d*x^2]*Arc
Cosh[c*x]^2*(-60*a + 45*b*Sinh[2*ArcCosh[c*x]] - 9*b*Sinh[4*ArcCosh[c*x]] +
 b*Sinh[6*ArcCosh[c*x]])))/(13824*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\left(cx\right)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.46, size = 1053, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/6*x*(-c^2*d*x^2+d)^(5/2)*a^2+1/6*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(
c*x-1)*c^6*arccosh(c*x)^2*x^7+5/24*a^2*d*x*(-c^2*d*x^2+d)^(3/2)+5/16*a^2*d^
2*x*(-c^2*d*x^2+d)^(1/2)+5/16*a^2*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/
(-c^2*d*x^2+d)^(1/2))-11/16*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c
*x-1)^(1/2)*c*x^2-1/18*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)
^(1/2)*c^5*x^6-17/24*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4*arc
cosh(c*x)^2*x^5+59/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*ar
ccosh(c*x)^2*x^3-11/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccos
h(c*x)*x+13/48*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^3*arccosh(c*x)*x^4-11/16*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*c*arccosh(c*x)*x^2-1/18*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*c^5*arccosh(c*x)*x^6-5/16*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)/c*arccosh(c*x)^2*d^2+13/48*a*b*(-d*(c^2*x^2-1))^(1/
2)*d^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3*x^4-299/1152*b^2*(-d*(c^2*x^2-1))^(1
/2)*d^2/(c*x+1)/(c*x-1)*x-17/12*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x
-1)*c^4*arccosh(c*x)*x^5+59/24*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-
1)*c^2*arccosh(c*x)*x^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*
c^6*arccosh(c*x)*x^7+299/1152*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/
(c*x-1)^(1/2)/c+299/1152*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)^(1/2)/(c*x-
1)^(1/2)/c*arccosh(c*x)-5/48*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)/c*arccosh(c*x)^3*d^2+1/108*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/
(c*x-1)*c^6*x^7-113/1728*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^4
*x^5+1091/3456*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*c^2*x^3-11/16
*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{48}\left(8\left(-c^2dx^2+d\right)^{\frac{5}{2}}x+10\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx+15\sqrt{-c^2dx^2+d}\,d^2x+\frac{15\,d^{\frac{5}{2}}\arcsin\left(cx\right)}{c}\right)a^2+\int\left(-c^2dx^2+d\right)^{\frac{5}{2}}b^2\,\mathrm{lo}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(8*(-c^2*d*x^2 + d)^(5/2)*x + 10*(-c^2*d*x^2 + d)^(3/2)*d*x + 15*sqrt(
-c^2*d*x^2 + d)*d^2*x + 15*d^(5/2)*arcsin(c*x)/c)*a^2 + integrate((-c^2*d*x
^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*(-c^2*d*x^2
+ d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}\,(c\,x))^2\,\left(d - c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a + b\,\mathrm{acosh}\,(cx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2, x)
```

3.190 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx$

Optimal. Leaf size=836

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{22bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^3}{45\sqrt{cx-1}\sqrt{cx+1}}-\frac{2}{27}b^2c^2d^2\sqrt{d-c^2dx^2}\,x^2-$$

```
[Out] 1/3*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2+1/5*(-c^2*d*x^2+d)^(5/2)*(a
+b*arccosh(c*x))^2+68/27*b^2*d^2*(-c^2*d*x^2+d)^(1/2)-2/27*b^2*c^2*d^2*x^2*
(-c^2*d*x^2+d)^(1/2)+16/75*b^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/(-c*x+
1)/(c*x+1)+8/225*b^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+
1)+2/125*b^2*d^2*(-c^2*x^2+1)^3*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)+d^2*(
a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)-2*a*b*c*d^2*x*(-c^2*d*x^2+d)^(1/2)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b^2*c*d^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-16/15*b*c*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^
2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+22/45*b*c^3*d^2*x^3*(a+b*arccosh(c*x
))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/25*b*c^5*d^2*x^5*(a+b
*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*d^2*(a+b*
arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I*b^2*d^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*I*b^2*d^2*
polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-2*I*b*d^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*I
*b*d^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 867, normalized size of antiderivative = 1.04, number of steps used = 25, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.586, Rules used = {5798, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460, 194, 520, 1247, 698}

$$-\frac{2bc^5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^5}{25\sqrt{cx-1}\sqrt{cx+1}}+\frac{22bc^3d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)x^3}{45\sqrt{cx-1}\sqrt{cx+1}}-\frac{2}{27}b^2c^2d^2\sqrt{d-c^2dx^2}\,x^2-$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] (68*b^2*d^2*Sqrt[d - c^2*d*x^2])/27 - (2*b^2*c^2*d^2*x^2*Sqrt[d - c^2*d*x^2
])/27 - (2*a*b*c*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (16*b^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(75*(1 - c*x)*(1 + c*x)) +
 (8*b^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(225*(1 - c*x)*(1 + c*x))
+ (2*b^2*d^2*(1 - c^2*x^2)^3*Sqrt[d - c^2*d*x^2])/(125*(1 - c*x)*(1 + c*x))
 - (2*b^2*c*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (16*b*c*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(15*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (22*b*c^3*d^2*x^3*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(45*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^5*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x]))/(25*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^2*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2 + (d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/3 + (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/5 - (2*d^2*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(2*I)*b*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcC
osh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((2*I)*b*d^2*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x
```

```
]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((2*I)*b^2*d^2*Sqrt[d - c^2*d*x^2]*Pol
yLog[3, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{1}{5}d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2 - \frac{\left(d^2\sqrt{d-c^2\right.}}{}$$

$$= -\frac{2bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4bc^3d^2x^3\sqrt{d-c^2dx^2}\left(a\right.}{15\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{22bc^3d^2x^3\sqrt{d-c^2dx^2}\left(\right.}{45\sqrt{-1+cx}}$$

$$= -\frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{16bcd^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{15\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

$$= -\frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2b^2cd^2x\sqrt{d-c^2}}{\sqrt{-1+cx}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{68}{27}b^2d^2\sqrt{d-c^2dx^2} - \frac{2}{27}b^2c^2d^2x^2\sqrt{d-c^2dx^2} - \frac{2abcd^2x\sqrt{d-c^2dx}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 7.25, size = 1031, normalized size = 1.23

$$a^2\log(cx)d^{5/2}-a^2\log\left(d+\sqrt{-d\left(c^2x^2-1\right)}\sqrt{d}\right)d^{5/2}+\frac{ab\sqrt{-d(cx-1)(cx+1)}\left(-12\left(\frac{cx-1}{cx+1}\right)^{3/2}\cosh^{-1}(cx)(cx+1)^3-\right.}{9\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x,x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((23*a^2*d^2)/15 - (11*a^2*c^2*d^2*x^2)/15 + (a^2
*c^4*d^2*x^4)/5) + (a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*((
-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]])
)/(9*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b^2*d^2*Sqrt[-(d*(-1 + c*x)*(
1 + c*x))]*(-26 + (27*c*x*ArcCosh[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*
x)) - 9*ArcCosh[c*x]^2 + (2 + 9*ArcCosh[c*x]^2)*Cosh[2*ArcCosh[c*x]] - (3*A
rcCosh[c*x]*Cosh[3*ArcCosh[c*x]])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))))/
27 + a^2*d^(5/2)*Log[c*x] - a^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*
x^2))]] + 2*a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^Arc
Cosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)) + (I*(PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))
/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))) + b^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 +
 c*x))]*(2 - (2*c*x*ArcCosh[c*x])/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) +
ArcCosh[c*x]^2 + (I*(ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x
]^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*
x]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^Arc
Cosh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x))) - (a*b*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-450*c*x + 450*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 25*Cosh[3*ArcCosh[c*x]] + 9*
Cosh[5*ArcCosh[c*x]] - 75*ArcCosh[c*x]*Sinh[3*ArcCosh[c*x]] - 45*ArcCosh[c*
x]*Sinh[5*ArcCosh[c*x]]))/(1800*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (b^
2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(13500*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x) - 13500*c*x*ArcCosh[c*x] + 6750*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]^2 + 750*ArcCosh[c*x]*Cosh[3*ArcCosh[c*x]] + 270*ArcCosh[c*x]*
Cosh[5*ArcCosh[c*x]] - 250*Sinh[3*ArcCosh[c*x]] - 1125*ArcCosh[c*x]^2*Sinh[
3*ArcCosh[c*x]] - 54*Sinh[5*ArcCosh[c*x]] - 675*ArcCosh[c*x]^2*Sinh[5*ArcCo
sh[c*x]]))/(54000*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4-2\,a^2c^2d^2x^2+a^2d^2+\left(b^2c^4d^2x^4-2\,b^2c^2d^2x^2+b^2d^2\right)\text{arcosh}\,(cx)^2+2\left(abc^4d^2x^4-2\,abc^2d^2x\right.\right.}{x}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="fricas"
)
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2 d\, x^2 + d\right)^{\frac{5}{2}} (a + b \operatorname{arccosh}(cx))^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{15}\left(15\, d^{\frac{5}{2}} \log\left(\frac{2\sqrt{-c^2dx^2 + d}\sqrt{d}}{|x|} + \frac{2\,d}{|x|}\right) - 3\left(-c^2dx^2 + d\right)^{\frac{5}{2}} - 5\left(-c^2dx^2 + d\right)^{\frac{3}{2}} d - 15\sqrt{-c^2dx^2 + d}\, d^2\right) a^2 + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x,x, algorithm="maxima"
)
```

```
[Out] -1/15*(15*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x)) -
 3*(-c^2*d*x^2 + d)^(5/2) - 5*(-c^2*d*x^2 + d)^(3/2)*d - 15*sqrt(-c^2*d*x^2
 + d)*d^2)*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/x + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\, x))^2 \left(d - c^2\, d\, x^2\right)^{5/2}}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx - 1)\,(cx + 1))^{\frac{5}{2}} (a + b \operatorname{acosh}(cx))^2}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x, x)
```

3.191 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx$

Optimal. Leaf size=607

$$-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2+\frac{5cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^3}{8b\sqrt{cx-1}\sqrt{cx+1}}+\frac{cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(c\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -5/4*c^2*d*x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-(-c^2*d*x^2+d)^(5/2)
*(a+b*arccosh(c*x))^2/x-31/64*b^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)-1/32*b^2*c
^2*d^2*x*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)-15/8*c^2*d^2*x*(a+b*arccosh(
c*x))^2*(-c^2*d*x^2+d)^(1/2)-89/64*b^2*c*d^2*arccosh(c*x)*(-c^2*d*x^2+d)^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15/8*b*c^3*d^2*x^2*(a+b*arccosh(c*x))*(-c^2
*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d^2*(-c^2*x^2+1)*(a+b*arcco
sh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*d^2*(-c^2
*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+c*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)+5/8*c*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2)/b/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2*b*c*d^2*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b^2*c*d^2*polylog
(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 638, normalized size of antiderivative = 1.05, number of steps used = 24, number of rules used = 16, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.552, Rules used = {5798, 5740, 5685, 5683, 5676, 5662, 90, 52, 5716, 38, 5727, 5660, 3718, 2190, 2279, 2391}

$$\frac{b^2cd^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{cx-1}\sqrt{cx+1}}+\frac{15bc^3d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{8\sqrt{cx-1}\sqrt{cx+1}}-\frac{15}{8}c^2d^2x\sqrt{d-c^2dx^2}\left(a+\right.$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (-31*b^2*c^2*d^2*x*Sqrt[d - c^2*d*x^2])/64 - (b^2*c^2*d^2*x*(1 - c*x)*(1 +
c*x)*Sqrt[d - c^2*d*x^2])/32 - (89*b^2*c*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c*
x])/(64*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*b*c^3*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*d^2*(1 - c
^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/
(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*c^2*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b
*ArcCosh[c*x])^2)/8 - (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(S
qrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x])^2)/4 - (d^2*(1 - c*x)^2*(1 + c*x)^2*Sqrt[d -
c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/x + (5*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^3)/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*c*d^2*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (b^2*c*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[2, -E^(2*ArcCosh[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

]

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{x} + \frac{\left(2bcd^2\sqrt{d}\right.}{}$$

$$= \frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{5}{4}c^2d^2x(1-cx)(1$$

$$= \frac{1}{8}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{bcd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{11}{16}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} + \frac{1}{}$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} -$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} -$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} -$$

$$= -\frac{31}{64}b^2c^2d^2x\sqrt{d-c^2dx^2} - \frac{1}{32}b^2c^2d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2} -$$

Mathematica [A] time = 5.94, size = 554, normalized size = 0.91

$$\frac{d^2\left(1440a^2c\sqrt{d}\,x\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + 96a^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2c^4x^4-9c^2x^2-8\right)\sqrt{d-c^2dx^2} - 76\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^2,x]
```

```
[Out] (d^2*(96*a^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[d - c^2*d*x^2]*(-8 -
 9*c^2*x^2 + 2*c^4*x^4) + 1440*a^2*c*Sqrt[d]*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 768*a
*b*Sqrt[d - c^2*d*x^2]*(2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]
 - c*x*(ArcCosh[c*x]^2 + 2*Log[c*x])) - 256*b^2*Sqrt[d - c^2*d*x^2]*(ArcCos
h[c*x]*(3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(ArcCosh[
c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 + E^(-2*ArcCosh[c*x])])) + 3*c*x*PolyLog[
2, -E^(-2*ArcCosh[c*x])]) + 384*a*b*c*x*Sqrt[d - c^2*d*x^2]*(Cosh[2*ArcCosh
[c*x]] + 2*ArcCosh[c*x]*(ArcCosh[c*x] - Sinh[2*ArcCosh[c*x]])) + 64*b^2*c*x
*Sqrt[d - c^2*d*x^2]*(4*ArcCosh[c*x]^3 + 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]
] - 3*(1 + 2*ArcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]) - 12*a*b*c*x*Sqrt[d - c^
```

```
2*d*x^2]*(8*ArcCosh[c*x]^2 + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*Sinh[4*A
rcCosh[c*x]]) - b^2*c*x*Sqrt[d - c^2*d*x^2]*(32*ArcCosh[c*x]^3 + 12*ArcCosh
[c*x]*Cosh[4*ArcCosh[c*x]] - 3*(1 + 8*ArcCosh[c*x]^2)*Sinh[4*ArcCosh[c*x]])
))/(768*x*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2d^2x\right.}{x^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.76, size = 1227, normalized size = 2.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x)
```

```
[Out] -11/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)*x^3+1
/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^2/(c*x+1)/(c*x-1)*arccosh(c*x)*x+9/8*b^
2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x
^2-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcco
sh(c*x)*x^4+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)*d^2/(c*x+1)/(c*x-1)/x
+15/8*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^2
*d^2*c-15/8*a^2*c^2*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/8*b^2*(-d*(c^2*x^2-1))^(1/
2)*d^2*c^2/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x-11/8*b^2*(-d*(c^2*x^2-1))^(1/2)
*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c-1
/8*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^4+9/8*a
*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2-2*a*b*(-d
*(c^2*x^2-1))^(1/2)*d^2*c/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)+2*b^2*(-
d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)*ln(1+(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(
c*x+1)/(c*x-1)*arccosh(c*x)^2*x^5-a^2/d/x*(-c^2*d*x^2+d)^(7/2)-a^2*c^2*x*(-
c^2*d*x^2+d)^(5/2)+b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*p
olylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c+b^2*(-d*(c^2*x^2-1))^(
1/2)*arccosh(c*x)^2*d^2/(c*x+1)/(c*x-1)/x-15/8*a^2*c^2*d^3/(c^2*d)^(1/2)*ar
ctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-5/4*a^2*c^2*d*x*(-c^2*d*x^2+d)^(
3/2)+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)*x^
```

```
5-b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)
^2+5/8*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^
3*d^2*c+1/32*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*x^5-35/64*b
^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*x^3+33/64*b^2*(-d*(c^2*x^
2-1))^(1/2)*d^2*c^2/(c*x+1)/(c*x-1)*x-33/64*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*
c/(c*x+1)^(1/2)/(c*x-1)^(1/2)-33/64*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c/(c*x+1
)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(10\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2dx+15\sqrt{-c^2dx^2+d}\,c^2d^2x+15\,cd^{\frac{5}{2}}\arcsin\left(cx\right)+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}}{x}\right)a^2+\int\frac{\left(-c^2dx^2+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^2,x, algorithm="maxim
a")
```

```
[Out] -1/8*(10*(-c^2*d*x^2 + d)^(3/2)*c^2*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^2*d^2*x
 + 15*c*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)/x)*a^2 + integrate((
-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^2 + 2*
(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**2,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**2, x)
```

3.192 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx$

Optimal. Leaf size=890

$$-\frac{2bd^2x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)c^5}{9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5}{27}b^2d^2x^2\sqrt{d-c^2dx^2}\,c^4+\frac{5b^2d^2x\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)c^3}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd^2x\sqrt{d-}}{}$$

```
[Out] -5/6*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2-1/2*(-c^2*d*x^2+d)^(5/
2)*(a+b*arccosh(c*x))^2/x^2-170/27*b^2*c^2*d^2*(-c^2*d*x^2+d)^(1/2)+5/27*b^
2*c^4*d^2*x^2*(-c^2*d*x^2+d)^(1/2)+5/3*b^2*c^2*d^2*(-c^2*x^2+1)*(-c^2*d*x^2
+d)^(1/2)/(-c*x+1)/(c*x+1)+1/9*b^2*c^2*d^2*(-c^2*x^2+1)^2*(-c^2*d*x^2+d)^(1
/2)/(-c*x+1)/(c*x+1)-5/2*c^2*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)+
5*a*b*c^3*d^2*x*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*b^2*c^3*
d^2*x*arccosh(c*x)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*c*d^2
*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*
b*c^3*d^2*x*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-2/9*b*c^5*d^2*x^3*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+5*c^2*d^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b*c^2*d^2*
(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x
^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5*I*b^2*c^2*d^2*polylog(3,I*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2
)-5*I*b*c^2*d^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5*I*b^2*c^2*d^2*po
lylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-b^2*c^2*d^2*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)
*(-c^2*d*x^2+d)^(1/2)/(-c*x+1)/(c*x+1)
```

Rubi [A] time = 1.99, antiderivative size = 921, normalized size of antiderivative = 1.03, number of steps used = 27, number of rules used = 21, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.724, Rules used = {5798, 5740, 5745, 5743, 5761, 4180, 2531, 2282, 6589, 5654, 74, 5680, 12, 460, 270, 5731, 520, 1251, 897, 1153, 205}

$$-\frac{2bd^2x^3\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)c^5}{9\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5}{27}b^2d^2x^2\sqrt{d-c^2dx^2}\,c^4+\frac{5b^2d^2x\sqrt{d-c^2dx^2}\,\cosh^{-1}(cx)c^3}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{bd^2x\sqrt{d-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] (-170*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2])/27 + (5*b^2*c^4*d^2*x^2*Sqrt[d - c^2
*d*x^2])/27 + (5*a*b*c^3*d^2*x*Sqrt[d - c^2*d*x^2])/(Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) + (5*b^2*c^2*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2])/(3*(1 - c*x)*(1
 + c*x)) + (b^2*c^2*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2])/(9*(1 - c*x)*(
1 + c*x)) + (5*b^2*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*ArcCosh[c*x])/(Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (b*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(x*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c^3*d^2*x*Sqrt[d - c^2*d*x^2]*(a + b*Arc
Cosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c^5*d^2*x^3*Sqrt[d - c^
2*d*x^2]*(a + b*ArcCosh[c*x]))/(9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*c^2*d^
2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/2 - (5*c^2*d^2*(1 - c*x)*(1 +
 c*x)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/6 - (d^2*(1 - c*x)^2*(1 +
 c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(2*x^2) + (5*c^2*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (b^2*c^2*d^2*Sqrt[-1 + c^2*x^2]*Sqrt[d - c^2*d*x^2]*
ArcTan[Sqrt[-1 + c^2*x^2]])/((1 - c*x)*(1 + c*x)) - ((5*I)*b*c^2*d^2*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + ((5*I)*b*c^2*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
```

```
sh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((5
*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - ((5*I)*b^2*c^2*d^2*Sqrt[d - c^2*d*x^2]*PolyLog[3
, I*E^ArcCosh[c*x]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
```

```
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5731

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dist
[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c
^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5743

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*Sqrt[(d1_)
+ (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[((f*x)^(m + 1)*
Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(f*(m + 2)), x] + (
-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((m + 2)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x
], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2)*Sqrt[1 + c*
x]*Sqrt[-1 + c*x]), Int[(f*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1), x], x])
 /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e
2 + c*d2, 0] && GtQ[n, 0] &&  !LtQ[m, -1] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5745

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 2*p + 1)), x]
 + (Dist[(2*d1*d2*p)/(m + 2*p + 1), Int[(f*x)^m*(d1 + e1*x)^(p - 1)*(d2 + e
2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p - 1
/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 2*p + 1)*Sqrt[1 + c*x]*Sqrt[-1
 + c*x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(
n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] &&  !LtQ[m, -1] && In
tegerQ[p - 1/2] && (RationalQ[m] || EqQ[n, 1])
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^3}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2} + \frac{\left(bcd^2\sqrt{d-c^2\right.}{}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2bc^3d^2x\sqrt{d-c^2dx^2}\left(a+b\cos\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3d^2x\sqrt{d-c^2dx^2}\left(a+b\cosh\right.}{3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bcd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{bc^3d^2x}{}$$

$$= \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2dx^2}}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5b^2c^3d^2x\sqrt{d-c^2dx}}{\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= -\frac{170}{27}b^2c^2d^2\sqrt{d-c^2dx^2} + \frac{5}{27}b^2c^4d^2x^2\sqrt{d-c^2dx^2} + \frac{5abc^3d^2x\sqrt{d-c^2}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

Mathematica [A] time = 90.31, size = 1384, normalized size = 1.56

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^3,x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*((-7*a^2*c^2*d^2)/3 - (a^2*d^2)/(2*x^2) + (a^2*c^
4*d^2*x^2)/3) - (a*b*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-9*c*x - 12*(
(-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*ArcCosh[c*x] + Cosh[3*ArcCosh[c*x]]
))/(18*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) - (5*a^2*c^2*d^(5/2)*Log[x])/2
 + (5*a^2*c^2*d^(5/2)*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/2 - 4*a*b
*c^2*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]*(-((c*x)/(Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x))) + ArcCosh[c*x] + (I*ArcCosh[c*x]*(Log[1 - I/E^ArcCosh[c*x]]
- Log[1 + I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (I*(
PolyLog[2, (-I)/E^ArcCosh[c*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/(Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x))) + (I*a*b*c^2*d^3*(((-I)*Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x))/(c*x) - (I*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x])/(c^2*x^2) +
```

```
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]]
 - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*
x]] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]]
- Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[
-(d*(-1 + c*x)*(1 + c*x))] + (b^2*d^2*Sqrt[d - c^2*d*x^2]*((244*c^2)/(-1 +
c*x) - (244*c^3*x)/(-1 + c*x) - (4*c^4*x^2)/(-1 + c*x) + (4*c^5*x^3)/(-1 +
c*x) - (54*c^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (54*c*ArcCo
sh[c*x])/(x*(-1 + c*x)^(3/2)*Sqrt[1 + c*x]) - (252*c^3*x*ArcCosh[c*x])/((-1
 + c*x)^(3/2)*Sqrt[1 + c*x]) + (252*c^4*x^2*ArcCosh[c*x])/((-1 + c*x)^(3/2)
*Sqrt[1 + c*x]) + (12*c^5*x^3*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]
) - (12*c^6*x^4*ArcCosh[c*x])/((-1 + c*x)^(3/2)*Sqrt[1 + c*x]) + (126*c^2*A
rcCosh[c*x]^2)/(-1 + c*x) + (27*ArcCosh[c*x]^2)/(x^2*(-1 + c*x)) - (126*c^3
*x*ArcCosh[c*x]^2)/(-1 + c*x) - (18*c^4*x^2*ArcCosh[c*x]^2)/(-1 + c*x) + (1
8*c^5*x^3*ArcCosh[c*x]^2)/(-1 + c*x) + (27*c*ArcCosh[c*x]^2)/(x - c*x^2) +
(54*c^2*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - (54
*c^3*x*ArcTan[1/Sqrt[-1 + c^2*x^2]])/((-1 + c*x)*Sqrt[-1 + c^2*x^2]) - ((13
5*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]
])/(-1 + c*x) + ((135*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]^2*Log[
1 + I/E^ArcCosh[c*x]])/(-1 + c*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]
*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]])/(-1 + c*x) + ((270*I)*c^2*Sq
rt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]])/(-1 + c
*x) - ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, (-I)/E^ArcCosh[c*x
]])/(-1 + c*x) + ((270*I)*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*PolyLog[3, I/E^Arc
Cosh[c*x]])/(-1 + c*x)))/54
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arcosh}\left(cx\right)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2\right.\right.}{x^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2 + d\right)^{\frac{5}{2}} \left(a + b\,\mathrm{arccosh}\left(cx\right)\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)
```

[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(15\,c^2d^{\frac{5}{2}}\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)-3\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2-5\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d-15\sqrt{-c^2dx^2+d}\,c^2d^2-\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^3,x, algorithm="maxim
a")

[Out] 1/6*(15*c^2*d^(5/2)*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))
 - 3*(-c^2*d*x^2 + d)^(5/2)*c^2 - 5*(-c^2*d*x^2 + d)^(3/2)*c^2*d - 15*sqrt(
-c^2*d*x^2 + d)*c^2*d^2 - 3*(-c^2*d*x^2 + d)^(7/2)/(d*x^2))*a^2 + integrate
((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^3 +
2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3,x)

[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}\,(a+b\,\mathrm{acosh}\,(cx))^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**3,x)

[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**3, x)

3.193 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx$

Optimal. Leaf size=638

$$-\frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5c^2d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x}-\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+\right.}{3x^3}$$

```
[Out] 5/3*c^2*d*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/x-1/3*(-c^2*d*x^2+d)^(5
/2)*(a+b*arccosh(c*x))^2/x^3+7/12*b^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+1/3*b^
2*c^2*d^2*(-c*x+1)*(c*x+1)*(-c^2*d*x^2+d)^(1/2)/x+5/2*c^4*d^2*x*(a+b*arccos
h(c*x))^2*(-c^2*d*x^2+d)^(1/2)+23/12*b^2*c^3*d^2*arccosh(c*x)*(-c^2*d*x^2+d
)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/2*b*c^5*d^2*x^2*(a+b*arccosh(c*x))*(-
c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-7/3*b*c^3*d^2*(-c^2*x^2+1)*(
a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3*b*c*
d^2*(-c^2*x^2+1)^2*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/x^2/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-7/3*c^3*d^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)-5/6*c^3*d^2*(a+b*arccosh(c*x))^3*(-c^2*d*x^2+d)^(1/2
)/b/(c*x-1)^(1/2)/(c*x+1)^(1/2)-14/3*b*c^3*d^2*(a+b*arccosh(c*x))*ln(1+1/(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+7/3*b^2*c^3*d^2*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*
(-c^2*d*x^2+d)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.62, antiderivative size = 669, normalized size of antiderivative = 1.05, number of steps used = 29, number of rules used = 17, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.586$, Rules used = {5798, 5740, 5683, 5676, 5662, 90, 52, 5727, 5660, 3718, 2190, 2279, 2391, 38, 5729, 97, 12}

$$-\frac{7b^2c^3d^2\sqrt{d-c^2dx^2}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{cx-1}\sqrt{cx+1}}-\frac{5bc^5d^2x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{5}{2}c^4d^2x\sqrt{d-c^2dx^2}$$

Warning: Unable to verify antiderivative.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (7*b^2*c^4*d^2*x*Sqrt[d - c^2*d*x^2])/12 + (b^2*c^2*d^2*(1 - c*x)*(1 + c*x)
*Sqrt[d - c^2*d*x^2])/(3*x) + (23*b^2*c^3*d^2*Sqrt[d - c^2*d*x^2]*ArcCosh[c
*x])/(12*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*b*c^5*d^2*x^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x]))/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b*c^3*d^2*(1
 - c^2*x^2)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]))/(3*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) - (b*c*d^2*(1 - c^2*x^2)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x]))/(3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^4*d^2*x*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^2)/2 + (7*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCos
h[c*x])^2)/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*c^2*d^2*(1 - c*x)*(1 + c*x
)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x) - (d^2*(1 - c*x)^2*(1 +
 c*x)^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2)/(3*x^3) - (5*c^3*d^2*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^3)/(6*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (14*b*c^3*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*Arc
Cosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*b^2*c^3*d^2*Sqrt[d - c^2
*d*x^2]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 38

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> Simp[(x
*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a
```

```
+ b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
*c + a*d, 0] && IGtQ[m + 1/2, 0]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 97

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*
(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*
(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ
[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5727

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.))/(x_),
 x_Symbol] :> Simp[((d + e*x^2)^p*(a + b*ArcCosh[c*x]))/(2*p), x] + (Dist[d
, Int[((d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]))/x, x], x] - Dist[(b*c*(-d)
^p)/(2*p), Int[(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x]) /; FreeQ[{
a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5729

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_
)^2)^(p_.), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^p*(a + b*ArcCosh[c
*x]))/(f*(m + 1)), x] + (-Dist[(b*c*(-d)^p)/(f*(m + 1)), Int[(f*x)^(m + 1)*
(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2), x], x] - Dist[(2*e*p)/(f^2*(m + 1
)), Int[(f*x)^(m + 2)*(d + e*x^2)^(p - 1)*(a + b*ArcCosh[c*x]), x], x]) /;
FreeQ[{a, b, c, d, e, f}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && ILtQ[(m
+ 1)/2, 0]
```

Rule 5740

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n)/(f*(m + 1)), x] + (-D
ist[(2*e1*e2*p)/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(d1 + e1*x)^(p - 1)*(d2 +
e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p -
1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(f*(m + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*
x]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n - 1
), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] &
& EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && IntegerQ[p -
 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}{x^4}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{d^2(1-cx)^2(1+cx)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3} + \frac{\left(2bcd^2\sqrt{d-c\right.}{}$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{5c^2d^2(1-cx)(1+}{}$$

$$= \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} - \frac{7bc^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}\left(a\right.}{3\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{7}{6}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} - \frac{5bc^5d^2}{}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{7b^2c^3d^2}{6}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3}{12}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3}{12}$$

$$= \frac{7}{12}b^2c^4d^2x\sqrt{d-c^2dx^2} + \frac{b^2c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}}{3x} + \frac{23b^2c^3}{12}$$

Mathematica [A] time = 3.17, size = 803, normalized size = 1.26

$$-12a^2c^6d^3\sqrt{\frac{cx-1}{cx+1}}\,x^6 + 6abc^4d^3\cosh\left(2\cosh^{-1}(cx)\right)x^4 + 112abc^4d^3\log(cx)x^4 - 3b^2c^4d^3\sinh\left(2\cosh^{-1}(cx)\right)x^4 -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2)/x^4,x]
```

```
[Out] (-8*a*b*c*d^3*x + 8*a*b*c^2*d^3*x^2 - 8*a^2*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]
+ 64*a^2*c^2*d^3*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*b^2*c^2*d^3*x^2*Sqrt[(-
1 + c*x)/(1 + c*x)] - 44*a^2*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 8*b^2
*c^4*d^3*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 12*a^2*c^6*d^3*x^6*Sqrt[(-1 + c*x
)/(1 + c*x)] + 20*b^2*c^3*d^3*x^3*(-1 + c*x)*ArcCosh[c*x]^3 - 60*a^2*c^3*d^
(5/2)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d
 - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] - 6*a*b*c^3*d^3*x^3*Cosh[2*ArcCosh
```

```
[c*x]] + 6*a*b*c^4*d^3*x^4*Cosh[2*ArcCosh[c*x]] - 112*a*b*c^3*d^3*x^3*Log[c
*x] + 112*a*b*c^4*d^3*x^4*Log[c*x] - 56*b^2*c^3*d^3*x^3*(-1 + c*x)*PolyLog[
2, -E^(-2*ArcCosh[c*x])] + 3*b^2*c^3*d^3*x^3*Sinh[2*ArcCosh[c*x]] - 3*b^2*c
^4*d^3*x^4*Sinh[2*ArcCosh[c*x]] + 2*b*d^3*(-1 + c*x)*ArcCosh[c*x]*(4*b*c*x
+ 8*a*Sqrt[(-1 + c*x)/(1 + c*x)] + 8*a*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*
a*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] - 56*a*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*
x)] + 3*b*c^3*x^3*Cosh[2*ArcCosh[c*x]] + 56*b*c^3*x^3*Log[1 + E^(-2*ArcCosh
[c*x])] - 6*a*c^3*x^3*Sinh[2*ArcCosh[c*x]]) - 2*b*d^3*(-1 + c*x)*ArcCosh[c*
x]^2*(-30*a*c^3*x^3 + 4*b*(-Sqrt[(-1 + c*x)/(1 + c*x)] - c*x*Sqrt[(-1 + c*x
)/(1 + c*x)] + 7*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 7*c^3*x^3*(-1 + Sqrt[
(-1 + c*x)/(1 + c*x)])) + 3*b*c^3*x^3*Sinh[2*ArcCosh[c*x]]))/(24*x^3*Sqrt[(
-1 + c*x)/(1 + c*x)]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\operatorname{arcosh}(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2\right.}{x^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="frica
s")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)/x^4, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.03, size = 3431, normalized size = 5.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x)
```

```
[Out] 70*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/
2)/(c*x-1)^(1/2)*arccosh(c*x)*c^5-294*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^
4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^7+294*a*
b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*
arccosh(c*x)*c^8-406*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+
1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)*c^6+380/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^
2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-46/3*a*b*(-d
*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh
(c*x)*c^2+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(
c*x+1)/(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^
2+1)/x/(c*x+1)/(c*x-1)*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-1
5*c^2*x^2+1)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2-7/3*b^2*(-d*(c^2*x^2-1))^(1
/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^
```

```
2*c^3+5*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1
/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-1/2*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^5/(
c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*x^2-21*b^2*(-d*(c^2*x^2-1))^(1/2)*d
^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^7+5*b^2*(-d*
(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^
(1/2)*c^5+28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arcco
sh(c*x)*d^2*c^3-5/2*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*
arccosh(c*x)^2*d^2*c^3-14/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3-1/2*a*b*(-d*(c^2*
x^2-1))^(1/2)*d^2*c^5/(c*x+1)^(1/2)/(c*x-1)^(1/2)*x^2+5*a*b*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+5/3*a
^2*c^4*d*x*(-c^2*d*x^2+d)^(3/2)+5/2*a^2*c^4*d^2*x*(-c^2*d*x^2+d)^(1/2)+5/2*
a^2*c^4*d^3/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+1/2*
b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x^3-1/2*b
^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*x)^2*x+56/3*b^2
*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*c
^8-14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)
*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*d^2*c^3-71/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+4/3*a^2*c^2/d
/x*(-c^2*d*x^2+d)^(7/2)+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)*x^3*c^6-49/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^
2+1)*x^3*arccosh(c*x)*c^6+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15
*c^2*x^2+1)*x*arccosh(c*x)*c^4-49/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*
x^4-15*c^2*x^2+1)*x^3*c^6+7/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15
*c^2*x^2+1)*x*c^4+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^3/(c*x+1)^(1/2)/(c*x
-1)^(1/2)*arccosh(c*x)-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^
2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c^3+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2
*c^3/(c*x+1)^(1/2)/(c*x-1)^(1/2)+14/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)*arccosh(c*x)^2*d^2*c^3-5/6*b^2*(-d*(c^2*x^2-1))^(1/2)/(c*
x-1)^(1/2)/(c*x+1)^(1/2)*arccosh(c*x)^3*d^2*c^3-7/3*b^2*(-d*(c^2*x^2-1))^(1
/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)*d^2*c^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*x^3-1/4
*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*x-1/3*a^2/d/x^3*(-c^2*d
*x^2+d)^(7/2)+4/3*a^2*c^4*x*(-c^2*d*x^2+d)^(5/2)-1/3*b^2*(-d*(c^2*x^2-1))^(
1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(
c*x)*c-147*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^4/(c*
x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2*c^7+35*b^2*(-d*(c^2*x^2-1))^(1/2)*d
^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)^2
*c^5-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x^2/(c*x+
1)^(1/2)/(c*x-1)^(1/2)*c-14/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15
*c^2*x^2+1)/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arccosh(c*x)*c^3-21*a*b*(-d*(c^2*x^
2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*c
^5+190/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)
/(c*x-1)*arccosh(c*x)^2*c^4+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-
15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*arccosh(c*x)*c^4-23/3*b^2*(-d*(c^2*x^2-1))^
(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)/x/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^2+14
7*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x+1)/(c*x
-1)*arccosh(c*x)^2*c^8+49/3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c
^2*x^2+1)*x^5/(c*x+1)/(c*x-1)*arccosh(c*x)*c^8-203*b^2*(-d*(c^2*x^2-1))^(1/
2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*arccosh(c*x)^2*c^6-56/
3*b^2*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x
-1)*arccosh(c*x)*c^6+a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^6/(c*x+1)/(c*x-1)*arc
cosh(c*x)*x^3-a*b*(-d*(c^2*x^2-1))^(1/2)*d^2*c^4/(c*x+1)/(c*x-1)*arccosh(c*
x)*x+49/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^5/(c*x
+1)/(c*x-1)*c^8+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1
)/x^3/(c*x+1)/(c*x-1)*arccosh(c*x)-56/3*a*b*(-d*(c^2*x^2-1))^(1/2)*d^2/(63*
c^4*x^4-15*c^2*x^2+1)*x^3/(c*x+1)/(c*x-1)*c^6+7/3*a*b*(-d*(c^2*x^2-1))^(1/2
)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x/(c*x+1)/(c*x-1)*c^4-21*b^2*(-d*(c^2*x^2-1
))^(1/2)*d^2/(63*c^4*x^4-15*c^2*x^2+1)*x^2/(c*x+1)^(1/2)/(c*x-1)^(1/2)*arcc
```

```
osh(c*x)*c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\left(10\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4dx+15\sqrt{-c^2dx^2+d}\,c^4d^2x+15\,c^3d^{\frac{5}{2}}\arcsin\left(cx\right)+\frac{8\left(-c^2dx^2+d\right)^{\frac{5}{2}}c^2}{x}-\frac{2\left(-c^2dx^2+d\right)}{dx^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/x^4,x, algorithm="maxim
a")
```

```
[Out] 1/6*(10*(-c^2*d*x^2 + d)^(3/2)*c^4*d*x + 15*sqrt(-c^2*d*x^2 + d)*c^4*d^2*x
+ 15*c^3*d^(5/2)*arcsin(c*x) + 8*(-c^2*d*x^2 + d)^(5/2)*c^2/x - 2*(-c^2*d*x
^2 + d)^(7/2)/(d*x^3))*a^2 + integrate((-c^2*d*x^2 + d)^(5/2)*b^2*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1))^2/x^4 + 2*(-c^2*d*x^2 + d)^(5/2)*a*b*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2/x**4,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2/x**4, x)
```

3.194 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=421

$$-\frac{2bx^5\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}}-\frac{x^4\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2d}-\frac{8\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{15c^6d}$$

```
[Out] -4144/3375*b^2*(-c*x+1)*(c*x+1)/c^6/(-c^2*d*x^2+d)^(1/2)-272/3375*b^2*x^2*(
-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-2/125*b^2*x^4*(-c*x+1)*(c*x+1)/c^2
/(-c^2*d*x^2+d)^(1/2)-16/15*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*d*x
^2+d)^(1/2)-16/15*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/(-c^2*
d*x^2+d)^(1/2)-8/45*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
3/(-c^2*d*x^2+d)^(1/2)-2/25*b*x^5*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c/(-c^2*d*x^2+d)^(1/2)-8/15*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)
/c^6/d-4/15*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/5*x^4*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 1.13, antiderivative size = 445, normalized size of antiderivative = 1.06, number of steps used = 17, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{16abx\sqrt{cx-1}\,\sqrt{cx+1}}{15c^5\sqrt{d-c^2dx^2}}-\frac{2bx^5\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}}-\frac{x^4(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2\sqrt{d-c^2dx^2}}-\frac{8bx^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (41
44*b^2*(1 - c*x)*(1 + c*x))/(3375*c^6*Sqrt[d - c^2*d*x^2]) - (272*b^2*x^2*(
1 - c*x)*(1 + c*x))/(3375*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^4*(1 - c*x)*(
1 + c*x))/(125*c^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(15*c^5*Sqrt[d - c^2*d*x^2]) - (8*b*x^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(45*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^5
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(25*c*Sqrt[d - c^2*d*x^
2]) - (8*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(15*c^6*Sqrt[d - c^2*d
*x^2]) - (4*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(15*c^4*Sqrt[d
- c^2*d*x^2]) - (x^4*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(5*c^2*Sqr
t[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
```

```
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5 \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^4(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{5c^2\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{2bx^5\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{25c\sqrt{d-c^2dx^2}} - \frac{4x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(\right.}{15c^4\sqrt{d-c^2dx^2}}$$

$$= -\frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{8bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{45c^3\sqrt{d-c^2dx^2}} - \frac{2bx^5\sqrt{-1}}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{8b^2x^2(1-cx)(1+cx)}{135c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{8bx}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^4(1-cx)(1+cx)}{125c^2\sqrt{d-c^2dx^2}} - \frac{1}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{32b^2(1-cx)(1+cx)}{27c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} - \frac{2b}{}$$

$$= -\frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{15c^5\sqrt{d-c^2dx^2}} - \frac{4144b^2(1-cx)(1+cx)}{3375c^6\sqrt{d-c^2dx^2}} - \frac{272b^2x^2(1-cx)(1+cx)}{3375c^4\sqrt{d-c^2dx^2}} -$$

Mathematica [A] time = 0.55, size = 255, normalized size = 0.61

$$\frac{\sqrt{d-c^2dx^2}\,\left(-225a^2\left(3c^6x^6+c^4x^4+4c^2x^2-8\right)+30abcx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4x^4+20c^2x^2+120\right)+30b\cosh^{-1}(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(30*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(120 + 20*c^2
*x^2 + 9*c^4*x^4) - 225*a^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c^6*x^6) - 2*b^2*
(-2072 + 1936*c^2*x^2 + 109*c^4*x^4 + 27*c^6*x^6) + 30*b*(b*c*x*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(120 + 20*c^2*x^2 + 9*c^4*x^4) - 15*a*(-8 + 4*c^2*x^2 + c
^4*x^4 + 3*c^6*x^6))*ArcCosh[c*x] - 225*b^2*(-8 + 4*c^2*x^2 + c^4*x^4 + 3*c
^6*x^6)*ArcCosh[c*x]^2))/(3375*c^6*d*(-1 + c*x)*(1 + c*x))
```

fricas [A] time = 0.48, size = 348, normalized size = 0.83

$$-\frac{225\left(3\,b^2c^6x^6+b^2c^4x^4+4\,b^2c^2x^2-8\,b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-30\left(9\,abc^5x^5+20\,abc^3x^3+120\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/3375*(225*(3*b^2*c^6*x^6 + b^2*c^4*x^4 + 4*b^2*c^2*x^2 - 8*b^2)*sqrt(-c^
2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 - 30*(9*a*b*c^5*x^5 + 20*a*b*c^
3*x^3 + 120*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1) - 30*((9*b^2*c^
```

```
5*x^5 + 20*b^2*c^3*x^3 + 120*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1
) - 15*(3*a*b*c^6*x^6 + a*b*c^4*x^4 + 4*a*b*c^2*x^2 - 8*a*b)*sqrt(-c^2*d*x^
2 + d))*log(c*x + sqrt(c^2*x^2 - 1)) + (27*(25*a^2 + 2*b^2)*c^6*x^6 + (225*
a^2 + 218*b^2)*c^4*x^4 + 4*(225*a^2 + 968*b^2)*c^2*x^2 - 1800*a^2 - 4144*b^
2)*sqrt(-c^2*d*x^2 + d))/(c^8*d*x^2 - c^6*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.80, size = 1314, normalized size = 3.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/5*x^4/c^2/d*(-c^2*d*x^2+d)^(1/2)+4/5/c^2*(-1/3*x^2/c^2/d*(-c^2*d*x^
2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2)))+b^2*(-1/4000*(-d*(c^2*x^2-1))^(
1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x
^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-1)*(25*arccosh(c*x)^2-10*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*
x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*arccosh(c*x)+2)/c^6/
d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+
c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^
2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^
2+2*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-5/864*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c
-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^6/d/(c^2*x^2-1)-1/4000*
(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+
20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x*c+13*c^2*x^2-1)*(25*arccosh(c*x)^2+10*arccosh(c*x)+2)/c^6/d/(c^2*x^2
-1))+2*a*b*(-1/800*(-d*(c^2*x^2-1))^(1/2)*(16*c^6*x^6-28*c^4*x^4+16*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^5*c^5+13*c^2*x^2-20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*
c^3+5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-1)*(-1+5*arccosh(c*x))/c^6/d/(c^2*x^2
-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*arccosh(c*x))/c
^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^6/d/(c^2*x^2-1)-5/16*(-d*(c^2*x^2-1))^(1/
2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^6/d/(c^2
*x^2-1)-5/288*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^
3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*arccosh(c*x
))/c^6/d/(c^2*x^2-1)-1/800*(-d*(c^2*x^2-1))^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^
4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)*(1+5*arccosh(c*x))/c^6/d/
(c^2*x^2-1))
```

maxima [A] time = 1.03, size = 407, normalized size = 0.97

$$-\frac{1}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-c^2dx^2+d}\,x^2}{c^4d}+\frac{8\sqrt{-c^2dx^2+d}}{c^6d}\right)b^2\operatorname{arcosh}(cx)^2-\frac{2}{15}\left(\frac{3\sqrt{-c^2dx^2+d}\,x^4}{c^2d}+\frac{4\sqrt{-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4
*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d))*b^2*arccosh(c*x)^2 - 2/15*(3*sqrt(-c^
2*d*x^2 + d)*x^4/(c^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2
*d*x^2 + d)/(c^6*d))*a*b*arccosh(c*x) - 1/15*(3*sqrt(-c^2*d*x^2 + d)*x^4/(c
^2*d) + 4*sqrt(-c^2*d*x^2 + d)*x^2/(c^4*d) + 8*sqrt(-c^2*d*x^2 + d)/(c^6*d)
)*a^2 - 2/3375*b^2*((27*sqrt(c^2*x^2 - 1)*c^2*sqrt(-d)*x^4 + 136*sqrt(c^2*x
^2 - 1)*sqrt(-d)*x^2 + 2072*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^4*d) - 15*(9
*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(-d)*x)*arccosh(c*x)/(c^5
*d)) + 2/225*(9*c^4*sqrt(-d)*x^5 + 20*c^2*sqrt(-d)*x^3 + 120*sqrt(-d)*x)*a*
b/(c^5*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,(a+b\,\mathrm{acosh}\,(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\,(a+b\,\mathrm{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.195 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=355

$$-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}}-\frac{x^3\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2d}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}\right.}{8bc^5\sqrt{d-c^2dx^2}}$$

```
[Out] -15/64*b^2*x*(-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-1/32*b^2*x^3*(-c*x+1
)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)+15/64*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*b*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/8*b*x^4*(a+b*arccosh(c*x))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/8*(a+b*arccosh(c*x))^3*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/(-c^2*d*x^2+d)^(1/2)-3/8*x*(a+b*arccosh(c*x
))^2*(-c^2*d*x^2+d)^(1/2)/c^4/d-1/4*x^3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)
^(1/2)/c^2/d
```

Rubi [A] time = 1.02, antiderivative size = 371, normalized size of antiderivative = 1.05, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{bx^4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}}-\frac{x^3(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2\sqrt{d-c^2dx^2}}-\frac{3bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\right.}{8c^3\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-15*b^2*x*(1 - c*x)*(1 + c*x))/(64*c^4*Sqrt[d - c^2*d*x^2]) - (b^2*x^3*(1
- c*x)*(1 + c*x))/(32*c^2*Sqrt[d - c^2*d*x^2]) + (15*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*ArcCosh[c*x])/(64*c^5*Sqrt[d - c^2*d*x^2]) - (3*b*x^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c^3*Sqrt[d - c^2*d*x^2]) - (b*x
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(8*c*Sqrt[d - c^2*d*x
^2]) - (3*x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(8*c^4*Sqrt[d - c^2
*d*x^2]) - (x^3*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(4*c^2*Sqrt[d -
 c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(8*b*c
^5*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^3(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{4c^2\sqrt{d-c^2dx^2}} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\,\sqrt{1}}}{4c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^4\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{8c\sqrt{d-c^2dx^2}} - \frac{3x(1-cx)(1+cx)\left(a+b\cosh^{-1}\right.}{8c^4\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} - \frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{8c^3\sqrt{d-c^2dx^2}} - \frac{bx^4\sqrt{-1}}{}$$

$$= -\frac{3b^2x(1-cx)(1+cx)}{16c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} - \frac{3bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\right.}{8c^3\sqrt{d-c^2dx^2}}$$

$$= -\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} + \frac{3b^2\sqrt{-1+cx}\,\sqrt{1+cx}\cosh^{-1}}{16c^5\sqrt{d-c^2dx^2}}$$

$$= -\frac{15b^2x(1-cx)(1+cx)}{64c^4\sqrt{d-c^2dx^2}} - \frac{b^2x^3(1-cx)(1+cx)}{32c^2\sqrt{d-c^2dx^2}} + \frac{15b^2\sqrt{-1+cx}\,\sqrt{1+cx}\cosh}{64c^5\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.54, size = 295, normalized size = 0.83

$$32a^2c\sqrt{d}\,x\left(c^2x^2-1\right)\left(2c^2x^2+3\right) - 96a^2\sqrt{d-c^2dx^2}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) - 4ab\sqrt{d}\,\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(16\cosh\left(2\,c\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (32*a^2*c*Sqrt[d]*x*(-1 + c^2*x^2)*(3 + 2*c^2*x^2) - 96*a^2*Sqrt[d - c^2*d*
x^2]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + b^2*Sqrt[
d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(32*ArcCosh[c*x]^3 - 4*ArcCosh[c*x]
*(16*Cosh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]]) + 32*Sinh[2*ArcCosh[c*x]]
 + Sinh[4*ArcCosh[c*x]] + 8*ArcCosh[c*x]^2*(8*Sinh[2*ArcCosh[c*x]] + Sinh[4
*ArcCosh[c*x]])) - 4*a*b*Sqrt[d]*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(16*C
osh[2*ArcCosh[c*x]] + Cosh[4*ArcCosh[c*x]] - 4*ArcCosh[c*x]*(6*ArcCosh[c*x]
 + 8*Sinh[2*ArcCosh[c*x]] + Sinh[4*ArcCosh[c*x]])))/(256*c^5*Sqrt[d]*Sqrt[d
 - c^2*d*x^2])
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^4\,\text{arcosh}\,(cx)^2 + 2\,abx^4\,\text{arcosh}\,(cx) + a^2x^4\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2x^4}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.95, size = 887, normalized size = 2.50

$$-\frac{a^2x^3\sqrt{-c^2d\,x^2+d}}{4c^2d}-\frac{3a^2x\sqrt{-c^2d\,x^2+d}}{8c^4d}+\frac{3a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{8c^4\sqrt{c^2d}}+\frac{3b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{c}}{8d\,c^3\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/4*a^2*x^3/c^2/d*(-c^2*d*x^2+d)^(1/2)-3/8*a^2/c^4*x/d*(-c^2*d*x^2+d)^(1/2
)+3/8*a^2/c^4/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))+3/
8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*x^2-1/32*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*x^5-13/64*b^
2*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*x^3+15/64*b^2*(-d*(c^2*x^2-1))^(
1/2)/d/c^4/(c^2*x^2-1)*x-1/8*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)^3+1/8*b^2*(-d*(c^2*x^2-1))^(1/2)/d/
c/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4-15/64*b^2*(-d*(c
^2*x^2-1))^(1/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*x^5-1/8*b^2*(-
d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)^2*x^3+3/8*b^2*(-d*(c^2*
x^2-1))^(1/2)/d/c^4/(c^2*x^2-1)*arccosh(c*x)^2*x-3/8*a*b*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c^5/(c^2*x^2-1)*arccosh(c*x)^2+3/8*a*b*(
-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-1/2
*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*x^5-1/4*a*b*(-d*(c^2
*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3+3/4*a*b*(-d*(c^2*x^2-1))^
(1/2)/d/c^4/(c^2*x^2-1)*arccosh(c*x)*x-15/64*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c
^5/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+1/8*a*b*(-d*(c^2*x^2-1))^(1/2)/d
/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}a^2\left(\frac{2\sqrt{-c^2dx^2+d}\,x^3}{c^2d}+\frac{3\sqrt{-c^2dx^2+d}\,x}{c^4d}-\frac{3\arcsin(cx)}{c^5\sqrt{d}}\right)+\int\frac{b^2x^4\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}}+\frac{2abx^4\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/8*a^2*(2*sqrt(-c^2*d*x^2 + d)*x^3/(c^2*d) + 3*sqrt(-c^2*d*x^2 + d)*x/(c^
4*d) - 3*arcsin(c*x)/(c^5*sqrt(d))) + integrate(b^2*x^4*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*x^4*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 (a + b \operatorname{acosh}(cx))^2}{\sqrt{-d (cx - 1) (cx + 1)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.196 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=292

$$-\frac{x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d}-\frac{2bx^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}}-\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d}$$

```
[Out] -40/27*b^2*(-c*x+1)*(c*x+1)/c^4/(-c^2*d*x^2+d)^(1/2)-2/27*b^2*x^2*(-c*x+1)*
(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)-4/3*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/
(-c^2*d*x^2+d)^(1/2)-4/3*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3
/(-c^2*d*x^2+d)^(1/2)-2/9*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/c/(-c^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^
4/d-1/3*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.77, antiderivative size = 308, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{4abx\sqrt{cx-1}\sqrt{cx+1}}{3c^3\sqrt{d-c^2dx^2}}-\frac{2bx^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}}-\frac{x^2(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2\sqrt{d-c^2dx^2}}-\frac{2(1-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (40*b
^2*(1 - c*x)*(1 + c*x))/(27*c^4*Sqrt[d - c^2*d*x^2]) - (2*b^2*x^2*(1 - c*x)
*(1 + c*x))/(27*c^2*Sqrt[d - c^2*d*x^2]) - (4*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*ArcCosh[c*x])/(3*c^3*Sqrt[d - c^2*d*x^2]) - (2*b*x^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c*Sqrt[d - c^2*d*x^2]) - (2*(1 - c*x)
*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^4*Sqrt[d - c^2*d*x^2]) - (x^2*(1 -
c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x^2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{3c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{9c\sqrt{d-c^2dx^2}} - \frac{2(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{3c^4\sqrt{d-c^2dx^2}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b}{9c\sqrt{d-c^2dx^2}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh}{3c^3\sqrt{d-c^2dx^2}}$$

$$= -\frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^3\sqrt{d-c^2dx^2}} - \frac{40b^2(1-cx)(1+cx)}{27c^4\sqrt{d-c^2dx^2}} - \frac{2b^2x^2(1-cx)(1+cx)}{27c^2\sqrt{d-c^2dx^2}} - \frac{4b^2x}{}$$

Mathematica [A] time = 0.45, size = 201, normalized size = 0.69

$$\frac{\sqrt{d-c^2dx^2}\left(-9a^2\left(c^4x^4+c^2x^2-2\right)+6abcx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2+6\right)+6b\cosh^{-1}(cx)\left(bcx\sqrt{cx-1}\,\sqrt{cx+1}\,(c\right.\right.}{27c^4d(cx-1)(cx+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(6*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2)
- 9*a^2*(-2 + c^2*x^2 + c^4*x^4) - 2*b^2*(-20 + 19*c^2*x^2 + c^4*x^4) + 6*b
*(b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(6 + c^2*x^2) - 3*a*(-2 + c^2*x^2 + c^
4*x^4))*ArcCosh[c*x] - 9*b^2*(-2 + c^2*x^2 + c^4*x^4)*ArcCosh[c*x]^2))/(27*
c^4*d*(-1 + c*x)*(1 + c*x))
```

fricas [A] time = 0.62, size = 282, normalized size = 0.97

$$-\frac{9\left(b^2c^4x^4+b^2c^2x^2-2\,b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2-6\left(abc^3x^3+6\,abcx\right)\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}-6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] -1/27*(9*(b^2*c^4*x^4 + b^2*c^2*x^2 - 2*b^2)*sqrt(-c^2*d*x^2 + d)*log(c*x +
 sqrt(c^2*x^2 - 1))^2 - 6*(a*b*c^3*x^3 + 6*a*b*c*x)*sqrt(-c^2*d*x^2 + d)*sq
rt(c^2*x^2 - 1) - 6*((b^2*c^3*x^3 + 6*b^2*c*x)*sqrt(-c^2*d*x^2 + d)*sqrt(c^
2*x^2 - 1) - 3*(a*b*c^4*x^4 + a*b*c^2*x^2 - 2*a*b)*sqrt(-c^2*d*x^2 + d))*lo
g(c*x + sqrt(c^2*x^2 - 1)) + ((9*a^2 + 2*b^2)*c^4*x^4 + (9*a^2 + 38*b^2)*c^
2*x^2 - 18*a^2 - 40*b^2)*sqrt(-c^2*d*x^2 + d))/(c^6*d*x^2 - c^4*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.70, size = 752, normalized size = 2.58

$$a^2\left(-\frac{x^2\sqrt{-c^2d\,x^2+d}}{3c^2d}-\frac{2\sqrt{-c^2d\,x^2+d}}{3d\,c^4}\right)+b^2\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(4c^4x^4-5c^2x^2+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3-3\right.}{216c^4d\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2*(-1/3*x^2/c^2/d*(-c^2*d*x^2+d)^(1/2)-2/3/d/c^4*(-c^2*d*x^2+d)^(1/2))+b^
2*(-1/216*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(9*arccosh(c*x)^2-6*a
rccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^4/d/(c^2*x
^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^4/d/(c^2*x^2-1)-1/216*(-d*(c^2*x^2-1
))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(9*arccosh(c*x)^2+6*arccosh(c*x)+2)/c^4/d/(c
^2*x^2-1))+2*a*b*(-1/72*(-d*(c^2*x^2-1))^(1/2)*(4*c^4*x^4-5*c^2*x^2+4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+1)*(-1+3*a
rccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccosh(c*x))/c^4/d/(c^2*x^2-1)-3/8*(-d*(c^2
*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x)
)/c^4/d/(c^2*x^2-1)-1/72*(-d*(c^2*x^2-1))^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)*(1+3*
arccosh(c*x))/c^4/d/(c^2*x^2-1))
```

maxima [A] time = 1.02, size = 279, normalized size = 0.96

$$-\frac{1}{3}\,b^2\left(\frac{\sqrt{-c^2dx^2+d}\,x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right)\operatorname{arcosh}\left(cx\right)^2-\frac{2}{3}\,ab\left(\frac{\sqrt{-c^2dx^2+d}\,x^2}{c^2d}+\frac{2\,\sqrt{-c^2dx^2+d}}{c^4d}\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*b^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)
)*arccosh(c*x)^2 - 2/3*a*b*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d) + 2*sqrt(-c^2*
d*x^2 + d)/(c^4*d))*arccosh(c*x) - 1/3*a^2*(sqrt(-c^2*d*x^2 + d)*x^2/(c^2*d
) + 2*sqrt(-c^2*d*x^2 + d)/(c^4*d)) - 2/27*b^2*((sqrt(c^2*x^2 - 1)*sqrt(-d)
*x^2 + 20*sqrt(c^2*x^2 - 1)*sqrt(-d)/c^2)/(c^2*d) - 3*(c^2*sqrt(-d)*x^3 + 6
*sqrt(-d)*x)*arccosh(c*x)/(c^3*d)) + 2/9*(c^2*sqrt(-d)*x^3 + 6*sqrt(-d)*x)*
a*b/(c^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))^2}{\sqrt{-d(cx-1)(cx+1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x**3*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.197 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=226

$$-\frac{x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2d}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(\right.}{6bc^3\sqrt{d-c^2dx^2}}$$

```
[Out] -1/4*b^2*x*(-c*x+1)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)+1/4*b^2*arccosh(c*x)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*b*x^2*(a+b*arccosh(
c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)+1/6*(a+b*arccosh(c
*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(-c^2*d*x^2+d)^(1/2)-1/2*x*(a+b*ar
ccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2/d
```

Rubi [A] time = 0.63, antiderivative size = 234, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {5798, 5759, 5676, 5662, 90, 52}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{6bc^3\sqrt{d-c^2dx^2}}-\frac{x(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh\right.}{2c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -(b^2*x*(1 - c*x)*(1 + c*x))/(4*c^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^3*Sqrt[d - c^2*d*x^2]) - (b*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c*Sqrt[d - c^2*d*x^2]) - (x
*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*c^2*Sqrt[d - c^2*d*x^2]) +
(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(6*b*c^3*Sqrt[d - c^2
*d*x^2])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
```

```
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{2c\sqrt{d-c^2dx^2}} - \frac{x(1-cx)(1+}{2c^2}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{4c^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\cosh^{-1}(cx)}{4c^3\sqrt{d-c^2dx^2}} - \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2c\sqrt{d-c}}$$

Mathematica [A] time = 0.88, size = 228, normalized size = 1.01

$$\frac{-\frac{12a^2cx\sqrt{d-c^2dx^2}}{d} - \frac{12a^2\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)}{\sqrt{d}} + \frac{6ab\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2\cosh^{-1}(cx)\left(\cosh^{-1}(cx)+\sinh\left(2\cosh^{-1}(cx)\right)\right)-\cosh\left(2\cosh^{-1}(cx)\right)\right)}{\sqrt{d-c^2dx^2}} + b^2\sqrt{\cdots}}{24c^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] ((-12*a^2*c*x*Sqrt[d - c^2*d*x^2])/d - (12*a^2*ArcTan[(c*x*Sqrt[d - c^2*d*x
^2])/(Sqrt[d]*(-1 + c^2*x^2))])/Sqrt[d] + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*(4*ArcCosh[c*x]^3 - 6*ArcCosh[c*x]*Cosh[2*ArcCosh[c*x]] + (3 + 6*A
```

```
rcCosh[c*x]^2)*Sinh[2*ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (6*a*b*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-Cosh[2*ArcCosh[c*x]] + 2*ArcCosh[c*x]*(ArcCo
sh[c*x] + Sinh[2*ArcCosh[c*x]])))/Sqrt[d - c^2*d*x^2])/(24*c^3)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^2\,\text{arcosh}\,(cx)^2+2\,abx^2\,\text{arcosh}\,(cx)+a^2x^2\right)\sqrt{-c^2dx^2+d}}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\,\text{arcosh}\,(cx)+a\right)^2x^2}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.72, size = 624, normalized size = 2.76

$$-\frac{a^2x\sqrt{-c^2d\,x^2+d}}{2c^2d}+\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{2c^2\sqrt{c^2d}}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\text{arccosh}\,(cx)^3}{6d\,c^3\left(c^2x^2-1\right)}-\frac{b^2\sqrt{-d\left(c^2x^2\right.}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -1/2*a^2*x/c^2/d*(-c^2*d*x^2+d)^(1/2)+1/2*a^2/c^2/(c^2*d)^(1/2)*arctan((c^2
*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/6*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)^3-1/2*b^2*(-d*(c^2*x^2-1))
^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*x^3+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^
2/(c^2*x^2-1)*arccosh(c*x)^2*x+1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-
1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2-1/4*b^2*(-d*(c^2*x^2-1))^(1
/2)/d/(c^2*x^2-1)*x^3+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*x-1/
4*b^2*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)-1/2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/c
^3/(c^2*x^2-1)*arccosh(c*x)^2-a*b*(-d*(c^2*x^2-1))^(1/2)/d/(c^2*x^2-1)*arcc
osh(c*x)*x^3+1/2*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c/(c^2*x^2-1)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^2+a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^2/(c^2*x^2-1)*arccosh(c*x)*
x-1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/c^3/(c^2*x^2-1)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a^2\left(\frac{\sqrt{-c^2dx^2+d}\,x}{c^2d}-\frac{\arcsin\,(cx)}{c^3\sqrt{d}}\right)+\int\frac{b^2x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}}+\frac{2\,abx^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-\right.}}{\sqrt{-c^2dx^2+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*(sqrt(-c^2*d*x^2 + d)*x/(c^2*d) - arcsin(c*x)/(c^3*sqrt(d))) + int
egrate(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/sqrt(-c^2*d*x^2 + d
) + 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(-c^2*d*x^2 + d),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.198 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=155

$$-\frac{2abx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d}-\frac{2b^2(1-cx)(cx+1)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh^{-1}(}{c\sqrt{d-c^2dx^2}}$$

```
[Out] -2*b^2*(-c*x+1)*(c*x+1)/c^2/(-c^2*d*x^2+d)^(1/2)-2*a*b*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-2*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/(-c^2*d*x^2+d)^(1/2)-(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^2
/d
```

Rubi [A] time = 0.34, antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.148, Rules used = {5798, 5718, 5654, 74}

$$-\frac{2abx\sqrt{cx-1}\,\sqrt{cx+1}}{c\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2(1-cx)(cx+1)}{c^2\sqrt{d-c^2dx^2}}-\frac{2b^2x\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh}{c\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (-2*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*Sqrt[d - c^2*d*x^2]) - (2*b^2*(1
 - c*x)*(1 + c*x))/(c^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*ArcCosh[c*x])/(c*Sqrt[d - c^2*d*x^2]) - ((1 - c*x)*(1 + c*x)*(a +
 b*ArcCosh[c*x])^2)/(c^2*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int\left(a+b\cosh^{-1}(cx\right.}{c\sqrt{d-c^2dx^2}}$$
$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{c^2\sqrt{d-c^2dx^2}} - \frac{\left(2b^2\sqrt{-1+cx}\right.}{c}$$
$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{c\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(\right.}{c^2\sqrt{d}}$$
$$= -\frac{2abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^2\sqrt{d-c^2dx^2}} - \frac{2b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(c}{c\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.39, size = 149, normalized size = 0.96

$$\frac{\sqrt{d-c^2dx^2}\left(a^2\left(1-c^2x^2\right)+2b\cosh^{-1}(cx)\left(-ac^2x^2+a+bcx\sqrt{cx-1}\,\sqrt{cx+1}\right)+2abcx\sqrt{cx-1}\,\sqrt{cx+1}-2b^2\left(c\right.\right.}{c^2d(cx-1)(cx+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(2*a*b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + a^2*(1 - c^2
*x^2) - 2*b^2*(-1 + c^2*x^2) + 2*b*(a - a*c^2*x^2 + b*c*x*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])*ArcCosh[c*x] + b^2*(1 - c^2*x^2)*ArcCosh[c*x]^2))/(c^2*d*(-1 +
 c*x)*(1 + c*x))
```

fricas [A] time = 1.11, size = 218, normalized size = 1.41

$$\frac{2\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,abcx-\left(b^2c^2x^2-b^2\right)\sqrt{-c^2dx^2+d}\,\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\left(\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,b\right.}{c^4dx^2-c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a*b*c*x - (b^2*c^2*x^2 - b^2)*sqr
t(-c^2*d*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 2*(sqrt(-c^2*d*x^2 + d)*
sqrt(c^2*x^2 - 1)*b^2*c*x - (a*b*c^2*x^2 - a*b)*sqrt(-c^2*d*x^2 + d))*log(c
*x + sqrt(c^2*x^2 - 1)) - ((a^2 + 2*b^2)*c^2*x^2 - a^2 - 2*b^2)*sqrt(-c^2*d
*x^2 + d))/(c^4*d*x^2 - c^2*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arcosh}(cx)+a\right)^2 x}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2*x/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.31, size = 314, normalized size = 2.03

$$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{c^2d}+b^2\left(-\frac{\sqrt{-d\left(c^2x^2-1\right)}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\left(\operatorname{arccosh}(cx)^2-2\operatorname{arccosh}(cx)+2\right.}{2c^2d\left(c^2x^2-1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
[Out] -a^2/c^2/d*(-c^2*d*x^2+d)^(1/2)+b^2*(-1/2*(-d*(c^2*x^2-1))^(1/2)*((c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(arccosh(c*x)^2-2*arccosh(c*x)+2)/c^2/d/(
c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2
*x^2-1)*(arccosh(c*x)^2+2*arccosh(c*x)+2)/c^2/d/(c^2*x^2-1))+2*a*b*(-1/2*(-
d*(c^2*x^2-1))^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-1+arccos
h(c*x))/c^2/d/(c^2*x^2-1)-1/2*(-d*(c^2*x^2-1))^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)*(1+arccosh(c*x))/c^2/d/(c^2*x^2-1))
```

maxima [A] time = 0.39, size = 145, normalized size = 0.94

$$2\,b^2\left(\frac{\sqrt{-d}\,x\operatorname{arcosh}(cx)}{cd}-\frac{\sqrt{c^2x^2-1}\,\sqrt{-d}}{c^2d}\right)+\frac{2\,ab\sqrt{-d}\,x}{cd}-\frac{\sqrt{-c^2dx^2+d}\,b^2\operatorname{arcosh}(cx)^2}{c^2d}-\frac{2\,\sqrt{-c^2dx^2+d}\,ab\operatorname{arc}}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
[Out] 2*b^2*(sqrt(-d)*x*arccosh(c*x)/(c*d) - sqrt(c^2*x^2 - 1)*sqrt(-d)/(c^2*d))
+ 2*a*b*sqrt(-d)*x/(c*d) - sqrt(-c^2*d*x^2 + d)*b^2*arccosh(c*x)^2/(c^2*d)
- 2*sqrt(-c^2*d*x^2 + d)*a*b*arccosh(c*x)/(c^2*d) - sqrt(-c^2*d*x^2 + d)*a^
2/(c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}(cx)\right)^2}{\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.199 $\int \frac{(a+b\cosh^{-1}(cx))^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=53

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*d*x^2+d)^(1/
2)
```

Rubi [A] time = 0.20, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c*Sqrt[d - c^2*d
*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(-c^2*d*x^2 + d), x)
```

maple [B] time = 0.09, size = 149, normalized size = 2.81

$$\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{\sqrt{c^2d}}-\frac{b^2\sqrt{-(cx-1)(cx+1)d}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^3}{3cd\left(c^2x^2-1\right)}-\frac{ab\sqrt{-(cx-1)(cx+1)d}\,\sqrt{c}}{cd\left(c^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] a^2/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-1/3*b^2*(-(c
*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*x^2-1)*arccosh(
c*x)^3-a*b*(-(c*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(c^2*
x^2-1)*arccosh(c*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2\arcsin(cx)}{c\sqrt{d}}+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}}+\frac{2\,ab\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arcsin(c*x)/(c*sqrt(d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))^2/sqrt(-c^2*d*x^2 + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{(a+b\operatorname{acosh}(c\,x))^2}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.200 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=273

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{\sqrt{d-c^2dx^2}}$$

```
[Out] 2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-2*I*b*(a+b*arccosh(c*x))*polylog(2,-I*
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+
d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2
+d)^(1/2)-2*I*b^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {5798, 5761, 4180, 2531, 2282, 6589}

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx}\right.}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/Sqrt[d - c^2*d*x^2] + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3
, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
```

```
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\operatorname{Subst}\left(\int (a+bx)^2\operatorname{sech}(x)\,dx, x, \cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{\left(2ib\sqrt{-1+cx}\,\sqrt{1+cx}\right.}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\,\sqrt{1+cx}\,(}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\,\sqrt{1+cx}\,(}{}$$

$$= \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}} - \frac{2ib\sqrt{-1+cx}\,\sqrt{1+cx}\,(}{}$$

Mathematica [A] time = 0.63, size = 315, normalized size = 1.15

$$-\frac{a^2\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)}{\sqrt{d}}+\frac{a^2\log(cx)}{\sqrt{d}}-\frac{2iab\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(\operatorname{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right)-\operatorname{Li}_2\left(ie^{-\cosh^{-1}(cx)}\right)+\cosh^{-1}\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a^2*Log[c*x])/Sqrt[d] - (a^2*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d]
 - ((2*I)*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(Log[1 - I
/E^ArcCosh[c*x]] - Log[1 + I/E^ArcCosh[c*x]]) + PolyLog[2, (-I)/E^ArcCosh[c
*x]] - PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2] + (I*b^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*(1 + c*x)*(-(ArcCosh[c*x]^2*(Log[1 - I/E^ArcCosh[c*x]] -
 Log[1 + I/E^ArcCosh[c*x]])) - 2*ArcCosh[c*x]*(PolyLog[2, (-I)/E^ArcCosh[c*
x]] - PolyLog[2, I/E^ArcCosh[c*x]]) - 2*PolyLog[3, (-I)/E^ArcCosh[c*x]] + 2
*PolyLog[3, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2*d*x^2]
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2\right)}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^3 - d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x), x)
```

maple [F] time = 0.51, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}{x\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{\sqrt{d}}+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}\,x}+\frac{2\,ab\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) + inte
grate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x)
 + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}{x\sqrt{-d\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.201 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{dx}-\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}-\frac{2bc\sqrt{cx-1}\sqrt{cx+1}\log\left(e^{-2\cosh^{-1}(cx)}\right.}{\sqrt{d-c^2dx}}$$

```
[Out] -c*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-2*
b*c*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+b^2*c*polylog(2,-1/(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-(a+b
*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.52, antiderivative size = 194, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5798, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{b^2c\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{\sqrt{d-c^2dx^2}}-\frac{(1-cx)(cx+1)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}}+\frac{c\sqrt{cx-1}\sqrt{cx+1}\left(a+\right.}{\sqrt{d-c^2d}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2]
 - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(x*Sqrt[d - c^2*d*x^2]) - (
2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCos
h[c*x])])/Sqrt[d - c^2*d*x^2] - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog
[2, -E^(2*ArcCosh[c*x])])/Sqrt[d - c^2*d*x^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)\,\text{ta}\ldots\right.}{\sqrt{d-c^2dx^2}}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{(4\ldots}{\ldots}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2b\ldots}{\ldots}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2b\ldots}{\ldots}$$

$$= \frac{c\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{d-c^2dx^2}} - \frac{2b\ldots}{\ldots}$$

Mathematica [A] time = 0.86, size = 179, normalized size = 0.96

$$\frac{\frac{a^2\left(c^2x^2-1\right)}{x} - \frac{2ab\left(c^2x^2-1\right)\left(\frac{cx\log(cx)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\cosh^{-1}(cx)\right)}{x} + b^2c\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(\text{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+\cosh^{-1}(cx)\left(\frac{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\cosh\ldots}{cx}\right.\right.}{\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] ((a^2*(-1 + c^2*x^2))/x - (2*a*b*(-1 + c^2*x^2)*(-ArcCosh[c*x] + (c*x*Log[c
*x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])))/x + b^2*c*Sqrt[(-1 + c*x)/(1 + c*x)]*
(1 + c*x)*(ArcCosh[c*x]*(-ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*ArcCosh[c*x])/(c*x) - 2*Log[1 + E^(-2*ArcCosh[c*x])]) + PolyLog[2, -E^(
-2*ArcCosh[c*x])]))/Sqrt[d - c^2*d*x^2]
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2\right)}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^4 - d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}{\sqrt{-c^2dx^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

maple [B] time = 0.48, size = 513, normalized size = 2.76

$$-\frac{a^2\sqrt{-c^2d\,x^2+d}}{dx}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}\left(cx\right)^2c}{d\left(c^2x^2-1\right)}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}\left(cx\right)^2x\,c^2}{\left(c^2x^2-1\right)d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] -a^2/d/x*(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c-b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh
(c*x)^2*x/(c^2*x^2-1)/d*c^2+b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^
2*x^2-1)/d+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*
x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)*c-2*a*b*(-d*(c^2*x^2-1))^(1/2)*ar
ccosh(c*x)*x/(c^2*x^2-1)/d*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/x/
(c^2*x^2-1)/d+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c
^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^2d\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right)+i\,(-1)^{-2\,c^2dx^2+2\,d}\,\sqrt{d}\,\log\left(-2\,c^2d+\frac{2\,d}{x^2}\right)\right)abc}{d}+b^2\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}\,x^2}\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(c^2*d*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2) + I*(-1)^(-2*c^2*d*x^2 + 2*d)*sqr
t(d)*log(-2*c^2*d + 2*d/x^2))*a*b*c/d + b^2*integrate(log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^2), x) - 2*sqrt(-c^2*d*x^2 + d)
*a*b*arccosh(c*x)/(d*x) - sqrt(-c^2*d*x^2 + d)*a^2/(d*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{x^2\,\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{x^{2} \sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.202 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=430

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{\sqrt{d-c^2dx^2}}$$

```
[Out] b*c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x/(-c^2*d*x^2+d)^(1/2)+c
^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^
(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b*c^2*(a+b*arccos
h(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+I*b*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/
2)+I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*
arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x^2
```

Rubi [A] time = 0.88, antiderivative size = 438, normalized size of antiderivative = 1.02, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5748, 5761, 4180, 2531, 2282, 6589, 5662, 92, 205}

$$-\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{d-c^2dx^2}}+\frac{ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(\right.}{\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(x*Sqrt[d - c^2*d*x
^2]) - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*x^2*Sqrt[d - c^2*d*x
^2]) + (c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^Ar
cCosh[c*x]])/Sqrt[d - c^2*d*x^2] - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Ar
cTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/Sqrt[d - c^2*d*x^2] - (I*b*c^2*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/
Sqrt[d - c^2*d*x^2] + (I*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[
c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2] + (I*b^2*c^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/Sqrt[d - c^2*d*x^2]
 - (I*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/Sq
rt[d - c^2*d*x^2]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{x\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{2x^2\sqrt{d-c^2dx^2}} +$$

Mathematica [A] time = 84.83, size = 551, normalized size = 1.28

$$\frac{1}{2}a\left(-\frac{a\sqrt{d-c^2dx^2}}{dx^2} - \frac{ac^2\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right)}{\sqrt{d}} + \frac{ac^2\log(x)}{\sqrt{d}} + \frac{2b(cx+1)\left(-ic^2x^2\sqrt{\frac{cx-1}{cx+1}}\,\mathrm{Li}_2\left(-ie^{-\cosh^{-1}(cx}\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (a*(-((a*Sqrt[d - c^2*d*x^2])/(d*x^2)) + (a*c^2*Log[x])/Sqrt[d] - (a*c^2*Lo
g[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]])/Sqrt[d] + (2*b*(1 + c*x)*(c*x*Sqrt[(-1
+ c*x)/(1 + c*x)] - ArcCosh[c*x] + c*x*ArcCosh[c*x] - I*c^2*x^2*Sqrt[(-1 +
c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1
 + c*x)/(1 + c*x)]*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] - I*c^2*x^2*Sqrt[
(-1 + c*x)/(1 + c*x)]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + I*c^2*x^2*Sqrt[(-1
+ c*x)/(1 + c*x)]*PolyLog[2, I/E^ArcCosh[c*x]]))/(x^2*Sqrt[d - c^2*d*x^2]))
)/2 - (b^2*c^2*Sqrt[d - c^2*d*x^2]*((2*Sqrt[(-1 + c*x)/(1 + c*x)]*ArcCosh[c
*x])/(-(c*x) + c^2*x^2) + ArcCosh[c*x]^2/(c^2*x^2) - (I*((-4*I)*ArcTan[Tanh
[ArcCosh[c*x]/2]] + ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - ArcCosh[c*x]
^2*Log[1 + I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x
]] - 2*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 2*PolyLog[3, (-I)/E^ArcC
```

```
osh[c*x]] - 2*PolyLog[3, I/E^ArcCosh[c*x]]))/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x))))/(2*d)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^5-dx^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^5 - d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int\frac{(a+b\,\mathrm{arccosh}(cx))^2}{x^3\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{c^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{\sqrt{d}}+\frac{\sqrt{-c^2dx^2+d}}{dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\sqrt{-c^2dx^2+d}\,x^3}+\frac{2\,ab\log\left(cx+\sqrt{cx+1}\right.}{\sqrt{-c^2dx^2+d\,}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*(c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/sqrt(d) +
 sqrt(-c^2*d*x^2 + d)/(d*x^2))*a^2 + integrate(b^2*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x^3) + 2*a*b*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))/(sqrt(-c^2*d*x^2 + d)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{(a+b\,\mathrm{acosh}(c\,x))^2}{x^3\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{x^3 \sqrt{-d(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(x**3*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.203 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=328

$$-\frac{2c^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3dx}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}}-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3}-$$

```
[Out] 1/3*b^2*c^2*(-c*x+1)*(c*x+1)/x/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+b*arccosh(c*
x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2/(-c^2*d*x^2+d)^(1/2)-2/3*c^3*(a+b*arcco
sh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-4/3*b*c^3*(a+b*
arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)+2/3*b^2*c^3*polylog(2,-1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-c^2*d*x^2+d)^(1/2)-1/3*(a+
b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/d/x^3-2/3*c^2*(a+b*arccosh(c*x))^2*(
-c^2*d*x^2+d)^(1/2)/d/x
```

Rubi [A] time = 0.87, antiderivative size = 344, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {5798, 5748, 5724, 5660, 3718, 2190, 2279, 2391, 5662, 95}

$$-\frac{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3\sqrt{d-c^2dx^2}}+\frac{2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3\sqrt{d-c^2dx^2}}-\frac{2c^2(1-cx)(cx+1)}{3x\sqrt{d\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*x^2*Sqrt[d - c^2*d*x^2]) + (2*c^3
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*Sqrt[d - c^2*d*x^2
]) - ((1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*x^3*Sqrt[d - c^2*d*x^2
]) - (2*c^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*x*Sqrt[d - c^2*d
*x^2]) - (4*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 +
 E^(2*ArcCosh[c*x])])/(3*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*Sqrt[d - c^2*d*x^2])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
```

```
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

$$= -\frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{3\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a+b\cosh^{-1}(cx)\right)^2}{3x^3\sqrt{d-c^2dx^2}} - 2c$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} - \frac{(1-cx)(1+cx)\left(a\right.}{3x^3\sqrt{d-}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\,\sqrt{1+}}{3\sqrt{}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\,\sqrt{1+}}{3\sqrt{}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\,\sqrt{1+}}{3\sqrt{}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3x\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3x^2\sqrt{d-c^2dx^2}} + \frac{2c^3\sqrt{-1+cx}\,\sqrt{1+}}{3\sqrt{}}$$

Mathematica [A] time = 1.64, size = 346, normalized size = 1.05

$$(cx-1)\sqrt{cx+1}\left(a^2\sqrt{cx-1}\,\sqrt{cx+1}\left(2c^2x^2+1\right)-4abc^3x^3\log(cx)+abcx-b^2c^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\right)+b\sqrt{cx-1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (-(b^2*Sqrt[-1 + c*x]*(1 + c*x)*(1 - c*x + 2*c^2*x^2 + 2*c^3*x^3*(-1 + Sqrt
[(-1 + c*x)/(1 + c*x)]))*ArcCosh[c*x]^2) + b*Sqrt[-1 + c*x]*(1 + c*x)*ArcCo
sh[c*x]*(b*c*x*Sqrt[(-1 + c*x)/(1 + c*x)] + 2*a*(-1 + c*x - 2*c^2*x^2 + 2*c
^3*x^3) - 4*b*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*Log[1 + E^(-2*ArcCosh[c*x]
)]) + (-1 + c*x)*Sqrt[1 + c*x]*(a*b*c*x - b^2*c^2*x^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x] + a^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 + 2*c^2*x^2) - 4*a*b*c^3*x^3*
Log[c*x]) + (2*b^2*c^3*x^3*(-1 + c*x)^(3/2)*PolyLog[2, -E^(-2*ArcCosh[c*x])
])/Sqrt[(-1 + c*x)/(1 + c*x)])/(3*x^3*Sqrt[-1 + c*x]*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^2dx^6-dx^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^2*d*x^6 - d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\sqrt{-c^2dx^2+d}\,x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(sqrt(-c^2*d*x^2 + d)*x^4), x)
```

maple [B] time = 0.90, size = 2198, normalized size = 6.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] 2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8+4*a*b*(-d*
(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*c^5-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)
*x*arccosh(c*x)*(c*x+1)*(c*x-1)*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4
*x^4-2*c^2*x^2-1)/x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+4/3*b^2*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)
*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-4/3*a*b*(-d*(c^2*x^2-1))^(1/
2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-2/3*a*b*(-d*(c^2*x^2-1
))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*(c*x+1)*(c*x-1)*c^4+4/3*a*b*(-d*(c^2*x
^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(
1/2)*c^3-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*c+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-1/3*b^
2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*c^2+1/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)^2-2/3*b^2*(-d*(c^2*
x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-1/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+4/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)/x*arccosh(c*x)^2*c^2
+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*c^8-2/3*a*b*(
-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*c^6-2/3*a*b*(-d*(c^2*x^
2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d/(3*c^4*x^4-2*c^2*x^2-1)/x^3*arccosh(c*x)-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d
/(3*c^4*x^4-2*c^2*x^2-1)*x^3*(c*x+1)*(c*x-1)*c^6-b^2*(-d*(c^2*x^2-1))^(1/2)
/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+2/3*b^2*(-d*
(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arccosh(c*x)^2*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*c^3-b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*arcco
sh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*arccosh(c*x)^2*c^3+2/3*b^2*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(c^2*x^2-1)*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3-4*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2
*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^
4-2*c^2*x^2-1)*x*arccosh(c*x)*c^4-a*b*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2
*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d/(3*c^4*x^4-2*c^2*x^2-1)/x*arccosh(c*x)*c^2-2/3*a^2*c^2/d/x*(-c^2*d*x^2+d)
^(1/2)-1/3*a^2/d/x^3*(-c^2*d*x^2+d)^(1/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/
```

```
(3*c^4*x^4-2*c^2*x^2-1)*x^3*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4
-2*c^2*x^2-1)*x*c^4-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d/(c^2*x^2-1)*arccosh(c*x)*c^3-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*
x^4-2*c^2*x^2-1)*x^3*arccosh(c*x)*(c*x+1)*(c*x-1)*c^6+2*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^2*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*c^5+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^5*arcc
osh(c*x)*c^8-2*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^3*arc
cosh(c*x)^2*c^6-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)*x^
3*arccosh(c*x)*c^6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d/(3*c^4*x^4-2*c^2*x^2-1)
*x*arccosh(c*x)^2*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{4\,c^2\sqrt{-d}\,\log(x)}{d}-\frac{\sqrt{-d}}{dx^2}\right)abc-\frac{2}{3}\,ab\left(\frac{2\,\sqrt{-c^2dx^2+d}\,c^2}{dx}+\frac{\sqrt{-c^2dx^2+d}}{dx^3}\right)\operatorname{arcosh}(cx)-\frac{1}{3}\,a^2\left(\frac{2\,\sqrt{-c^2dx^2+d}\,c^2}{dx}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(4*c^2*sqrt(-d)*log(x)/d - sqrt(-d)/(d*x^2))*a*b*c - 2/3*a*b*(2*sqrt(-c
^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3))*arccosh(c*x) - 1/3*
a^2*(2*sqrt(-c^2*d*x^2 + d)*c^2/(d*x) + sqrt(-c^2*d*x^2 + d)/(d*x^3)) + b^2
*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(sqrt(-c^2*d*x^2 + d)*x
^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{x^4\,\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{x^{4} \sqrt{- d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.204 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=556

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^6d^2}+\frac{4b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{c^6d\sqrt{d-c^2dx^2}}$$

```
[Out] 94/27*b^2*(-c*x+1)*(c*x+1)/c^6/d/(-c^2*d*x^2+d)^(1/2)+2/27*b^2*x^2*(-c*x+1)
*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^4*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*
x^2+d)^(1/2)+16/3*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1
/2)+16/3*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d
)^(1/2)-2*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*
x^2+d)^(1/2)+2/9*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d
/(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d/
(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^6/d/(-c^2*d*x^2+d)^(1/2)+8/3*(a+b*arccosh(c*x))^2*
(-c^2*d*x^2+d)^(1/2)/c^6/d^2+4/3*x^2*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1
/2)/c^4/d^2
```

Rubi [A] time = 1.37, antiderivative size = 578, normalized size of antiderivative = 1.04, number of steps used = 23, number of rules used = 14, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.483, Rules used = {5798, 5752, 5759, 5718, 5654, 74, 5662, 100, 12, 5766, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^6d\sqrt{d-c^2dx^2}}+\frac{16abx\sqrt{cx-1}\sqrt{c}}{3c^5d\sqrt{d-c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) + (94
*b^2*(1 - c*x)*(1 + c*x))/(27*c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*x^2*(1 -
c*x)*(1 + c*x))/(27*c^4*d*Sqrt[d - c^2*d*x^2]) + (16*b^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^5*d*Sqrt[d - c^2*d*x^2]) + (2*
b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3*d*Sqrt[d -
c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (8
*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^6*d*Sqrt[d - c^2*d*x^2])
+ (4*x^2*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^4*d*Sqrt[d - c^2*
d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^
ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(c^6*d*Sqrt[d - c^2*d
*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
```

```
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+\right.}{}$$

$$= -\frac{2bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3c^3d\sqrt{d-c^2dx^2}} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4x^2(1-}{}$$

$$= -\frac{2b^2x^2(1-cx)(1+cx)}{9c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^5d\sqrt{d-c^2dx^2}} + \frac{2bx^3\sqrt{-1+}{}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-}{}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} - \frac{22b^2(1-cx)(1+cx)}{9c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x}{}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x}{}$$

$$= \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d\sqrt{d-c^2dx^2}} + \frac{94b^2(1-cx)(1+cx)}{27c^6d\sqrt{d-c^2dx^2}} + \frac{2b^2x^2(1-cx)(1+cx)}{27c^4d\sqrt{d-c^2dx^2}} + \frac{16b^2x}{}$$

Mathematica [A] time = 3.84, size = 358, normalized size = 0.64

$$\frac{-36a^2\left(c^4x^4+4c^2x^2-8\right)+3ab\left(-60\cosh\left(2\cosh^{-1}(cx)\right)\cosh^{-1}(cx)-3\cosh\left(4\cosh^{-1}(cx)\right)\cosh^{-1}(cx)+135\,\mathrm{c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-36*a^2*(-8 + 4*c^2*x^2 + c^4*x^4) + 3*a*b*(135*ArcCosh[c*x] - 60*ArcCosh[
c*x]*Cosh[2*ArcCosh[c*x]] - 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 72*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 62*Sinh[2*ArcCos
h[c*x]] + Sinh[4*ArcCosh[c*x]]) - b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*
(378*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - 378*c*x*ArcCosh[c*x] + 189*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*ArcCosh[c*x]*Cosh[3*Arc
Cosh[c*x]] - 54*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] + 216*ArcCosh[c*x]*Log[
1 - E^(-ArcCosh[c*x])] - 216*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 216*
PolyLog[2, -E^(-ArcCosh[c*x])] - 216*PolyLog[2, E^(-ArcCosh[c*x])] + 2*Sinh
[3*ArcCosh[c*x]] + 9*ArcCosh[c*x]^2*Sinh[3*ArcCosh[c*x]] + 54*ArcCosh[c*x]^
2*Tanh[ArcCosh[c*x]/2]))/(108*c^6*d*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2x^5\,\mathrm{arcosh}\,(cx)^2+2\,abx^5\,\mathrm{arcosh}\,(cx)+a^2x^5\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.86, size = 1099, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/3*a^2*x^4/c^2/d/(-c^2*d*x^2+d)^(1/2)-4/3*a^2/c^4*x^2/d/(-c^2*d*x^2+d)^(1
/2)+8/3*a^2/c^6/d/(-c^2*d*x^2+d)^(1/2)-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d
^2/(c^2*x^2-1)*arccosh(c*x)^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))-2/9*b^2*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*arccosh(c*x)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*x^3-10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^5/d^2/(c^2*x
^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x-2*b^2*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))+2/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2
-1)*x^4+92/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*x^2+2*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*polylog(
2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-94/27*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d^2
/(c^2*x^2-1)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x
)^2*x^4+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x
^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^
2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*(c^2*x^2-1))^(1/
2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))-2/9*a*b*(-d*(c^2*x^2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^3-10/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^5/d^2/(c^2*x^2-1)*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^2/(c^2*x
^2-1)*arccosh(c*x)+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arcco
sh(c*x)*x^4+8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)
*x^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(c^2*
x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{x^4}{\sqrt{-c^2dx^2+d}\,c^2d}+\frac{4x^2}{\sqrt{-c^2dx^2+d}\,c^4d}-\frac{8}{\sqrt{-c^2dx^2+d}\,c^6d}\right)+\frac{\left(b^2c^4\sqrt{d}\,x^4+4\,b^2c^2\sqrt{d}\,x^2-8\,b^2\sqrt{d}\right)\sqrt{cx+}}{3\left(c^8d^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*a^2*(x^4/(sqrt(-c^2*d*x^2 + d)*c^2*d) + 4*x^2/(sqrt(-c^2*d*x^2 + d)*c^
4*d) - 8/(sqrt(-c^2*d*x^2 + d)*c^6*d)) + 1/3*(b^2*c^4*sqrt(d)*x^4 + 4*b^2*c
^2*sqrt(d)*x^2 - 8*b^2*sqrt(d))*sqrt(c*x + 1)*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))^2/(c^8*d^2*x^2 - c^6*d^2) + integrate(-2/3*((4*b^2
*c^3*x^3 - (3*a*b*c^5 - b^2*c^5)*x^5 - 8*b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) +
 (3*b^2*c^4*x^4 - (3*a*b*c^6 - b^2*c^6)*x^6 - 12*b^2*c^2*x^2 + 8*b^2)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(c^10*d^(3/
2)*x^5 - 2*c^8*d^(3/2)*x^3 + c^6*d^(3/2)*x + (c^9*d^(3/2)*x^4 - 2*c^7*d^(3/
2)*x^2 + c^5*d^(3/2))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.205 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=440

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^3}{2bc^5d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^5d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-}}{}$$

```
[Out] 1/4*b^2*x*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^3*(a+b*arccosh(c*x)
)^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-1/4*b^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+1/2*b*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^3*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/b/c^5/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(c*x))*ln(
1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-
c^2*d*x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5/d/(-c^2*d*x^2+d)^(1/2)+3/2*x*(a+b*arccosh(c*x))^
2*(-c^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] time = 1.21, antiderivative size = 451, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5759, 5676, 5662, 90, 52, 5766, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^5d\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right.}{2c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (b^2*x*(1 - c*x)*(1 + c*x))/(4*c^4*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(4*c^5*d*Sqrt[d - c^2*d*x^2]) + (b*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(2*c^3*d*Sqrt[d - c^2*d*x^2
]) + (x^3*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^5*d*Sqrt[d - c^2*d*x^2]) + (3
*x*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(2*c^4*d*Sqrt[d - c^2*d*x^2]
) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(2*b*c^5*d*Sqrt[d
 - c^2*d*x^2]) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log
[1 - E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^5*d*Sqrt[d - c^2*d*x^2])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
```

```
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+\right.}{}$$

$$= -\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{3x(1-cx)}{}$$

$$= -\frac{b^2x(1-cx)(1+cx)}{2c^4d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{2c^3d\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh\right.}{c^2d\sqrt{d-c}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{2c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(\right.}{2c^3d\sqrt{d-c}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(\right.}{2c^3d\sqrt{d-c}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(\right.}{2c^3d\sqrt{d-c}}$$

$$= \frac{b^2x(1-cx)(1+cx)}{4c^4d\sqrt{d-c^2dx^2}} - \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{4c^5d\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(\right.}{2c^3d\sqrt{d-c}}$$

Mathematica [A] time = 2.00, size = 343, normalized size = 0.78

$$-4a^2c\sqrt{d}\,x\left(c^2x^2-3\right)+12a^2\sqrt{d-c^2dx^2}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right)+2ab\sqrt{d}\,\left(8cx\cosh^{-1}(cx)-\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(8\log\left(\sqrt{\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-4*a^2*c*Sqrt[d]*x*(-3 + c^2*x^2) + 12*a^2*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x
*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 2*a*b*Sqrt[d]*(8*c*x*ArcC
osh[c*x] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(6*ArcCosh[c*x]^2 - Cosh[2*
ArcCosh[c*x]] + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)] + 2*ArcCosh[c*x
]*Sinh[2*ArcCosh[c*x]])) + b^2*Sqrt[d]*(8*c*x*ArcCosh[c*x]^2 + 8*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])] - Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(4*ArcCosh[c*x]^3 - 2*ArcCosh[c*x]*(Cosh[2*ArcCosh[c
*x]] - 8*Log[1 - E^(-2*ArcCosh[c*x])]) + Sinh[2*ArcCosh[c*x]] + 2*ArcCosh[c
*x]^2*(4 + Sinh[2*ArcCosh[c*x]]))))/(8*c^5*d^(3/2)*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2x^4\,\mathrm{arcosh}\,(cx)^2+2\,abx^4\,\mathrm{arcosh}\,(cx)+a^2x^4\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 1.01, size = 1141, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -1/2*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(1/2)+3/2*a^2/c^4*x/d/(-c^2*d*x^2+d)^(1/2
)-3/2*a^2/c^4/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-
1/2*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^2+1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arc
cosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^
2/c^5/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/2*b^
2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)^2*x^3-3/2*b^2*(-d
*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arccosh(c*x)^2*x+1/2*b^2*(-d*(c^2*x
^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)^3
+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-
1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/4*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^2/c^2/(c^2*x^2-1)*x^3-1/4*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1
)*x-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2
-1)*arccosh(c*x)^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^2/c^5/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+3/2*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^5/(c^2*x^2-1)*arccos
h(c*x)^2+a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^2/(c^2*x^2-1)*arccosh(c*x)*x^3-1/
2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)
^(1/2)*(c*x-1)^(1/2)-3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^4/(c^2*x^2-1)*arcco
sh(c*x)*x+1/4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/c^5/(c^2*x^2-1)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/
c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a^2\left(\frac{x^3}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{3\,x}{\sqrt{-c^2dx^2+d}\,c^4d}+\frac{3\,\arcsin\left(cx\right)}{c^5d^{\frac{3}{2}}}\right)+\int\frac{b^2x^4\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx^4\,\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a^2*(x^3/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 3*x/(sqrt(-c^2*d*x^2 + d)*c^4*
d) + 3*arcsin(c*x)/(c^5*d^(3/2))) + integrate(b^2*x^4*log(c*x + sqrt(c*x +
```

```
1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x^4*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \left(a + b \operatorname{acosh}\left(c\,x\right)\right)^2}{\left(d - c^2\,d\,x^2\right)^{3/2}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \left(a + b \operatorname{acosh}\left(cx\right)\right)^2}{\left(-d\left(cx - 1\right)\left(cx + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.206 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=413

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2}+\frac{4b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh\right.}{c^4d\sqrt{d-c^2dx^2}}$$

```
[Out] 2*b^2*(-c*x+1)*(c*x+1)/c^4/d/(-c^2*d*x^2+d)^(1/2)+x^2*(a+b*arccosh(c*x))^2/
c^2/d/(-c^2*d*x^2+d)^(1/2)+4*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*
d*x^2+d)^(1/2)+4*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2
*d*x^2+d)^(1/2)-2*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/
(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*poly
log(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d/(
-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^4/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))^2*(-c
^2*d*x^2+d)^(1/2)/c^4/d^2
```

Rubi [A] time = 0.92, antiderivative size = 424, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.345, Rules used = {5798, 5752, 5718, 5654, 74, 5766, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^4d\sqrt{d-c^2dx^2}}+\frac{4abx\sqrt{cx-1}\sqrt{cx}}{c^3d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (4*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*(1 - c*x)*(1 + c*x))/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b^2*x*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcCosh[c*x])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (2*b*x*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(c^3*d*Sqrt[d - c^2*d*x^2]) + (x^
2*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (2*(1 - c*x)*(1 + c
*x)*(a + b*ArcCosh[c*x])^2)/(c^4*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^4*d*Sqrt[
d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh
[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*P
olyLog[2, E^ArcCosh[c*x]])/(c^4*d*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1\right.}{}}{}$$

$$= -\frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c^3d\sqrt{d-c^2dx^2}} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{2(1-c}{}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} - \frac{2bx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\right.}{c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} - \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-}}{c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-}}{c^3d\sqrt{d-c^2dx^2}}$$

$$= \frac{4abx\sqrt{-1+cx}\,\sqrt{1+cx}}{c^3d\sqrt{d-c^2dx^2}} + \frac{2b^2(1-cx)(1+cx)}{c^4d\sqrt{d-c^2dx^2}} + \frac{4b^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-}}{c^3d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.64, size = 302, normalized size = 0.73

$$\frac{-2a^2\left(c^2x^2-2\right)+2ab\left(-\cosh^{-1}(cx)\left(\cosh\left(2\cosh^{-1}(cx)\right)-3\right)+\sinh\left(2\cosh^{-1}(cx)\right)-2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\log\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (-2*a^2*(-2 + c^2*x^2) + 2*a*b*(-(ArcCosh[c*x]*(-3 + Cosh[2*ArcCosh[c*x]]))
 - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + Sinh[
2*ArcCosh[c*x]]) + b^2*(2 + 3*ArcCosh[c*x]^2 - 2*Cosh[2*ArcCosh[c*x]] - Arc
Cosh[c*x]^2*Cosh[2*ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
rcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]
*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]]
))/(2*c^4*d*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(b^2x^3\operatorname{arcosh}(cx)^2+2\,abx^3\operatorname{arcosh}(cx)+a^2x^3\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.69, size = 836, normalized size = 2.02

$$-\frac{a^2x^2}{c^2d\sqrt{-c^2d\,x^2+d}}+\frac{2a^2}{d\,c^4\sqrt{-c^2d\,x^2+d}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}\left(cx\right)^2x^2}{c^2d^2\left(c^2x^2-1\right)}-\frac{2b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}\left(cx\right)\sqrt{c}}{c^3d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2*x^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+2*a^2/d/c^4/(-c^2*d*x^2+d)^(1/2)+b^2*(-
d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)^2*x^2-2*b^2*(-d*(c^2*
x^2-1))^(1/2)/c^3/d^2/(c^2*x^2-1)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*x^2-2*b^2*(-d*(c^2*x^2-1
))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^
4/d^2/(c^2*x^2-1)+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*po
lylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^4/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*a*b*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*x^2-2*a*b*(-d*(c^2*x^2-
1))^(1/2)/c^3/d^2/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x-4*a*b*(-d*(c^2*
x^2-1))^(1/2)/c^4/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^(1/2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^
2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-abc\left(\frac{2\sqrt{-d}\,x}{c^4d^2}+\frac{\sqrt{-d}\,\log\left(cx+1\right)}{c^5d^2}-\frac{\sqrt{-d}\,\log\left(cx-1\right)}{c^5d^2}\right)-2\,ab\left(\frac{x^2}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{2}{\sqrt{-c^2dx^2+d}\,c^4d}\right)\operatorname{arcosh}\left(cx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -a*b*c*(2*sqrt(-d)*x/(c^4*d^2) + sqrt(-d)*log(c*x + 1)/(c^5*d^2) - sqrt(-d)
*log(c*x - 1)/(c^5*d^2)) - 2*a*b*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) - 2/(sqr
t(-c^2*d*x^2 + d)*c^4*d))*arccosh(c*x) - a^2*(x^2/(sqrt(-c^2*d*x^2 + d)*c^2
```

```
*d) - 2/(sqrt(-c^2*d*x^2 + d)*c^4*d)) - b^2*((c^2*x^2 - 2)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))^2/(sqrt(c*x + 1)*sqrt(-c*x + 1)*c^4*d^(3/2)) - integ
rate(2*(c^4*x^4 - 3*c^2*x^2 + (c^3*x^3 - 2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
 + 2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c^5*d^(3/2)*x^2 - c^3*d^(3/
2))*(c*x + 1)*sqrt(c*x - 1) + (c^6*d^(3/2)*x^3 - c^4*d^(3/2)*x)*sqrt(c*x +
1))*sqrt(-c*x + 1)), x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.207 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=257

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\,\sqrt{}}{}$$

```
[Out] x*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x))^3
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(
c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3/d/(-c^2*d*x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {5798, 5752, 5676, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{c^3d\sqrt{d-c^2dx^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(c^2*d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(c^3*d*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^3*d*Sqrt[d - c^2*d*x^2]
) - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*Arc
Cosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*PolyLog[2, E^(2*ArcCosh[c*x])])/(c^3*d*Sqrt[d - c^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
```

```
erQ[4*k] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\right.}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^3d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{\right.}}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+}}{3bc^3}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+}}{3bc^3}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+}}{3bc^3}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}{c^3d\sqrt{d-c^2dx^2}} - \frac{\sqrt{-1+cx}\,\sqrt{1+}}{3bc^3}$$

Mathematica [A] time = 2.05, size = 270, normalized size = 1.05

$$3a^2\sqrt{d}\,\sqrt{d-c^2dx^2}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + 3a^2cdx + 3abd\left(2cx\cosh^{-1}(cx) - \sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(2\log\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\right)\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (3*a^2*c*d*x + 3*a^2*Sqrt[d]*Sqrt[d - c^2*d*x^2]*ArcTan[(c*x*Sqrt[d - c^2*d
*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + 3*a*b*d*(2*c*x*ArcCosh[c*x] - Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]^2 + 2*Log[Sqrt[(-1 + c*x)/(1 + c*x
)]*(1 + c*x)])) - b^2*d*(ArcCosh[c*x]*(-3*c*x*ArcCosh[c*x] + Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*(ArcCosh[c*x]*(3 + ArcCosh[c*x]) + 6*Log[1 - E^(-2*A
rcCosh[c*x])])) - 3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*A
rcCosh[c*x])]))/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2x^2\,\text{arcosh}\,(cx)^2 + 2\,abx^2\,\text{arcosh}\,(cx) + a^2x^2\right)\sqrt{-c^2dx^2+d}}{c^4d^2x^4 - 2\,c^2d^2x^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral((b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(-
c^2*d*x^2 + d)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\,(cx) + a\right)^2 x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.66, size = 738, normalized size = 2.87

$$\frac{a^2x}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{a^2\arctan\left(\frac{\sqrt{c^2d}\,x}{\sqrt{-c^2d\,x^2+d}}\right)}{c^2d\sqrt{c^2d}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{arccosh}\left(cx\right)^3}{3d^2c^3\left(c^2x^2-1\right)}-\frac{b^2\sqrt{-d\left(c^2x^2-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2*x/c^2/d/(-c^2*d*x^2+d)^(1/2)-a^2/c^2/d/(c^2*d)^(1/2)*arctan((c^2*d)^(1/
2)*x/(-c^2*d*x^2+d)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^3-b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)^2/d^2/c^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1
/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(
c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*
b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*
arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))+a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)^2-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*arccosh(c*x)-2*a*b*(-d*(c^2*x^2-1))^
(1/2)*arccosh(c*x)/d^2/c^2/(c^2*x^2-1)*x+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^2/c^3/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\frac{x}{\sqrt{-c^2dx^2+d}\,c^2d}-\frac{\arcsin\left(cx\right)}{c^3d^{\frac{3}{2}}}\right)+\int\frac{b^2x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] a^2*(x/(sqrt(-c^2*d*x^2 + d)*c^2*d) - arcsin(c*x)/(c^3*d^(3/2))) + integrat
e(b^2*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) +
 2*a*b*x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^2}{(-d (cx - 1) (cx + 1))^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.208 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}}+\frac{4b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(-e^{c}\right.}{c^2d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(1/2)+4*b*(a+b*arccosh(c*x))*arct
anh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^
2*d*x^2+d)^(1/2)+2*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5798, 5718, 5694, 4182, 2279, 2391}

$$\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}(cx\right.}{c^2d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(c^2*d*Sqrt[d - c^2*d*x^2]) + (4*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c
^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]
])/(c^2*d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^ArcCosh[c*x]])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{-1+c^2x^2}\,dx}{cd\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)\text{csch}(x)\,dx,x,\cosh\right.}{c^2d\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{c^2d\sqrt{d-c^2dx^2}} + \frac{4b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{c^2d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.01, size = 210, normalized size = 1.07

$$\frac{a^2+2ab\cosh^{-1}(cx)-2ab\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right)-2b^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\text{Li}_2\left(-e^{-\cosh^{-1}(cx)}\right)+2b^2}{c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (a^2 + 2*a*b*ArcCosh[c*x] + b^2*ArcCosh[c*x]*(ArcCosh[c*x] - 2*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[
c*x])])) - 2*a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]
/2]] - 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c
*x])] + 2*b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c
*x])])/(c^2*d*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2x\operatorname{arcosh}(cx)^2+2\,abx\operatorname{arcosh}(cx)+a^2x\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.34, size = 542, normalized size = 2.77

$$\frac{a^2}{c^2d\sqrt{-c^2d\,x^2+d}}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\operatorname{arccosh}(cx)^2}{c^2d^2\left(c^2x^2-1\right)}+\frac{2b^2\sqrt{-d\left(c^2x^2-1\right)}\sqrt{cx-1}\sqrt{cx+1}\operatorname{arccosh}(cx)\ln\left(1-\right.}{c^2d^2\left(c^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2/c^2/d/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-
1)*arccosh(c*x)^2+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^
2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*po
lylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))-2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^2/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*
(c^2*x^2-1))^(1/2)/c^2/d^2/(c^2*x^2-1)*arccosh(c*x)+2*a*b*(-d*(c^2*x^2-1))^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)-1)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^2/d^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2}{\sqrt{-c^2dx^2+d}\,c^2d}+\int\frac{b^2x\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] a^2/(sqrt(-c^2*d*x^2 + d)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2) + 2*a*b*x*log(c*x + sqrt(c*x + 1)*s
qrt(c*x - 1))/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a+b\,\mathrm{acosh}\,(c\,x))^2}{\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a+b\,\mathrm{acosh}\,(cx))^2}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.209 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=198

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(1-e^{2\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{cd\sqrt{d-c^2dx^2}}$$

```
[Out] x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+(a+b*arccosh(c*x))^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)-2*b*(a+b*arccosh(c*x))*ln(1-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d/(-c^2*d*
x^2+d)^(1/2)-b^2*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5713, 5688, 5715, 3716, 2190, 2279, 2391}

$$-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{cd\sqrt{d-c^2dx^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^2)/(c*d*Sqrt[d - c^2*d*x^2]) - (2*b*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(c*d*S
qrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*Arc
Cosh[c*x])])/(c*d*Sqrt[d - c^2*d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{1-c^2x^2}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(2b\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int(a+bx)\coth(x)\,dx,x,\cosh\right.}{cd\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} + \frac{\left(4b\sqrt{-1+cx}\,\sqrt{1\right.}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+}{}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}{cd\sqrt{d-c^2dx^2}} - \frac{2b\sqrt{-1+cx}\,\sqrt{1+}{}$$

Mathematica [A] time = 0.46, size = 126, normalized size = 0.64

$$\frac{\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(\left(a+b\cosh^{-1}(cx)\right)\left(a+b\cosh^{-1}(cx)-2b\log\left(1-e^{\cosh^{-1}(cx)}\right)-2b\log\left(e^{\cosh^{-1}(cx)}+1\right)\right)-2b^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(cx)}\right)-2b^2\mathrm{Li}_2\left(e^{\cosh^{-1}(cx)}\right)\right)}{c}+x\left(a\right.}{d\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x])^2 + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c
*x])*(a + b*ArcCosh[c*x] - 2*b*Log[1 - E^ArcCosh[c*x]] - 2*b*Log[1 + E^ArcC
osh[c*x]]) - 2*b^2*PolyLog[2, -E^ArcCosh[c*x]] - 2*b^2*PolyLog[2, E^ArcCosh
[c*x]]))/c)/(d*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [B] time = 0.31, size = 578, normalized size = 2.92

$$\frac{a^2x}{d\sqrt{-c^2d\,x^2+d}}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2}{d^2c\left(c^2x^2-1\right)}-\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)^2\,x}{d^2\left(c^2x^2-1\right)}+\frac{2b^2\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] a^2/d*x/(-c^2*d*x^2+d)^(1/2)-b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)^2-b^2*(-d*(c^2*x^2-1))^(1/2)*arccos
h(c*x)^2/d^2/(c^2*x^2-1)*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-
1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/d^2/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*arccosh(c
*x)-2*a*b*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)/d^2/(c^2*x^2-1)*x+2*a*b*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/c/(c^2*x^2-1)*ln((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{abc\sqrt{-\frac{1}{c^4d}}\,\log\left(x^2-\frac{1}{c^2}\right)}{d}+b^2\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx+\frac{2\,abx\operatorname{arcosh}(cx)}{\sqrt{-c^2dx^2+d}\,d}+\frac{a^2x}{\sqrt{-c^2dx^2+d}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*b*c*sqrt(-1/(c^4*d))*log(x^2 - 1/c^2)/d + b^2*integrate(log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(3/2), x) + 2*a*b*x*arccosh(c*x)/
(sqrt(-c^2*d*x^2 + d)*d) + a^2*x/(sqrt(-c^2*d*x^2 + d)*d)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))^2}{\left(d - c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{(-d\,(cx - 1)\,(cx + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.210 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=471

$$-\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{d\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+2*(a+b*arccosh(c*x))^2*arctan(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d
)^(1/2)+4*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*polylog(2,-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-2
*I*b*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*po
lylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/
(-c^2*d*x^2+d)^(1/2)-2*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/
2)-2*I*b^2*polylog(3,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.96, antiderivative size = 471, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391}

$$-\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\,\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx}\right.}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a + b*ArcCosh[c*x])^2/(d*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]
) + (4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCos
h[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[2, -E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (
(2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*
Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I
*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
```

```
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\right.}{}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^2\text{sech}(x)\,dx, x, \cosh\right.}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2\tan^{-1}\left(e^{\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 3.62, size = 577, normalized size = 1.23

$$-\frac{\frac{a^2\sqrt{d-c^2dx^2}}{c^2x^2-1} + a^2\sqrt{d}\,\log\left(\sqrt{d}\,\sqrt{d-c^2dx^2}+d\right) + a^2\left(-\sqrt{d}\right)\log(cx) + \frac{2iabd\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\text{Li}_2\left(-ie^{-\cosh^{-1}(cx)}\right) - \sqrt{\frac{cx-1}{cx+1}}\,(cx+\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -(((a^2*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2) - a^2*Sqrt[d]*Log[c*x] + a^2*Sq
rt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] + ((2*I)*a*b*d*(I*ArcCosh[c*x] +
 Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]
```

```
] - Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c
*x]] - I*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]]))/Sqrt[d - c^2
*d*x^2] + (b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((Sqrt[(-1 + c*x)/(1
+ c*x)]*ArcCosh[c*x]^2)/(1 - c*x) + 2*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x]
)] + I*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] - I*ArcCosh[c*x]^2*Log[1 +
I/E^ArcCosh[c*x]] - 2*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 2*PolyLog[2
, -E^(-ArcCosh[c*x])] + (2*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]]
- (2*I)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 2*PolyLog[2, E^(-ArcCos
h[c*x])] + (2*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (2*I)*PolyLog[3, I/E^Arc
Cosh[c*x]]))/Sqrt[d - c^2*d*x^2])/d^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a^2\left(\frac{\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{d^{\frac{3}{2}}}-\frac{1}{\sqrt{-c^2dx^2+d}\,d}\right)+\int \frac{b^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}+\frac{2\,ab\log\left(cx+\sqrt{cx+1}\sqrt{c\right.}}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -a^2*(log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2) - 1/(
sqrt(-c^2*d*x^2 + d)*d)) + integrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))^2/((-c^2*d*x^2 + d)^(3/2)*x) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.211 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}+\frac{2c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}}-\frac{4bc\sqrt{cx-1}\,\sqrt{cx+1}\,\log}{}$$

```
[Out] -(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*c^2*x*(a+b*arccosh(c*x))^2
/d/(-c^2*d*x^2+d)^(1/2)+2*c*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(-c^2*d*x^2+d)^(1/2)-4*b*c*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-4*b*
c*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.94, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 11, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.379, Rules used = {5798, 5748, 5688, 5715, 3716, 2190, 2279, 2391, 5721, 5461, 4182}

$$-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{2c^2x\left(a+b\cosh\right.}{d\sqrt{d-c^2dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])^2/(d*x*Sqrt[d - c^2*d*x^2])) + (2*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])^2)/(d*Sqrt[d - c^2*d*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d
*x^2]) - (4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E
^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*d*x^2]) - (b^2*c*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt[d - c^2*
d*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(-1+c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(2c^2\sqrt{-1+cx}\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} - \frac{\left(4bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\right.}{d\sqrt{d}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh\right.}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{dx\sqrt{d-c^2dx^2}} + \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{d\sqrt{d-c^2dx^2}} + \frac{2c\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh\right.}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.69, size = 315, normalized size = 0.92

$$\frac{a^2\left(2c^2x^2-1\right)+2ab\left(c^2x^2\cosh^{-1}(cx)+\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\cosh^{-1}(cx)-cx\left(\log(cx)+\log\left(\sqrt{\frac{cx-1}{cx+1}}\,(c\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 + 2*c^2*x^2) + 2*a*b*(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 +
c*x)]*(1 + c*x)*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(L
og[c*x] + Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(ArcCosh[c*x]*
(c^2*x^2*ArcCosh[c*x] + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - 2*c*x*(ArcCosh[c*x] + Log[1 - E^(-2*
ArcCosh[c*x])] + Log[1 + E^(-2*ArcCosh[c*x])]))) + c*x*Sqrt[(-1 + c*x)/(1 +
 c*x)]*(1 + c*x)*PolyLog[2, -E^(-2*ArcCosh[c*x])] + c*x*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(d*x*Sqrt[d - c^2*d*x^2
])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{(-c^2dx^2 + d)^{\frac{3}{2}} x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

maple [B] time = 0.49, size = 826, normalized size = 2.42

$$-\frac{a^2}{dx\sqrt{-c^2d\,x^2+d}}+\frac{2a^2c^2x}{d\sqrt{-c^2d\,x^2+d}}-\frac{2b^2\sqrt{-d\left(c^2x^2-1\right)}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\operatorname{arccosh}(cx)^2 c}{\left(c^2x^2-1\right)d^2}-\frac{2b^2\sqrt{-d\left(c^2x^2-1\right)}}{\left(c^2x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] -a^2/d/x/(-c^2*d*x^2+d)^(1/2)+2*a^2*c^2/d*x/(-c^2*d*x^2+d)^(1/2)-2*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)
^2*c-2*b^2*(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2*x/(c^2*x^2-1)/d^2*c^2+b^2*
(-d*(c^2*x^2-1))^(1/2)*arccosh(c*x)^2/x/(c^2*x^2-1)/d^2+2*b^2*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1+(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c+b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2)*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
/d^2*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*polylog(2,c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*
polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c-4*a*b*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*arccosh(c*x)*c-4*a*b*(-d*(c^2*x
^2-1))^(1/2)*arccosh(c*x)*x/(c^2*x^2-1)/d^2*c^2+2*a*b*(-d*(c^2*x^2-1))^(1/2
)*arccosh(c*x)/x/(c^2*x^2-1)/d^2+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/(c^2*x^2-1)/d^2*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^4-1)*c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$abc\left(\frac{\sqrt{-d}\,\log(cx+1)}{d^2}+\frac{\sqrt{-d}\,\log(cx-1)}{d^2}+\frac{2\sqrt{-d}\,\log(x)}{d^2}\right)+2\left(\frac{2c^2x}{\sqrt{-c^2dx^2+d}\,d}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx}\right)ab\operatorname{arcosh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] a*b*c*(sqrt(-d)*log(c*x + 1)/d^2 + sqrt(-d)*log(c*x - 1)/d^2 + 2*sqrt(-d)*l
og(x)/d^2) + 2*(2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d*x^2 + d)*
```

```
d*x))*a*b*arccosh(c*x) + (2*c^2*x/(sqrt(-c^2*d*x^2 + d)*d) - 1/(sqrt(-c^2*d
*x^2 + d)*d*x))*a^2 + b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^
2/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x^2\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.212 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=650

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cos\right.}{d\sqrt{d-c^2dx^2}}$$

```
[Out] 3/2*c^2*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)-1/2*(a+b*arccosh(c*x))^
2/d/x^2/(-c^2*d*x^2+d)^(1/2)+b*c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/d/x/(-c^2*d*x^2+d)^(1/2)+3*c^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^
2*c^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c
^2*d*x^2+d)^(1/2)+4*b*c^2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+2*b^2*c^2*pol
ylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^
2*d*x^2+d)^(1/2)-3*I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)+3*I*b
*c^2*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-2*b^2*c^2*polylog(2,c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
+3*I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-3*I*b^2*c^2*polylog(3,I*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.49, antiderivative size = 650, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5748, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5746, 92, 205}

$$-\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d-c^2dx^2}}+\frac{3ibc^2\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh}\right.}{d\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(d*x*Sqrt[d - c^2*d
*x^2]) + (3*c^2*(a + b*ArcCosh[c*x])^2)/(2*d*Sqrt[d - c^2*d*x^2]) - (a + b*
ArcCosh[c*x])^2/(2*d*x^2*Sqrt[d - c^2*d*x^2]) + (3*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x
^2]) - (b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + (2*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(d*Sqrt[d -
c^2*d*x^2]) - ((3*I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
)*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b*c^2*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x
]])/(d*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyL
og[2, E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2]) + ((3*I)*b^2*c^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d*Sqrt[d - c^2*d*x^2])
- ((3*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])
/(d*Sqrt[d - c^2*d*x^2])
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
```

```
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2dx^2\sqrt{d-c^2dx^2}} + \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(-1+c^2x^2\right)}\,dx}{d\sqrt{d-c^2dx^2}} - \frac{\left(3c^2\sqrt{-1+cx}\,\sqrt{\right.}{2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{dx\sqrt{d-c^2dx^2}} + \frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^2}{2d\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(\right.}{2dx^2\sqrt{d-c^2}}$$

Mathematica [A] time = 92.28, size = 979, normalized size = 1.51

$$\frac{b^2\sqrt{d-c^2dx^2}\left(-2\cosh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)\cosh^{-1}(cx)^2 + 2\sinh^2\left(\frac{1}{2}\cosh^{-1}(cx)\right)\cosh^{-1}(cx)^2 + \left(\frac{1}{c^2x^2}-1\right)\cosh^{-1}(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*Sqrt[d - c^2*d*x^2]*((-2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x])/(c*x) + (-1 + 1/(c^2*x^2))*ArcCosh[c*x]^2 + 4*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 2*ArcCosh[c*x]^2*Cosh[ArcC
osh[c*x]/2]^2 + 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 -
 E^(-ArcCosh[c*x])] + (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x]^2*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)
*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] - 4*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] + 4*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])] + (6*I)*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (6*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] - 4
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-ArcCosh[c*x])] + (6*I)
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, (-I)/E^ArcCosh[c*x]] - (6*
I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[3, I/E^ArcCosh[c*x]] + 2*Ar
cCosh[c*x]^2*Sinh[ArcCosh[c*x]/2]^2))/(2*d^2*(-1 + c^2*x^2)) + (a*(-((a*(-1
```

```
 + 3*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(x^2*(-1 + c^2*x^2))) + 3*a*c^2*Sqrt[d]*
Log[x] - 3*a*c^2*Sqrt[d]*Log[d + Sqrt[d]*Sqrt[d - c^2*d*x^2]] - (2*b*c^2*d*
(-((Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x)) + (-1 + 1/(c^2*x^2))*ArcCo
sh[c*x] - 2*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 + (3*I)*Sqrt[(-1 + c*x)/(1
+ c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]] + 2*Sqrt[
(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + (3*I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] - (3*I)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] + 2*ArcCosh[c*x]
*Sinh[ArcCosh[c*x]/2]^2))/Sqrt[d - c^2*d*x^2]))/(2*d^2)
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^7-2\,c^2d^2x^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^7 - 2*c^2*d^2*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^3), x)
```

maple [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(a+b\operatorname{arccosh}(cx))^2}{x^3\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\left(\frac{3\,c^2\log\left(\frac{2\,\sqrt{-c^2dx^2+d}\,\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)}{d^{\frac{3}{2}}}-\frac{3\,c^2}{\sqrt{-c^2dx^2+d}\,d}+\frac{1}{\sqrt{-c^2dx^2+d}\,dx^2}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*(3*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(3/2)
 - 3*c^2/(sqrt(-c^2*d*x^2 + d)*d) + 1/(sqrt(-c^2*d*x^2 + d)*d*x^2))*a^2 + i
ntegrate(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(3/
2)*x^3) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3
/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x^3\,{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x^3\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.213 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=496

$$-\frac{4c^2\left(a+b\cosh^{-1}(cx)\right)^2}{3dx\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\cosh^{-1}(cx\right.}{3d\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*b^2*c^2*(-c*x+1)*(c*x+1)/d/x/(-c^2*d*x^2+d)^(1/2)-1/3*(a+b*arccosh(c*x)
)^2/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*c^2*(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2
+d)^(1/2)+8/3*c^4*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+
b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2/(-c^2*d*x^2+d)^(1/2)+8/3*
c^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)
-20/3*b*c^3*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-16/3*b*c^3*(a+b*arccosh
(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*c^3*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^2*d*x^2+d)^(1/2)-b^2*c^3*polylo
g(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/(-c^
2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.46, antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5748, 5688, 5715, 3716, 2190, 2279, 2391, 5721, 5461, 4182, 5746, 95}

$$-\frac{5b^2c^3\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d\sqrt{d-c^2dx^2}}-\frac{b^2c^3\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{d\sqrt{d-c^2dx^2}}+\frac{8c^4x\left(a+b\right.}{3d\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (b^2*c^2*(1 - c*x)*(1 + c*x))/(3*d*x*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d*x^2*Sqrt[d - c^2*d*x^2]) - (a
 + b*ArcCosh[c*x])^2/(3*d*x^3*Sqrt[d - c^2*d*x^2]) - (4*c^2*(a + b*ArcCosh[
c*x])^2)/(3*d*x*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*
d*Sqrt[d - c^2*d*x^2]) + (8*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^2)/(3*d*Sqrt[d - c^2*d*x^2]) - (20*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2
]) - (16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^
(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (5*b^2*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*d*Sqrt[d - c^2*d*x^2]) - (b
^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(d*Sqrt
[d - c^2*d*x^2])
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(-1+c^2x^2\right)}\,dx}{3d\sqrt{d-c^2dx^2}} - \frac{\left(4c^2\sqrt{-1+cx}\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\sqrt{d-c^2dx^2}} - \frac{4c^2\left(a+b\cosh^{-1}(\right.}{3dx\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

$$= \frac{b^2c^2(1-cx)(1+cx)}{3dx\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3dx^2\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right.}{3dx^3\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 2.42, size = 529, normalized size = 1.07

$$\frac{a^2\left(8c^4x^4-4c^2x^2-1\right)+ab\left(6c^4x^4\cosh^{-1}(cx)+2c^2x^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(5\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\cosh^{-1}(cx)-cx\left(5\log(cx)\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] (a^2*(-1 - 4*c^2*x^2 + 8*c^4*x^4) + a*b*(6*c^4*x^4*ArcCosh[c*x] + Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*(c*x + 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*A
rcCosh[c*x]) + 2*c^2*x^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(5*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] - c*x*(5*Log[c*x] + 3*Log[Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)]))) + b^2*(c^2*x^2 - c^4*x^4 + c*x*Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x] + 3*c^4*x^4*ArcCosh[c*x]^2 + (-1 + c*
x)*(1 + c*x)*ArcCosh[c*x]^2 + 5*c^2*x^2*(-1 + c*x)*(1 + c*x)*ArcCosh[c*x]^2
 - 8*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2 - 6*c^3*x^
3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-2*ArcCosh[c
*x])] - 10*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1
+ E^(-2*ArcCosh[c*x])] + 5*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Pol
yLog[2, -E^(-2*ArcCosh[c*x])] + 3*c^3*x^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c
*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d*x^3*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^4d^2x^8-2\,c^2d^2x^6+d^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)/(c^4*d^2*x^8 - 2*c^2*d^2*x^6 + d^2*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{(-c^2dx^2 + d)^{\frac{3}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

maple [B] time = 0.95, size = 2868, normalized size = 5.78

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] 40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8+2*a*b*(
-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln((c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c^3-128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(
8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+16*a*b*(-d*(c^2*x^2-1))^(1/2)/d
^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4-8/3*a*b*(-d*(c^2*x^2-1))^(1/2
)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8*a*b*(-d*(c^
2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)*c^2-16/3*b^2*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x
)^2*c^3+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*
x-1)*(c*x+1)*c^8-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
*x^3*(c*x-1)*(c*x+1)*c^6-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^
2*x^2-1)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*
x+1)^(1/2))*c^3+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^
2/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+8/3*b^2*(-d*(c
^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*arccosh(c*x)^2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)*c^3-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)
*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3+5/3*b^2*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*polylog(2,-(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c^3-1/3*a^2/d/x^3/(-c^2*d*x^2+d)^(1/2)-4/3*a^2*c^2/d/x/(
-c^2*d*x^2+d)^(1/2)+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x
^2-1)*x^2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5-7/3*b^2*(-d*(c^2*x^2
-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*c^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^
4*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)^2-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(
8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10+8/3*a^2*c^4/d*x/(-c^2*d*x^2+d)^(1/2)-8/3*b^
2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*c^4+4*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x*arccosh(c*x)^2*c^2-
64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^7*c^10+32*a*b
*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*c^8-8*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*c^6-8/3*a*b*(-d*(c^2*x^2-1))^
(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*c^4+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/
(8*c^4*x^4-7*c^2*x^2-1)/x^3*arccosh(c*x)-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2
```

```
/(8*c^4*x^4-7*c^2*x^2-1)/x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c-32/3*a*b*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*c^
3+64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*(c*x-1)*(
c*x+1)*c^8-32/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*
(c*x-1)*(c*x+1)*c^6-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2
-1)*x*(c*x-1)*(c*x+1)*c^4+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*
c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^3+64/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*arccosh(c*x)*(c*x-1)*(c*x+1)*c
^8-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(
c*x)*(c*x-1)*(c*x+1)*c^6+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c
^2*x^2-1)*x^2*arccosh(c*x)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^5-8/3*b^2*(-d*(c
^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)*(c*x-1)*(c*x+1)
*c^4-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)/x^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*c^3+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c^3+2
*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*arc
cosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c^3+10/3*a*b*(-d*(c^2*x^2-1
))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2)*c^3-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2
*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^2
/(8*c^4*x^4-7*c^2*x^2-1)*x^7*arccosh(c*x)*c^10+32*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^5*arccosh(c*x)*c^8-64/3*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)^2*c^6-8*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x^3*arccosh(c*x)*c^6+8*b^2*(-d*(c^
2*x^2-1))^(1/2)/d^2/(8*c^4*x^4-7*c^2*x^2-1)*x*arccosh(c*x)^2*c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(\frac{8\,c^4x}{\sqrt{-c^2dx^2+d}\,d}-\frac{4\,c^2}{\sqrt{-c^2dx^2+d}\,dx}-\frac{1}{\sqrt{-c^2dx^2+d}\,dx^3}\right)a^2+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^4}+\frac{2\,ab\log\left(cx+\right.}{\left(-c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(8*c^4*x/(sqrt(-c^2*d*x^2 + d)*d) - 4*c^2/(sqrt(-c^2*d*x^2 + d)*d*x) -
1/(sqrt(-c^2*d*x^2 + d)*d*x^3))*a^2 + integrate(b^2*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(3/2)*x^4) + 2*a*b*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(3/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x^4\,\left(d-c^2\,d\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x^4\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.214 $\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=568

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{8\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^6d^3}-\frac{22b\sqrt{cx-1}\sqrt{cx+1}\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{3c^6d^2\sqrt{d-c^2dx^2}}$$

[Out] 1/3*x^4*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*x^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-7/3*b^2*(-c*x+1)*(c*x+1)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-4/3*x^2*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*a*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*b^2*x*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+11/3*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-22/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-11/3*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)+11/3*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/c^6/d^3

Rubi [A] time = 1.48, antiderivative size = 594, normalized size of antiderivative = 1.05, number of steps used = 27, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5718, 5654, 74, 5766, 5694, 4182, 2279, 2391, 5750, 98, 21}

$$-\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}+\frac{11b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^6d^2\sqrt{d-c^2dx^2}}-\frac{16abx\sqrt{cx-1}\sqrt{c}}{3c^5d^2\sqrt{d-c^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) - (16*a*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*(1 - c*x)*(1 + c*x))/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) - (16*b^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (11*b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (4*x^2*(a + b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^4*(a + b*ArcCosh[c*x])^2)/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (8*(1 - c*x)*(1 + c*x)*(a + b*ArcCosh[c*x])^2)/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) - (22*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) - (11*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2]) + (11*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^6*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
 + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x], x] /; FreeQ[{a,
 b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5766

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(e*(m + 2*p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(c*(m +
2*p + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a +
b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(f^2*(m - 1))/(c^2*(m + 2*p + 1)), I
nt[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a,
 b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[m, 1] && Ne
Q[m + 2*p + 1, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(4\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,d}{3c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^3\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{4x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^4}{3c^2d^2(1}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{11bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^3\sqrt{-1+}}{3c^3d}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{11b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} + \frac{11bx}{}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{3b^2(1-cx)(1+cx)}{c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x}{}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x}{}$$

$$= -\frac{b^2x^2}{3c^4d^2\sqrt{d-c^2dx^2}} - \frac{16abx\sqrt{-1+cx}\,\sqrt{1+cx}}{3c^5d^2\sqrt{d-c^2dx^2}} - \frac{7b^2(1-cx)(1+cx)}{3c^6d^2\sqrt{d-c^2dx^2}} - \frac{16b^2x}{}$$

Mathematica [A] time = 5.55, size = 437, normalized size = 0.77

$$-\frac{8a^2\left(3c^4x^4-12c^2x^2+8\right)+2ab\left(-36\cosh\left(2\cosh^{-1}(cx)\right)\cosh^{-1}(cx)+3\cosh\left(4\cosh^{-1}(cx)\right)\cosh^{-1}(cx)+2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -1/24*(8*a^2*(8 - 12*c^2*x^2 + 3*c^4*x^4) + 2*a*b*(25*ArcCosh[c*x] - 36*Arc
Cosh[c*x]*Cosh[2*ArcCosh[c*x]] + 3*ArcCosh[c*x]*Cosh[4*ArcCosh[c*x]] - 33*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] + 4*Sinh[2*Ar
cCosh[c*x]] + 11*Log[Tanh[ArcCosh[c*x]/2]]*Sinh[3*ArcCosh[c*x]] - 3*Sinh[4*
ArcCosh[c*x]]) + b^2*(22 + 25*ArcCosh[c*x]^2 - 4*(7 + 9*ArcCosh[c*x]^2)*Cos
h[2*ArcCosh[c*x]] + 3*(2 + ArcCosh[c*x]^2)*Cosh[4*ArcCosh[c*x]] - 66*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 66
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x]
)] + 88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*
x])] - 88*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c
*x])] + 8*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + 22*ArcCosh[c*x]*Log[1 - E^(-A
rcCosh[c*x])]*Sinh[3*ArcCosh[c*x]] - 22*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*
x])]*Sinh[3*ArcCosh[c*x]] - 6*ArcCosh[c*x]*Sinh[4*ArcCosh[c*x]]))/(c^6*d*(d
 - c^2*d*x^2)^(3/2))
```

fricas [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^5\operatorname{arcosh}\left(cx\right)^2+2\,abx^5\operatorname{arcosh}\left(cx\right)+a^2x^5\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^5*arccosh(c*x)^2 + 2*a*b*x^5*arccosh(c*x) + a^2*x^5)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.89, size = 1211, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 4*a^2/c^4*x^2/d/(-c^2*d*x^2+d)^(3/2)+2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(
c^2*x^2-1)+2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)-10
/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^6*arccosh(c*x)-b^2*(-d*(c
^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*arccosh(c*x)^2*x^2+2*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)^2*x^2-8/3*a^2/c^6/d/(-c^2*d*x^
2+d)^(3/2)+11/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/
d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-11/3*b^2*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*polylog(2,
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-2*a*b*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*
x^2-1)*arccosh(c*x)*x^2+4*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*
arccosh(c*x)*x^2-2*b^2*(-d*(c^2*x^2-1))^(1/2)/c^4/d^3/(c^2*x^2-1)*x^2+b^2*(
-d*(c^2*x^2-1))^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)^2+1/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*x^2-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(
c^2*x^2-1)^2/c^6*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2
-1)^2/c^6-a^2*x^4/c^2/d/(-c^2*d*x^2+d)^(3/2)-11/3*b^2*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*
x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/
c^5*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)
/c^5/d^3/(c^2*x^2-1)*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x+11/3*b^2*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*arccos
h(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+11/3*a*b*(-d*(c^2*x^2-1))^(1/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^6/d^3/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))-11/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
6/d^3/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)+2*a*b*(-d*(c^2*x^2-
1))^(1/2)/c^5/d^3/(c^2*x^2-1)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*a*b*(-d*(c^
2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{3x^4}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{12x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}+\frac{8}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^6d}\right)-\frac{\left(3b^2c^4\sqrt{d}x^4-12b^2c^2\sqrt{d}x^2+8b^2\sqrt{d}\right)\sqrt{}}{3\left(c^{10}d^3x^4-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/3*a^2*(3*x^4/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 12*x^2/((-c^2*d*x^2 + d)^(
3/2)*c^4*d) + 8/((-c^2*d*x^2 + d)^(3/2)*c^6*d)) - 1/3*(3*b^2*c^4*sqrt(d)*x^
4 - 12*b^2*c^2*sqrt(d)*x^2 + 8*b^2*sqrt(d))*sqrt(c*x + 1)*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(c^10*d^3*x^4 - 2*c^8*d^3*x^2 + c^6*
d^3) - integrate(2/3*((12*b^2*c^3*x^3 + 3*(a*b*c^5 - b^2*c^5)*x^5 - 8*b^2*c
*x)*(c*x + 1)*sqrt(c*x - 1) + (15*b^2*c^4*x^4 + 3*(a*b*c^6 - b^2*c^6)*x^6 -
 20*b^2*c^2*x^2 + 8*b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(c^12*d^(5/2)*x^7 - 3*c^10*d^(5/2)*x^5 + 3*c^8*d^(5/2)*x
^3 - c^6*d^(5/2)*x + (c^11*d^(5/2)*x^6 - 3*c^9*d^(5/2)*x^4 + 3*c^7*d^(5/2)*
x^2 - c^5*d^(5/2))*sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^5*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\left(a+b\operatorname{acosh}(cx)\right)^2}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**5*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.215 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=482

$$\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^3}{3bc^5d^2\sqrt{d-c^2dx^2}}-\frac{4\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{8b\sqrt{cx-}}{}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^5/d^2/(-c
^2*d*x^2+d)^(1/2)+1/3*b^2*(-c*x+1)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)-x*(a+b*arcc
osh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x^2*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-4/3*(a+
b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+
1/3*(a+b*arccosh(c*x))^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^5/d^2/(-c^2*d*x^2+
d)^(1/2)+8/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(-c^2*d*x^2+d)^(1/2)+4/3*b^2*polylog(2
,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5/d^2/(
-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 497, normalized size of antiderivative = 1.03, number of steps used = 19, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5752, 5676, 5715, 3716, 2190, 2279, 2391, 5750, 89, 12, 78, 52}

$$\frac{4b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^5d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh\right.}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2d}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^5*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^5*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (x*(a + b*ArcCosh[c*x])
^2)/(c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x])^2)/(3*c^2*d^2
*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])^2)/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^3)/(3*b*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (
8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[
c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2]) + (4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*PolyLog[2, E^(2*ArcCosh[c*x])])/(3*c^5*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^3\,(}{3c^2d^2(1-}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cos\right.}{c^4d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{x\left(a+b\cos\right.}{c^4d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}\right.}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} +$$

$$= -\frac{b^2}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^5d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh^{-1}(cx)}{3c^5d^2\sqrt{d-c^2dx^2}} +$$

Mathematica [A] time = 2.56, size = 382, normalized size = 0.79

$$\frac{\frac{a^2cx\left(4c^2x^2-3\right)\sqrt{d-c^2dx^2}}{\left(c^2x^2-1\right)^2} - 3a^2\sqrt{d}\,\tan^{-1}\left(\frac{cx\sqrt{d-c^2dx^2}}{\sqrt{d}\left(c^2x^2-1\right)}\right) + \frac{abd\left(-\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)+2cx\cosh^{-1}(cx)}{c^2x^2-1} - 8cx\cosh^{-1}(cx) + \sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(8\log\left(\sqrt{\frac{cx-}{cx+}}\right.\right.\right.}{\sqrt{d-c^2dx^2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 4*c^2*x^2)*Sqrt[d - c^2*d*x^2])/(-1 + c^2*x^2)^2 - 3*a^2*Sq
rt[d]*ArcTan[(c*x*Sqrt[d - c^2*d*x^2])/(Sqrt[d]*(-1 + c^2*x^2))] + (a*b*d*(
-8*c*x*ArcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh
[c*x])/(-1 + c^2*x^2) + Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*ArcCosh[c*x
]^2 + 8*Log[Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)])))/Sqrt[d - c^2*d*x^2] +
(b^2*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((c*x*(-1 + c^2*x^2 + (-3 + 4
*c^2*x^2)*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3)) + Ar
cCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x]*(4 + ArcCosh[c*x]) + 8*Log[1
- E^(-2*ArcCosh[c*x])]) - 4*PolyLog[2, E^(-2*ArcCosh[c*x])]))/Sqrt[d - c^2*
d*x^2])/(3*c^5*d^3)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^4\operatorname{arcosh}(cx)^2 + 2\,abx^4\operatorname{arcosh}(cx) + a^2x^4\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 x^4}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^4/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 1.01, size = 4074, normalized size = 8.45

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -44/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*
x^2+16)/d^3*x^5-76*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^
4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)^2*x^5-44/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+20/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+
16)*c^2/d^3*x^7+43/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*
c^4*x^4-71*c^2*x^2+16)/c^2/d^3*x^3-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8
-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*x-32*b^2*(-d*(c^2*x^2-1))^(1
/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*arccosh(c*x)^2*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87
*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*x+64/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)^2
*(c*x+1)^(1/2)*(c*x-1)^(1/2)+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87
*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x
-1)^(1/2)-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^
4-71*c^2*x^2+16)/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^5-1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^3-8*
b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16
)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*
c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x+1)*(c*x-1)*x^3+2
1*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+
16)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^2-8/3*b^2*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*polylog(2,
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+64*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8
-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7-16/3*a*b*(-
d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*
(c*x+1)*(c*x-1)*x^5+362/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6
+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)*x^3-32*a*b*(-d*(c^2*x^2-1)
)^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c
*x)*x+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71
*c^2*x^2+16)/c^5/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-8/3*b^2*(-d*(c^2*x^2-1))^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^
```

```
(1/2)*(c*x+1)^(1/2))+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+
118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x+1)*(c*x-1)*x-55/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^2+1/3*a^2*x^3/c^2/d/(-c^2*d*x^2+d)^(3/2)+a^2/c^4/d^2
/(c^2*d)^(1/2)*arctan((c^2*d)^(1/2)*x/(-c^2*d*x^2+d)^(1/2))-17*b^2*(-d*(c^2
*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*x^5+16
/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-
1)*arccosh(c*x)+28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*
c^4*x^4-71*c^2*x^2+16)/c^2/d^3*(c*x+1)*(c*x-1)*x^3+8*a*b*(-d*(c^2*x^2-1))^(
1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^4-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c
^4*x^4-71*c^2*x^2+16)/c^4/d^3*(c*x+1)*(c*x-1)*x-13*a*b*(-d*(c^2*x^2-1))^(1/
2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+1
18*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+
84*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2
+16)/c/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+28/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*ar
ccosh(c*x)*(c*x+1)*(c*x-1)*x^3-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1
)-a^2/c^4/d^2*x/(-c^2*d*x^2+d)^(1/2)-64*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*
x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^6+168*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-
440/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*
x^2+16)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*b^2*(-d*(c
^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*
x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))-a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(c^2*x^2-1)*arccosh(c*x)^2+16/3*b^2*(-d*(c^2*x
^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^5/d^3*(c*x
-1)^(1/2)*(c*x+1)^(1/2)+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x
^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*(c*x+1)*(c*x-1)*x^5+32*b^2*(-d*(c^2*x^2-1
))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*arccosh(
c*x)^2*x^7+16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x
^4-71*c^2*x^2+16)*c^2/d^3*arccosh(c*x)*x^7+181/3*b^2*(-d*(c^2*x^2-1))^(1/2)
/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d^3*arccosh(c*x)^2*x
^3+40/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^
2*x^2+16)/c^2/d^3*arccosh(c*x)*x^3-16*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^
8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)^2*x-4*b^2*(-d*
(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^4/d^
3*arccosh(c*x)*x+16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118
*c^4*x^4-71*c^2*x^2+16)*c^2/d^3*x^7-152*a*b*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*
x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/d^3*arccosh(c*x)*x^5+40/3*a*b*(-d
*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^2/d
^3*x^3-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^5/(
c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-13*b^2*(-d*(c
^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*
arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+8*b^2*(-d*(c^2*x^2-1))^(1/2)/(
24*c^8*x^8-87*c^6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c/d^3*arccosh(c*x)*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^4-220/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^
6*x^6+118*c^4*x^4-71*c^2*x^2+16)/c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(24*c^8*x^8-87*c^6*x^6+118*c^4*x^
4-71*c^2*x^2+16)/c^4/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\left(x\left(\frac{3\,x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)-\frac{x}{\sqrt{-c^2dx^2+d}\,c^4d^2}+\frac{3\,\arcsin\left(cx\right)}{c^5d^{\frac{5}{2}}}\right)a^2+\int\frac{b^2x^4\log\left(cx+\sqrt{cx}\right.}{\left(-c^2dx^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*(x*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^
4*d)) - x/(sqrt(-c^2*d*x^2 + d)*c^4*d^2) + 3*arcsin(c*x)/(c^5*d^(5/2)))*a^2
 + integrate(b^2*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 +
 d)^(5/2) + 2*a*b*x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(-c^2*d*x^2 +
d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.216 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=336

$$\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d\left(d-c^2dx^2\right)^{3/2}}-\frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}}-\frac{10b\sqrt{cx-1}\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{b\ldots}{\ldots}$$

```
[Out] 1/3*x^2*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^4/d^2/(-c
^2*d*x^2+d)^(1/2)-2/3*(a+b*arccosh(c*x))^2/c^4/d^2/(-c^2*d*x^2+d)^(1/2)+1/3
*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*x^2+1)/(-
c^2*d*x^2+d)^(1/2)-10/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2
*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
4/d^2/(-c^2*d*x^2+d)^(1/2)+5/3*b^2*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.97, antiderivative size = 351, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.310, Rules used = {5798, 5752, 5718, 5694, 4182, 2279, 2391, 5750, 74}

$$-\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{5b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^4d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\sqrt{cx\ldots}}{3c^3d^2\left(1-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c^3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*(a +
 b*ArcCosh[c*x])^2)/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (x^2*(a + b*ArcCosh[c
*x])^2)/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (10*b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3*c^4*
d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -
E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2]) + (5*b^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^4*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 5752

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[(f*(f*x)^(m -
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e
2*(p + 1)), x] + (-Dist[(f^2*(m - 1))/(2*e1*e2*(p + 1)), Int[(f*x)^(m - 2)*
(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*f*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPar
t[p])/(2*c*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)
^(m - 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /;
FreeQ[{a, b, c, d1, e1, d2, e2, f}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d
2, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3c^2d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh^{-1}(cx)\right)^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{x^2\left(a\right.}{3c^2d^2(1-c}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh\right.}{3c^4d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh\right.}{3c^4d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh\right.}{3c^4d^2\sqrt{d-}}$$

$$= -\frac{b^2}{3c^4d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3c^3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2\left(a+b\cosh\right.}{3c^4d^2\sqrt{d-}}$$

Mathematica [A] time = 4.54, size = 341, normalized size = 1.01

$$4a^2\left(3c^2x^2-2\right) - ab\left(\cosh^{-1}(cx)\left(4-12\cosh\left(2\cosh^{-1}(cx)\right)\right) - 2\sinh\left(2\cosh^{-1}(cx)\right) + 5\left(\sinh\left(3\cosh^{-1}(cx\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2*(-2 + 3*c^2*x^2) - b^2*(2 + 2*ArcCosh[c*x]^2 - 2*(1 + 3*ArcCosh[c*x]
^2)*Cosh[2*ArcCosh[c*x]] - 15*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[
c*x]*(Log[1 - E^(-ArcCosh[c*x])] - Log[1 + E^(-ArcCosh[c*x])]) + 20*((-1 +
c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] - 20*((-1
+ c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] - 2*ArcCo
sh[c*x]*Sinh[2*ArcCosh[c*x]] + 5*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])]*Si
nh[3*ArcCosh[c*x]] - 5*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*ArcCo
sh[c*x]]) - a*b*(ArcCosh[c*x]*(4 - 12*Cosh[2*ArcCosh[c*x]]) - 2*Sinh[2*ArcC
osh[c*x]] + 5*Log[Tanh[ArcCosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 +
 c*x) + Sinh[3*ArcCosh[c*x]])))/(12*c^4*d*(d - c^2*d*x^2)^(3/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^3\operatorname{arcosh}(cx)^2 + 2\,abx^3\operatorname{arcosh}(cx) + a^2x^3\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 835, normalized size = 2.49

$$\frac{a^2x^2}{c^2d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}-\frac{2a^2}{3d\,c^4\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)^2\,x^2}{d^3\left(c^2x^2-1\right)^2c^2}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\,\sqrt{}}{3d^3\left(c^2x^2-1\right)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] a^2*x^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-2/3*a^2/d/c^4/(-c^2*d*x^2+d)^(3/2)+b^2*(
-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)^2*x^2+1/3*b^2*(-d*
(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*x^2-2/3*b^2*(
-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arccosh(c*x)^2-1/3*b^2*(-d*(c^2
*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))-5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^3/c^4/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b^2*(-d
*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*arccosh
(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+5/3*b^2*(-d*(c^2*x^2-1))^(1/2)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh
(c*x)*x^2+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^3*(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^4*arcc
osh(c*x)-5/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4
/(c^2*x^2-1)*ln(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)-1)+5/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^4/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6}\,abc\left(\frac{2\sqrt{-d}\,x}{c^6d^3x^2-c^4d^3}+\frac{5\sqrt{-d}\,\log\left(cx+1\right)}{c^5d^3}-\frac{5\sqrt{-d}\,\log\left(cx-1\right)}{c^5d^3}\right)+\frac{2}{3}\,ab\left(\frac{3\,x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}-\frac{2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^4d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/6*a*b*c*(2*sqrt(-d)*x/(c^6*d^3*x^2 - c^4*d^3) + 5*sqrt(-d)*log(c*x + 1)/(
c^5*d^3) - 5*sqrt(-d)*log(c*x - 1)/(c^5*d^3)) + 2/3*a*b*(3*x^2/((-c^2*d*x^2
 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d))*arccosh(c*x) + 1/3*a
^2*(3*x^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) - 2/((-c^2*d*x^2 + d)^(3/2)*c^4*d)
```

```
) + b^2*integrate(x^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2
+ d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{\left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.217 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=389

$$\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d\left(d-c^2dx^2\right)^{3/2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-}}{}$$

```
[Out] 1/3*x^3*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^3/d^2/(-c^2*d
*x^2+d)^(1/2)+1/3*b^2*(-c*x+1)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*arccosh
(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*x^2*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*x^2+1)/(-c^2*d*x^2
+d)^(1/2)-1/3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^
2*d*x^2+d)^(1/2)+2/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b^2*p
olylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
^3/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 404, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 12, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.414, Rules used = {5798, 5724, 5750, 89, 12, 78, 52, 5715, 3716, 2190, 2279, 2391}

$$\frac{b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3c^3d^2\sqrt{d-c^2dx^2}}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*(1 - c*x))/(3*c^3*d^2*Sqrt[d -
c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcCosh[c*x])/(3*c^3*d^2*Sq
rt[d - c^2*d*x^2]) + (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (x^3*(a + b*ArcCosh[c*x])
^2)/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) - (Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (2*b*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])
/(3*c^3*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLo
g[2, E^(2*ArcCosh[c*x])])/(3*c^3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5750

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*(a
+ b*ArcCosh[c*x])^n)/(2*e*(p + 1)), x] + (-Dist[(b*f*n*(-d)^p)/(2*c*(p + 1)
), Int[(f*x)^(m - 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x] - Dist[(f^2*(m - 1))/(2*e*(p + 1)), Int[(f*x)^(m -
2)*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d
, e, f}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && GtQ[m, 1] &&
 IntegerQ[p]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(-1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(b^2\ldots}{\ldots}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\,c\ldots\right.}{3d^2(1-cx)(1+\ldots}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{x^3\left(a+b\,c\ldots\right.}{3d^2(1-cx)(1+\ldots}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(c\ldots\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\ldots}{\ldots}$$

$$= -\frac{b^2}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2(1-cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{b^2\sqrt{-1+cx}\,\sqrt{1+cx}\cosh^{-1}(cx)}{3c^3d^2\sqrt{d-c^2dx^2}} + \frac{bx^2\ldots}{\ldots}$$

Mathematica [A] time = 1.78, size = 264, normalized size = 0.68

$$\frac{\frac{a^2c^3x^3}{1-c^2x^2} + ab\left(\frac{\sqrt{\frac{cx-1}{cx+1}}\left(2\left(c^2x^2-1\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\right)-1\right)}{cx-1} + \frac{2c^3x^3\cosh^{-1}(cx)}{1-c^2x^2}\right) + b^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(-\frac{cx\left(c^2x^2+c^2x^2\cosh^{-1}(cx)^2-1\right)}{\left(\frac{cx-1}{cx+1}\right)^{3/2}(cx+1)^3} + \mathrm{co}\ldots\right.}{3c^3d^2\sqrt{d-c^2dx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c^3*x^3)/(1 - c^2*x^2) + a*b*((2*c^3*x^3*ArcCosh[c*x])/(1 - c^2*x^2)
+ (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + 2*(-1 + c^2*x^2)*Log[Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)*(-((c*x*(-1 + c^2*x^2 + c^2*x^2*ArcCosh[c*x]^2))/(((-1 + c*x)/(1 + c*x))^
(3/2)*(1 + c*x)^3)) + ArcCosh[c*x]*((1 - c^2*x^2)^(-1) + ArcCosh[c*x] + 2*L
og[1 - E^(-2*ArcCosh[c*x])]) - PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c^3*d^2
*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^2x^2\operatorname{arcosh}(cx)^2+2\,abx^2\operatorname{arcosh}(cx)+a^2x^2\right)\sqrt{-c^2dx^2+d}}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2)*sqrt(
-c^2*d*x^2 + d)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(b\operatorname{arcosh}(cx)+a\right)^2x^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x^2/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.76, size = 3445, normalized size = 8.86

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)
*c^4/d^3*x^7-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4
-5*c^2*x^2+1)*c^2/d^3*x^5+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x
^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3+b^2*(-d*(c^2*x^2-1))^(1/2)/
(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)^2*x^7+1/3
*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^
4/d^3*arccosh(c*x)*x^7-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c
^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)^2*x^5-2/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)*x^5+1/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c
^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^
8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*(c*x+1)*(c*x-1)*x^3-2/3*b^2*(-d*(c^
2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*polylog(2,c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*arccosh(c*x)*x^7-2*a*b*(-d*(c^2*x^2-
1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2/d^3*arccosh(c*x)
```

```
*x^5+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x
^2+1)/d^3*(c*x+1)*(c*x-1)*x^3+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c
^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2/3*b^2*
(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arcc
osh(c*x)^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5
*c^2*x^2+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x
^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^
6+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^3-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5
*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+1/3*b^2*(-d*(c^2*x^2-1))^
(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^2/d^3*(c*x+1)*(c*x-1)*
x-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+
1)/c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*
c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)+1/3*b^2
*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*ar
ccosh(c*x)^2*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4
*x^4-5*c^2*x^2+1)/d^3*x^3+4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6
+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4
-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1
)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-2*a*b*(-d*(c^2*x^2-1))
^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*arccosh(c*x)*(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x^6+1/3*a^2/c^2/d*x/(-c^2*d*x^2+d)^(3/2)-1/3*a^2/
c^2/d^2*x/(-c^2*d*x^2+d)^(1/2)+1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*
c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*x^3+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c
^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/d^3*arccosh(c*x)*x^3+2/3*b^2*(-d*(
c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^4/d^3*x^7-
b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c^2
/d^3*x^5+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3
/(c^2*x^2-1)*arccosh(c*x)-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+1
0*c^4*x^4-5*c^2*x^2+1)*c^3/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^6-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)*c^2/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^5+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(
3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)^2*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^4+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^
4*x^4-5*c^2*x^2+1)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-b^2*(
-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*ar
ccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/d^3/c^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2
+1)*c^2/d^3*(c*x+1)*(c*x-1)*x^5+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6
*x^6+10*c^4*x^4-5*c^2*x^2+1)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-a*b*(-d*
(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2+2/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^8*x^8-9*c^6*
x^6+10*c^4*x^4-5*c^2*x^2+1)/c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2
)-2/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^3/(c^2*x
^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)-4/3*b^2*(-d*(c^2*x^2-1))^(1
/2)/(3*c^8*x^8-9*c^6*x^6+10*c^4*x^4-5*c^2*x^2+1)/c/d^3*arccosh(c*x)^2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,abc\left(\frac{\sqrt{-d}}{c^{6}d^{3}x^{2}-c^{4}d^{3}}-\frac{\sqrt{-d}\,\log\left(cx+1\right)}{c^{4}d^{3}}-\frac{\sqrt{-d}\,\log\left(cx-1\right)}{c^{4}d^{3}}\right)-\frac{2}{3}\,ab\left(\frac{x}{\sqrt{-c^{2}dx^{2}+d}\,c^{2}d^{2}}-\frac{x}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}c^{2}d}\right)\mathrm{arc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^6*d^3*x^2 - c^4*d^3) - sqrt(-d)*log(c*x + 1)/(c^4*d^
3) - sqrt(-d)*log(c*x - 1)/(c^4*d^3)) - 2/3*a*b*(x/(sqrt(-c^2*d*x^2 + d)*c^
2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d))*arccosh(c*x) - 1/3*a^2*(x/(sqrt(
-c^2*d*x^2 + d)*c^2*d^2) - x/((-c^2*d*x^2 + d)^(3/2)*c^2*d)) + b^2*integrat
e(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.218 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=298

$$\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\tanh^{-1}\left(e^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{\left(a+b\cosh^{-1}\right.}{3c^2d\left(d-c^2\right.}$$

```
[Out] 1/3*(a+b*arccosh(c*x))^2/c^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/c^2/d^2/(-c^2*d
*x^2+d)^(1/2)+1/3*b*x*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/
(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+2/3*b*(a+b*arccosh(c*x))*arctanh(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)
^(1/2)+1/3*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)-1/3*b^2*polylog(2,c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 313, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.296, Rules used = {5798, 5718, 5689, 74, 5694, 4182, 2279, 2391}

$$\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}-\frac{b^2\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{\cosh^{-1}(cx)}\right)}{3c^2d^2\sqrt{d-c^2dx^2}}+\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a\right.}{3cd^2\left(1-c^2x^2\right)\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -b^2/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*Ar
cCosh[c*x])^2/(3*c^2*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(3
*c^2*d^2*Sqrt[d - c^2*d*x^2]) + (b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2
, -E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2]) - (b^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*c^2*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
```

```
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{\left(2b\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{\left(-1+c^2x^2\right)^2}dx}{3cd^2\sqrt{d-c^2dx^2}}$$

$$= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3c^2d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{(b^2}{}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cos\right.}{3c^2d^2(1-cx)(1+}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cos\right.}{3c^2d^2(1-cx)(1+}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cos\right.}{3c^2d^2(1-cx)(1+}$$

$$= -\frac{b^2}{3c^2d^2\sqrt{d-c^2dx^2}} + \frac{bx\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cos\right.}{3c^2d^2(1-cx)(1+}$$

Mathematica [A] time = 2.54, size = 332, normalized size = 1.11

$$4a^2 + ab\left(8\cosh^{-1}(cx) + 2\sinh\left(2\cosh^{-1}(cx)\right) + \left(\sinh\left(3\cosh^{-1}(cx)\right) - 3\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)\log\left(\tanh\left(\frac{1}{2}\cosh^{-1}(c\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (4*a^2 + b^2*(-2 + 4*ArcCosh[c*x]^2 + 2*Cosh[2*ArcCosh[c*x]] - 3*Sqrt[(-1 +
 c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] + 3*Sqrt
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] +
4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, -E^(-ArcCosh[c*x])] -
 4*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3*PolyLog[2, E^(-ArcCosh[c*x])] +
 2*ArcCosh[c*x]*Sinh[2*ArcCosh[c*x]] + ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x
])]*Sinh[3*ArcCosh[c*x]] - ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])]*Sinh[3*A
rcCosh[c*x]]) + a*b*(8*ArcCosh[c*x] + 2*Sinh[2*ArcCosh[c*x]] + Log[Tanh[Arc
Cosh[c*x]/2]]*(-3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sinh[3*ArcCosh[c*x
]])))/(12*c^2*d*(d - c^2*d*x^2)^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2x\,\text{arcosh}\,(cx)^2 + 2\,abx\,\text{arcosh}\,(cx) + a^2x\right)}{c^6d^3x^6 - 3\,c^4d^3x^4 + 3\,c^2d^3x^2 - d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
 + a^2*x)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2x}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*x/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.43, size = 720, normalized size = 2.42

$$\frac{a^2}{3c^2d\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,\operatorname{arccosh}(cx)\sqrt{cx+1}\,\sqrt{cx-1}\,x}{3d^3\left(c^2x^2-1\right)^2c}+\frac{b^2\sqrt{-d\left(c^2x^2-1\right)}\,x^2}{3d^3\left(c^2x^2-1\right)^2}+\frac{b^2\sqrt{-d\left(c^2x^2\right.}}{3d^3\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] 1/3*a^2/c^2/d/(-c^2*d*x^2+d)^(3/2)+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*
x^2-1)^2/c*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+1/3*b^2*(-d*(c^2*x^2-
1))^(1/2)/d^3/(c^2*x^2-1)^2*x^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2
-1)^2/c^2*arccosh(c*x)^2-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c
^2+1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*
x^2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b^2*(-d*(c^2*
x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*polylog(2,c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))-1/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^
2/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*a*b*(-d*(c^2*
x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x+2/3*a*b*(-d
*(c^2*x^2-1))^(1/2)/d^3/(c^2*x^2-1)^2/c^2*arccosh(c*x)+1/3*a*b*(-d*(c^2*x^2
-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c^2/(c^2*x^2-1)*ln(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2)-1)-1/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/d^3/c^2/(c^2*x^2-1)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2}{3\left(-c^2dx^2+d\right)^{\frac{3}{2}}c^2d}+\int \frac{b^2x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}+\frac{2\,abx\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*a^2/((-c^2*d*x^2 + d)^(3/2)*c^2*d) + integrate(b^2*x*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2) + 2*a*b*x*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))/(-c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}{\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x))^2)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}{\left(-d\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.219 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=331

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^2}{3cd^2\sqrt{d-c^2dx^2}}-\frac{4b\sqrt{cx}}{}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*x/d^2/(-c^2*d*x^2
+d)^(1/2)+2/3*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*(a+b*ar
ccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(
1/2)+2/3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2
+d)^(1/2)-4/3*b*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^2*d*x^2+d)^(1/2)-2/3*b^2*polylog(2,
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/d^2/(-c^
2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 346, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39}

$$-\frac{2b^2\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3cd^2\sqrt{d-c^2dx^2}}+\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] -(b^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*c*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (2*x*(a + b
*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (x*(a + b*ArcCosh[c*x])^2)/
(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^2)/(3*c*d^2*Sqrt[d - c^2*d*x^2]) - (4*b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*
c*d^2*Sqrt[d - c^2*d*x^2]) - (2*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 E^(2*ArcCosh[c*x])])/(3*c*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{3d^2\sqrt{d-c^2dx^2}} + \cdots$$

$$= \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{x\left(a+b\cdots\right.}{3d^2(1-cx)(1\cdots}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(c\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(c\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(c\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(c\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{b\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3cd^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{2x\left(a+b\cosh^{-1}(c\cdots\right.}{3d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.52, size = 289, normalized size = 0.87

$$\frac{\frac{a^2cx\left(2c^2x^2-3\right)}{c^2x^2-1} + ab\left(\frac{\sqrt{\frac{cx-1}{cx+1}}\left(\left(4-4c^2x^2\right)\log\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\right)-1\right)}{cx-1} + 2cx\left(\frac{1}{1-c^2x^2}+2\right)\cosh^{-1}(cx)\right) + b^2\left(-\frac{\cosh^{-1}(cx)\left(\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cdots\right.}{c^2x^2-1}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] ((a^2*c*x*(-3 + 2*c^2*x^2))/(-1 + c^2*x^2) + a*b*(2*c*x*(2 + (1 - c^2*x^2)^
(-1))*ArcCosh[c*x] + (Sqrt[(-1 + c*x)/(1 + c*x)]*(-1 + (4 - 4*c^2*x^2)*Log[
Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)]))/(-1 + c*x)) + b^2*(-((ArcCosh[c*x]*
(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + c*x*ArcCosh[c*x]))/(-1 + c^2*x^2))
+ c*x*(-1 + 2*ArcCosh[c*x]^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcC
osh[c*x]*(ArcCosh[c*x] + 2*Log[1 - E^(-2*ArcCosh[c*x])]) + 2*Sqrt[(-1 + c*x
)/(1 + c*x)]*(1 + c*x)*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*c*d^2*Sqrt[d -
c^2*d*x^2])
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{\left(-c^2 d x^2 + d\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [B] time = 0.44, size = 3050, normalized size = 9.21

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3
*x^7+14/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^
4/d^3*x^5-16/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-
4)*c^2/d^3*x^3-8*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)/d^3*arccosh(c*x)*x+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^
4+11*c^2*x^2-4)/d^3*(c*x+1)*(c*x-1)*x-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*arccosh(c*x)*x^7-2*b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)^2*x^5+14/
3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*ar
ccosh(c*x)*x^5+17/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2
*x^2-4)*c^2/d^3*arccosh(c*x)^2*x^3-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x
^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*x^3-4/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1
/2)+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*x-
28/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*a
rccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^4+34/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)*c^2/d^3*arccosh(c*x)*x^3+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^
4*x^4+11*c^2*x^2-4)/d^3*(c*x+1)*(c*x-1)*x-4/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3
*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*b^2*(
-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*arccosh(c*x)*
(c*x+1)*(c*x-1)*x+7/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c
^2*x^2-4)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+8/3*b^2*(-d*(c^2*x^2-1))^(1
/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)^2*(c*x+1)^(1/2)*
(c*x-1)^(1/2)-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x
^2-4)*c^2/d^3*(c*x+1)*(c*x-1)*x^3-b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*
c^4*x^4+11*c^2*x^2-4)*c^3/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4+4/3*b^2*(-d*(
c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*polylog(2,c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)^2-4/3*b^2*(-d*(c^2*x^2-1))^(
1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(
c*x-1)^(1/2)+2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^
2-4)*c^4/d^3*(c*x+1)*(c*x-1)*x^5+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))-4*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^
3*arccosh(c*x)*x^5+1/3*a^2/d*x/(-c^2*d*x^2+d)^(3/2)+2/3*a^2/d^2*x/(-c^2*d*x
```

```
^2+d)^(1/2)-4*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4
)/d^3*arccosh(c*x)^2*x+2*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+1
1*c^2*x^2-4)/d^3*arccosh(c*x)*x-2/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-1
0*c^4*x^4+11*c^2*x^2-4)*c^6/d^3*x^7+3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6
-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*x^5-13/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^
6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*x^3+2*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*
c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/d^3*x+b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^
6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
^2+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^
2-1)*arccosh(c*x)*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))+4/3*b^2*(-d*(c^2*x^
2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)*ln(1
+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))+4/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6
-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^5+2*b^2*(-
d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^3/d^3*arccosh(c*
x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4-10/3*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6
*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d^3*arccosh(c*x)*(c*x+1)*(c*x-1)*x^3-14/3
*b^2*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c/d^3*arcco
sh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2-8/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/c/(c^2*x^2-1)*arccosh(c*x)+4/3*a*b*(-d*(c^2*x
^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^4/d^3*(c*x+1)*(c*x-1)*x^
5-10/3*a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)*c^2/d
^3*(c*x+1)*(c*x-1)*x^3+a*b*(-d*(c^2*x^2-1))^(1/2)/(3*c^6*x^6-10*c^4*x^4+11*
c^2*x^2-4)*c/d^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2+16/3*a*b*(-d*(c^2*x^2-1))^
(1/2)/(3*c^6*x^6-10*c^4*x^4+11*c^2*x^2-4)/c/d^3*arccosh(c*x)*(c*x+1)^(1/2)*
(c*x-1)^(1/2)+4/3*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^
3/c/(c^2*x^2-1)*ln((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,abc\left(\frac{\sqrt{-d}}{c^4d^3x^2-c^2d^3}+\frac{2\,\sqrt{-d}\,\log\left(cx+1\right)}{c^2d^3}+\frac{2\,\sqrt{-d}\,\log\left(cx-1\right)}{c^2d^3}\right)+\frac{2}{3}\,ab\left(\frac{2\,x}{\sqrt{-c^2dx^2+d}\,d^2}+\frac{x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*b*c*(sqrt(-d)/(c^4*d^3*x^2 - c^2*d^3) + 2*sqrt(-d)*log(c*x + 1)/(c^2*
d^3) + 2*sqrt(-d)*log(c*x - 1)/(c^2*d^3)) + 2/3*a*b*(2*x/(sqrt(-c^2*d*x^2 +
 d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d))*arccosh(c*x) + 1/3*a^2*(2*x/(sqrt(
-c^2*d*x^2 + d)*d^2) + x/((-c^2*d*x^2 + d)^(3/2)*d)) + b^2*integrate(log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(-c^2*d*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{{\left(d-c^2\,d\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d - c^2*d*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{\left(-d \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(5/2), x)
```

3.220 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=597

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{Li}_2\left(ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}\right.}{d^2\sqrt{d-c^2dx^2}}$$

```
[Out] 1/3*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2/d^2/(-c^2*d*x^2+d)^
(1/2)+(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*x*(a+b*arccosh(
c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+2*(
a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+14/3*b*(a+b*arccosh(c*x))*arctanh(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d
)^(1/2)+7/3*b^2*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b*(a+b*arccosh(c*x))*polylog(2,-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d
*x^2+d)^(1/2)+2*I*b*(a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-7/3*b^2*pol
ylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c
^2*d*x^2+d)^(1/2)+2*I*b^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-2*I*b^2*polylog(3,I*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2
+d)^(1/2)
```

Rubi [A] time = 1.33, antiderivative size = 612, normalized size of antiderivative = 1.03, number of steps used = 25, number of rules used = 13, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.448, Rules used = {5798, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5689, 74}

$$-\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(cx)}\right)\left(a+b\cosh^{-1}(cx)\right)}{d^2\sqrt{d-c^2dx^2}}+\frac{2ib\sqrt{cx-1}\sqrt{cx+1}\,\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}(cx}\right.}{d^2\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -b^2/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a +
 b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCos
h[c*x])^2/(d^2*Sqrt[d - c^2*d*x^2]) + (a + b*ArcCosh[c*x])^2/(3*d^2*(1 - c*
x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x])^2*ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + (14*b*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^ArcCosh[c*x]])/(
3*d^2*Sqrt[d - c^2*d*x^2]) + (7*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2,
 -E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt
[d - c^2*d*x^2]) + ((2*I)*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x
])*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (7*b^2*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2
]) + ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]
])/(d^2*Sqrt[d - c^2*d*x^2]) - ((2*I)*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Poly
Log[3, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
```

```
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{(2bc}{}$$

$$= \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh\right.}{3d^2(1-cx)(1+c}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bcx\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 10.45, size = 806, normalized size = 1.35

$$\frac{\log(cx)a^2}{d^{5/2}} - \frac{\log\left(d+\sqrt{-d\left(c^2x^2-1\right)}\sqrt{d}\right)a^2}{d^{5/2}} + \frac{b\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(-\frac{1}{2}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\cosh^{-1}(cx)\operatorname{csch}^4\left(\frac{1}{2}\cosh^{-1}(cx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(a^2/(3*d^3*(-1 + c^2*x^2)^2) - a^2/(d^3*(-1 + c^
2*x^2))) + (a^2*Log[c*x])/d^(5/2) - (a^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2
*x^2))]])/d^(5/2) + (a*b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(14*ArcCosh[c
*x]*Coth[ArcCosh[c*x]/2] - Csch[ArcCosh[c*x]/2]^2 - (Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^4)/2 - (24*I)*ArcCosh[c*x]
*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]]
- 28*Log[Tanh[ArcCosh[c*x]/2]] - (24*I)*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (
24*I)*PolyLog[2, I/E^ArcCosh[c*x]] - Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*
x]*Sinh[ArcCosh[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) - 14*
ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2]))/(12*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]
) + (b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-4*Coth[ArcCosh[c*x]/2] + 14
*ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2] - 2*ArcCosh[c*x]*Csch[ArcCosh[c*x]/2]^
2 - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Csch[ArcCosh[c*x]/
2]^4)/2 - 56*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (24*I)*ArcCosh[c*x]^
2*Log[1 - I/E^ArcCosh[c*x]] + (24*I)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x
]] + 56*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x])] - 56*PolyLog[2, -E^(-ArcCos
```

```
h[c*x])] - (48*I)*ArcCosh[c*x]*PolyLog[2, (-I)/E^ArcCosh[c*x]] + (48*I)*Arc
Cosh[c*x]*PolyLog[2, I/E^ArcCosh[c*x]] + 56*PolyLog[2, E^(-ArcCosh[c*x])] -
 (48*I)*PolyLog[3, (-I)/E^ArcCosh[c*x]] + (48*I)*PolyLog[3, I/E^ArcCosh[c*x
]] - 2*ArcCosh[c*x]*Sech[ArcCosh[c*x]/2]^2 - (8*ArcCosh[c*x]^2*Sinh[ArcCosh
[c*x]/2]^4)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)^3) + 4*Tanh[ArcCosh[c*x
]/2] - 14*ArcCosh[c*x]^2*Tanh[ArcCosh[c*x]/2]))/(24*d^2*Sqrt[-(d*(-1 + c*x)
*(1 + c*x))])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^7-3\,c^4d^3x^5+3\,c^2d^3x^3-d^3x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^7 - 3*c^4*d^3*x^5 + 3*c^2*d^3*x^3 - d^3*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}(cx))^2}{x\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{3}a^2\left(\frac{3\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2d}{|x|}\right)}{d^{\frac{5}{2}}}-\frac{3}{\sqrt{-c^2dx^2+d}\,d^2}-\frac{1}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}\right)+\int\frac{b^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxima"
)
```

```
[Out] -1/3*a^2*(3*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^(5/2)
 - 3/(sqrt(-c^2*d*x^2 + d)*d^2) - 1/((-c^2*d*x^2 + d)^(3/2)*d)) + integrate
(b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x) +
2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))^2}{x\,\left(d - c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)),x)
[Out] int((a + b*acosh(c*x))^2/(x*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{x\,(-d\,(cx - 1)\,(cx + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x/(-c**2*d*x**2+d)**(5/2),x)
[Out] Integral((a + b*acosh(c*x))**2/(x*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.221 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=476

$$\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}+\frac{8c\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2\sqrt{d-c^2dx^2}}-16$$

```
[Out] -(a+b*arccosh(c*x))^2/d/x/(-c^2*d*x^2+d)^(3/2)+4/3*c^2*x*(a+b*arccosh(c*x))
^2/d/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*c^2*x/d^2/(-c^2*d*x^2+d)^(1/2)+8/3*c^2*x*
(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*c*(a+b*arccosh(c*x))*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)+8/3*c*(a+b*
arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-4*b*c*
(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-16/3*b*c*(a+b*arccosh(c*x))*ln(1-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*
d*x^2+d)^(1/2)-b^2*c*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5/3*b^2*c*polylog(2,(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)
^(1/2)
```

Rubi [A] time = 1.19, antiderivative size = 506, normalized size of antiderivative = 1.06, number of steps used = 20, number of rules used = 15, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.517, Rules used = {5798, 5748, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 5754, 5721, 5461, 4182}

$$-\frac{b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{d^2\sqrt{d-c^2dx^2}}-\frac{5b^2c\sqrt{cx-1}\,\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{bc\sqrt{cx-1}\,\sqrt{}}{3d^2\left(1\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2*x)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (8*c^2*
x*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x]
)^2/(d^2*x*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (4*c^2*x*(a + b*ArcCo
sh[c*x])^2)/(3*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) -
(4*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCo
sh[c*x])])/(d^2*Sqrt[d - c^2*d*x^2]) - (16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^
2]) - (b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/
(d^2*Sqrt[d - c^2*d*x^2]) - (5*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2
, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x\left(-1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{\left(4c^2\right.}{}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d^2x(1-cx)(1+cx)\sqrt{d-c^2dx^2}} + \frac{4c}{3d^2(1}$$

$$= \frac{b^2c^2x}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(c\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(c\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(c\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(c\right.}{3d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{b^2c^2x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{8c^2x\left(a+b\cosh^{-1}(c\right.}{3d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 3.08, size = 457, normalized size = 0.96

$$c\left(\frac{a^2\left(8c^4x^4-12c^2x^2+3\right)}{cx\left(c^2x^2-1\right)} + ab\left(-\frac{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)+2cx\cosh^{-1}(cx)}{c^2x^2-1} + 10cx\cosh^{-1}(cx) - 2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(3\log(cx) + 5\log\left(\sqrt{\frac{cx-}{cx+}}\right.\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^2*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (c*((a^2*(3 - 12*c^2*x^2 + 8*c^4*x^4))/(c*x*(-1 + c^2*x^2)) + a*b*(10*c*x*A
rcCosh[c*x] - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + 2*c*x*ArcCosh[c*x])/(
-1 + c^2*x^2) - 2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((-3*Sqrt[(-1 + c*x)
/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x])/(c*x) + 3*Log[c*x] + 5*Log[Sqrt[(-1 + c
*x)/(1 + c*x)]*(1 + c*x)])) + b^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((c*
x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) + ArcCosh[c*x]/(1 - c^2*x^2) - 8*Ar
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (5*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (3*Sq
rt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 10*ArcCosh[c*x]*
Log[1 - E^(-2*ArcCosh[c*x])] - 6*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])]
+ 3*PolyLog[2, -E^(-2*ArcCosh[c*x])] + 5*PolyLog[2, E^(-2*ArcCosh[c*x])])))
/(3*d^2*Sqrt[d - c^2*d*x^2])
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^8-3\,c^4d^3x^6+3\,c^2d^3x^4-d^3x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^8 - 3*c^4*d^3*x^6 + 3*c^2*d^3*x^4 - d^3*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

maple [B] time = 0.60, size = 3798, normalized size = 7.98

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] -160/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x
^5*c^6-64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2
-9)*x^9*arccosh(c*x)*c^10+224/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-2
5*c^4*x^4+26*c^2*x^2-9)*x^7*arccosh(c*x)*c^8-64/3*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)^2*c^6-3*b^2*(-d*
(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*c-8/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c
^2*x^2-9)*x^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5+17/3*b^2*(-d*(c^2*x^2-1))^(1/
2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
c^3-272/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9
)*x^2*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+24*b^2*(-d*(c^2*x^2-1))^
(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*arccosh(c*x)^2*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*c+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/d^3/(c^2*x^2-1)*polylog(2,-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+b^2*(-d*(c^2
*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog(2,-(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c-16/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)^2*c+80/3*b^2*(-d*(c^2*x^2-1
))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x+1)*(c*x-1)*c^4-a^
2/d/x/(-c^2*d*x^2+d)^(3/2)+29*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*
c^4*x^4+26*c^2*x^2-9)*x^3*c^4-5*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-2
5*c^4*x^4+26*c^2*x^2-9)*x*c^2+9*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-2
5*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)^2-32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^9*c^10+40*b^2*(-d*(c^2*x^2-1))^(1/2
)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*c^8+112*a*b*(-d*(c^2*x^2-1))^
(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)*c^4-88*a*b*(
-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x
)*c^2-88/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-
```

```
9)*x^5*(c*x+1)*(c*x-1)*c^6-128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-
25*c^4*x^4+26*c^2*x^2-9)*x^5*arccosh(c*x)*c^6-3*a*b*(-d*(c^2*x^2-1))^(1/2)/
d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+8/3*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*arcco
sh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^3+8/3*a^2*c^2/d^2*x/(-c^2*d*x^2+d)^(1
/2)-3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*ar
ccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*arccosh(c*x)*(c*x+1)*(c*x-1)*c^8-
160/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^
5*arccosh(c*x)*(c*x+1)*(c*x-1)*c^6+64/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^4*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*c^5+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^
2*x^2-1)*arccosh(c*x)*ln(1+c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+10/3*b^2*(-d*
(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)
*ln(1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*c+2*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)*c-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4
+26*c^2*x^2-9)*x*arccosh(c*x)*(c*x+1)*(c*x-1)*c^2-32/3*a*b*(-d*(c^2*x^2-1))
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*arccosh(c*x)*c+64/3*a*b*
(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^7*(c*x+1)*
(c*x-1)*c^8-160/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c
^2*x^2-9)*x^5*(c*x+1)*(c*x-1)*c^6+40*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*
x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*(c*x+1)*(c*x-1)*c^4-8*a*b*(-d*(c^2*x^2-1))
^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x+1)*(c*x-1)*c^2+8/3*a*
b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*(c*x+1
)^(1/2)*(c*x-1)^(1/2)*c^3+48*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c
^4*x^4+26*c^2*x^2-9)*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c+10/3*a*b*(-
d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2-1)*c+2*a*b*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*c
+32/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^
7*(c*x+1)*(c*x-1)*c^8+4/3*a^2*c^2/d*x/(-c^2*d*x^2+d)^(3/2)-8*b^2*(-d*(c^2*x
^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*(c*x+1)*(c*x-1)*c^2+
40*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*a
rccosh(c*x)*(c*x+1)*(c*x-1)*c^4+10/3*b^2*(-d*(c^2*x^2-1))^(1/2)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d^3/(c^2*x^2-1)*polylog(2,c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*c-64/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*
x^9*c^10+224/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*
x^2-9)*x^7*c^8-280/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+2
6*c^2*x^2-9)*x^5*arccosh(c*x)*c^6-280/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c
^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^5*c^6+48*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(
8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*c^4-8*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3
/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x*c^2+18*a*b*(-d*(c^2*x^2-1))^(1/2)/d^
3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)/x*arccosh(c*x)+56*b^2*(-d*(c^2*x^2-1)
)^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arccosh(c*x)^2*c^4+48*b
^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^3*arcco
sh(c*x)*c^4-44*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*
x^2-9)*x*arccosh(c*x)^2*c^2-8*b^2*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*
c^4*x^4+26*c^2*x^2-9)*x*arccosh(c*x)*c^2-136/3*b^2*(-d*(c^2*x^2-1))^(1/2)/d
^3/(8*c^6*x^6-25*c^4*x^4+26*c^2*x^2-9)*x^2*arccosh(c*x)^2*(c*x+1)^(1/2)*(c*
x-1)^(1/2)*c^3+128/3*a*b*(-d*(c^2*x^2-1))^(1/2)/d^3/(8*c^6*x^6-25*c^4*x^4+2
6*c^2*x^2-9)*x^4*arccosh(c*x)*(c*x+1)^(1/2)*(c*x-1)^(1/2)*c^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^{2}\left(\frac{8\,c^{2}x}{\sqrt{-c^{2}dx^{2}+d}\,d^{2}}+\frac{4\,c^{2}x}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}d}-\frac{3}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}dx}\right)+\int\frac{b^{2}\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^{2}}{\left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}x^{2}}+\frac{2\,ab\log\left(cx+\right.}{\left(-c^{2}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a^2*(8*c^2*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 4*c^2*x/((-c^2*d*x^2 + d)^(3/
2)*d) - 3/((-c^2*d*x^2 + d)^(3/2)*d*x)) + integrate(b^2*log(c*x + sqrt(c*x
+ 1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x^2) + 2*a*b*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x^2\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^2*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x^2\,(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**2*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.222 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=796

$$-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)c^3}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{2d^2\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{6d\left(d-c^2dx^2\right)^{3/2}}+\frac{5\sqrt{cx-1}\sqrt{cx+}}{}$$

```
[Out] 5/6*c^2*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+d)^(3/2)-1/2*(a+b*arccosh(c*x))^
2/d/x^2/(-c^2*d*x^2+d)^(3/2)-1/3*b^2*c^2/d^2/(-c^2*d*x^2+d)^(1/2)+5/2*c^2*(
a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+b*c*(a+b*arccosh(c*x))*(c*x-1)
^(1/2)*(c*x+1)^(1/2)/d^2/x/(-c^2*x^2+1)/(-c^2*d*x^2+d)^(1/2)-2/3*b*c^3*x*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*x^2+1)/(-c^2*d*x^2+d
)^(1/2)+5*c^2*(a+b*arccosh(c*x))^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-b^2*c^2*arctan((c*x-1)
^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+
26/3*b*c^2*(a+b*arccosh(c*x))*arctanh(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x
-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+13/3*b^2*c^2*polylog(2,-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+
d)^(1/2)-5*I*b*c^2*(a+b*arccosh(c*x))*polylog(2,-I*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)+5*I*b*c^2*(
a+b*arccosh(c*x))*polylog(2,I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-13/3*b^2*c^2*polylog(2,c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)+5*I*b^2*c^2*polylog(3,-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-5*I*b^2*c^2*polylog(3,I*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2)))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)
```

Rubi [A] time = 1.94, antiderivative size = 826, normalized size of antiderivative = 1.04, number of steps used = 39, number of rules used = 19, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.655, Rules used = {5798, 5748, 5756, 5761, 4180, 2531, 2282, 6589, 5694, 4182, 2279, 2391, 5689, 74, 5746, 104, 21, 92, 205}

$$-\frac{2bx\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)c^3}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{6d^2(1-cx)(cx+1)\sqrt{d-c^2dx^2}}+\frac{5\left(a+b\cosh^{-1}(cx)\right)^2c^2}{2d^2\sqrt{d-c^2dx^2}}+\frac{5\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] -(b^2*c^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x]))/(d^2*x*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) - (2*b*c^3*
x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*(1 - c^2*x^2)*S
qrt[d - c^2*d*x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(2*d^2*Sqrt[d - c^2*d*
x^2]) + (5*c^2*(a + b*ArcCosh[c*x])^2)/(6*d^2*(1 - c*x)*(1 + c*x)*Sqrt[d -
c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(2*d^2*x^2*(1 - c*x)*(1 + c*x)*Sqrt[d
- c^2*d*x^2]) + (5*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^2*
ArcTan[E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - (b^2*c^2*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/(d^2*Sqrt[d - c^2*d*x^
2]) + (26*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E
^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + (13*b^2*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*PolyLog[2, -E^ArcCosh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) - ((5*
I)*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, (-I)*
E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) + ((5*I)*b*c^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x])*PolyLog[2, I*E^ArcCosh[c*x]])/(d^2*Sqrt[d
```

```
- c^2*d*x^2]) - (13*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, E^ArcCo
sh[c*x]])/(3*d^2*Sqrt[d - c^2*d*x^2]) + ((5*I)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*PolyLog[3, (-I)*E^ArcCosh[c*x]])/(d^2*Sqrt[d - c^2*d*x^2]) - ((5*I
)*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[3, I*E^ArcCosh[c*x]])/(d^2*S
qrt[d - c^2*d*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ
[2*m, 2*n, 2*p]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
```

```
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5756

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[((f*x)^(m +
1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2
*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d1*d2*(p + 1)), Int[(f*x)^m*(d1 +
e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/
(2*f*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m +
1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[
{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2,
0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] && (IntegerQ[m] || EqQ[n, 1])
&& IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2d^2x^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(-1+c^2x^2\right)^2}\,dx}{d^2\sqrt{d-c^2dx^2}} + \frac{(5c \cdots}{\cdots}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{5c^2\left(a+b\cosh^{-1}(cx)\right)^2}{6d^2(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{(\cdots}{2d^2x^2(\cdots}$$

$$= -\frac{b^2c^2}{d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+c\cdots}}{3d^2\left(1-c^2x^2\right)\cdots}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+\cdots}}{3d^2\left(1-c^2x^2\cdots\right.}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+\cdots}}{3d^2\left(1-c^2x^2\cdots\right.}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+\cdots}}{3d^2\left(1-c^2x^2\cdots\right.}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+\cdots}}{3d^2\left(1-c^2x^2\cdots\right.}$$

$$= -\frac{b^2c^2}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}{d^2x\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{2bc^3x\sqrt{-1+cx}\,\sqrt{1+\cdots}}{3d^2\left(1-c^2x^2\cdots\right.}$$

Mathematica [A] time = 99.16, size = 1181, normalized size = 1.48

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^3*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] Sqrt[-(d*(-1 + c^2*x^2))]*(-1/2*a^2/(d^3*x^2) + (a^2*c^2)/(3*d^3*(-1 + c^2*
x^2)^2) - (2*a^2*c^2)/(d^3*(-1 + c^2*x^2))) + (5*a^2*c^2*Log[x])/(2*d^(5/2)
) - (5*a^2*c^2*Log[d + Sqrt[d]*Sqrt[-(d*(-1 + c^2*x^2))]])/(2*d^(5/2)) + (a
*b*c^2*((6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(c*x) + (6*(-1 + c*x)*(1 +
 c*x)*ArcCosh[c*x])/(c^2*x^2) + 26*ArcCosh[c*x]*Cosh[ArcCosh[c*x]/2]^2 - Co
th[ArcCosh[c*x]/2] - ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]^2 - (30*I)*Sqrt[(-1
+ c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 - I/E^ArcCosh[c*x]] + (30*I)
*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + I/E^ArcCosh[c*x]
] - 26*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[Tanh[ArcCosh[c*x]/2]] - (30
*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, (-I)/E^ArcCosh[c*x]] +
(30*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, I/E^ArcCosh[c*x]] -
26*ArcCosh[c*x]*Sinh[ArcCosh[c*x]/2]^2 - Tanh[ArcCosh[c*x]/2] - ArcCosh[c*x
]*Tanh[ArcCosh[c*x]/2]^2))/(6*d^2*Sqrt[-(d*(-1 + c*x)*(1 + c*x))]) - (b^2*c
^2*Sqrt[d - c^2*d*x^2]*((12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*
x])/(c*x) + 6*(1 - 1/(c^2*x^2))*ArcCosh[c*x]^2 - 24*Sqrt[(-1 + c*x)/(1 + c*
x)]*(1 + c*x)*ArcTan[Tanh[ArcCosh[c*x]/2]] - 4*Cosh[ArcCosh[c*x]/2]^2 + 26*
```

```
ArcCosh[c*x]^2*Cosh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Coth[ArcCosh[c*x]/2]
 - ArcCosh[c*x]^2*Coth[ArcCosh[c*x]/2]^2 - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1
 + c*x)*ArcCosh[c*x]*Log[1 - E^(-ArcCosh[c*x])] - (30*I)*Sqrt[(-1 + c*x)/(1
 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 - I/E^ArcCosh[c*x]] + (30*I)*Sqrt[(
-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2*Log[1 + I/E^ArcCosh[c*x]] + 5
2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*Log[1 + E^(-ArcCosh[c*x
])] - 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*PolyLog[2, -E^(-ArcCosh[c*x])
] - (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]*PolyLog[2, (-I
)/E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x
]*PolyLog[2, I/E^ArcCosh[c*x]] + 52*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Po
lyLog[2, E^(-ArcCosh[c*x])] - (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*P
olyLog[3, (-I)/E^ArcCosh[c*x]] + (60*I)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x
)*PolyLog[3, I/E^ArcCosh[c*x]] + 4*Sinh[ArcCosh[c*x]/2]^2 - 26*ArcCosh[c*x]
^2*Sinh[ArcCosh[c*x]/2]^2 - 2*ArcCosh[c*x]*Tanh[ArcCosh[c*x]/2] - ArcCosh[c
*x]^2*Tanh[ArcCosh[c*x]/2]^2))/(12*d^3*(-1 + c^2*x^2))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^9-3\,c^4d^3x^7+3\,c^2d^3x^5-d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^9 - 3*c^4*d^3*x^7 + 3*c^2*d^3*x^5 - d^3*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

maple [F] time = 0.97, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{arccosh}(cx))^2}{x^3\left(-c^2d\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{6}a^2\left(\frac{15\,c^2\log\left(\frac{2\sqrt{-c^2dx^2+d}\sqrt{d}}{|x|}+\frac{2\,d}{|x|}\right)}{d^{\frac{5}{2}}}-\frac{15\,c^2}{\sqrt{-c^2dx^2+d}\,d^2}-\frac{5\,c^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}d}+\frac{3}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}dx^2}\right)+\int\frac{b^2\log\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^3/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] -1/6*a^2*(15*c^2*log(2*sqrt(-c^2*d*x^2 + d)*sqrt(d)/abs(x) + 2*d/abs(x))/d^
(5/2) - 15*c^2/(sqrt(-c^2*d*x^2 + d)*d^2) - 5*c^2/((-c^2*d*x^2 + d)^(3/2)*d
) + 3/((-c^2*d*x^2 + d)^(3/2)*d*x^2)) + integrate(b^2*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1))^2/((-c^2*d*x^2 + d)^(5/2)*x^3) + 2*a*b*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))/((-c^2*d*x^2 + d)^(5/2)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))^2}{x^3\,\left(d - c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^3*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))^2}{x^3\,(-d\,(cx - 1)\,(cx + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**3/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**3*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.223 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=562

$$\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}}-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)^2}{dx\left(d-c^2dx^2\right)^{3/2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3dx^3\left(d-c^2dx^2\right)^{3/2}}+\frac{16c^4x\left(a+b\cosh^{-1}(cx\right.}{3d^2\sqrt{d-c^2dx^2}}$$

```
[Out] -1/3*(a+b*arccosh(c*x))^2/d/x^3/(-c^2*d*x^2+d)^(3/2)-2*c^2*(a+b*arccosh(c*x
))^2/d/x/(-c^2*d*x^2+d)^(3/2)+8/3*c^4*x*(a+b*arccosh(c*x))^2/d/(-c^2*d*x^2+
d)^(3/2)+1/3*b^2*c^2/d^2/x/(-c^2*d*x^2+d)^(1/2)-2/3*b^2*c^4*x/d^2/(-c^2*d*x
^2+d)^(1/2)+16/3*c^4*x*(a+b*arccosh(c*x))^2/d^2/(-c^2*d*x^2+d)^(1/2)+1/3*b*
c*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/x^2/(-c^2*x^2+1)/(-c^2
*d*x^2+d)^(1/2)+16/3*c^3*(a+b*arccosh(c*x))^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d
^2/(-c^2*d*x^2+d)^(1/2)-32/3*b*c^3*(a+b*arccosh(c*x))*arctanh((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2
)-32/3*b*c^3*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*
d*x^2+d)^(1/2)-8/3*b^2*c^3*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/d^2/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 1.82, antiderivative size = 607, normalized size of antiderivative = 1.08, number of steps used = 34, number of rules used = 18, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.621, Rules used = {5798, 5748, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 5754, 5721, 5461, 4182, 5746, 103, 12}

$$-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,-e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}-\frac{8b^2c^3\sqrt{cx-1}\sqrt{cx+1}\,\text{PolyLog}\left(2,e^{2\cosh^{-1}(cx)}\right)}{3d^2\sqrt{d-c^2dx^2}}+\frac{16c^4x\left(a\right.}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] (b^2*c^2)/(3*d^2*x*Sqrt[d - c^2*d*x^2]) - (2*b^2*c^4*x)/(3*d^2*Sqrt[d - c^2
*d*x^2]) + (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(3*d^2*x
^2*(1 - c^2*x^2)*Sqrt[d - c^2*d*x^2]) + (16*c^4*x*(a + b*ArcCosh[c*x])^2)/(
3*d^2*Sqrt[d - c^2*d*x^2]) - (a + b*ArcCosh[c*x])^2/(3*d^2*x^3*(1 - c*x)*(1
 + c*x)*Sqrt[d - c^2*d*x^2]) - (2*c^2*(a + b*ArcCosh[c*x])^2)/(d^2*x*(1 - c
*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (8*c^4*x*(a + b*ArcCosh[c*x])^2)/(3*d^
2*(1 - c*x)*(1 + c*x)*Sqrt[d - c^2*d*x^2]) + (16*c^3*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(a + b*ArcCosh[c*x])^2)/(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])*ArcTanh[E^(2*ArcCosh[c*x])])/
(3*d^2*Sqrt[d - c^2*d*x^2]) - (32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])*Log[1 - E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2]) - (
8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[2, -E^(2*ArcCosh[c*x])])/(3*
d^2*Sqrt[d - c^2*d*x^2]) - (8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*PolyLog[
2, E^(2*ArcCosh[c*x])])/(3*d^2*Sqrt[d - c^2*d*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
```

```
Q[b*c + a*d, 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5461

```
Int[Csch[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sech[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csch[2*a + 2*b*x]
^n, x], x] /; FreeQ[{a, b, c, d}, x] && RationalQ[m] && IntegerQ[n]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
```

```
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5721

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((x_)*((d_) + (e_.)*(x_)^2)),
 x_Symbol] :> -Dist[d^(-1), Subst[Int[(a + b*x)^n/(Cosh[x]*Sinh[x]), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGt
Q[n, 0]
```

Rule 5746

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
b*ArcCosh[c*x])^n)/(d*f*(m + 1)), x] + (Dist[(b*c*n*(-d)^p)/(f*(m + 1)), In
t[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x
])^(n - 1), x], x] + Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m +
 2)*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e,
f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[m, -1] && IntegerQ[m] &&
 IntegerQ[p]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5754

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_
.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((f*x)^(m + 1)*(d + e*x^2)^(p + 1)*(a +
 b*ArcCosh[c*x])^n)/(2*d*f*(p + 1)), x] + (Dist[(m + 2*p + 3)/(2*d*(p + 1))
, Int[(f*x)^m*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x] - Dist[(b*
c*n*(-d)^p)/(2*f*(p + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p + 1/2)*(-1 + c*x)
^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d, e, f
, m}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] &&  !GtQ[m, 1] &&
IntegerQ[p]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{x^4(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{\left(2bc\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(-1+c^2x^2\right)^2}\,dx}{3d^2\sqrt{d-c^2dx^2}} +$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{3d^2x^3(1-cx)(1+cx)\sqrt{d-c^2dx^2}} - \frac{}{d^2}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{8bc^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1}}{3d^2x^2\left(1-\right.}$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} + \frac{8b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

$$= \frac{b^2c^2}{3d^2x\sqrt{d-c^2dx^2}} - \frac{2b^2c^4x}{3d^2\sqrt{d-c^2dx^2}} + \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{3d^2x^2\left(1-c^2x^2\right)\sqrt{d-c^2dx^2}} +$$

Mathematica [A] time = 3.68, size = 534, normalized size = 0.95

$$\frac{a^2\left(16c^6x^6-24c^4x^4+6c^2x^2+1\right)}{x^3\left(c^2x^2-1\right)} + abc^3\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(\frac{1}{1-c^2x^2}+\frac{1}{c^2x^2}+\frac{2\left(16c^6x^6-24c^4x^4+6c^2x^2+1\right)\left(\frac{cx-1}{cx+1}\right)^{3/2}\cosh^{-1}(cx)}{c^3x^3(cx-1)^3}-16\log(c\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(x^4*(d - c^2*d*x^2)^(5/2)),x]
```

```
[Out] ((a^2*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6))/(x^3*(-1 + c^2*x^2)) + a*b
*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(1/(c^2*x^2) + (1 - c^2*x^2)^(-1)
 + (2*((-1 + c*x)/(1 + c*x))^(3/2)*(1 + 6*c^2*x^2 - 24*c^4*x^4 + 16*c^6*x^6
)*ArcCosh[c*x])/(c^3*x^3*(-1 + c*x)^3) - 16*Log[c*x] - 16*Log[Sqrt[(-1 + c*
x)/(1 + c*x)]*(1 + c*x)]) + b^2*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*((
c*x*Sqrt[(-1 + c*x)/(1 + c*x)])/(1 - c*x) - (Sqrt[(-1 + c*x)/(1 + c*x)]*(1
+ c*x))/(c*x) + ArcCosh[c*x]/(c^2*x^2) + ArcCosh[c*x]/(1 - c^2*x^2) - 16*Ar
cCosh[c*x]^2 - (c*x*ArcCosh[c*x]^2)/(((-1 + c*x)/(1 + c*x))^(3/2)*(1 + c*x)
^3) + (8*c*x*ArcCosh[c*x]^2)/(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + (Sqrt
```

```
[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c^3*x^3) + (8*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x)*ArcCosh[c*x]^2)/(c*x) - 16*ArcCosh[c*x]*Log[1 - E
^(-2*ArcCosh[c*x])] - 16*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 8*Poly
Log[2, -E^(-2*ArcCosh[c*x])] + 8*PolyLog[2, E^(-2*ArcCosh[c*x])]))/(3*d^2*S
qrt[d - c^2*d*x^2])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)}{c^6d^3x^{10}-3\,c^4d^3x^8+3\,c^2d^3x^6-d^3x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)/(c^6*d^3*x^10 - 3*c^4*d^3*x^8 + 3*c^2*d^3*x^6 - d^3*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)^2}{\left(-c^2dx^2+d\right)^{\frac{5}{2}}x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

maple [B] time = 0.88, size = 5251, normalized size = 9.34

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,abc\left(\frac{8\,c^2\sqrt{-d}\,\log(cx+1)}{d^3}+\frac{8\,c^2\sqrt{-d}\,\log(cx-1)}{d^3}+\frac{16\,c^2\sqrt{-d}\,\log(x)}{d^3}+\frac{\sqrt{-d}}{c^2d^3x^4-d^3x^2}\right)+\frac{2}{3}\left(\frac{16\,c^4x}{\sqrt{-c^2dx^2+d}\,d^2}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/x^4/(-c^2*d*x^2+d)^(5/2),x, algorithm="maxim
a")
```

```
[Out] 1/3*a*b*c*(8*c^2*sqrt(-d)*log(c*x + 1)/d^3 + 8*c^2*sqrt(-d)*log(c*x - 1)/d^
3 + 16*c^2*sqrt(-d)*log(x)/d^3 + sqrt(-d)/(c^2*d^3*x^4 - d^3*x^2)) + 2/3*(1
6*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x^2 + d)^(3/2)*d) - 6
*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)^(3/2)*d*x^3))*a*b*a
rccosh(c*x) + 1/3*(16*c^4*x/(sqrt(-c^2*d*x^2 + d)*d^2) + 8*c^4*x/((-c^2*d*x
^2 + d)^(3/2)*d) - 6*c^2/((-c^2*d*x^2 + d)^(3/2)*d*x) - 1/((-c^2*d*x^2 + d)
^(3/2)*d*x^3))*a^2 + b^2*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2
/((-c^2*d*x^2 + d)^(5/2)*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{x^4\,\left(d-c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(x^4*(d - c^2*d*x^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^2}{x^4\,(-d\,(cx-1)\,(cx+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/x**4/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(x**4*(-d*(c*x - 1)*(c*x + 1))**(5/2)), x)
```

3.224 $\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=429

$$-\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^2}{15c^3\sqrt{c-a^2cx^2}}+\frac{8\sqrt{ax}}{}$$

```
[Out] 1/5*x*arccosh(a*x)^2/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)^2/c^2/(-a^2
*c*x^2+c)^(3/2)-1/3*x/c^3/(-a^2*c*x^2+c)^(1/2)-1/30*x/c^3/(-a*x+1)/(a*x+1)/
(-a^2*c*x^2+c)^(1/2)+8/15*x*arccosh(a*x)^2/c^3/(-a^2*c*x^2+c)^(1/2)+1/10*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)^2/(-a^2*c*x^2+c)^
(1/2)+4/15*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^
2*c*x^2+c)^(1/2)+8/15*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^
2*c*x^2+c)^(1/2)-16/15*arccosh(a*x)*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^
2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/15*polylog(2,(a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*
c*x^2+c)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 459, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2279, 2391, 5716, 39, 40}

$$-\frac{8\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{15ac^3\sqrt{c-a^2cx^2}}-\frac{x}{3c^3\sqrt{c-a^2cx^2}}-\frac{x}{30c^3(1-ax)(ax+1)\sqrt{c-a^2cx^2}}+\frac{8x\cosh^{-1}(ax)^2}{15c^3\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -x/(3*c^3*Sqrt[c - a^2*c*x^2]) - x/(30*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2
*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(10*a*c^3*(1 - a^2*x
^2)^2*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/
(15*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (8*x*ArcCosh[a*x]^2)/(15*c^3
*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^2)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*S
qrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x]^2)/(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt
[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(15*a*c^
3*Sqrt[c - a^2*c*x^2]) - (16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Log[
1 - E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/(15*a*c^3*Sqrt[c - a^2*c*x^2
])
```

Rule 39

```
Int[1/(((a_) + (b_.)*(x_))^(3/2)*((c_) + (d_.)*(x_))^(3/2)), x_Symbol] :> S
imp[x/(a*c*Sqrt[a + b*x]*Sqrt[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[b*c + a*d, 0]
```

Rule 40

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] :> -Simp[(
x*(a + b*x)^(m + 1)*(c + d*x)^(m + 1))/(2*a*c*(m + 1)), x] + Dist[(2*m + 3)
/(2*a*c*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(m + 1), x], x] /; FreeQ[
{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && ILtQ[m + 3/2, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
```

```
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(2a\sqrt{-1\right.}{}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^2}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}(a}{15c^3(1-ax)(1+ax)}$$

$$= -\frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4\sqrt{-1+ax}\,\sqrt{1+ax}}{15ac^3\left(1-a^2x^2\right)}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4}{}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4}{}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4}{}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4}{}$$

$$= -\frac{x}{3c^3\sqrt{c-a^2cx^2}} - \frac{x}{30c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{10ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{4}{}$$

Mathematica [A] time = 1.38, size = 220, normalized size = 0.51

$$-\frac{ax\left(\frac{1}{1-a^2x^2}+10\right)+2\left(ax\left(\frac{4}{a^2x^2-1}-\frac{3}{\left(a^2x^2-1\right)^2}+8\sqrt{\frac{ax-1}{ax+1}}-8\right)+8\sqrt{\frac{ax-1}{ax+1}}\right)\cosh^{-1}(ax)^2+\frac{\left(\frac{ax-1}{ax+1}\right)^{3/2}\cosh^{-1}(ax)\left(8a^2x^2+32(a\right.}{(ax}}{30ac^3\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/30*(a*x*(10 + (1 - a^2*x^2)^(-1)) + 2*(8*Sqrt[(-1 + a*x)/(1 + a*x)] + a*
x*(-8 + 8*Sqrt[(-1 + a*x)/(1 + a*x)] - 3/(-1 + a^2*x^2)^2 + 4/(-1 + a^2*x^2
)))*ArcCosh[a*x]^2 + (((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]*(-11 + 8*a^
2*x^2 + 32*(-1 + a^2*x^2)^2*Log[1 - E^(-2*ArcCosh[a*x])]))/(-1 + a*x)^3 - 1
6*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*PolyLog[2, E^(-2*ArcCosh[a*x])])/(a*
c^3*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^2}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^2/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{2,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4
]%%%}+%%%{2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[2,2,2]%%%}+%%%{-4,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{2,[0,1,0]%%%}+%%%{1,[0,0,0]%%
%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const ind
ex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.50, size = 794, normalized size = 1.85

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(8x^5a^5-20x^3a^3-8\sqrt{ax+1}\,\sqrt{ax-1}\,x^4a^4+15ax+16a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}-8\sqrt{ax-1}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/30*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*x^4*a^4+15*a*x+16*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(-64*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)*x^7*a^7-64*ar
ccosh(a*x)*x^8*a^8-32*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-32*x^8*a^8+248*ar
ccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5+280*arccosh(a*x)*x^6*a^6+126
*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5+142*x^6*a^6+80*a^4*x^4*arccosh(a*x)^2-
340*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3-456*a^4*x^4*arccosh(a*
x)-156*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-265*x^4*a^4-190*a^2*x^2*arccosh(
a*x)^2+165*arccosh(a*x)*a*x*(a*x-1)^(1/2)*(a*x+1)^(1/2)+328*a^2*x^2*arccosh
(a*x)+62*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+235*a^2*x^2+128*arccosh(a*x)^2-88*
arccosh(a*x)-80)/(40*a^10*x^10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*
x^2-64)/a/c^4-16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/
a/(a^2*x^2-1)*arccosh(a*x)^2+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2
-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1
/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^
2-1)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1)
^(1/2)*(-c*(a^2*x^2-1))^(1/2)/c^4/a/(a^2*x^2-1)*arccosh(a*x)*ln(1+a*x+(a*x-
```

```
1)^(1/2)*(a*x+1)^(1/2))+16/15*(a*x+1)^(1/2)*(a*x-1)^(1/2)*(-c*(a^2*x^2-1))^
(1/2)/c^4/a/(a^2*x^2-1)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^2/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^2}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(c - a^2*c*x^2)^(7/2),x)
[Out] int(acosh(a*x)^2/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{2}(ax)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*c*x**2+c)**(7/2),x)
[Out] Integral(acosh(a*x)**2/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.225 $\int \frac{x^4 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=243

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^3}{8a^5\sqrt{1-ax}}+\frac{15\sqrt{ax-1}\cosh^{-1}(ax)}{64a^5\sqrt{1-ax}}-\frac{15x\sqrt{1-ax}\sqrt{ax+1}}{64a^4}-\frac{3x^2\sqrt{ax-1}\cosh^{-1}(ax)}{8a^3\sqrt{1-ax}}-\frac{x^3\sqrt{1-ax}\sqrt{a}}{32a^2}$$

```
[Out] 15/64*arccosh(a*x)*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-3/8*x^2*arccosh(a*x)*(a
*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/8*x^4*arccosh(a*x)*(a*x-1)^(1/2)/a/(-a*x+1
)^(1/2)+1/8*arccosh(a*x)^3*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-15/64*x*(-a*x+1
)^(1/2)*(a*x+1)^(1/2)/a^4-1/32*x^3*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-3/8*x*a
rccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^4-1/4*x^3*arccosh(a*x)^2*(-a^2*x^2+1)^(1
/2)/a^2
```

Rubi [A] time = 0.79, antiderivative size = 329, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5759, 5676, 5662, 90, 52, 100, 12}

$$-\frac{x^3(1-ax)(ax+1)}{32a^2\sqrt{1-a^2x^2}}-\frac{15x(1-ax)(ax+1)}{64a^4\sqrt{1-a^2x^2}}-\frac{x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(ax)^2}{4a^2\sqrt{1-a^2x^2}}-3$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-15*x*(1 - a*x)*(1 + a*x))/(64*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1
+ a*x))/(32*a^2*Sqrt[1 - a^2*x^2]) + (15*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCo
sh[a*x])/(64*a^5*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x])/(8*a^3*Sqrt[1 - a^2*x^2]) - (x^4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x])/(8*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x)*(1 + a*x)*ArcCosh[a*
x]^2)/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^2)/
(4*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(
8*a^5*Sqrt[1 - a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
```

```
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^2}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1\right.}{}$$

$$= -\frac{x^4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+}{4a^2\sqrt{}}$$

$$= -\frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{8a\sqrt{1-a^2x^2}}$$

$$= -\frac{3x(1-ax)(1+ax)}{16a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{8a^3\sqrt{1-a^2x^2}} - \frac{x^4\sqrt{}}{}$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{16a^5\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{}}{}$$

$$= -\frac{15x(1-ax)(1+ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+ax)}{32a^2\sqrt{1-a^2x^2}} + \frac{15\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{64a^5\sqrt{1-a^2x^2}} - \frac{3x^2}{}$$

Mathematica [A] time = 0.26, size = 116, normalized size = 0.48

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(32\cosh^{-1}(ax)^3 - 4\left(16\cosh\left(2\cosh^{-1}(ax)\right) + \cosh\left(4\cosh^{-1}(ax)\right)\right)\cosh^{-1}(ax) + 8\cosh^{-1}(a\right.}{256a^5\sqrt{1-a^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(32*ArcCosh[a*x]^3 - 4*ArcCosh[a*x]*(
16*Cosh[2*ArcCosh[a*x]] + Cosh[4*ArcCosh[a*x]]) + 32*Sinh[2*ArcCosh[a*x]] +
 Sinh[4*ArcCosh[a*x]] + 8*ArcCosh[a*x]^2*(8*Sinh[2*ArcCosh[a*x]] + Sinh[4*A
rcCosh[a*x]])))/(256*a^5*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^4\,\mathrm{arcosh}\,(ax)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,\mathrm{arcosh}\,(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [B] time = 0.77, size = 488, normalized size = 2.01

$$-\frac{\sqrt{-a^{2}x^{2}+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}\left(ax\right)^{3}}{8a^{5}\left(a^{2}x^{2}-1\right)}-\frac{\sqrt{-a^{2}x^{2}+1}\,\left(8x^{5}a^{5}-12x^{3}a^{3}+8\sqrt{ax+1}\,\sqrt{ax-1}\,x^{4}a^{4}+4ax-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccosh
(a*x)^3-1/512*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x
-1)^(1/2)*x^4*a^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(8*arccosh(a*x)^2-4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)-1/16*(-
a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*
x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1)/a^5/(a^2*x^2-
1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)
^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arccosh(a*x)+1)/a^5
/(a^2*x^2-1)-1/512*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/2)
*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^
(1/2)*(a*x+1)^(1/2))*(8*arccosh(a*x)^2+4*arccosh(a*x)+1)/a^5/(a^2*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\mathrm{acosh}\left(a\,x\right)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\operatorname{acosh}^{2}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.226 $\int \frac{x^3 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\, dx$

Optimal. Leaf size=177

$$-\frac{40\sqrt{1-ax}\,\sqrt{ax+1}}{27a^4}-\frac{4x\sqrt{ax-1}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-ax}}-\frac{2x^2\sqrt{1-ax}\,\sqrt{ax+1}}{27a^2}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{3a^2}-\frac{2\sqrt{1-a^2x^2}}{3}$$

```
[Out] -4/3*x*arccosh(a*x)*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-2/9*x^3*arccosh(a*x)*(
a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-40/27*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-2/27*x
^2*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-2/3*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a
^4-1/3*x^2*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.59, antiderivative size = 237, normalized size of antiderivative = 1.34, number of steps used = 9, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5759, 5718, 5654, 74, 5662, 100, 12}

$$-\frac{2x^2(1-ax)(ax+1)}{27a^2\sqrt{1-a^2x^2}}-\frac{40(1-ax)(ax+1)}{27a^4\sqrt{1-a^2x^2}}-\frac{2x^3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax)^2}{3a^2\sqrt{1-a^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*(1 - a*x)*(1 + a*x))/(27*a^4*Sqrt[1 - a^2*x^2]) - (2*x^2*(1 - a*x)*(1
+ a*x))/(27*a^2*Sqrt[1 - a^2*x^2]) - (4*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x])/(3*a^3*Sqrt[1 - a^2*x^2]) - (2*x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x])/(9*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^
2)/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^2)/(3*
a^2*Sqrt[1 - a^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1+\right.}{}$$

$$= -\frac{2x^3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{9a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^2}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax}{3a^2\sqrt{1-}}$$

$$= -\frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh}{9a\sqrt{1-a^2x^2}}$$

$$= -\frac{4(1-ax)(1+ax)}{3a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1}}{}$$

$$= -\frac{40(1-ax)(1+ax)}{27a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)}{27a^2\sqrt{1-a^2x^2}} - \frac{4x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{3a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-}}{}$$

Mathematica [A] time = 0.15, size = 123, normalized size = 0.69

$$\left(-\frac{40}{27a^4}-\frac{2x^2}{27a^2}\right)\sqrt{1-a^2x^2}-\frac{\sqrt{1-a^2x^2}\left(a^2x^2+2\right)\cosh^{-1}(ax)^2}{3a^4}+\frac{2x\sqrt{1-a^2x^2}\left(a^2x^2+6\right)\cosh^{-1}(ax)}{9a^3\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40/(27*a^4) - (2*x^2)/(27*a^2))*Sqrt[1 - a^2*x^2] + (2*x*Sqrt[1 - a^2*x^2
]*(6 + a^2*x^2)*ArcCosh[a*x])/(9*a^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (Sqrt[
1 - a^2*x^2]*(2 + a^2*x^2)*ArcCosh[a*x]^2)/(3*a^4)
```

fricas [A] time = 0.62, size = 150, normalized size = 0.85

$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)^2-6\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(ax+\sqrt{a^2x^2-1}\right)}{27\left(a^6x^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^2 - 6*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1)) + 2*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1))/(a
^6*x^2 - a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.50, size = 343, normalized size = 1.94

$$-\frac{\sqrt{-a^2x^2+1}\left(4x^4a^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\sqrt{ax+1}-3\sqrt{ax+1}\sqrt{ax-1}\,ax+1\right)\left(9\operatorname{arccosh}(ax)^2-6\operatorname{arcc}\right.}{216a^4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^2-6*arccosh(
a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x+a^2*x^2-1)*(arccosh(a*x)^2-2*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-3/8*(-a^2
*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2
*arccosh(a*x)+2)/a^4/(a^2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*
x^2-4*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x
+1)*(9*arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

maxima [C] time = 0.47, size = 105, normalized size = 0.59

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^2+\frac{2\left(-i\sqrt{a^2x^2-1}\,x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)}{27\,a^2}+\frac{2\left(i\,a^2x^3+6i\,x\right)\operatorname{arcosh}}{9\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^2
 + 2/27*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)/a^2 + 2/9*(
I*a^2*x^3 + 6*I*x)*arccosh(a*x)/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\mathrm{acosh}\left(a\,x\right)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,\mathrm{acosh}^2\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.227 $\int \frac{x^2 \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^3}{6a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)}{4a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^2}{2a^2}-\frac{x\sqrt{1-ax}\,\sqrt{ax+1}}{4a^2}-\frac{x^2\sqrt{ax-1}\,\cosh}{2a\sqrt{1-ax}}$$

```
[Out] 1/4*arccosh(a*x)*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/2*x^2*arccosh(a*x)*(a*x
-1)^(1/2)/a/(-a*x+1)^(1/2)+1/6*arccosh(a*x)^3*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1
/2)-1/4*x*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-1/2*x*arccosh(a*x)^2*(-a^2*x^2+1
)^(1/2)/a^2
```

Rubi [A] time = 0.51, antiderivative size = 207, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5759, 5676, 5662, 90, 52}

$$-\frac{x(1-ax)(ax+1)}{4a^2\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{6a^3\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}}-\frac{x^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh}{2a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -(x*(1 - a*x)*(1 + a*x))/(4*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x])/(4*a^3*Sqrt[1 - a^2*x^2]) - (x^2*Sqrt[-1 + a*x]*Sqrt[
1 + a*x]*ArcCosh[a*x])/(2*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*(1 + a*x)*Arc
Cosh[a*x]^2)/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcC
osh[a*x]^3)/(6*a^3*Sqrt[1 - a^2*x^2])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
```

```
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+ax}\right.}{}$$
$$= -\frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}}{6a^3\sqrt{1-}}$$
$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2a^2\sqrt{1-a^2x^2}}$$
$$= -\frac{x(1-ax)(1+ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{4a^3\sqrt{1-a^2x^2}} - \frac{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}{2a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.18, size = 87, normalized size = 0.58

$$-\frac{\sqrt{-((ax-1)(ax+1))}\left(4\cosh^{-1}(ax)^3-6\cosh\left(2\cosh^{-1}(ax)\right)\cosh^{-1}(ax)+\left(6\cosh^{-1}(ax)^2+3\right)\sinh\left(2\cosh^{-1}(\right.\right.}{24a^3\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/24*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(4*ArcCosh[a*x]^3 - 6*ArcCosh[a*x]*Cos
h[2*ArcCosh[a*x]] + (3 + 6*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]]))/(a^3*Sqrt
[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\operatorname{arcosh}(ax)^2}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.46, size = 239, normalized size = 1.58

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^3}{6a^3\left(a^2x^2-1\right)}-\frac{\sqrt{-a^2x^2+1}\,\left(2x^3a^3-2ax+2a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-\right.}{16a^3\left(a^2x^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/6*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^3-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a
*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2-2*arccosh(a*x)+1
)/a^3/(a^2*x^2-1)-1/16*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x-2*a^2*x^2*(a*x-1
)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(2*arccosh(a*x)^2+2*arcc
osh(a*x)+1)/a^3/(a^2*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 \operatorname{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.228 $\int \frac{x \cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{a^2}-\frac{2\sqrt{1-ax}\sqrt{ax+1}}{a^2}-\frac{2x\sqrt{ax-1}\cosh^{-1}(ax)}{a\sqrt{1-ax}}$$

```
[Out] -2*x*arccosh(a*x)*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-2*(-a*x+1)^(1/2)*(a*x+1)^(
1/2)/a^2-arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.27, antiderivative size = 109, normalized size of antiderivative = 1.38, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {5798, 5718, 5654, 74}

$$-\frac{2(1-ax)(ax+1)}{a^2\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}-\frac{2x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-2*(1 - a*x)*(1 + a*x))/(a^2*Sqrt[1 - a^2*x^2]) - (2*x*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*ArcCosh[a*x])/(a*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 + a*x)*ArcCos
h[a*x]^2)/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} - \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \cosh^{-1}(ax)\,dx}{a\sqrt{1-a^2x^2}}$$
$$= -\frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+\right.}{\sqrt{}}$$
$$= -\frac{2(1-ax)(1+ax)}{a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{a^2\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.68

$$\frac{\sqrt{1-a^2x^2}\left(-\cosh^{-1}(ax)^2 + \frac{2ax\cosh^{-1}(ax)}{\sqrt{ax-1}\,\sqrt{ax+1}} - 2\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(-2 + (2*a*x*ArcCosh[a*x])/(Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - ArcCosh[a*x]^2))/a^2
```

fricas [A] time = 0.52, size = 114, normalized size = 1.44

$$\frac{2\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2-1}\right) + \left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^2 - 2\left(a^2x^2-1\right)\sqrt{-a^2x^{}}}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1)) +
(-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^2 - 2*(a^2*x^2 - 1)*sqrt(
-a^2*x^2 + 1))/(a^4*x^2 - a^2)
```

giac [C] time = 0.56, size = 76, normalized size = 0.96

$$-\frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2}{a^2} - \frac{2i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right) - \frac{\sqrt{a^2x^2-1}}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^2/a^2 - 2*I*(x*log(a*x + s
qrt(a^2*x^2 - 1)) - sqrt(a^2*x^2 - 1)/a)/a
```

maple [A] time = 0.20, size = 139, normalized size = 1.76

$$-\frac{\sqrt{-a^2x^2+1}\,\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)\left(\operatorname{arccosh}(ax)^2-2\operatorname{arccosh}(ax)+2\right)}{2a^2\left(a^2x^2-1\right)} - \frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^2-2*arccosh(a*x)+2)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1)^(1/2)*(a^2*x^2-
(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^2+2*arccosh(a*x)+2)/a^2/(a
^2*x^2-1)
```

maxima [C] time = 0.38, size = 50, normalized size = 0.63

$$\frac{2i\,x\operatorname{arcosh}(ax)}{a}-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)^2}{a^2}-\frac{2i\,\sqrt{a^2x^2-1}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*I*x*arccosh(a*x)/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/a^2 - 2*I*sqrt(a^2
*x^2 - 1)/a^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\operatorname{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*acosh(a*x)^2)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.229 $\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^3}{3a\sqrt{1-ax}}$$

```
[Out] 1/3*arccosh(a*x)^3*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3}{3a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^2/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/(3*a*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)^2}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.06, size = 51, normalized size = 1.59

$$-\frac{\sqrt{-(ax-1)(ax+1)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^3}{3a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x)
[Out] -1/3*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^2/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(a\,x)^2}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2),x)
[Out] int(acosh(a*x)^2/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**2/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.230 $\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=183

$$-\frac{2i\sqrt{ax-1}\cosh^{-1}(ax)\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\cosh^{-1}(ax)\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{2i\sqrt{ax-1}\,\text{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}$$

```
[Out] 2*arccosh(a*x)^2*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*
x+1)^(1/2)-2*I*arccosh(a*x)*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))
*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+2*I*arccosh(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+2*I*polylog(3,-I*(a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-2*I*polylog(3,I*(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 248, normalized size of antiderivative = 1.36, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5761, 4180, 2531, 2282, 6589}

$$-\frac{2i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{2i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt
[1 - a^2*x^2] - ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2,
 (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((2*I)*S
qrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x
^2] - ((2*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])/Sqr
t[1 - a^2*x^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\text{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(2i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x\log\right.}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\text{Li}_2\left(\right.}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\text{Li}_2\left(\right.}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)\text{Li}_2\left(\right.}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.19, size = 151, normalized size = 0.83

$$\frac{i\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-2\cosh^{-1}(ax)\left(\text{Li}_2\left(-ie^{-\cosh^{-1}(ax)}\right)-\text{Li}_2\left(ie^{-\cosh^{-1}(ax)}\right)\right)-2\text{Li}_3\left(-ie^{-\cosh^{-1}(ax)}\right)+2\text{Li}_3\left(ie^{-\cosh^{-1}\right.}{\sqrt{1-a^2x^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (I*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-(ArcCosh[a*x]^2*(Log[1 - I/E^ArcC
osh[a*x]] - Log[1 + I/E^ArcCosh[a*x]])) - 2*ArcCosh[a*x]*(PolyLog[2, (-I)/E
^ArcCosh[a*x]] - PolyLog[2, I/E^ArcCosh[a*x]]) - 2*PolyLog[3, (-I)/E^ArcCos
h[a*x]] + 2*PolyLog[3, I/E^ArcCosh[a*x]]))/Sqrt[1 - a^2*x^2]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\ \operatorname{arcosh}(ax)^2}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^2}{x\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
[Out] int(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^2/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**2/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.231 $\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=124

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}{x}-\frac{a\sqrt{ax-1}\,\mathrm{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{a\sqrt{ax-1}\cosh^{-1}(ax)^2}{\sqrt{1-ax}}-\frac{2a\sqrt{ax-1}\cosh^{-1}(ax)\log\left(e^{2\cosh^{-1}(ax)}\right.}{\sqrt{1-ax}}$$

```
[Out] a*arccosh(a*x)^2*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-2*a*arccosh(a*x)*ln(1+(a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-a*polylog(2,-(a*
x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-arccosh(a*x)
^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.44, antiderivative size = 174, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5724, 5660, 3718, 2190, 2279, 2391}

$$-\frac{a\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}}-2$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/Sqrt[1 - a^2*x^2] - ((1 - a
*x)*(1 + a*x)*ArcCosh[a*x]^2)/(x*Sqrt[1 - a^2*x^2]) - (2*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]*Log[1 + E^(2*ArcCosh[a*x])])/Sqrt[1 - a^2*x^2] -
(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, -E^(2*ArcCosh[a*x])])/Sqrt[1 - a
^2*x^2]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(2a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x\tanh(x)\,dx, x, \cosh^{-1}\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{\left(4a\sqrt{-1+ax}\,\sqrt{1+a\right.}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{x\sqrt{1-a^2x^2}} - \frac{2a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

Mathematica [A] time = 0.48, size = 111, normalized size = 0.90

$$\frac{a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(\mathrm{Li}_2\left(-e^{-2\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)\left(\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)}{ax}-\cosh^{-1}(ax)-2\log\left(e^{-2\cosh^{-1}(ax)}+\right.\right.\right.}{\sqrt{-((ax-1)(ax+1))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(ArcCosh[a*x]*(-ArcCosh[a*x] + (Sqr
t[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 2*Log[1 + E^(-2*Arc
Cosh[a*x])]) + PolyLog[2, -E^(-2*ArcCosh[a*x])]))/Sqrt[-((-1 + a*x)*(1 + a*
x))]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\ \operatorname{arcosh}(ax)^2}{a^2x^4-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.36, size = 241, normalized size = 1.94

$$-\frac{\sqrt{-a^2x^2+1}\ \left(a^2x^2-\sqrt{ax+1}\ \sqrt{ax-1}\ ax-1\right)\operatorname{arccosh}(ax)^2}{x\left(a^2x^2-1\right)}-\frac{2\sqrt{-a^2x^2+1}\ \sqrt{ax-1}\ \sqrt{ax+1}\ \operatorname{arccosh}(ax)^2\,a}{a^2x^2-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^2/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^2*a+2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+(-a^2*x^2+1
)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(2,-(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))^2)*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(a^2x^2-1\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^2}{\sqrt{ax+1}\,\sqrt{-ax+1}\,x}-\int\frac{2\left(a^3x^2+\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x-a\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)}{\left(\sqrt{ax+1}\,ax^2+(ax+1)\sqrt{ax-1}\,x\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(2*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((sqrt(a*x + 1)*a*x^2 + (a*x + 1)*
sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^2}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^2/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**2/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.232 $\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=296

$$-\frac{ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{ia^2\sqrt{ax-1}\,\mathrm{Li}_3\left(-ie^{\cosh^{-1}(ax)}\right.}{\sqrt{1-ax}}$$

```
[Out] a*arccosh(a*x)*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)+a^2*arccosh(a*x)^2*arctan(a*x
+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-a^2*arctan((a*x-
1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-I*a^2*arccosh(a*x)*pol
ylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+I
*a^2*arccosh(a*x)*polylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1
/2)/(-a*x+1)^(1/2)+I*a^2*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a
*x-1)^(1/2)/(-a*x+1)^(1/2)-I*a^2*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*arccosh(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 0.72, antiderivative size = 398, normalized size of antiderivative = 1.34, number of steps used = 13, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5798, 5748, 5761, 4180, 2531, 2282, 6589, 5662, 92, 205}

$$-\frac{ia^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{ia^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,ie^{\cosh^{-1}}\right.}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x])/(x*Sqrt[1 - a^2*x^2]) - ((1 -
 a*x)*(1 + a*x)*ArcCosh[a*x]^2)/(2*x^2*Sqrt[1 - a^2*x^2]) + (a^2*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
 - (a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcTan[Sqrt[-1 + a*x]*Sqrt[1 + a*x]])/
Sqrt[1 - a^2*x^2] - (I*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLo
g[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + (I*a^2*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*ArcCosh[a*x]*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + (I*a
^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a
^2*x^2] - (I*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, I*E^ArcCosh[a*x]])
/Sqrt[1 - a^2*x^2]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)}{x^2}\,dx}{\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+\right.}{2\sqrt{}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+ax}\right)S}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{co}}{\sqrt{1}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{co}}{\sqrt{1}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{co}}{\sqrt{1}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)}{x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^2}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\text{co}}{\sqrt{1}}$$

Mathematica [A] time = 1.01, size = 233, normalized size = 0.79

$$\frac{ia^2\sqrt{-((ax-1)(ax+1))}\left(\frac{i\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)^2}{a^2x^2} + 2\cosh^{-1}(ax)\text{Li}_2\left(-ie^{-\cosh^{-1}(ax)}\right) - 2\cosh^{-1}(ax)\text{Li}_2\left(ie^{-\cosh^{-1}(ax}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^2/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/2)*a^2*Sqrt[-((-1 + a*x)*(1 + a*x))]*(((2*I)*ArcCosh[a*x])/(a*x) + (I*S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^2)/(a^2*x^2) - (4*I)*ArcTa
n[Tanh[ArcCosh[a*x]/2]] + ArcCosh[a*x]^2*Log[1 - I/E^ArcCosh[a*x]] - ArcCos
h[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] + 2*ArcCosh[a*x]*PolyLog[2, (-I)/E^ArcCo
sh[a*x]] - 2*ArcCosh[a*x]*PolyLog[2, I/E^ArcCosh[a*x]] + 2*PolyLog[3, (-I)/
E^ArcCosh[a*x]] - 2*PolyLog[3, I/E^ArcCosh[a*x]]))/(Sqrt[(-1 + a*x)/(1 + a*
x)]*(1 + a*x))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^2}{a^2x^5-x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^2/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [F] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^2}{x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
[Out] int(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^2}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^2/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^2/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^2}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^2/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^2(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**2/x**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**2/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.233 $\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=1154

$$\frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2(fx)^{m+1}}{f(m+6)}+\frac{5d\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2(fx)^{m+1}}{f(m+4)(m+6)}+\frac{15d^2\sqrt{d-c^2dx^2}\,(a+}{f(m+6)(}$$

```
[Out] 5*d*(f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/f/(4+m)/(6+m)+(f*
x)^(1+m)*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2/f/(6+m)-10*b^2*c^2*d^2*(
f*x)^(3+m)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3/(6+m)-2*b^2*c^2*d^2*(m^2+15*m+5
2)*(f*x)^(3+m)*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^2/(6+m)^3/(-c*x+
1)/(c*x+1)+2*b^2*c^4*d^2*(f*x)^(5+m)*(-c^2*x^2+1)*(-c^2*d*x^2+d)^(1/2)/f^5/
(6+m)^3/(-c*x+1)/(c*x+1)+15*d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))^2*(-c^2*d*x^
2+d)^(1/2)/f/(6+m)/(m^2+6*m+8)-2*b*c*d^2*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c
^2*d*x^2+d)^(1/2)/f^2/(2+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-30*b*c*d^2*(f
*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(6+m)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-10*b*c*d^2*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*
d*x^2+d)^(1/2)/f^2/(6+m)/(m^2+6*m+8)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+10*b*c^3*d
^2*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^4/(4+m)^2/(6+m)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+4*b*c^3*d^2*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*
d*x^2+d)^(1/2)/f^4/(4+m)/(6+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b*c^5*d^2*(f*x
)^(6+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^6/(6+m)^2/(c*x-1)^(1/2)/(
c*x+1)^(1/2)-30*b^2*c^2*d^2*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2
*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(4+m
)/(6+m)/(-c*x+1)/(c*x+1)-10*b^2*c^2*d^2*(10+3*m)*(f*x)^(3+m)*hypergeom([1/2
, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f
^3/(4+m)^3/(6+m)/(m^2+5*m+6)/(-c*x+1)/(c*x+1)-2*b^2*c^2*d^2*(15*m^2+130*m+2
64)*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1
)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^2/(6+m)^3/(m^2+5*m+6)/(-c*x+1)/(c*x+
1)+15*d^3*Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
/(6+m)/(m^2+6*m+8)
```

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (d^2*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2
)*(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.59, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2c^4d^2x^4 - 2\,a^2c^2d^2x^2 + a^2d^2 + \left(b^2c^4d^2x^4 - 2\,b^2c^2d^2x^2 + b^2d^2\right)\text{arcosh}\,(cx)^2 + 2\left(abc^4d^2x^4 - 2\,abc^2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral((a^2*c^4*d^2*x^4 - 2*a^2*c^2*d^2*x^2 + a^2*d^2 + (b^2*c^4*d^2*x^4
- 2*b^2*c^2*d^2*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*c^4*d^2*x^4 - 2*a*b*
c^2*d^2*x^2 + a*b*d^2)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 2.23, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2}d\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\text{arccosh}\,(cx)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^{2}dx^{2}+d\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)^{2}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\text{acosh}\,(c\,x)\right)^{2}\left(d-c^{2}\,d\,x^{2}\right)^{5/2}\left(f\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(5/2)*(f*x)^m, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**2,x)
[Out] Timed out
```

3.234 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=584

$$\frac{3d^2\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)}{m^2+6m+8}-\frac{2bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}\left(a+b\cosh^{-1}(cx)\right)}{f^2(m+2)(m+4)\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{6bcd\sqrt{d-c^2dx^2}\,(fx)^{m+2}\left(a+b\cosh\right.}{f^2(m+2)^2(m+4)\sqrt{cx-1}\,\sqrt{}}$$

```
[Out] (f*x)^(1+m)*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2/f/(4+m)-2*b^2*c^2*d*(
f*x)^(3+m)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3+3*d*(f*x)^(1+m)*(a+b*arccosh(c*
x))^2*(-c^2*d*x^2+d)^(1/2)/f/(m^2+6*m+8)-6*b*c*d*(f*x)^(2+m)*(a+b*arccosh(c
*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(4+m)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2*b
*c*d*(f*x)^(2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)/(4+m)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*c^3*d*(f*x)^(4+m)*(a+b*arccosh(c*x))*(-c^2*d*
x^2+d)^(1/2)/f^4/(4+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-6*b^2*c^2*d*(f*x)^(3+m
)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*
d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(4+m)/(-c*x+1)/(c*x+1)-2*b^2*c^2*d*(10+3*m
)*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^
(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(4+m)^3/(m^2+5*m+6)/(-c*x+1)/(c*x+1)+3*d^2*U
nintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)/(m^2+6*m+8
)
```

Rubi [A] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2
)*(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.53, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2c^2dx^2-a^2d+\left(b^2c^2dx^2-b^2d\right)\text{arcosh}\left(cx\right)^2+2\left(abc^2dx^2-abd\right)\text{arcosh}\left(cx\right)\right)\sqrt{-c^2dx^2+d}\,\left(fx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(-(a^2*c^2*d*x^2 - a^2*d + (b^2*c^2*d*x^2 - b^2*d)*arccosh(c*x)^2 +
 2*(a*b*c^2*d*x^2 - a*b*d)*arccosh(c*x))*sqrt(-c^2*d*x^2 + d)*(f*x)^m, x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 1.75, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{2}\left(fx\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.235 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$

Optimal. Leaf size=240

$$\frac{d\mathrm{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)}{m+2}-\frac{2bc\sqrt{d-c^2dx^2}\,(fx)^{m+2}\left(a+b\cosh^{-1}(cx)\right)}{f^2(m+2)^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{d-c^2dx^2}\,(fx)^{m+1}\left(a+b\cosh^{-1}(c\right.}{f(m+2)}$$

```
[Out] (f*x)^(1+m)*(a+b*arccosh(c*x))^2*(-c^2*d*x^2+d)^(1/2)/f/(2+m)-2*b*c*(f*x)^(
2+m)*(a+b*arccosh(c*x))*(-c^2*d*x^2+d)^(1/2)/f^2/(2+m)^2/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-2*b^2*c^2*(f*x)^(3+m)*hypergeom([1/2, 3/2+1/2*m],[5/2+1/2*m],c^2*
x^2)*(-c^2*x^2+1)^(1/2)*(-c^2*d*x^2+d)^(1/2)/f^3/(2+m)^2/(3+m)/(-c*x+1)/(c*
x+1)+d*Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)/(2
+m)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx=\frac{\sqrt{d-c^2dx^2}\,\int (fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2 \, dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^2 \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-c^2dx^2+d}\left(b^2\,\mathrm{arcosh}\,(cx)^2+2\,ab\,\mathrm{arcosh}\,(cx)+a^2\right)\left(fx\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="g
iac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.54, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-c^2 d x^2 + d}\, (a + b\,\mathrm{arccosh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^2 d x^2 + d}\,(b\,\mathrm{arcosh}(cx) + a)^2 (fx)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^2,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^2*(f*x)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\,\mathrm{acosh}(c\,x))^2 \sqrt{d - c^2\,d\,x^2}\, (f\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \sqrt{-d\,(cx - 1)\,(cx + 1)}\, (a + b\,\mathrm{acosh}(cx))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2, x)
```

3.236 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.47, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 3.54, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/Sqrt[d - c^2*d*x^2], x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)\left(fx\right)^m}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^2*d*x^2 - d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^2\left(fx\right)^m}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a + b \operatorname{arccosh}\left(cx\right)\right)^{2}}{\sqrt{-c^{2} d\, x^{2} + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b \operatorname{arcosh}\left(cx\right) + a\right)^{2} \left(fx\right)^{m}}{\sqrt{-c^{2} dx^{2} + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/sqrt(-c^2*d*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(f\,x\right)}^m}{\sqrt{d-c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}{\sqrt{- d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.237 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\mathrm{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)
/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 4.59, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)\left(fx\right)^m}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2)*(f*x)^m/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 (fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b\operatorname{arccosh}(cx))^2}{(-c^2d\,x^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)^2 (fx)^m}{(-c^2dx^2 + d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\operatorname{acosh}(c\,x))^2 (f\,x)^m}{(d - c^2\,d\,x^2)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b\operatorname{acosh}(cx))^2}{(-d(cx-1)(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**2/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.238 $\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^2)/(
(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)), x])/(d^2*Sqrt[d - c^2*d*x^2])
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^2}{(-1+cx)^{5/2}(1+cx)^{5/2}}\,dx}{d^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 4.78, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)^2}{\left(d-c^2dx^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^2)/(d - c^2*d*x^2)^(5/2), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)(fx)^m}{c^6d^3x^6-3\,c^4d^3x^4+3\,c^2d^3x^2-d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a
^2)*(f*x)^m/(c^6*d^3*x^6 - 3*c^4*d^3*x^4 + 3*c^2*d^3*x^2 - d^3), x)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^2 \left(fx\right)^m}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="g
iac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

maple [**A**] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m (a + b\,\mathrm{arccosh}\,(cx))^2}{\left(-c^2d\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^2 \left(fx\right)^m}{\left(-c^2dx^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^2/(-c^2*d*x^2+d)^(5/2),x, algorithm="m
axima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2*(f*x)^m/(-c^2*d*x^2 + d)^(5/2), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))^2 \left(f\,x\right)^m}{\left(d - c^2\,d\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^2*(f*x)^m)/(d - c^2*d*x^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**2/(-c**2*d*x**2+d)**(5/2),x)
```

```
[Out] Timed out
```

3.239 $\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}} dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\cosh^{-1}(cx)^2(fx)^m}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*ArcCosh[c*x]^2)/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}} dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \cosh^{-1}(cx)^2}{\sqrt{1-c^2x^2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*ArcCosh[c*x]^2)/Sqrt[1 - c^2*x^2], x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(fx)^m \operatorname{arcosh}(cx)^2}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*arccosh(c*x)^2/(c^2*x^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{arcosh}(cx)^2}{\sqrt{-c^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

maple [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{arccosh}(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
[Out] int((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{arcosh}(cx)^2}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*arccosh(c*x)^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((f*x)^m*arccosh(c*x)^2/sqrt(-c^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\operatorname{acosh}(c\,x)^2\,(f\,x)^m}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
[Out] int((acosh(c*x)^2*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \operatorname{acosh}^2(cx)}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*acosh(c*x)**2/(-c**2*x**2+1)**(1/2),x)
[Out] Integral((f*x)**m*acosh(c*x)**2/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.240 $\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=505

$$-\frac{6}{343}a^6c^3x^7\cosh^{-1}(ax)+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{6c^3\left(1-a^2x^2\right)^4}{2401a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2664c^3}{214375a\sqrt{}}$$

```
[Out] 4322/1225*c^3*x*arccosh(a*x)-1514/3675*a^2*c^3*x^3*arccosh(a*x)+702/6125*a^
4*c^3*x^5*arccosh(a*x)-6/343*a^6*c^3*x^7*arccosh(a*x)+8/35*c^3*(a*x-1)^(3/2
)*(a*x+1)^(3/2)*arccosh(a*x)^2/a-18/175*c^3*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arc
cosh(a*x)^2/a+3/49*c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)*arccosh(a*x)^2/a+16/35*c
^3*x*arccosh(a*x)^3+8/35*c^3*x*(-a^2*x^2+1)*arccosh(a*x)^3+6/35*c^3*x*(-a^2
*x^2+1)^2*arccosh(a*x)^3+1/7*c^3*x*(-a^2*x^2+1)^3*arccosh(a*x)^3+7104/42875
*c^3*(-a^2*x^2+1)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1184/42875*c^3*(-a^2*x^2+1)
^2/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2664/214375*c^3*(-a^2*x^2+1)^3/a/(a*x-1)^(
1/2)/(a*x+1)^(1/2)+6/2401*c^3*(-a^2*x^2+1)^4/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-
976/315*c^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+16/315*a*c^3*x^2*(a*x-1)^(1/2)*(a
*x+1)^(1/2)-48/35*c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 1.41, antiderivative size = 505, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 14, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {5681, 5718, 194, 5680, 12, 1610, 1799, 1850, 520, 1247, 698, 460, 74, 5654}

$$\frac{6c^3\left(1-a^2x^2\right)^4}{2401a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2664c^3\left(1-a^2x^2\right)^3}{214375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1184c^3\left(1-a^2x^2\right)^2}{42875a\sqrt{ax-1}\sqrt{ax+1}}+\frac{7104c^3\left(1-a^2x^2\right)}{42875a\sqrt{ax-1}\sqrt{ax+1}}-\frac{6}{343}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```

```
[Out] (-976*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(315*a) + (16*a*c^3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x])/315 + (7104*c^3*(1 - a^2*x^2))/(42875*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (1184*c^3*(1 - a^2*x^2)^2)/(42875*a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]) + (2664*c^3*(1 - a^2*x^2)^3)/(214375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 + (6*c^3*(1 - a^2*x^2)^4)/(2401*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (4322*c^
3*x*ArcCosh[a*x])/1225 - (1514*a^2*c^3*x^3*ArcCosh[a*x])/3675 + (702*a^4*c^
3*x^5*ArcCosh[a*x])/6125 - (6*a^6*c^3*x^7*ArcCosh[a*x])/343 - (48*c^3*Sqrt[
-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(35*a) + (8*c^3*(-1 + a*x)^(3/2)*(1
 + a*x)^(3/2)*ArcCosh[a*x]^2)/(35*a) - (18*c^3*(-1 + a*x)^(5/2)*(1 + a*x)^(
5/2)*ArcCosh[a*x]^2)/(175*a) + (3*c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2)*ArcC
osh[a*x]^2)/(49*a) + (16*c^3*x*ArcCosh[a*x]^3)/35 + (8*c^3*x*(1 - a^2*x^2)*
ArcCosh[a*x]^3)/35 + (6*c^3*x*(1 - a^2*x^2)^2*ArcCosh[a*x]^3)/35 + (c^3*x*(
1 - a^2*x^2)^3*ArcCosh[a*x]^3)/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
```

```
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^3 \cosh^{-1}(ax)^3\,dx = \frac{1}{7}c^3x\left(1-a^2x^2\right)^3\cosh^{-1}(ax)^3+\frac{1}{7}(6c)\int \left(c-a^2cx^2\right)^2\cosh^{-1}(ax)^3\,dx+\frac{1}{7}\left(3ac^3\right.$$

$$=\frac{3c^3(-1+ax)^{7/2}(1+ax)^{7/2}\cosh^{-1}(ax)^2}{49a}+\frac{6}{35}c^3x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3+\frac{1}{7}c^3x$$

$$=\frac{6}{49}c^3x\cosh^{-1}(ax)-\frac{6}{49}a^2c^3x^3\cosh^{-1}(ax)+\frac{18}{245}a^4c^3x^5\cosh^{-1}(ax)-\frac{6}{343}a^6c^3x$$

$$=\frac{402c^3x\cosh^{-1}(ax)}{1225}-\frac{318a^2c^3x^3\cosh^{-1}(ax)}{1225}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a^6c$$

$$=\frac{962c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a^6$$

$$=\frac{4322c^3x\cosh^{-1}(ax)}{1225}-\frac{1514a^2c^3x^3\cosh^{-1}(ax)}{3675}+\frac{702a^4c^3x^5\cosh^{-1}(ax)}{6125}-\frac{6}{343}a$$

$$=-\frac{96c^3\sqrt{-1+ax}\sqrt{1+ax}}{35a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{4322c^3x\cosh^{-1}(ax)}{1225}$$

$$=-\frac{976c^3\sqrt{-1+ax}\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{96c^3\left(1-a^2x^2\right.}{1715a\sqrt{-1+ax}\sqrt{1}}$$

$$=-\frac{976c^3\sqrt{-1+ax}\sqrt{1+ax}}{315a}+\frac{16}{315}ac^3x^2\sqrt{-1+ax}\sqrt{1+ax}+\frac{7104c^3\left(1-a^2\right.}{42875a\sqrt{-1+ax}}$$

Mathematica [A] time = 0.42, size = 179, normalized size = 0.35

$$\frac{c^3\left(2\sqrt{ax-1}\sqrt{ax+1}\left(16875a^6x^6-134541a^4x^4+747937a^2x^2-22329151\right)-385875ax\left(5a^6x^6-21a^4x^4+35a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3*ArcCosh[a*x]^3,x]
```

```
[Out] (c^3*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-22329151 + 747937*a^2*x^2 - 134541*a
^4*x^4 + 16875*a^6*x^6) - 210*a*x*(-226905 + 26495*a^2*x^2 - 7371*a^4*x^4 +
 1125*a^6*x^6)*ArcCosh[a*x] + 11025*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-2161 + 7
57*a^2*x^2 - 351*a^4*x^4 + 75*a^6*x^6)*ArcCosh[a*x]^2 - 385875*a*x*(-35 + 3
5*a^2*x^2 - 21*a^4*x^4 + 5*a^6*x^6)*ArcCosh[a*x]^3))/(13505625*a)
```

fricas [A] time = 0.47, size = 248, normalized size = 0.49

$$-\frac{385875\left(5\,a^7c^3x^7-21\,a^5c^3x^5+35\,a^3c^3x^3-35\,ac^3x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-11025\left(75\,a^6c^3x^6-351\,a^4c^3x^4+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/13505625*(385875*(5*a^7*c^3*x^7 - 21*a^5*c^3*x^5 + 35*a^3*c^3*x^3 - 35*a
*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 11025*(75*a^6*c^3*x^6 - 351*a^4*c^
3*x^4 + 757*a^2*c^3*x^2 - 2161*c^3)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^
2 - 1))^2 + 210*(1125*a^7*c^3*x^7 - 7371*a^5*c^3*x^5 + 26495*a^3*c^3*x^3 -
226905*a*c^3*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(16875*a^6*c^3*x^6 - 13454
1*a^4*c^3*x^4 + 747937*a^2*c^3*x^2 - 22329151*c^3)*sqrt(a^2*x^2 - 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.13, size = 294, normalized size = 0.58

$$-\frac{c^3\left(1929375\operatorname{arccosh}(ax)^3 a^7x^7 - 826875\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}\,a^6x^6 - 8103375\operatorname{arccosh}(ax)^3 a^5x^5\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x)
```

```
[Out] -1/13505625/a*c^3*(1929375*arccosh(a*x)^3*a^7*x^7-826875*arccosh(a*x)^2*(a*
x-1)^(1/2)*(a*x+1)^(1/2)*a^6*x^6-8103375*arccosh(a*x)^3*a^5*x^5+3869775*arc
cosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^4*x^4+236250*arccosh(a*x)*a^7*x^7
-33750*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^6*x^6+13505625*a^3*x^3*arccosh(a*x)^3-
8345925*arccosh(a*x)^2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1547910*arccosh(
a*x)*a^5*x^5+269082*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4-13505625*a*x*arccos
h(a*x)^3+23825025*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+5563950*a^3*x^
3*arccosh(a*x)-1495874*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-47650050*a*x*arc
cosh(a*x)+44658302*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

maxima [A] time = 0.50, size = 276, normalized size = 0.55

$$\frac{1}{1225}\left(75\sqrt{a^2x^2-1}\,a^4c^3x^6 - 351\sqrt{a^2x^2-1}\,a^2c^3x^4 + 757\sqrt{a^2x^2-1}\,c^3x^2 - \frac{2161\sqrt{a^2x^2-1}\,c^3}{a^2}\right)a\operatorname{arcosh}(ax)^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/1225*(75*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 351*sqrt(a^2*x^2 - 1)*a^2*c^3*x^
4 + 757*sqrt(a^2*x^2 - 1)*c^3*x^2 - 2161*sqrt(a^2*x^2 - 1)*c^3/a^2)*a*arcco
sh(a*x)^2 - 1/35*(5*a^6*c^3*x^7 - 21*a^4*c^3*x^5 + 35*a^2*c^3*x^3 - 35*c^3*
x)*arccosh(a*x)^3 + 2/13505625*(16875*sqrt(a^2*x^2 - 1)*a^4*c^3*x^6 - 13454
1*sqrt(a^2*x^2 - 1)*a^2*c^3*x^4 + 747937*sqrt(a^2*x^2 - 1)*c^3*x^2 - 223291
51*sqrt(a^2*x^2 - 1)*c^3/a^2 - 105*(1125*a^6*c^3*x^7 - 7371*a^4*c^3*x^5 + 2
6495*a^2*c^3*x^3 - 226905*c^3*x)*arccosh(a*x)/a)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^3\left(c-a^2\,c\,x^2\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^3, x)
```

sympy [A] time = 17.36, size = 367, normalized size = 0.73

$$\begin{cases} -\frac{a^6c^3x^7\operatorname{acosh}^3(ax)}{7} - \frac{6a^6c^3x^7\operatorname{acosh}(ax)}{343} + \frac{3a^5c^3x^6\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{49} + \frac{6a^5c^3x^6\sqrt{a^2x^2-1}}{2401} + \frac{3a^4c^3x^5\operatorname{acosh}^3(ax)}{5} + \frac{702a^4c^3x^5\operatorname{acosh}}{6125} \\ -\frac{i\pi^3c^3x}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**6*c**3*x**7*acosh(a*x)**3/7 - 6*a**6*c**3*x**7*acosh(a*x)/34
3 + 3*a**5*c**3*x**6*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/49 + 6*a**5*c**3*x**
6*sqrt(a**2*x**2 - 1)/2401 + 3*a**4*c**3*x**5*acosh(a*x)**3/5 + 702*a**4*c*
*3*x**5*acosh(a*x)/6125 - 351*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)
**2/1225 - 29898*a**3*c**3*x**4*sqrt(a**2*x**2 - 1)/1500625 - a**2*c**3*x**
3*acosh(a*x)**3 - 1514*a**2*c**3*x**3*acosh(a*x)/3675 + 757*a*c**3*x**2*sqr
t(a**2*x**2 - 1)*acosh(a*x)**2/1225 + 1495874*a*c**3*x**2*sqrt(a**2*x**2 -
1)/13505625 + c**3*x*acosh(a*x)**3 + 4322*c**3*x*acosh(a*x)/1225 - 2161*c**
3*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(1225*a) - 44658302*c**3*sqrt(a**2*x**2
 - 1)/(13505625*a), Ne(a, 0)), (-I*pi**3*c**3*x/8, True))
```

3.241 $\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=388

$$\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)+\frac{6c^2\left(1-a^2x^2\right)^3}{625a\sqrt{ax-1}\sqrt{ax+1}}+\frac{8c^2\left(1-a^2x^2\right)^2}{375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{16c^2\left(1-a^2x^2\right)}{125a\sqrt{ax-1}}$$

```
[Out] 298/75*c^2*x*arccosh(a*x)-76/225*a^2*c^2*x^3*arccosh(a*x)+6/125*a^4*c^2*x^5
*arccosh(a*x)+4/15*c^2*(a*x-1)^(3/2)*(a*x+1)^(3/2)*arccosh(a*x)^2/a-3/25*c^
2*(a*x-1)^(5/2)*(a*x+1)^(5/2)*arccosh(a*x)^2/a+8/15*c^2*x*arccosh(a*x)^3+4/
15*c^2*x*(-a^2*x^2+1)*arccosh(a*x)^3+1/5*c^2*x*(-a^2*x^2+1)^2*arccosh(a*x)^
3+16/125*c^2*(-a^2*x^2+1)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+8/375*c^2*(-a^2*x^2
+1)^2/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+6/625*c^2*(-a^2*x^2+1)^3/a/(a*x-1)^(1/2
)/(a*x+1)^(1/2)-488/135*c^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+8/135*a*c^2*x^2*(
a*x-1)^(1/2)*(a*x+1)^(1/2)-8/5*c^2*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a
```

Rubi [A] time = 0.84, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5681, 5718, 194, 5680, 12, 520, 1247, 698, 460, 74, 5654}

$$\frac{6c^2\left(1-a^2x^2\right)^3}{625a\sqrt{ax-1}\sqrt{ax+1}}+\frac{8c^2\left(1-a^2x^2\right)^2}{375a\sqrt{ax-1}\sqrt{ax+1}}+\frac{16c^2\left(1-a^2x^2\right)}{125a\sqrt{ax-1}\sqrt{ax+1}}+\frac{6}{125}a^4c^2x^5\cosh^{-1}(ax)-\frac{76}{225}a^2c^2x^3\cosh^{-1}(ax)$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```

```
[Out] (-488*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(135*a) + (8*a*c^2*x^2*Sqrt[-1 + a*
x]*Sqrt[1 + a*x])/135 + (16*c^2*(1 - a^2*x^2))/(125*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]) + (8*c^2*(1 - a^2*x^2)^2)/(375*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (
6*c^2*(1 - a^2*x^2)^3)/(625*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (298*c^2*x*Ar
cCosh[a*x])/75 - (76*a^2*c^2*x^3*ArcCosh[a*x])/225 + (6*a^4*c^2*x^5*ArcCosh
[a*x])/125 - (8*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a) + (4
*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(15*a) - (3*c^2*(-1 +
 a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]^2)/(25*a) + (8*c^2*x*ArcCosh[a*x]^
3)/15 + (4*c^2*x*(1 - a^2*x^2)*ArcCosh[a*x]^3)/15 + (c^2*x*(1 - a^2*x^2)^2*
ArcCosh[a*x]^3)/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
```

```
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^3\,dx = \frac{1}{5}c^2x\left(1-a^2x^2\right)^2\cosh^{-1}(ax)^3 + \frac{1}{5}(4c)\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx - \frac{1}{5}\left(3a\right.$$

$$= -\frac{3c^2(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^2}{25a} + \frac{4}{15}c^2x\left(1-a^2x^2\right)\cosh^{-1}(ax)^3 + \frac{1}{5}$$

$$= \frac{6}{25}c^2x\cosh^{-1}(ax) - \frac{4}{25}a^2c^2x^3\cosh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\cosh^{-1}(ax) + \frac{4c^2(-1}{}$$

$$= \frac{58}{75}c^2x\cosh^{-1}(ax) - \frac{76}{225}a^2c^2x^3\cosh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\cosh^{-1}(ax) - \frac{8c^2\sqrt{}}{}$$

$$= \frac{298}{75}c^2x\cosh^{-1}(ax) - \frac{76}{225}a^2c^2x^3\cosh^{-1}(ax) + \frac{6}{125}a^4c^2x^5\cosh^{-1}(ax) - \frac{8c^2\sqrt{}}{}$$

$$= -\frac{16c^2\sqrt{-1+ax}\sqrt{1+ax}}{5a} + \frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax} + \frac{298}{75}c^2x\cosh^{-1}(a$$

$$= -\frac{488c^2\sqrt{-1+ax}\sqrt{1+ax}}{135a} + \frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax} + \frac{298}{75}c^2x\cosh^{-1}($$

$$= -\frac{488c^2\sqrt{-1+ax}\sqrt{1+ax}}{135a} + \frac{8}{135}ac^2x^2\sqrt{-1+ax}\sqrt{1+ax} + \frac{16c^2\left(1-a^2\right.}{125a\sqrt{-1+ax}}$$

Mathematica [A] time = 0.22, size = 147, normalized size = 0.38

$$\frac{c^2\left(-2\sqrt{ax-1}\sqrt{ax+1}\left(81a^4x^4-842a^2x^2+31841\right)+1125ax\left(3a^4x^4-10a^2x^2+15\right)\cosh^{-1}(ax)^3-225\sqrt{ax-}\right.}{16875a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*(-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(31841 - 842*a^2*x^2 + 81*a^4*x^4) +
30*a*x*(2235 - 190*a^2*x^2 + 27*a^4*x^4)*ArcCosh[a*x] - 225*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*(149 - 38*a^2*x^2 + 9*a^4*x^4)*ArcCosh[a*x]^2 + 1125*a*x*(15
- 10*a^2*x^2 + 3*a^4*x^4)*ArcCosh[a*x]^3))/(16875*a)
```

fricas [A] time = 0.62, size = 204, normalized size = 0.53

$$\frac{1125\left(3\,a^5c^2x^5-10\,a^3c^2x^3+15\,ac^2x\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-225\left(9\,a^4c^2x^4-38\,a^2c^2x^2+149\,c^2\right)\sqrt{a^2x^2-1}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(1125*(3*a^5*c^2*x^5 - 10*a^3*c^2*x^3 + 15*a*c^2*x)*log(a*x + sqrt(
a^2*x^2 - 1))^3 - 225*(9*a^4*c^2*x^4 - 38*a^2*c^2*x^2 + 149*c^2)*sqrt(a^2*x
^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 30*(27*a^5*c^2*x^5 - 190*a^3*c^2*x
```

```
^3 + 2235*a*c^2*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(81*a^4*c^2*x^4 - 842*a
^2*c^2*x^2 + 31841*c^2)*sqrt(a^2*x^2 - 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 218, normalized size = 0.56

$$\frac{c^2\left(3375\mathrm{arccosh}\left(ax\right)^3a^5x^5-2025\mathrm{arccosh}\left(ax\right)^2\sqrt{ax-1}\,\sqrt{ax+1}\,a^4x^4-11250a^3x^3\mathrm{arccosh}\left(ax\right)^3+8550\mathrm{arccos}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x)
```

```
[Out] 1/16875/a*c^2*(3375*arccosh(a*x)^3*a^5*x^5-2025*arccosh(a*x)^2*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*a^4*x^4-11250*a^3*x^3*arccosh(a*x)^3+8550*arccosh(a*x)^2*a^
2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+810*arccosh(a*x)*a^5*x^5-162*(a*x+1)^(1/2
)*(a*x-1)^(1/2)*x^4*a^4+16875*a*x*arccosh(a*x)^3-33525*arccosh(a*x)^2*(a*x-
1)^(1/2)*(a*x+1)^(1/2)-5700*a^3*x^3*arccosh(a*x)+1684*a^2*x^2*(a*x-1)^(1/2)
*(a*x+1)^(1/2)+67050*a*x*arccosh(a*x)-63682*(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

maxima [A] time = 0.42, size = 210, normalized size = 0.54

$$-\frac{1}{75}\left(9\sqrt{a^2x^2-1}\,a^2c^2x^4-38\sqrt{a^2x^2-1}\,c^2x^2+\frac{149\sqrt{a^2x^2-1}\,c^2}{a^2}\right)a\,\mathrm{arcosh}\left(ax\right)^2+\frac{1}{15}\left(3\,a^4c^2x^5-10\,a^2c^2x^3+15\,c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] -1/75*(9*sqrt(a^2*x^2 - 1)*a^2*c^2*x^4 - 38*sqrt(a^2*x^2 - 1)*c^2*x^2 + 149
*sqrt(a^2*x^2 - 1)*c^2/a^2)*a*arccosh(a*x)^2 + 1/15*(3*a^4*c^2*x^5 - 10*a^2
*c^2*x^3 + 15*c^2*x)*arccosh(a*x)^3 - 2/16875*(81*sqrt(a^2*x^2 - 1)*a^2*c^2
*x^4 - 842*sqrt(a^2*x^2 - 1)*c^2*x^2 - 15*(27*a^4*c^2*x^5 - 190*a^2*c^2*x^3
 + 2235*c^2*x)*arccosh(a*x)/a + 31841*sqrt(a^2*x^2 - 1)*c^2/a^2)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)^3\,{\left(c-a^2\,c\,x^2\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^2, x)
```

sympy [A] time = 6.34, size = 274, normalized size = 0.71

$$\begin{cases} \frac{a^4c^2x^5\,\mathrm{acosh}^3\left(ax\right)}{5}+\frac{6a^4c^2x^5\,\mathrm{acosh}\left(ax\right)}{125}-\frac{3a^3c^2x^4\sqrt{a^2x^2-1}\,\mathrm{acosh}^2\left(ax\right)}{25}-\frac{6a^3c^2x^4\sqrt{a^2x^2-1}}{625}-\frac{2a^2c^2x^3\,\mathrm{acosh}^3\left(ax\right)}{3}-\frac{76a^2c^2x^3\,\mathrm{acosh}\left(ax\right)}{225}+ \\ -\frac{i\pi^3c^2x}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2*acosh(a*x)**3,x)
```

```
[Out] Piecewise((a**4*c**2*x**5*acosh(a*x)**3/5 + 6*a**4*c**2*x**5*acosh(a*x)/125
 - 3*a**3*c**2*x**4*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/25 - 6*a**3*c**2*x**4
*sqrt(a**2*x**2 - 1)/625 - 2*a**2*c**2*x**3*acosh(a*x)**3/3 - 76*a**2*c**2*
x**3*acosh(a*x)/225 + 38*a*c**2*x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/75 +
 1684*a*c**2*x**2*sqrt(a**2*x**2 - 1)/16875 + c**2*x*acosh(a*x)**3 + 298*c*
*2*x*acosh(a*x)/75 - 149*c**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/(75*a) - 63
682*c**2*sqrt(a**2*x**2 - 1)/(16875*a), Ne(a, 0)), (-I*pi**3*c**2*x/8, True
))
```

3.242 $\int \left(c-a^2 c x^2\right) \cosh^{-1}(ax)^3 \, dx$

Optimal. Leaf size=175

$$-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{2}{27}acx^2\sqrt{ax-1}\,\sqrt{ax+1}-\frac{122c\sqrt{ax-1}\,\sqrt{ax+1}}{27a}+\frac{2}{3}cx\cosh^{-1}($$

```
[Out] 14/3*c*x*arccosh(a*x)-2/9*a^2*c*x^3*arccosh(a*x)+1/3*c*(a*x-1)^(3/2)*(a*x+1
)^(3/2)*arccosh(a*x)^2/a+2/3*c*x*arccosh(a*x)^3+1/3*c*x*(-a^2*x^2+1)*arccos
h(a*x)^3-122/27*c*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a+2/27*a*c*x^2*(a*x-1)^(1/2)*
(a*x+1)^(1/2)-2*c*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a
```

Rubi [A] time = 0.48, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5681, 5718, 5680, 12, 460, 74, 5654}

$$-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)+\frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{2}{27}acx^2\sqrt{ax-1}\,\sqrt{ax+1}-\frac{122c\sqrt{ax-1}\,\sqrt{ax+1}}{27a}+\frac{2}{3}cx\cosh^{-1}($$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-122*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(27*a) + (2*a*c*x^2*Sqrt[-1 + a*x]*Sq
rt[1 + a*x])/27 + (14*c*x*ArcCosh[a*x])/3 - (2*a^2*c*x^3*ArcCosh[a*x])/9 -
(2*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/a + (c*(-1 + a*x)^(3/2)*(
1 + a*x)^(3/2)*ArcCosh[a*x]^2)/(3*a) + (2*c*x*ArcCosh[a*x]^3)/3 + (c*x*(1 -
 a^2*x^2)*ArcCosh[a*x]^3)/3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5680

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
```

```
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0]
```

Rule 5681

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Simp[(x*(d + e*x^2)^p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] +
(-Dist[(b*c*n*(-d)^p)/(2*p + 1), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p -
1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*d*p)/(2*p + 1), Int[(d
+ e*x^2)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ[{a, b, c, d, e},
x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)\cosh^{-1}(ax)^3\,dx = \frac{1}{3}cx\left(1-a^2x^2\right)\cosh^{-1}(ax)^3+\frac{1}{3}(2c)\int \cosh^{-1}(ax)^3\,dx+(ac)\int x\sqrt{-1+ax}\,\sqrt{\cdots}$$
$$=\frac{c(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^2}{3a}+\frac{2}{3}cx\cosh^{-1}(ax)^3+\frac{1}{3}cx\left(1-a^2x^2\right)\cos\cdots$$
$$=\frac{2}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a}+\cdots$$
$$=\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a^2cx^3\cosh^{-1}(ax)-\frac{2c\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a}+\cdots$$
$$=-\frac{4c\sqrt{-1+ax}\,\sqrt{1+ax}}{a}+\frac{2}{27}acx^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(ax)-\frac{2}{9}a\cdots$$
$$=-\frac{122c\sqrt{-1+ax}\,\sqrt{1+ax}}{27a}+\frac{2}{27}acx^2\sqrt{-1+ax}\,\sqrt{1+ax}+\frac{14}{3}cx\cosh^{-1}(ax)-\cdots$$

Mathematica [A] time = 0.12, size = 109, normalized size = 0.62

$$\frac{c\left(2\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2x^2-61\right)-9ax\left(a^2x^2-3\right)\cosh^{-1}(ax)^3+9\sqrt{ax-1}\,\sqrt{ax+1}\,\left(a^2x^2-7\right)\cosh^{-1}(ax)^2-\cdots\right.}{27a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c*(2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-61 + a^2*x^2) - 6*a*x*(-21 + a^2*x^2)*
ArcCosh[a*x] + 9*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(-7 + a^2*x^2)*ArcCosh[a*x]^2
 - 9*a*x*(-3 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a)
```

fricas [A] time = 0.63, size = 140, normalized size = 0.80

$$-\frac{9\left(a^3cx^3-3\,acx\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^3-9\left(a^2cx^2-7\,c\right)\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^2+6\left(a^3cx^3-21\,a\cdots\right.}{27\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^3*c*x^3 - 3*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1))^3 - 9*(a^2*c*x^
2 - 7*c)*sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 - 1))^2 + 6*(a^3*c*x^3 -
21*a*c*x)*log(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^2*c*x^2 - 61*c)*sqrt(a^2*x^2
- 1))/a
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.09, size = 140, normalized size = 0.80

$$-\frac{c\left(-9\operatorname{arccosh}(ax)^2 a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}+9a^3x^3\operatorname{arccosh}(ax)^3+63\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}-2a^2x^2\sqrt{}\right.}{27a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)*arccosh(a*x)^3,x)
```

```
[Out] -1/27/a*c*(-9*arccosh(a*x)^2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+9*a^3*x^3*
arccosh(a*x)^3+63*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-2*a^2*x^2*(a*x
-1)^(1/2)*(a*x+1)^(1/2)-27*a*x*arccosh(a*x)^3+6*a^3*x^3*arccosh(a*x)+122*(a
*x-1)^(1/2)*(a*x+1)^(1/2)-126*a*x*arccosh(a*x))
```

maxima [A] time = 0.46, size = 124, normalized size = 0.71

$$\frac{1}{3}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{7\sqrt{a^2x^2-1}\,c}{a^2}\right)a\operatorname{arcosh}(ax)^2-\frac{1}{3}\left(a^2cx^3-3\,cx\right)\operatorname{arcosh}(ax)^3+\frac{2}{27}\left(\sqrt{a^2x^2-1}\,cx^2-\frac{3\left(a^2cx^3-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(sqrt(a^2*x^2 - 1)*c*x^2 - 7*sqrt(a^2*x^2 - 1)*c/a^2)*a*arccosh(a*x)^2
- 1/3*(a^2*c*x^3 - 3*c*x)*arccosh(a*x)^3 + 2/27*(sqrt(a^2*x^2 - 1)*c*x^2 -
3*(a^2*c*x^3 - 21*c*x)*arccosh(a*x)/a - 61*sqrt(a^2*x^2 - 1)*c/a^2)*a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \operatorname{acosh}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2), x)
```

sympy [A] time = 2.00, size = 160, normalized size = 0.91

$$\begin{cases} -\frac{a^2cx^3\operatorname{acosh}^3(ax)}{3}-\frac{2a^2cx^3\operatorname{acosh}(ax)}{9}+\frac{acx^2\sqrt{a^2x^2-1}\operatorname{acosh}^2(ax)}{3}+\frac{2acx^2\sqrt{a^2x^2-1}}{27}+cx\operatorname{acosh}^3(ax)+\frac{14cx\operatorname{acosh}(ax)}{3}-\frac{7c\sqrt{a^2x^2-}}{} \\ -\frac{i\pi^3cx}{8} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)*acosh(a*x)**3,x)
```

```
[Out] Piecewise((-a**2*c*x**3*acosh(a*x)**3/3 - 2*a**2*c*x**3*acosh(a*x)/9 + a*c*
x**2*sqrt(a**2*x**2 - 1)*acosh(a*x)**2/3 + 2*a*c*x**2*sqrt(a**2*x**2 - 1)/2
7 + c*x*acosh(a*x)**3 + 14*c*x*acosh(a*x)/3 - 7*c*sqrt(a**2*x**2 - 1)*acosh
(a*x)**2/(3*a) - 122*c*sqrt(a**2*x**2 - 1)/(27*a), Ne(a, 0)), (-I*pi**3*c*x
/8, True))
```

3.243 $\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx$

Optimal. Leaf size=144

$$\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\cosh^{-1}(ax)\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{6\cosh^{-1}(ax)\text{Li}_3\left(\right.}{ac}$$

```
[Out] 2*arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+3*arccosh(a*x
)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-3*arccosh(a*x)^2*polylo
g(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*arccosh(a*x)*polylog(3,-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*
x+1)^(1/2))/a/c+6*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*polylog
(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c
```

Rubi [A] time = 0.13, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5694, 4182, 2531, 6609, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{6\cosh^{-1}(ax)\text{PolyLog}\left(3,-e^{\cosh^{-1}}\right.}{ac}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```

```
[Out] (2*ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c) + (3*ArcCosh[a*x]^2*PolyLo
g[2, -E^ArcCosh[a*x]])/(a*c) - (3*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]]
)/(a*c) - (6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(a*c) + (6*ArcCosh[a
*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c) + (6*PolyLog[4, -E^ArcCosh[a*x]])/(a*
c) - (6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
```

```
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx &= -\frac{\text{Subst}\left(\int x^3\text{csch}(x)\,dx,x,\cosh^{-1}(ax)\right)}{ac}\\
&= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\,\text{Subst}\left(\int x^2\log\left(1-e^x\right)\,dx,x,\cosh^{-1}(ax)\right)}{ac}-\frac{3\,\text{Su}}{}\\
&= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2}{ac}\\
&= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2}{ac}\\
&= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2}{ac}\\
&= \frac{2\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac}+\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{ac}-\frac{3\cosh^{-1}(ax)^2\text{Li}_2}{ac}\end{aligned}$$

Mathematica [A] time = 0.11, size = 129, normalized size = 0.90

$$\frac{3\cosh^{-1}(ax)^2\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)-3\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)-6\cosh^{-1}(ax)\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)+6\cosh^{-1}(ax}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2),x]
```

```
[Out] (-(ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]]) + ArcCosh[a*x]^3*Log[1 + E^ArcCo
sh[a*x]] + 3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*
PolyLog[2, E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]] + 6
*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 6*PolyLog[4, -E^ArcCosh[a*x]] -
6*PolyLog[4, E^ArcCosh[a*x]])/(a*c)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\text{arcosh}\,(ax)^3}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arcosh}(ax)^3}{a^2cx^2 - c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

maple [A] time = 0.09, size = 273, normalized size = 1.90

$$-\frac{\operatorname{arccosh}(ax)^3 \ln\left(1 - ax - \sqrt{ax-1}\,\sqrt{ax+1}\right)}{ac} - \frac{3\operatorname{arccosh}(ax)^2 \operatorname{polylog}\left(2, ax + \sqrt{ax-1}\,\sqrt{ax+1}\right)}{ac} + \frac{6\operatorname{arccosh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c),x)
```

```
[Out] -1/a/c*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-3*arccosh(a*x)^
2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*arccosh(a*x)*polylog(3,a
*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))/a/c+1/a/c*arccosh(a*x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3*arcc
osh(a*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c-6*arccosh(a*x)*p
olylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c+6*polylog(4,-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))/a/c
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\log(ax+1) - \log(ax-1)\right)\log\left(ax + \sqrt{ax+1}\,\sqrt{ax-1}\right)^3}{2\,ac} - \int \frac{3\left(\left(ax\log(ax+1) - ax\log(ax-1)\right)\sqrt{ax+1}\,\sqrt{ax}\right.}{2\left(a^3c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c),x, algorithm="maxima")
```

```
[Out] 1/2*(log(a*x + 1) - log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/
(a*c) - integrate(3/2*((a*x*log(a*x + 1) - a*x*log(a*x - 1))*sqrt(a*x + 1)*
sqrt(a*x - 1) + (a^2*x^2 - 1)*log(a*x + 1) - (a^2*x^2 - 1)*log(a*x - 1))*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/(a^3*c*x^3 - a*c*x + (a^2*c*x^2 - c)
*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^3}{c - a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^3(ax)}{a^2x^2-1}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c),x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**2*x**2 - 1), x)/c
```

3.244 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^2}\,dx$

Optimal. Leaf size=260

$$\frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)}+\frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)\mathrm{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)}{ac^2}+\cdots$$

```
[Out] 1/2*x*arccosh(a*x)^3/c^2/(-a^2*x^2+1)-6*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2))/a/c^2+arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2))/a/c^2-3*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3/2*arccosh
(a*x)^2*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3*polylog(2,a*x+(
a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/2*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^
(1/2)*(a*x+1)^(1/2))/a/c^2-3*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x
+1)^(1/2))/a/c^2+3*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/
a/c^2+3*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*polylog(4,a*x+(
a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/2*arccosh(a*x)^2/a/c^2/(a*x-1)^(1/2)/(a
*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5689, 5718, 5694, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{3\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{2ac^2}-\frac{3\cosh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{ac^2}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-3*ArcCosh[a*x]^2)/(2*a*c^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x
]^3)/(2*c^2*(1 - a^2*x^2)) - (6*ArcCosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^
2) + (ArcCosh[a*x]^3*ArcTanh[E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[2, -E^Ar
cCosh[a*x]])/(a*c^2) + (3*ArcCosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*
c^2) + (3*PolyLog[2, E^ArcCosh[a*x]])/(a*c^2) - (3*ArcCosh[a*x]^2*PolyLog[2
, E^ArcCosh[a*x]])/(2*a*c^2) - (3*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])
/(a*c^2) + (3*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]])/(a*c^2) + (3*PolyLog
[4, -E^ArcCosh[a*x]])/(a*c^2) - (3*PolyLog[4, E^ArcCosh[a*x]])/(a*c^2)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^2}\,dx = \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{2c^2} + \frac{\int \frac{\cosh^{-1}(ax)^3}{c-a^2cx^2}\,dx}{2c}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{3\int \frac{\cosh^{-1}(ax)}{-1+a^2x^2}\,dx}{c^2} - \frac{\text{Subst}\left(\int x^3\text{csch}(x)\,dx, x, c\right.}{2ac^2}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^3\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{3\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax}{}$$

$$= -\frac{3\cosh^{-1}(ax)^2}{2ac^2\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax)^3}{2c^2\left(1-a^2x^2\right)} - \frac{6\cosh^{-1}(ax)\tanh^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{ac^2} + \frac{\cosh^{-1}(ax}{}$$

Mathematica [A] time = 2.16, size = 276, normalized size = 1.06

$$\frac{-24\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right) - 48\cosh^{-1}(ax)\text{Li}_3\left(-e^{-\cosh^{-1}(ax)}\right) + 48\cosh^{-1}(ax)\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right) - 24\left(\cosh^{-1}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^2,x]
```

```
[Out] (-Pi^4 + 2*ArcCosh[a*x]^4 - 12*ArcCosh[a*x]^2*Coth[ArcCosh[a*x]/2] - 2*ArcC
osh[a*x]^3*Csch[ArcCosh[a*x]/2]^2 + 48*ArcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x
])] - 48*ArcCosh[a*x]*Log[1 + E^(-ArcCosh[a*x])] + 8*ArcCosh[a*x]^3*Log[1 +
 E^(-ArcCosh[a*x])] - 8*ArcCosh[a*x]^3*Log[1 - E^ArcCosh[a*x]] - 24*(-2 + A
rcCosh[a*x]^2)*PolyLog[2, -E^(-ArcCosh[a*x])] - 48*PolyLog[2, E^(-ArcCosh[a
*x])] - 24*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]] - 48*ArcCosh[a*x]*Poly
Log[3, -E^(-ArcCosh[a*x])] + 48*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] - 4
8*PolyLog[4, -E^(-ArcCosh[a*x])] - 48*PolyLog[4, E^ArcCosh[a*x]] - 2*ArcCos
h[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + 12*ArcCosh[a*x]^2*Tanh[ArcCosh[a*x]/2])/(
16*a*c^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\text{arcosh}\,(ax)^3}{a^4c^2x^4 - 2\,a^2c^2x^2 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="fricas")
```

```
[Out] integral(arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{(a^2cx^2-c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(a^2*c*x^2 - c)^2, x)
```

maple [A] time = 0.27, size = 464, normalized size = 1.78

$$-\frac{\operatorname{arccosh}(ax)^3 x}{2(a^2x^2-1)c^2}-\frac{3\operatorname{arccosh}(ax)^2\sqrt{ax-1}\,\sqrt{ax+1}}{2a(a^2x^2-1)c^2}-\frac{\operatorname{arccosh}(ax)^3\ln\left(1-ax-\sqrt{ax-1}\,\sqrt{ax+1}\right)}{2ac^2}-\frac{3\operatorname{arccos}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x)
```

```
[Out] -1/2/(a^2*x^2-1)*arccosh(a*x)^3/c^2*x-3/2/a/(a^2*x^2-1)*arccosh(a*x)^2/c^2*
(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/2/a/c^2*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/2)
*(a*x+1)^(1/2))-3/2*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2
))/a/c^2+3*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*
polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+1/2/a/c^2*arccosh(a*x)^3*l
n(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+3/2*arccosh(a*x)^2*polylog(2,-a*x-(a*x
-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*
(a*x+1)^(1/2))/a/c^2+3*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2+3/
a/c^2*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+3*polylog(2,a*x+(a
*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2-3/a/c^2*arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))-3*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(2\,ax-\left(a^2x^2-1\right)\log\left(ax+1\right)+\left(a^2x^2-1\right)\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^3}{4\left(a^3c^2x^2-ac^2\right)}-\int -\frac{3\left(2\,a^3x^3+\left(2\,a^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a*x - (a^2*x^2 - 1)*log(a*x + 1) + (a^2*x^2 - 1)*log(a*x - 1))*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(a^3*c^2*x^2 - a*c^2) - integrate(-3/4
*(2*a^3*x^3 + (2*a^2*x^2 - (a^3*x^3 - a*x)*log(a*x + 1) + (a^3*x^3 - a*x)*l
og(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x - 1) - 2*a*x - (a^4*x^4 - 2*a^2*x^2 + 1
)*log(a*x + 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))^2/(a^5*c^2*x^5 - 2*a^3*c^2*x^3 + a*c^2*x + (a^4*c^2*x^
4 - 2*a^2*c^2*x^2 + c^2)*sqrt(a*x + 1)*sqrt(a*x - 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^2,x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\operatorname{acosh}^{3}(ax)}{a^{4}x^{4}-2a^{2}x^{2}+1}\,dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**2,x)
```

```
[Out] Integral(acosh(a*x)**3/(a**4*x**4 - 2*a**2*x**2 + 1), x)/c**2
```

3.245 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^3}\,dx$

Optimal. Leaf size=387

$$\frac{3x\cosh^{-1}(ax)^3}{8c^3\left(1-a^2x^2\right)}+\frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2}-\frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)}+\frac{9\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)^2\mathrm{Li}_2\left(e^{\cosh^{-1}(ax)}\right)}{8ac^3}$$

```
[Out] -1/4*x*arccosh(a*x)/c^3/(-a^2*x^2+1)+1/4*arccosh(a*x)^2/a/c^3/(a*x-1)^(3/2)
/(a*x+1)^(3/2)+1/4*x*arccosh(a*x)^3/c^3/(-a^2*x^2+1)^2+3/8*x*arccosh(a*x)^3
/c^3/(-a^2*x^2+1)-5*arccosh(a*x)*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a
/c^3+3/4*arccosh(a*x)^3*arctanh(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/2*
polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/8*arccosh(a*x)^2*polylo
g(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+5/2*polylog(2,a*x+(a*x-1)^(1/2)
*(a*x+1)^(1/2))/a/c^3-9/8*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1
)^(1/2))/a/c^3-9/4*arccosh(a*x)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
/a/c^3+9/4*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/
4*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-9/4*polylog(4,a*x+(a*x-
1)^(1/2)*(a*x+1)^(1/2))/a/c^3+1/4/a/c^3/(a*x-1)^(1/2)/(a*x+1)^(1/2)-9/8*arc
cosh(a*x)^2/a/c^3/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {5689, 5718, 74, 5694, 4182, 2279, 2391, 2531, 6609, 2282, 6589}

$$\frac{9\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,e^{\cosh^{-1}(ax)}\right)}{8ac^3}-\frac{9\cosh^{-1}(ax)\mathrm{PolyLog}\left(3,-e^{\cosh^{-1}(ax)}\right)}{4ac^3}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```

```
[Out] 1/(4*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (x*ArcCosh[a*x])/(4*c^3*(1 - a^2
*x^2)) + ArcCosh[a*x]^2/(4*a*c^3*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)) - (9*Arc
Cosh[a*x]^2)/(8*a*c^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*ArcCosh[a*x]^3)/(4
*c^3*(1 - a^2*x^2)^2) + (3*x*ArcCosh[a*x]^3)/(8*c^3*(1 - a^2*x^2)) - (5*Arc
Cosh[a*x]*ArcTanh[E^ArcCosh[a*x]])/(a*c^3) + (3*ArcCosh[a*x]^3*ArcTanh[E^Ar
cCosh[a*x]])/(4*a*c^3) - (5*PolyLog[2, -E^ArcCosh[a*x]])/(2*a*c^3) + (9*Arc
Cosh[a*x]^2*PolyLog[2, -E^ArcCosh[a*x]])/(8*a*c^3) + (5*PolyLog[2, E^ArcCos
h[a*x]])/(2*a*c^3) - (9*ArcCosh[a*x]^2*PolyLog[2, E^ArcCosh[a*x]])/(8*a*c^3
) - (9*ArcCosh[a*x]*PolyLog[3, -E^ArcCosh[a*x]])/(4*a*c^3) + (9*ArcCosh[a*x
]*PolyLog[3, E^ArcCosh[a*x]])/(4*a*c^3) + (9*PolyLog[4, -E^ArcCosh[a*x]])/(
4*a*c^3) - (9*PolyLog[4, E^ArcCosh[a*x]])/(4*a*c^3)
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4182

```
Int[csc[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)])/(f*fz*I), x]
 + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 - E^(-(I*e) + f*fz*x)
], x], x] + Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1 + E^(-(I*e) +
f*fz*x)], x], x]) /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5689

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> -Simp[(x*(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d*(p +
1)), x] + (-Dist[(b*c*n*(-d)^p)/(2*(p + 1)), Int[x*(1 + c*x)^(p + 1/2)*(-1
+ c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] + Dist[(2*p + 3)/(2*d
*(p + 1)), Int[(d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n, x], x]) /; FreeQ
[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && LtQ[p, -1] && Int
egerQ[p]
```

Rule 5694

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(c*d)^(-1), Subst[Int[(a + b*x)^n*Csch[x], x], x, ArcCosh[c*x]
], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^3}\,dx = \frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2} - \frac{(3a)\int \frac{x\cosh^{-1}(ax)^2}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{4c^3} + \frac{3\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^2}\,dx}{4c}$$

$$= \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} + \frac{x\cosh^{-1}(ax)^3}{4c^3\left(1-a^2x^2\right)^2} + \frac{3x\cosh^{-1}(ax)^3}{8c^3\left(1-a^2x^2\right)} - \frac{\int \frac{\cosh^{-1}(ax)}{\left(-1+a^2x^2\right)^2}\,dx}{2c^3} + \frac{(9a)}{}$$

$$= -\frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(ax)^2}{8ac^3\sqrt{-1+ax}\sqrt{1+ax}} + \frac{x\cosh^{-1}(ax}{4c^3\left(1-a^2x^2\right.}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(}{8ac^3\sqrt{-1+ax}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(}{8ac^3\sqrt{-1+ax}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(}{8ac^3\sqrt{-1+ax}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(}{8ac^3\sqrt{-1+ax}}$$

$$= \frac{1}{4ac^3\sqrt{-1+ax}\sqrt{1+ax}} - \frac{x\cosh^{-1}(ax)}{4c^3\left(1-a^2x^2\right)} + \frac{\cosh^{-1}(ax)^2}{4ac^3(-1+ax)^{3/2}(1+ax)^{3/2}} - \frac{9\cosh^{-1}(}{8ac^3\sqrt{-1+ax}}$$

Mathematica [A] time = 8.19, size = 455, normalized size = 1.18

$$-\frac{72\cosh^{-1}(ax)^2\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right) + 144\cosh^{-1}(ax)\text{Li}_3\left(-e^{-\cosh^{-1}(ax)}\right) - 144\cosh^{-1}(ax)\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right) + 8\left(9\,c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^3,x]
```

```
[Out] -1/64*(3*Pi^4 - 6*ArcCosh[a*x]^4 - 8*Coth[ArcCosh[a*x]/2] + 40*ArcCosh[a*x]
^2*Coth[ArcCosh[a*x]/2] - 4*ArcCosh[a*x]*Csch[ArcCosh[a*x]/2]^2 + 6*ArcCosh
[a*x]^3*Csch[ArcCosh[a*x]/2]^2 - Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCo
```

```
sh[a*x]^2*Csch[ArcCosh[a*x]/2]^4 - ArcCosh[a*x]^3*Csch[ArcCosh[a*x]/2]^4 -
160*ArcCosh[a*x]*Log[1 - E^(-ArcCosh[a*x])] + 160*ArcCosh[a*x]*Log[1 + E^(-
ArcCosh[a*x])] - 24*ArcCosh[a*x]^3*Log[1 + E^(-ArcCosh[a*x])] + 24*ArcCosh[
a*x]^3*Log[1 - E^ArcCosh[a*x]] + 8*(-20 + 9*ArcCosh[a*x]^2)*PolyLog[2, -E^(
-ArcCosh[a*x])] + 160*PolyLog[2, E^(-ArcCosh[a*x])] + 72*ArcCosh[a*x]^2*Pol
yLog[2, E^ArcCosh[a*x]] + 144*ArcCosh[a*x]*PolyLog[3, -E^(-ArcCosh[a*x])] -
 144*ArcCosh[a*x]*PolyLog[3, E^ArcCosh[a*x]] + 144*PolyLog[4, -E^(-ArcCosh[
a*x])] + 144*PolyLog[4, E^ArcCosh[a*x]] - 4*ArcCosh[a*x]*Sech[ArcCosh[a*x]/
2]^2 + 6*ArcCosh[a*x]^3*Sech[ArcCosh[a*x]/2]^2 + ArcCosh[a*x]^3*Sech[ArcCos
h[a*x]/2]^4 - (16*ArcCosh[a*x]^2*Sinh[ArcCosh[a*x]/2]^4)/(((-1 + a*x)/(1 +
a*x))^(3/2)*(1 + a*x)^3) + 8*Tanh[ArcCosh[a*x]/2] - 40*ArcCosh[a*x]^2*Tanh[
ArcCosh[a*x]/2])/(a*c^3)
```

fricas [F] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\operatorname{arcosh}\left(ax\right)^{3}}{a^{6}c^{3}x^{6}-3\,a^{4}c^{3}x^{4}+3\,a^{2}c^{3}x^{2}-c^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\operatorname{arcosh}\left(ax\right)^{3}}{\left(a^{2}cx^{2}-c\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="giac")
```

```
[Out] integrate(-arccosh(a*x)^3/(a^2*c*x^2 - c)^3, x)
```

maple [A] time = 0.40, size = 710, normalized size = 1.83

$$-\frac{3a^{2}\operatorname{arccosh}\left(ax\right)^{3}x^{3}}{8\left(x^{4}a^{4}-2a^{2}x^{2}+1\right)c^{3}}-\frac{9a\operatorname{arccosh}\left(ax\right)^{2}\sqrt{ax-1}\,\sqrt{ax+1}\,x^{2}}{8\left(x^{4}a^{4}-2a^{2}x^{2}+1\right)c^{3}}+\frac{a^{2}x^{3}\operatorname{arccosh}\left(ax\right)}{4\left(x^{4}a^{4}-2a^{2}x^{2}+1\right)c^{3}}+\frac{a\sqrt{ax+1}\,\sqrt{ax-1}\,x^{2}}{4\left(x^{4}a^{4}-2a^{2}x^{2}+1\right)c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x)
```

```
[Out] -3/8*a^2/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^3*x^3-9/8*a/(a^4*x^4-2*a^2*
x^2+1)/c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*x^2+1/4*a^2/(a^4*x^4-
2*a^2*x^2+1)/c^3*x^3*arccosh(a*x)+1/4*a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a*x+1)^(
1/2)*(a*x-1)^(1/2)*x^2+5/8/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^3*x+11/8/
a/(a^4*x^4-2*a^2*x^2+1)/c^3*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-1/4/
(a^4*x^4-2*a^2*x^2+1)/c^3*x*arccosh(a*x)-1/4/a/(a^4*x^4-2*a^2*x^2+1)/c^3*(a
*x-1)^(1/2)*(a*x+1)^(1/2)+5/2/a/c^3*arccosh(a*x)*ln(1-a*x-(a*x-1)^(1/2)*(a*
x+1)^(1/2))+5/2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-5/2/a/c^3*
arccosh(a*x)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-5/2*polylog(2,-a*x-(a*x-
1)^(1/2)*(a*x+1)^(1/2))/a/c^3-3/8/a/c^3*arccosh(a*x)^3*ln(1-a*x-(a*x-1)^(1/
2)*(a*x+1)^(1/2))-9/8*arccosh(a*x)^2*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1
/2))/a/c^3+9/4*arccosh(a*x)*polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^
3-9/4*polylog(4,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3+3/8/a/c^3*arccosh(a*
x)^3*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+9/8*arccosh(a*x)^2*polylog(2,-a*
x-(a*x-1)^(1/2)*(a*x+1)^(1/2))/a/c^3-9/4*arccosh(a*x)*polylog(3,-a*x-(a*x-1
```

```
)^(1/2)*(a*x+1)^(1/2))/a/c^3+9/4*polylog(4,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)
)/a/c^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(6\,a^3x^3-10\,ax-3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax+1\right)+3\left(a^4x^4-2\,a^2x^2+1\right)\log\left(ax-1\right)\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{}\right.}{16\left(a^5c^3x^4-2\,a^3c^3x^2+ac^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(6*a^3*x^3 - 10*a*x - 3*(a^4*x^4 - 2*a^2*x^2 + 1)*log(a*x + 1) + 3*(a
^4*x^4 - 2*a^2*x^2 + 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)
)^3/(a^5*c^3*x^4 - 2*a^3*c^3*x^2 + a*c^3) - integrate(-3/16*(6*a^5*x^5 - 16
*a^3*x^3 + (6*a^4*x^4 - 10*a^2*x^2 - 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x
+ 1) + 3*(a^5*x^5 - 2*a^3*x^3 + a*x)*log(a*x - 1))*sqrt(a*x + 1)*sqrt(a*x -
 1) + 10*a*x - 3*(a^6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x + 1) + 3*(a^
6*x^6 - 3*a^4*x^4 + 3*a^2*x^2 - 1)*log(a*x - 1))*log(a*x + sqrt(a*x + 1)*sq
rt(a*x - 1))^2/(a^7*c^3*x^7 - 3*a^5*c^3*x^5 + 3*a^3*c^3*x^3 - a*c^3*x + (a^
6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)*sqrt(a*x + 1)*sqrt(a*x - 1
)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\mathrm{acosh}\left(a\,x\right)^3}{{\left(c-a^2\,c\,x^2\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^3,x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\operatorname{acosh}^{3}{\left(ax \right)}}{a^{6}x^{6}-3a^{4}x^{4}+3a^{2}x^{2}-1}\, dx}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**3,x)
```

```
[Out] -Integral(acosh(a*x)**3/(a**6*x**6 - 3*a**4*x**4 + 3*a**2*x**2 - 1), x)/c**
3
```

3.246 $\int \left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=605

$$-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}+\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15ac^2x^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32\sqrt{ax-1}\sqrt{ax+1}}+\frac{5}{16}c^2x\sqrt{c-a^2cx^2}\cosh^{-1}($$

```
[Out] 5/24*c*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3+1/6*x*(-a^2*c*x^2+c)^(5/2)*arc
cosh(a*x)^3+245/384*c^2*x*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+65/576*c^2*x*(-
a*x+1)*(a*x+1)*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+1/36*c^2*x*(-a*x+1)^2*(a*x
+1)^2*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+5/16*c^2*x*arccosh(a*x)^3*(-a^2*c*x
^2+c)^(1/2)-865/2304*a*c^2*x^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(
1/2)+65/2304*a^3*c^2*x^4*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1
/216*c^2*(-a^2*x^2+1)^3*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+
115/768*c^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/
2)-15/32*a*c^2*x^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1
)^(1/2)+5/32*c^2*(-a^2*x^2+1)^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)+1/12*c^2*(-a^2*x^2+1)^3*arccosh(a*x)^2*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/64*c^2*arccosh(a*x)^4*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 1.50, antiderivative size = 636, normalized size of antiderivative = 1.05, number of steps used = 25, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.454, Rules used = {5713, 5685, 5683, 5676, 5662, 5759, 30, 5716, 14, 261}

$$\frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}-\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{ax-1}\sqrt{ax+1}}+\frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15ac^2x^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{32\sqrt{ax-1}\sqrt{ax+1}}+\frac{5}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-865*a*c^2*x^2*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(65*a^3*c^2*x^4*Sqrt[c - a^2*c*x^2])/(2304*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) +
(c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2])/(216*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) + (245*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/384 + (65*c^2*x*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/576 + (c^2*x*(1 - a*x)^2*(1 +
 a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/36 + (115*c^2*Sqrt[c - a^2*c*x^2]
*ArcCosh[a*x]^2)/(768*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c^2*x^2*Sqrt[
c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*(
1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(32*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c^2*(1 - a^2*x^2)^3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (5*c^2*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/16 + (5*c^2*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3)/2
4 + (c^2*x*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3)/6 -
(5*c^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^3\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{6}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 - \frac{\left(5c^2\sqrt{c-a^2cx^2}\right)\int(-1+ax}{6\sqrt{-1+a}}$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{12a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{5}{24}c^2x(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\mathrm{co}$$

$$= \frac{1}{36}c^2x(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{5c^2\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}}{32a\sqrt{-1+ax}\sqrt{1+}}$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{65}{576}c^2x(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax) +$$

$$= \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2cx^2}}{216a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{245}{384}c^2x\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{65}{576}c^2x(1-ax)($$

$$= -\frac{865ac^2x^2\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\sqrt{1+ax}} + \frac{65a^3c^2x^4\sqrt{c-a^2cx^2}}{2304\sqrt{-1+ax}\sqrt{1+ax}} + \frac{c^2\left(1-a^2x^2\right)^3\sqrt{c-a^2}}{216a\sqrt{-1+ax}\sqrt{1+}}$$

Mathematica [A] time = 1.20, size = 189, normalized size = 0.31

$$\frac{c^2\sqrt{c-a^2cx^2}\left(-4320\cosh^{-1}(ax)^4-72\left(270\cosh\left(2\cosh^{-1}(ax)\right)-27\cosh\left(4\cosh^{-1}(ax)\right)+2\cosh\left(6\cosh^{-1}(a\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-4320*ArcCosh[a*x]^4 - 9720*Cosh[2*ArcCosh[a*x]]
+ 243*Cosh[4*ArcCosh[a*x]] - 8*Cosh[6*ArcCosh[a*x]] - 72*ArcCosh[a*x]^2*(27
0*Cosh[2*ArcCosh[a*x]] - 27*Cosh[4*ArcCosh[a*x]] + 2*Cosh[6*ArcCosh[a*x]])
+ 288*ArcCosh[a*x]^3*(45*Sinh[2*ArcCosh[a*x]] - 9*Sinh[4*ArcCosh[a*x]] + Si
nh[6*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(1620*Sinh[2*ArcCosh[a*x]] - 81*Sinh[
4*ArcCosh[a*x]] + 4*Sinh[6*ArcCosh[a*x]])))/(55296*a*Sqrt[(-1 + a*x)/(1 + a
*x)]*(1 + a*x))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*sqrt(-a^2*c*x^2 + c)*arccosh(a
*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.51, size = 887, normalized size = 1.47

$$-\frac{5\sqrt{-c\left(a^2x^2-1\right)}\,\mathrm{arccosh}\left(ax\right)^4 c^2}{64\sqrt{ax-1}\,\sqrt{ax+1}\,a}+\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(32x^7a^7-64x^5a^5+32\sqrt{ax-1}\,\sqrt{ax+1}\,a^6x^6+38x^3a^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x)
```

```
[Out] -5/64*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
^2+1/13824*(-c*(a^2*x^2-1))^(1/2)*(32*x^7*a^7-64*x^5*a^5+32*(a*x-1)^(1/2)*(
a*x+1)^(1/2)*a^6*x^6+38*x^3*a^3-48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4-6*a*
x+18*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(36*a
rccosh(a*x)^3-18*arccosh(a*x)^2+6*arccosh(a*x)-1)*c^2/(a*x-1)/(a*x+1)/a-3/4
096*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x-1)^(1
/2)*x^4*a^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+
1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c^2/(a*x-
1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-
1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c^2/(a*x-1)/(a*x+1)/a+15/512*(-c*(a^2*x^2-1))
^(1/2)*(2*x^3*a^3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c^2/(a
*x-1)/(a*x+1)/a-3/4096*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+
1)^(1/2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-
(a*x-1)^(1/2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccos
h(a*x)+3)*c^2/(a*x-1)/(a*x+1)/a+1/13824*(-c*(a^2*x^2-1))^(1/2)*(-32*(a*x-1)
^(1/2)*(a*x+1)^(1/2)*a^6*x^6+32*x^7*a^7+48*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4*
a^4-64*x^5*a^5-18*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+38*x^3*a^3+(a*x-1)^(1
/2)*(a*x+1)^(1/2)-6*a*x)*(36*arccosh(a*x)^3+18*arccosh(a*x)^2+6*arccosh(a*x
)+1)*c^2/(a*x-1)/(a*x+1)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}(a\,x)^3\,\left(c-a^2\,c\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(5/2),x)
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)*acosh(a*x)**3,x)
[Out] Timed out
```

3.247 $\int \left(c-a^2cx^2\right)^{3/2} \cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=402

$$-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{16\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{1}{4}x\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3+\frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3+45/64*c*x*arccosh(a*x)*(-a^2*c*x^
2+c)^(1/2)+3/32*c*x*(-a*x+1)*(a*x+1)*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+3/8*
c*x*arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2)-51/128*a*c*x^2*(-a^2*c*x^2+c)^(1/2)
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*a^3*c*x^4*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(
1/2)/(a*x+1)^(1/2)+27/128*c*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(
1/2)/(a*x+1)^(1/2)-9/16*a*c*x^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)
^(1/2)/(a*x+1)^(1/2)+3/16*c*(-a^2*x^2+1)^2*arccosh(a*x)^2*(-a^2*c*x^2+c)^(1
/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/32*c*arccosh(a*x)^4*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 414, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5713, 5685, 5683, 5676, 5662, 5759, 30, 5716, 14}

$$\frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{9acx^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^2}{16\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3+\frac{1}{4}c$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] (-51*a*c*x^2*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*a
^3*c*x^4*Sqrt[c - a^2*c*x^2])/(128*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (45*c*x*
Sqrt[c - a^2*c*x^2]*ArcCosh[a*x])/64 + (3*c*x*(1 - a*x)*(1 + a*x)*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x])/32 + (27*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(12
8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^2)/(16*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a
^2*c*x^2]*ArcCosh[a*x]^2)/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x]^3)/8 + (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c
*x^2]*ArcCosh[a*x]^3)/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^3\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}}{4\sqrt{-1+ax}}$$

$$= \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{16a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 + \frac{1}{4}c$$

$$= \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax) - \frac{9acx^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax}{16\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{3}{32}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)$$

$$= -\frac{51acx^2\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3a^3cx^4\sqrt{c-a^2cx^2}}{128\sqrt{-1+ax}\sqrt{1+ax}} + \frac{45}{64}cx\sqrt{c-a^2cx^2}\cosh$$

Mathematica [A] time = 0.44, size = 148, normalized size = 0.37

$$-\frac{c\sqrt{c-a^2cx^2}\left(96\cosh^{-1}(ax)^4 - 24\left(\cosh\left(4\cosh^{-1}(ax)\right) - 16\cosh\left(2\cosh^{-1}(ax)\right)\right)\cosh^{-1}(ax)^2 - 3\left(\cosh\left(4\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^3,x]
```

```
[Out] -1/1024*(c*Sqrt[c - a^2*c*x^2]*(96*ArcCosh[a*x]^4 - 3*(-64*Cosh[2*ArcCosh[a
*x]] + Cosh[4*ArcCosh[a*x]]) - 24*ArcCosh[a*x]^2*(-16*Cosh[2*ArcCosh[a*x]]
+ Cosh[4*ArcCosh[a*x]]) + 12*ArcCosh[a*x]*(-32*Sinh[2*ArcCosh[a*x]] + Sinh[
4*ArcCosh[a*x]]) + 32*ArcCosh[a*x]^3*(-8*Sinh[2*ArcCosh[a*x]] + Sinh[4*ArcC
osh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2cx^2-c\right)\sqrt{-a^2cx^2+c}\,\text{arcosh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)*sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.35, size = 536, normalized size = 1.33

$$-\frac{3\sqrt{-c\left(a^2x^2-1\right)}\,\mathrm{arccosh}\left(ax\right)^4c}{32\sqrt{ax-1}\,\sqrt{ax+1}\,a}-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(8x^5a^5-12x^3a^3+8\sqrt{ax+1}\,\sqrt{ax-1}\,x^4a^4+4ax-8a^2x^2\sqrt{a}\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x)
```

```
[Out] -3/32*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4*c
-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a*x-1
)^(1/2)*x^4*a^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(
a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)*c/(a*
x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-
1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arc
cosh(a*x)^2+6*arccosh(a*x)-3)*c/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/
2)*(2*x^3*a^3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*
x+1)^(1/2))*(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)*c/(a*x-1)/
(a*x+1)/a-1/2048*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/
2)*(a*x-1)^(1/2)*x^4*a^4+4*a*x+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1
)^(1/2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)
+3)*c/(a*x-1)/(a*x+1)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)^3\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \operatorname{acosh}^{3}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**3,x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*acosh(a*x)**3, x)
```

3.248 $\int \sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^3\,dx$

Optimal. Leaf size=231

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{8a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3-\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{ax-1}\sqrt{ax+1}}+3$$

```
[Out] 3/4*x*arccosh(a*x)*(-a^2*c*x^2+c)^(1/2)+1/2*x*arccosh(a*x)^3*(-a^2*c*x^2+c)
^(1/2)-3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/8*arcco
sh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/4*a*x^2*arcc
osh(a*x)^2*(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/8*arccosh(a*x
)^4*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {5713, 5683, 5676, 5662, 5759, 30}

$$-\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^4}{8a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3-\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{ax-1}\sqrt{ax+1}}+3$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```

```
[Out] (-3*a*x^2*Sqrt[c - a^2*c*x^2])/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*x*Sqrt
[c - a^2*c*x^2]*ArcCosh[a*x])/4 + (3*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^2)/(8
*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x
]^2)/(4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]
^3)/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^4)/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
```

```
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3a\sqrt{c-a^2cx^2}\right)\ldots}{2\sqrt{-1+a\ldots}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^3 - \frac{\sqrt{c-a^2cx^2}\cosh\ldots}{8a\sqrt{-1+ax}\sqrt{1\ldots}}$$

$$= \frac{3}{4}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax) - \frac{3ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{4\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh\ldots$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{4}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax) + \frac{3\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^2}{8a\sqrt{-1+ax}\sqrt{1+ax}}\ldots$$

Mathematica [A] time = 0.21, size = 98, normalized size = 0.42

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(2\cosh^{-1}(ax)^4+\left(6\cosh^{-1}(ax)^2+3\right)\cosh\left(2\cosh^{-1}(ax)\right)-2\left(2\cosh^{-1}(ax)^2+3\right)\cosh^{-1}\ldots\right.}{16a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^3,x]
```

```
[Out] -1/16*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(2*ArcCosh[a*x]^4 + (3 + 6*ArcCosh[a
*x]^2)*Cosh[2*ArcCosh[a*x]] - 2*ArcCosh[a*x]*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*
ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-a^2cx^2+c}\,\text{arcosh}\left(ax\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.33, size = 256, normalized size = 1.11

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\mathrm{arccosh}\left(ax\right)^{4}}{8\sqrt{ax-1}\,\sqrt{ax+1}\,a}+\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(2x^{3}a^{3}-2ax+2a^{2}x^{2}\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-1}\,\sqrt{ax+1}\right.}{32\left(ax-1\right)\left(ax+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/8*(-c*(a^2*x^2-1))^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)/a*arccosh(a*x)^4+1/
32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^
(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2+6*arc
cosh(a*x)-3)/(a*x-1)/(a*x+1)/a+1/32*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-2*a*x
-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/(a*x-1)/(a*x+1)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3*(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)^3\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^3*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(ax-1\right)\left(ax+1\right)}\,\mathrm{acosh}^{3}\left(ax\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3*(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**3, x)
```

3.249 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=46

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

[Out] 1/4*arccosh(a*x)^4*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)

Rubi [A] time = 0.16, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Int[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]

[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[ArcCosh[a*x]^3/Sqrt[c - a^2*c*x^2],x]

[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[c - a^2*c*x^2])

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}\left(ax\right)^3}{a^2cx^2-c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^2*c*x^2 - c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.06, size = 55, normalized size = 1.20

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)c}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}\left(ax\right)^4}{4ac\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] -1/4*(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(a^2*x^2-1)
*arccosh(a*x)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(ax\right)^3}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(a\,x\right)^3}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}\left(ax\right)}{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**3/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.250 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\mathrm{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\,\sqrt{}}{a}$$

```
[Out] x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(1/2)+arccosh(a*x)^3*(a*x-1)^(1/2)*(a*x+1
)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)-3*arccosh(a*x)^2*ln(1-(a*x+(a*x-1)^(1/2)*(
a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)-3*arc
cosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x
+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)+3/2*polylog(3,(a*x+(a*x-1)^(1/2)*(a*x+1)
^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5713, 5688, 5715, 3716, 2190, 2531, 2282, 6589}

$$-\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\mathrm{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac\sqrt{c-a^2cx^2}}+\frac{3\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{2ac\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(}{c\sqrt{c-a^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^3)/(c*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*
ArcCosh[a*x]^3)/(a*c*Sqrt[c - a^2*c*x^2]) - (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c*Sqrt[c - a^2*c*x^2]) - (3
*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])])/
(a*c*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E^(2
*ArcCosh[a*x])])/(2*a*c*Sqrt[c - a^2*c*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^2}{1-a^2x^2}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} - \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\coth(x)\,dx, x, \cosh^{-1}(ax)\right)}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} + \frac{\left(6\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{e^{2x}x^2}{1-e^{2x}}\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\log\left(\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\log\left(\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\log\left(\right.}{ac\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{c\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{ac\sqrt{c-a^2cx^2}} - \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\log\left(\right.}{ac\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 0.26, size = 145, normalized size = 0.60

$$\frac{\frac{\sqrt{ax-1}\,\sqrt{ax+1}\left(-6\cosh^{-1}(ax)\text{Li}_2\left(-e^{\cosh^{-1}(ax)}\right)-6\cosh^{-1}(ax)\text{Li}_2\left(e^{\cosh^{-1}(ax)}\right)+6\text{Li}_3\left(-e^{\cosh^{-1}(ax)}\right)+6\text{Li}_3\left(e^{\cosh^{-1}(ax)}\right)+\cosh^{-1}(ax)^3-3\cosh^{-1}(ax)^2\log\right.}{a}}{c\sqrt{c-a^2cx^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^3 + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(ArcCosh[a*x]^3 - 3*ArcCo
sh[a*x]^2*Log[1 - E^ArcCosh[a*x]] - 3*ArcCosh[a*x]^2*Log[1 + E^ArcCosh[a*x]
] - 6*ArcCosh[a*x]*PolyLog[2, -E^ArcCosh[a*x]] - 6*ArcCosh[a*x]*PolyLog[2,
E^ArcCosh[a*x]] + 6*PolyLog[3, -E^ArcCosh[a*x]] + 6*PolyLog[3, E^ArcCosh[a*
x]]))/a)/(c*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3}{a^4c^2x^4-2\,a^2c^2x^2+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^4*c^2*x^4 - 2*a^2*c^2*x^2 +
 c^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [B] time = 0.39, size = 548, normalized size = 2.27

$$-\frac{\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\left(-\sqrt{ax-1}\,\sqrt{ax+1}+ax\right)\operatorname{arccosh}\left(ax\right)^{3}}{c^{2}a\left(a^{2}x^{2}-1\right)}-\frac{2\sqrt{-c\left(a^{2}x^{2}-1\right)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}\left(ax\right)^{3}}{c^{2}a\left(a^{2}x^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x)
```

```
[Out] -(-c*(a^2*x^2-1))^(1/2)*(-(a*x-1)^(1/2)*(a*x+1)^(1/2)+a*x)*arccosh(a*x)^3/c
^2/a/(a^2*x^2-1)-2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a
/(a^2*x^2-1)*arccosh(a*x)^3+3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2)
)+6*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*ar
ccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(-c*(a^2*x^2-1))^(1
/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))+3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/
a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+6*(-c*(a
^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*arccosh(a*x)
*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-6*(-c*(a^2*x^2-1))^(1/2)*(a*x-
1)^(1/2)*(a*x+1)^(1/2)/c^2/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(1/2)*(a*x+
1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(ax\right)^{3}}{\left(-a^{2}cx^{2}+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{acosh}\left(a\,x\right)}^3}{{\left(c-a^2\,c\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}{\left(ax \right)}}{\left(- c \left(ax - 1\right) \left(ax + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(3/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.251 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx$

Optimal. Leaf size=413

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\text{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\log\left(1-\right.}{2ac^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/3*x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(3/2)-x*arccosh(a*x)/c^2/(-a^2*c*x^2+
c)^(1/2)+2/3*x*arccosh(a*x)^3/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*arccosh(a*x)^2*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)+2/3*arcc
osh(a*x)^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*arccosh
(a*x)^2*ln(1-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a/c^2/(-a^2*c*x^2+c)^(1/2)+1/2*ln(-a^2*x^2+1)*(a*x-1)^(1/2)*(a*x+1)^(1/2
)/a/c^2/(-a^2*c*x^2+c)^(1/2)-2*arccosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a
*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^2/(-a^2*c*x^2+c)^(1/2)+poly
log(3,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^
2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 428, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2531, 2282, 6589, 5716, 260}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{ac^2\sqrt{c-a^2cx^2}}+\frac{\sqrt{ax-1}\sqrt{}}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] -((x*ArcCosh[a*x])/(c^2*Sqrt[c - a^2*c*x^2])) + (Sqrt[-1 + a*x]*Sqrt[1 + a*
x]*ArcCosh[a*x]^2)/(2*a*c^2*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]) + (2*x*ArcCo
sh[a*x]^3)/(3*c^2*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]^3)/(3*c^2*(1 - a*x
)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[
a*x]^3)/(3*a*c^2*Sqrt[c - a^2*c*x^2]) - (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Arc
Cosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqr
t[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(2*a*c^2*Sqrt[c - a^2*c*x^2]) -
 (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])
])/(a*c^2*Sqrt[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, E
^(2*ArcCosh[a*x])])/(a*c^2*Sqrt[c - a^2*c*x^2])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{5/2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} - \frac{\left(2\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}} + \frac{\left(a\sqrt{-1+\right.}{}$$

$$= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{(\sqrt{}{}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

$$= -\frac{x\cosh^{-1}(ax)}{c^2\sqrt{c-a^2cx^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2ac^2\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} + \frac{2x\cosh^{-1}(ax)^3}{3c^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(}{3c^2(1-ax)(1+ax}$$

Mathematica [C] time = 1.00, size = 258, normalized size = 0.62

$$\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\frac{6\cosh^{-1}(ax)^2}{1-a^2x^2} - 24\cosh^{-1}(ax)\mathrm{Li}_2\left(e^{2\cosh^{-1}(ax)}\right) + 12\mathrm{Li}_3\left(e^{2\cosh^{-1}(ax)}\right) + 12\log\left(\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\right) - \frac{4}{}\right.}{12a}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((-I)*Pi^3 - (12*a*x*Sqrt[(-1 + a*x)/
(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) + (6*ArcCosh[a*x]^2)/(1 - a^2*x^2) + 8*
ArcCosh[a*x]^3 + (8*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*
x) - (4*a*x*((-1 + a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - 24*
ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])] + 12*Log[Sqrt[(-1 + a*x)/(1 + a*
x)]*(1 + a*x)] - 24*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] + 12*PolyLo
g[3, E^(2*ArcCosh[a*x])]))/(12*a*c^2*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2cx^2+c}\,\mathrm{arcosh}\left(ax\right)^3}{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^6*c^3*x^6 - 3*a^4*c^3*x^4
+ 3*a^2*c^3*x^2 - c^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{2,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4
]%%%}+%%%{2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[2,2,2]%%%}+%%%{-4,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{2,[0,1,0]%%%}+%%%{1,[0,0,0]%%
%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const ind
ex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.56, size = 955, normalized size = 2.31

$$-\frac{\sqrt{-c\left(a^2x^2-1\right)}\,\left(2x^3a^3-3ax-2a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}+2\sqrt{ax-1}\,\sqrt{ax+1}\right)\mathrm{arccosh}\left(ax\right)\left(6\,\mathrm{arccosh}\left(ax\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] -1/6*(-c*(a^2*x^2-1))^(1/2)*(2*x^3*a^3-3*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1
)^(1/2)+2*(a*x-1)^(1/2)*(a*x+1)^(1/2))*arccosh(a*x)*(6*arccosh(a*x)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)*a^3*x^3+6*a^4*x^4*arccosh(a*x)+6*a^3*x^3*(a*x-1)^(1/2)
*(a*x+1)^(1/2)+6*x^4*a^4+6*a^2*x^2*arccosh(a*x)^2-9*arccosh(a*x)*a*x*(a*x-1
)^(1/2)*(a*x+1)^(1/2)-12*a^2*x^2*arccosh(a*x)-6*(a*x+1)^(1/2)*(a*x-1)^(1/2)
*a*x-18*a^2*x^2-8*arccosh(a*x)^2+6*arccosh(a*x)+12)/(3*a^6*x^6-10*a^4*x^4+1
1*a^2*x^2-4)/a/c^3+2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3
/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-(-c*(a^2*x^2-1))^(1/2)*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(
1/2)-1)-(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1
)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-4/3*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^
(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^3+2*(-c*(a^2*x^2-1))^(1/
2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a
*x-1)^(1/2)*(a*x+1)^(1/2))+4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/
2))-4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*
```

```
polylog(3,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+2*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)
^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2
)*(a*x+1)^(1/2))+4*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a
/(a^2*x^2-1)*arccosh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-4*(-c
*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^3/a/(a^2*x^2-1)*polylog(3
,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{\left(c-a^2\,c\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(5/2),x)
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(5/2),x)
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.252 $\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{7/2}}\,dx$

Optimal. Leaf size=607

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{Li}_2\left(e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\sqrt{ax+1}\,\text{Li}_3\left(e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}-\frac{\sqrt{ax-1}\sqrt{ax+1}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-\ldots}}$$

```
[Out] 1/5*x*arccosh(a*x)^3/c/(-a^2*c*x^2+c)^(5/2)+4/15*x*arccosh(a*x)^3/c^2/(-a^2
*c*x^2+c)^(3/2)-x*arccosh(a*x)/c^3/(-a^2*c*x^2+c)^(1/2)-1/10*x*arccosh(a*x)
/c^3/(-a*x+1)/(a*x+1)/(-a^2*c*x^2+c)^(1/2)+8/15*x*arccosh(a*x)^3/c^3/(-a^2*
c*x^2+c)^(1/2)-1/20*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^2+1)/(-a^2*c*
x^2+c)^(1/2)+3/20*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*x^
2+1)^2/(-a^2*c*x^2+c)^(1/2)+2/5*arccosh(a*x)^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
a/c^3/(-a^2*x^2+1)/(-a^2*c*x^2+c)^(1/2)+8/15*arccosh(a*x)^3*(a*x-1)^(1/2)*(
a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)-8/5*arccosh(a*x)^2*ln(1-(a*x+(a*x-1
)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^
(1/2)+1/2*ln(-a^2*x^2+1)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(
1/2)-8/5*arccosh(a*x)*polylog(2,(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1
)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(1/2)+4/5*polylog(3,(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/c^3/(-a^2*c*x^2+c)^(
1/2)
```

Rubi [A] time = 1.08, antiderivative size = 637, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {5713, 5691, 5688, 5715, 3716, 2190, 2531, 2282, 6589, 5716, 260, 261}

$$-\frac{8\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}+\frac{4\sqrt{ax-1}\sqrt{ax+1}\,\text{PolyLog}\left(3,e^{2\cosh^{-1}(ax)}\right)}{5ac^3\sqrt{c-a^2cx^2}}-\frac{\ldots}{20ac^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(20*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2]
) - (x*ArcCosh[a*x])/(c^3*Sqrt[c - a^2*c*x^2]) - (x*ArcCosh[a*x])/(10*c^3*(
1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^2)/(20*a*c^3*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]) + (2*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(5*a*c^3*(1 - a^2*x^2)*Sqrt[c - a^2*c*
x^2]) + (8*x*ArcCosh[a*x]^3)/(15*c^3*Sqrt[c - a^2*c*x^2]) + (x*ArcCosh[a*x]
^3)/(5*c^3*(1 - a*x)^2*(1 + a*x)^2*Sqrt[c - a^2*c*x^2]) + (4*x*ArcCosh[a*x]
^3)/(15*c^3*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]*ArcCosh[a*x]^3)/(15*a*c^3*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a
*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 - E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt
[c - a^2*c*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[1 - a^2*x^2])/(2*a*c^3
*Sqrt[c - a^2*c*x^2]) - (8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLo
g[2, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]) + (4*Sqrt[-1 + a*x]
*Sqrt[1 + a*x]*PolyLog[3, E^(2*ArcCosh[a*x])])/(5*a*c^3*Sqrt[c - a^2*c*x^2]
)
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3716

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_
.)*(x_)], x_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2
*I, Int[((c + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(E^(2*I*k*Pi)*(1 + E^(2*(-(I*
e) + f*fz*x))/E^(2*I*k*Pi))), x], x] /; FreeQ[{c, d, e, f, fz}, x] && Integ
erQ[4*k] && IGtQ[m, 0]
```

Rule 5688

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(((d1_) + (e1_.)*(x_))^(3/2)*
((d2_) + (e2_.)*(x_))^(3/2)), x_Symbol] :> Simp[(x*(a + b*ArcCosh[c*x])^n)/
(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x] + Dist[(b*c*n*Sqrt[1 + c*x]*Sqr
t[-1 + c*x])/(d1*d2*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(x*(a + b*ArcCosh
[c*x])^(n - 1))/(1 - c^2*x^2), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0]
```

Rule 5691

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_)*((
d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> -Simp[(x*(d1 + e1*x)^(p + 1)*(d2 + e
2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*d1*d2*(p + 1)), x] + (Dist[(2*p + 3
)/(2*d1*d2*(p + 1)), Int[(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*Arc
Cosh[c*x])^n, x], x] - Dist[(b*c*n*(-(d1*d2))^(p + 1/2)*Sqrt[1 + c*x]*Sqrt[
-1 + c*x])/(2*(p + 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[x*(-1 + c^2*x^2
)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1
, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && LtQ[p, -
1] && NeQ[p, -3/2] && IntegerQ[p + 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5715

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_))/((d_) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[1/e, Subst[Int[(a + b*x)^n*Coth[x], x], x, ArcCosh[c*x]]
, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{\left(c-a^2cx^2\right)^{7/2}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{7/2}(1+ax)^{7/2}}\,dx}{c^3\sqrt{c-a^2cx^2}}$$

$$= \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{\left(4\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{5c^3\sqrt{c-a^2cx^2}} - \frac{\left(3a\sqrt{-1\right.}{}$$

$$= \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{x\cosh^{-1}(ax)^3}{5c^3(1-ax)^2(1+ax)^2\sqrt{c-a^2cx^2}} + \frac{4x\cosh^{-1}}{15c^3(1-ax)(1+a}$$

$$= -\frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{20ac^3\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}} + \frac{2\sqrt{-1+ax}\,\sqrt{1+}}{5ac^3\left(1-a^2x^2\right)}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

$$= -\frac{\sqrt{-1+ax}\,\sqrt{1+ax}}{20ac^3\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{c^3\sqrt{c-a^2cx^2}} - \frac{x\cosh^{-1}(ax)}{10c^3(1-ax)(1+ax)\sqrt{c-a^2cx^2}} + \frac{3\sqrt{-1}}{20a}$$

Mathematica [C] time = 1.85, size = 363, normalized size = 0.60

$$-\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(\frac{3}{1-a^2x^2} + \frac{24\cosh^{-1}(ax)^2}{a^2x^2-1} - \frac{9\cosh^{-1}(ax)^2}{\left(a^2x^2-1\right)^2} + 96\cosh^{-1}(ax)\mathrm{Li}_2\left(e^{2\cosh^{-1}(ax)}\right) - 48\mathrm{Li}_3\left(e^{2\cosh^{-1}(ax)}\right) - 60\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(c - a^2*c*x^2)^(7/2),x]
```

```
[Out] -1/60*(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*((4*I)*Pi^3 + 3/(1 - a^2*x^2) +
 (60*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x])/(-1 + a*x) - (6*a*x*((-1
+ a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x])/(-1 + a*x)^3 - (9*ArcCosh[a*x]^2)/(-1
 + a^2*x^2)^2 + (24*ArcCosh[a*x]^2)/(-1 + a^2*x^2) - 32*ArcCosh[a*x]^3 - (3
2*a*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3)/(-1 + a*x) + (16*a*x*((-1
+ a*x)/(1 + a*x))^(3/2)*ArcCosh[a*x]^3)/(-1 + a*x)^3 - (12*a*x*Sqrt[(-1 + a
*x)/(1 + a*x)]*ArcCosh[a*x]^3)/((-1 + a*x)^3*(1 + a*x)^2) + 96*ArcCosh[a*x]
^2*Log[1 - E^(2*ArcCosh[a*x])] - 60*Log[Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x
```

```
)] + 96*ArcCosh[a*x]*PolyLog[2, E^(2*ArcCosh[a*x])] - 48*PolyLog[3, E^(2*Ar
cCosh[a*x])]))/(a*c^3*Sqrt[c - a^2*c*x^2])
```

fricas [F] time = 1.30, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}(ax)^3}{a^8c^4x^8-4\,a^6c^4x^6+6\,a^4c^4x^4-4\,a^2c^4x^2+c^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^3/(a^8*c^4*x^8 - 4*a^6*c^4*x^6 +
 6*a^4*c^4*x^4 - 4*a^2*c^4*x^2 + c^4), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, choosing root of [1,0,%%%{2,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{-2,[0,1,0]%%%}+%%%{2,[0,0,0]%%%},0,%%%{1,[4,2,4
]%%%}+%%%{2,[4,1,4]%%%}+%%%{1,[4,0,4]%%%}+%%%{-2,[2,2,2]%%%}+%%%{-4,[2,1,2]
%%%}+%%%{-2,[2,0,2]%%%}+%%%{1,[0,2,0]%%%}+%%%{2,[0,1,0]%%%}+%%%{1,[0,0,0]%%
%}] at parameters values [86,-97,-82]sym2poly/r2sym(const gen & e,const ind
ex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.59, size = 1319, normalized size = 2.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x)
```

```
[Out] -1/60*(-c*(a^2*x^2-1))^(1/2)*(8*x^5*a^5-20*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^
(1/2)*x^4*a^4+15*a*x+16*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-8*(a*x-1)^(1/2)
*(a*x+1)^(1/2))*(24+24*x^8*a^8-96*x^6*a^6+144*x^4*a^4-96*a^2*x^2-192*arccos
h(a*x)^2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-192*(a*x-1)^(1/2)*(a*x+1)^(1/2
)*arccosh(a*x)*x^7*a^7-1368*a^4*x^4*arccosh(a*x)^2-192*arccosh(a*x)*x^8*a^8
+105*a^3*x^3*(a*x-1)^(1/2)*(a*x+1)^(1/2)-45*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x
-192*arccosh(a*x)^2*x^8*a^8-380*a^2*x^2*arccosh(a*x)^3+984*a^2*x^2*arccosh(
a*x)^2+744*arccosh(a*x)^2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5-1020*arccosh(
a*x)^2*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^3*a^3+852*arccosh(a*x)*x^6*a^6-1590*a^
4*x^4*arccosh(a*x)+1410*a^2*x^2*arccosh(a*x)+372*arccosh(a*x)*a*x*(a*x-1)^(
1/2)*(a*x+1)^(1/2)-936*arccosh(a*x)*(a*x-1)^(1/2)*(a*x+1)^(1/2)*a^3*x^3+24*
(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^7*a^7-84*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^5*a^5-
480*arccosh(a*x)-264*arccosh(a*x)^2+256*arccosh(a*x)^3+756*arccosh(a*x)*(a*
x-1)^(1/2)*(a*x+1)^(1/2)*a^5*x^5+495*arccosh(a*x)^2*a*x*(a*x-1)^(1/2)*(a*x+
1)^(1/2)+840*arccosh(a*x)^2*a^6*x^6+160*arccosh(a*x)^3*x^4*a^4)/(40*a^10*x^
10-215*a^8*x^8+469*a^6*x^6-517*a^4*x^4+287*a^2*x^2-64)/a/c^4+2*(-c*(a^2*x^2
-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/
2)*(a*x+1)^(1/2))-(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/
(a^2*x^2-1)*ln(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)-1)-(-c*(a^2*x^2-1))^(1/2)*(a
*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^
(1/2))-16/15*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*
x^2-1)*arccosh(a*x)^3+8/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2
)/c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))+16
```

```
/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*arc
cosh(a*x)*polylog(2,a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(-c*(a^2*x^2-1))^
(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,a*x+(a*x-1)^(
1/2)*(a*x+1)^(1/2))+8/5*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
c^4/a/(a^2*x^2-1)*arccosh(a*x)^2*ln(1+a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))+16/5
*(-c*(a^2*x^2-1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*arcco
sh(a*x)*polylog(2,-a*x-(a*x-1)^(1/2)*(a*x+1)^(1/2))-16/5*(-c*(a^2*x^2-1))^(
1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/c^4/a/(a^2*x^2-1)*polylog(3,-a*x-(a*x-1)^(
1/2)*(a*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\left(-a^2cx^2+c\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*c*x^2+c)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^3/(-a^2*c*x^2 + c)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{{\left(c-a^2\,c\,x^2\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(c - a^2*c*x^2)^(7/2),x)
```

```
[Out] int(acosh(a*x)^3/(c - a^2*c*x^2)^(7/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{3}(ax)}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*c*x**2+c)**(7/2),x)
```

```
[Out] Integral(acosh(a*x)**3/(-c*(a*x - 1)*(a*x + 1))**(7/2), x)
```

3.253 $\int \frac{x^4 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=315

$$\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^4}{32a^5\sqrt{1-ax}}+\frac{45\sqrt{ax-1}\cosh^{-1}(ax)^2}{128a^5\sqrt{1-ax}}-\frac{45x\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{64a^4}-\frac{45x^2\sqrt{ax-1}}{128a^3\sqrt{1-ax}}-\frac{9x^2\sqrt{ax}}{16}$$

```
[Out] -45/128*x^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/128*x^4*(a*x-1)^(1/2)/a/(-a*
x+1)^(1/2)+45/128*arccosh(a*x)^2*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)-9/16*x^2*
arccosh(a*x)^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/16*x^4*arccosh(a*x)^2*(a*
x-1)^(1/2)/a/(-a*x+1)^(1/2)+3/32*arccosh(a*x)^4*(a*x-1)^(1/2)/a^5/(-a*x+1)^
(1/2)-45/64*x*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^4-3/32*x^3*arccos
h(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-3/8*x*arccosh(a*x)^3*(-a^2*x^2+1)^(
1/2)/a^4-1/4*x^3*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 1.44, antiderivative size = 427, normalized size of antiderivative = 1.36, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {5798, 5759, 5676, 5662, 30}

$$-\frac{3x^4\sqrt{ax-1}\sqrt{ax+1}}{128a\sqrt{1-a^2x^2}}-\frac{45x^2\sqrt{ax-1}\sqrt{ax+1}}{128a^3\sqrt{1-a^2x^2}}-\frac{3x^4\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{16a\sqrt{1-a^2x^2}}-\frac{x^3(1-ax)(ax+1)\cosh^{-1}(}{4a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-45*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(128*a^3*Sqrt[1 - a^2*x^2]) - (3*x^4
*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(128*a*Sqrt[1 - a^2*x^2]) - (45*x*(1 - a*x)*
(1 + a*x)*ArcCosh[a*x])/(64*a^4*Sqrt[1 - a^2*x^2]) - (3*x^3*(1 - a*x)*(1 +
a*x)*ArcCosh[a*x])/(32*a^2*Sqrt[1 - a^2*x^2]) + (45*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^2)/(128*a^5*Sqrt[1 - a^2*x^2]) - (9*x^2*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(16*a^3*Sqrt[1 - a^2*x^2]) - (3*x^4*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(16*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a*x
)*(1 + a*x)*ArcCosh[a*x]^3)/(8*a^4*Sqrt[1 - a^2*x^2]) - (x^3*(1 - a*x)*(1 +
 a*x)*ArcCosh[a*x]^3)/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^4)/(32*a^5*Sqrt[1 - a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^4\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^3(1-ax)(1+ax)\cosh^{-1}(ax)^3}{4a^2\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{4a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+\right.}{}$$

$$= -\frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{16a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{8a^4\sqrt{1-a^2x^2}} - \frac{x^3(1-ax)(1+}{4a^2\sqrt{1}}$$

$$= -\frac{3x^3(1-ax)(1+ax)\cosh^{-1}(ax)}{32a^2\sqrt{1-a^2x^2}} - \frac{9x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{16a^3\sqrt{1-a^2x^2}} - \frac{3x^4\sqrt{-1+ax}}{16a}$$

$$= -\frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cosh^{-1}(ax)}{64a^4\sqrt{1-a^2x^2}} - \frac{3x^3(1-ax)(1+ax)\cosh^{-1}}{32a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{45x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{128a^3\sqrt{1-a^2x^2}} - \frac{3x^4\sqrt{-1+ax}\,\sqrt{1+ax}}{128a\sqrt{1-a^2x^2}} - \frac{45x(1-ax)(1+ax)\cosh^{-1}(ax)}{64a^4\sqrt{1-a^2x^2}} -$$

Mathematica [A] time = 0.45, size = 136, normalized size = 0.43

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(-192\left(2\cosh^{-1}(ax)^2+1\right)\cosh\left(2\cosh^{-1}(ax)\right)-3\left(8\cosh^{-1}(ax)^2+1\right)\cosh\left(4\cosh^{-1}(ax)\right)+4\,\mathrm{c}\right.}{1024a^5\sqrt{-(}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(-192*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*A
rcCosh[a*x]] - 3*(1 + 8*ArcCosh[a*x]^2)*Cosh[4*ArcCosh[a*x]] + 4*ArcCosh[a*
x]*(24*ArcCosh[a*x]^3 + 32*(3 + 2*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]] + (3
 + 8*ArcCosh[a*x]^2)*Sinh[4*ArcCosh[a*x]])))/(1024*a^5*Sqrt[-((-1 + a*x)*(1
 + a*x))])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^4\,\mathrm{arcosh}\,(ax)^3}{a^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

maple [B] time = 0.71, size = 520, normalized size = 1.65

$$-\frac{3\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^4}{32a^5\left(a^2x^2-1\right)}-\frac{\sqrt{-a^2x^2+1}\,\left(8x^5a^5-12x^3a^3+8\sqrt{ax+1}\,\sqrt{ax-1}\,x^4a^4+4a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -3/32*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*arccos
h(a*x)^4-1/2048*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3+8*(a*x+1)^(1/2)*(a
*x-1)^(1/2)*x^4*a^4+4*a*x-8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/
2)*(a*x+1)^(1/2))*(32*arccosh(a*x)^3-24*arccosh(a*x)^2+12*arccosh(a*x)-3)/a
^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(
1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh
(a*x)^2+6*arccosh(a*x)-3)/a^5/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^
3-2*a*x-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*
(4*arccosh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^5/(a^2*x^2-1)-1/2048
*(-a^2*x^2+1)^(1/2)*(8*x^5*a^5-12*x^3*a^3-8*(a*x+1)^(1/2)*(a*x-1)^(1/2)*x^4
*a^4+4*a*x+8*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2
))*(32*arccosh(a*x)^3+24*arccosh(a*x)^2+12*arccosh(a*x)+3)/a^5/(a^2*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4 \operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^4*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.254 $\int \frac{x^3 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=243

$$-\frac{40\sqrt{1-ax}\,\sqrt{ax+1}\,\cosh^{-1}(ax)}{9a^4}-\frac{40x\sqrt{ax-1}}{9a^3\sqrt{1-ax}}-\frac{2x\sqrt{ax-1}\,\cosh^{-1}(ax)^2}{a^3\sqrt{1-ax}}-\frac{x^2\sqrt{1-a^2x^2}\,\cosh^{-1}(ax)^3}{3a^2}-\frac{2x^2\sqrt{1-}}{}$$

```
[Out] -40/9*x*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-2/27*x^3*(a*x-1)^(1/2)/a/(-a*x+1)^
(1/2)-2*x*arccosh(a*x)^2*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-1/3*x^3*arccosh(a
*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-40/9*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+
1)^(1/2)/a^4-2/9*x^2*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-2/3*arcc
osh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^4-1/3*x^2*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)
/a^2
```

Rubi [A] time = 1.03, antiderivative size = 329, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5759, 5718, 5654, 8, 5662, 30}

$$-\frac{2x^3\sqrt{ax-1}\,\sqrt{ax+1}}{27a\sqrt{1-a^2x^2}}-\frac{40x\sqrt{ax-1}\,\sqrt{ax+1}}{9a^3\sqrt{1-a^2x^2}}-\frac{x^3\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2}{3a\sqrt{1-a^2x^2}}-\frac{x^2(1-ax)(ax+1)\cosh^{-1}(ax}{3a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-40*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(9*a^3*Sqrt[1 - a^2*x^2]) - (2*x^3*Sqr
t[-1 + a*x]*Sqrt[1 + a*x])/(27*a*Sqrt[1 - a^2*x^2]) - (40*(1 - a*x)*(1 + a*
x)*ArcCosh[a*x])/(9*a^4*Sqrt[1 - a^2*x^2]) - (2*x^2*(1 - a*x)*(1 + a*x)*Arc
Cosh[a*x])/(9*a^2*Sqrt[1 - a^2*x^2]) - (2*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Ar
cCosh[a*x]^2)/(a^3*Sqrt[1 - a^2*x^2]) - (x^3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*A
rcCosh[a*x]^2)/(3*a*Sqrt[1 - a^2*x^2]) - (2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x
]^3)/(3*a^4*Sqrt[1 - a^2*x^2]) - (x^2*(1 - a*x)*(1 + a*x)*ArcCosh[a*x]^3)/(
3*a^2*Sqrt[1 - a^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x^2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^2\sqrt{1-a^2x^2}} + \frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{3a^2\sqrt{1-a^2x^2}} - \frac{\left(\sqrt{-1+a\right.}}{}$$

$$= -\frac{x^3\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{3a\sqrt{1-a^2x^2}} - \frac{2(1-ax)(1+ax)\cosh^{-1}(ax)^3}{3a^4\sqrt{1-a^2x^2}} - \frac{x^2(1-ax)(1+ax}{3a^2\sqrt{1-}}$$

$$= -\frac{2x^2(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^2\sqrt{1-a^2x^2}} - \frac{2x\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^2}{a^3\sqrt{1-a^2x^2}} - \frac{x^3\sqrt{-1+ax}\sqrt{1}}{3a\sqrt{1}}$$

$$= -\frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}} - \frac{2x^2(1-ax)(1+ax)\cosh^{-1}(a}{9a^2\sqrt{1-a^2x^2}}$$

$$= -\frac{40x\sqrt{-1+ax}\sqrt{1+ax}}{9a^3\sqrt{1-a^2x^2}} - \frac{2x^3\sqrt{-1+ax}\sqrt{1+ax}}{27a\sqrt{1-a^2x^2}} - \frac{40(1-ax)(1+ax)\cosh^{-1}(ax)}{9a^4\sqrt{1-a^2x^2}} - \frac{2x^3}{}$$

Mathematica [A] time = 0.17, size = 140, normalized size = 0.58

$$\frac{\sqrt{1-a^2x^2}\left(2ax\left(a^2x^2+60\right)-9\sqrt{ax-1}\sqrt{ax+1}\left(a^2x^2+2\right)\cosh^{-1}(ax)^3+9ax\left(a^2x^2+6\right)\cosh^{-1}(ax)^2-6\sqrt{ax-}\right.}{27a^4\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(2*a*x*(60 + a^2*x^2) - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*(
20 + a^2*x^2)*ArcCosh[a*x] + 9*a*x*(6 + a^2*x^2)*ArcCosh[a*x]^2 - 9*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*(2 + a^2*x^2)*ArcCosh[a*x]^3))/(27*a^4*Sqrt[-1 + a*x]
*Sqrt[1 + a*x])
```

fricas [A] time = 0.50, size = 205, normalized size = 0.84

$$-\frac{9\left(a^4x^4+a^2x^2-2\right)\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^3-9\left(a^3x^3+6\,ax\right)\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a\cdots\right.}{27\left(a\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/27*(9*(a^4*x^4 + a^2*x^2 - 2)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2
- 1))^3 - 9*(a^3*x^3 + 6*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x
+ sqrt(a^2*x^2 - 1))^2 + 6*(a^4*x^4 + 19*a^2*x^2 - 20)*sqrt(-a^2*x^2 + 1)*l
og(a*x + sqrt(a^2*x^2 - 1)) - 2*(a^3*x^3 + 60*a*x)*sqrt(a^2*x^2 - 1)*sqrt(-
a^2*x^2 + 1))/(a^6*x^2 - a^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.49, size = 375, normalized size = 1.54

$$-\frac{\sqrt{-a^2x^2+1}\,\left(4x^4a^4-5a^2x^2+4a^3x^3\sqrt{ax-1}\,\sqrt{ax+1}\,-3\sqrt{ax+1}\,\sqrt{ax-1}\,ax+1\right)\left(9\operatorname{arccosh}(ax)^3-9\operatorname{arcc}\cdots\right.}{216a^4\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2+4*a^3*x^3*(a*x-1)^(1/2)*(a*x
+1)^(1/2)-3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3-9*arccosh(
a*x)^2+6*arccosh(a*x)-2)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1
/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccosh(a*x)^3-3*arccosh(a*x)^2+6*arccosh
(a*x)-6)/a^4/(a^2*x^2-1)-3/8*(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x
-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arccosh(a*x)^2+6*arccosh(a*x)+6)/a^4/(a^
2*x^2-1)-1/216*(-a^2*x^2+1)^(1/2)*(4*x^4*a^4-5*a^2*x^2-4*a^3*x^3*(a*x-1)^(1
/2)*(a*x+1)^(1/2)+3*(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+1)*(9*arccosh(a*x)^3+9*
arccosh(a*x)^2+6*arccosh(a*x)+2)/a^4/(a^2*x^2-1)
```

maxima [C] time = 0.71, size = 131, normalized size = 0.54

$$-\frac{1}{3}\left(\frac{\sqrt{-a^2x^2+1}\,x^2}{a^2}+\frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\operatorname{arcosh}(ax)^3+\frac{2}{27}a\left(\frac{3\left(-i\sqrt{a^2x^2-1}\,x^2-\frac{20i\sqrt{a^2x^2-1}}{a^2}\right)\operatorname{arcosh}(ax)}{a^3}+\frac{i\,a^2\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 + 1)/a^4)*arccosh(a*x)^3
 + 2/27*a*(3*(-I*sqrt(a^2*x^2 - 1)*x^2 - 20*I*sqrt(a^2*x^2 - 1)/a^2)*arccos
h(a*x)/a^3 + (I*a^2*x^3 + 60*I*x)/a^4) + 1/3*(I*a^2*x^3 + 6*I*x)*arccosh(a*
x)^2/a^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\mathrm{acosh}\left(a\,x\right)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\operatorname{acosh}^{3}\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.255 $\int \frac{x^2 \cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} dx$

Optimal. Leaf size=188

$$\frac{\sqrt{ax-1}\cosh^{-1}(ax)^4}{8a^3\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\cosh^{-1}(ax)^2}{8a^3\sqrt{1-ax}}-\frac{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{2a^2}-\frac{3x\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{4a^2}-\frac{3x}{8a}$$

```
[Out] -3/8*x^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+3/8*arccosh(a*x)^2*(a*x-1)^(1/2)/a^
3/(-a*x+1)^(1/2)-3/4*x^2*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)+1/8*
arccosh(a*x)^4*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)-3/4*x*arccosh(a*x)*(-a*x+1)
^(1/2)*(a*x+1)^(1/2)/a^2-1/2*x*arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.77, antiderivative size = 257, normalized size of antiderivative = 1.37, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {5798, 5759, 5676, 5662, 30}

$$-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}}{8a\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^4}{8a^3\sqrt{1-a^2x^2}}-\frac{x(1-ax)(ax+1)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}}-\frac{3x^2\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-3*x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(8*a*Sqrt[1 - a^2*x^2]) - (3*x*(1 - a
*x)*(1 + a*x)*ArcCosh[a*x])/(4*a^2*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2)/(8*a^3*Sqrt[1 - a^2*x^2]) - (3*x^2*Sqrt[-1 + a
*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(4*a*Sqrt[1 - a^2*x^2]) - (x*(1 - a*x)*(1
 + a*x)*ArcCosh[a*x]^3)/(2*a^2*Sqrt[1 - a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^4)/(8*a^3*Sqrt[1 - a^2*x^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
```

```
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x^2\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{2a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\right.}{}$$

$$= -\frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2a^2\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\,\sqrt{1+}}{8a^3\sqrt{1}}$$

$$= -\frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} - \frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{4a\sqrt{1-a^2x^2}} - \frac{x(1-ax)(1+a}{2a^2\sqrt{1-}}$$

$$= -\frac{3x^2\sqrt{-1+ax}\,\sqrt{1+ax}}{8a\sqrt{1-a^2x^2}} - \frac{3x(1-ax)(1+ax)\cosh^{-1}(ax)}{4a^2\sqrt{1-a^2x^2}} + \frac{3\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(}{8a^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.22, size = 98, normalized size = 0.52

$$-\frac{\sqrt{-((ax-1)(ax+1))}\left(2\cosh^{-1}(ax)\left(\cosh^{-1}(ax)^3+\left(2\cosh^{-1}(ax)^2+3\right)\sinh\left(2\cosh^{-1}(ax)\right)\right)-3\left(2\cosh^{-1}(ax)\right.}{16a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] -1/16*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(-3*(1 + 2*ArcCosh[a*x]^2)*Cosh[2*ArcC
osh[a*x]] + 2*ArcCosh[a*x]*(ArcCosh[a*x]^3 + (3 + 2*ArcCosh[a*x]^2)*Sinh[2*
ArcCosh[a*x]])))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2\,\text{arcosh}\,(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.45, size = 255, normalized size = 1.36

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^4}{8a^3\left(a^2x^2-1\right)}-\frac{\sqrt{-a^2x^2+1}\,\left(2x^3a^3-2ax+2a^2x^2\sqrt{ax-1}\,\sqrt{ax+1}-\sqrt{ax-}\right.}{32a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*arccosh
(a*x)^4-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x+2*a^2*x^2*(a*x-1)^(1/2)*(a
*x+1)^(1/2)-(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arccosh(a*x)^3-6*arccosh(a*x)^2
+6*arccosh(a*x)-3)/a^3/(a^2*x^2-1)-1/32*(-a^2*x^2+1)^(1/2)*(2*x^3*a^3-2*a*x
-2*a^2*x^2*(a*x-1)^(1/2)*(a*x+1)^(1/2)+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(4*arcc
osh(a*x)^3+6*arccosh(a*x)^2+6*arccosh(a*x)+3)/a^3/(a^2*x^2-1)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\mathrm{acosh}\left(a\,x\right)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\,\mathrm{acosh}^3\left(ax\right)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.256 $\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{a^2}-\frac{6\sqrt{1-ax}\sqrt{ax+1}\cosh^{-1}(ax)}{a^2}-\frac{6x\sqrt{ax-1}}{a\sqrt{1-ax}}-\frac{3x\sqrt{ax-1}\cosh^{-1}(ax)^2}{a\sqrt{1-ax}}$$

```
[Out] -6*x*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)-3*x*arccosh(a*x)^2*(a*x-1)^(1/2)/a/(-a*
x+1)^(1/2)-6*arccosh(a*x)*(-a*x+1)^(1/2)*(a*x+1)^(1/2)/a^2-arccosh(a*x)^3*(
-a^2*x^2+1)^(1/2)/a^2
```

Rubi [A] time = 0.39, antiderivative size = 153, normalized size of antiderivative = 1.39, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {5798, 5718, 5654, 8}

$$-\frac{6x\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{1-a^2x^2}}-\frac{(1-ax)(ax+1)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}}-\frac{3x\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}}-\frac{6(1-ax)(ax+1)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (-6*x*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2]) - (6*(1 - a*x)*(1
 + a*x)*ArcCosh[a*x])/(a^2*Sqrt[1 - a^2*x^2]) - (3*x*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]*ArcCosh[a*x]^2)/(a*Sqrt[1 - a^2*x^2]) - ((1 - a*x)*(1 + a*x)*ArcCosh
[a*x]^3)/(a^2*Sqrt[1 - a^2*x^2])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} - \frac{\left(3\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \cosh^{-1}(ax)^2\,dx}{a\sqrt{1-a^2x^2}}$$
$$= -\frac{3x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{a^2\sqrt{1-a^2x^2}} + \frac{\left(6\sqrt{-1+ax}\,\sqrt{1\right.}{}$$
$$= -\frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{a\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)}{a^2\sqrt{1-}}$$
$$= -\frac{6x\sqrt{-1+ax}\,\sqrt{1+ax}}{a\sqrt{1-a^2x^2}} - \frac{6(1-ax)(1+ax)\cosh^{-1}(ax)}{a^2\sqrt{1-a^2x^2}} - \frac{3x\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}}{a\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.92

$$\frac{\sqrt{1-a^2x^2}\left(6ax-\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^3+3ax\cosh^{-1}(ax)^2-6\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)\right)}{a^2\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[1 - a^2*x^2]*(6*a*x - 6*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x] + 3
*a*x*ArcCosh[a*x]^2 - Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3))/(a^2*Sq
rt[-1 + a*x]*Sqrt[1 + a*x])
```

fricas [A] time = 0.55, size = 159, normalized size = 1.45

$$\frac{3\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}\,ax\log\left(ax+\sqrt{a^2x^2-1}\right)^2+\left(-a^2x^2+1\right)^{\frac{3}{2}}\log\left(ax+\sqrt{a^2x^2-1}\right)^3+6\sqrt{a^2x^2-1}\,\sqrt{-a^2}}{a^4x^2-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] (3*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*a*x*log(a*x + sqrt(a^2*x^2 - 1))^2
+ (-a^2*x^2 + 1)^(3/2)*log(a*x + sqrt(a^2*x^2 - 1))^3 + 6*sqrt(a^2*x^2 - 1)
*sqrt(-a^2*x^2 + 1)*a*x - 6*(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(a*x + sqrt
(a^2*x^2 - 1)))/(a^4*x^2 - a^2)
```

giac [C] time = 0.68, size = 103, normalized size = 0.94

$$-\frac{\sqrt{-a^2x^2+1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)^3}{a^2}-\frac{3i\left(x\log\left(ax+\sqrt{a^2x^2-1}\right)^2+2\,a\left(\frac{x}{a}-\frac{\sqrt{a^2x^2-1}\,\log\left(ax+\sqrt{a^2x^2-1}\right)}{a^2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(-a^2*x^2 + 1)*log(a*x + sqrt(a^2*x^2 - 1))^3/a^2 - 3*I*(x*log(a*x + s
qrt(a^2*x^2 - 1))^2 + 2*a*(x/a - sqrt(a^2*x^2 - 1)*log(a*x + sqrt(a^2*x^2 -
 1))/a^2))/a
```

maple [A] time = 0.20, size = 155, normalized size = 1.41

$$-\frac{\sqrt{-a^2x^2+1}\,\left(\sqrt{ax+1}\,\sqrt{ax-1}\,ax+a^2x^2-1\right)\left(\operatorname{arccosh}(ax)^3-3\operatorname{arccosh}(ax)^2+6\operatorname{arccosh}(ax)-6\right)}{2a^2\left(a^2x^2-1\right)}-\frac{\sqrt{-a^2x^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] -1/2*(-a^2*x^2+1)^(1/2)*((a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x+a^2*x^2-1)*(arccos
h(a*x)^3-3*arccosh(a*x)^2+6*arccosh(a*x)-6)/a^2/(a^2*x^2-1)-1/2*(-a^2*x^2+1
)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*(arccosh(a*x)^3+3*arcco
sh(a*x)^2+6*arccosh(a*x)+6)/a^2/(a^2*x^2-1)
```

maxima [C] time = 0.33, size = 65, normalized size = 0.59

$$\frac{3i\,x\operatorname{arcosh}(ax)^2}{a}-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)^3}{a^2}-\frac{3\left(-2i\,x+\frac{2i\,\sqrt{a^2x^2-1}\,\operatorname{arcosh}(ax)}{a}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] 3*I*x*arccosh(a*x)^2/a - sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/a^2 - 3*(-2*I*x
+ 2*I*sqrt(a^2*x^2 - 1)*arccosh(a*x)/a)/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\operatorname{acosh}(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2),x)
```

```
[Out] int((x*acosh(a*x)^3)/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\operatorname{acosh}^3(ax)}{\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x*acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.257 $\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\sqrt{ax-1}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-ax}}$$

```
[Out] 1/4*arccosh(a*x)^4*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 45, normalized size = 1.41

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^4}{4a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^3/Sqrt[1 - a^2*x^2],x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^4)/(4*a*Sqrt[1 - a^2*x^2])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^3}{a^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

maple [A] time = 0.06, size = 51, normalized size = 1.59

$$-\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{arccosh}\,(ax)^4}{4a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
[Out] -1/4*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*arc
cosh(a*x)^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/sqrt(-a^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\text{acosh}\,(a\,x)^3}{\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2),x)
[Out] int(acosh(a*x)^3/(1 - a^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{acosh}^3\,(ax)}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/sqrt(-(a*x - 1)*(a*x + 1)), x)
```

3.258 $\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=265

$$-\frac{3i\sqrt{ax-1}\cosh^{-1}(ax)^2\text{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3i\sqrt{ax-1}\cosh^{-1}(ax)^2\text{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{6i\sqrt{ax-1}\cosh^{-1}(ax)\text{Li}}{\sqrt{1-ax}}$$

```
[Out] 2*arccosh(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*
x+1)^(1/2)-3*I*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)
))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3*I*arccosh(a*x)^2*polylog(2,I*(a*x+(a*x-1)
^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+6*I*arccosh(a*x)*polylo
g(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-6*I*
arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(
-a*x+1)^(1/2)-6*I*polylog(4,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(
1/2)/(-a*x+1)^(1/2)+6*I*polylog(4,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x
-1)^(1/2)/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 356, normalized size of antiderivative = 1.34, number of steps used = 11, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5798, 5761, 4180, 2531, 6609, 2282, 6589}

$$-\frac{3i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{3i\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)^2\text{PolyLog}\left(2,ie^{\cos}\right.}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[a*x]])/Sqrt
[1 - a^2*x^2] - ((3*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2*PolyLog[
2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*ArcCosh[a*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((6*
I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]
])/Sqrt[1 - a^2*x^2] - ((6*I)*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*Pol
yLog[3, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((6*I)*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((6*I)*Sqrt[-1
 + a*x]*Sqrt[1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
```

```
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x^3\mathrm{sech}(x)\,dx, x, \cosh^{-1}(ax)\right)}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{\left(3i\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int x\right.}{\sqrt{1-a\right.}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\mathrm{L}}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\mathrm{L}}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\mathrm{L}}{\sqrt{1-a^2x^2}}$$
$$= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3\tan^{-1}\left(e^{\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}} - \frac{3i\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2\mathrm{L}}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.66, size = 488, normalized size = 1.84

$$\frac{i\sqrt{-((ax-1)(ax+1))}\left(192\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)+192i\pi\cosh^{-1}(ax)\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)+384\cosh^{-1}(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((I/64)*Sqrt[-((-1 + a*x)*(1 + a*x))]*(7*Pi^4 + (8*I)*Pi^3*ArcCosh[a*x] + 2
4*Pi^2*ArcCosh[a*x]^2 - (32*I)*Pi*ArcCosh[a*x]^3 - 16*ArcCosh[a*x]^4 + (8*I
)*Pi^3*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*ArcCosh[a*x]*Log[1 + I/E^ArcCosh
[a*x]] - (96*I)*Pi*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*ArcCosh[a*
x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a
*x]] + (96*I)*Pi*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I)*Pi^3*Log[
1 + I*E^ArcCosh[a*x]] + 64*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)
*Pi^3*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*(Pi - (2*I)*ArcCosh[a*x])^
2*PolyLog[2, (-I)/E^ArcCosh[a*x]] + 192*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^Ar
cCosh[a*x]] - 48*Pi^2*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*ArcCosh[a*x
]*PolyLog[2, I*E^ArcCosh[a*x]] + (192*I)*Pi*PolyLog[3, (-I)/E^ArcCosh[a*x]]
 + 384*ArcCosh[a*x]*PolyLog[3, (-I)/E^ArcCosh[a*x]] - 384*ArcCosh[a*x]*Poly
Log[3, (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*PolyLog[3, I*E^ArcCosh[a*x]] + 384
*PolyLog[4, (-I)/E^ArcCosh[a*x]] + 384*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/(S
qrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\mathrm{arcosh}\,(ax)^3}{a^2x^3-x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^3 - x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

maple [F] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^3}{x\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
[Out] int(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{x\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^3/(x*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/(x*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.259 $\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}{x}-\frac{3a\sqrt{ax-1}\cosh^{-1}(ax)\text{Li}_2\left(-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-ax}}+\frac{3a\sqrt{ax-1}\,\text{Li}_3\left(-e^{2\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{a\sqrt{ax-1}}{\sqrt{}}$$

```
[Out] a*arccosh(a*x)^3*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*a*arccosh(a*x)^2*ln(1+(a*x+
(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*a*arccosh(a*
x)*polylog(2,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-a*x+1)^(
1/2)+3/2*a*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)-arccosh(a*x)^3*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] time = 0.49, antiderivative size = 229, normalized size of antiderivative = 1.38, number of steps used = 8, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5798, 5724, 5660, 3718, 2190, 2531, 2282, 6589}

$$-\frac{3a\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)\text{PolyLog}\left(2,-e^{2\cosh^{-1}(ax)}\right)}{\sqrt{1-a^2x^2}}+\frac{3a\sqrt{ax-1}\sqrt{ax+1}\,\text{PolyLog}\left(3,-e^{2\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3)/Sqrt[1 - a^2*x^2] - ((1 - a
*x)*(1 + a*x)*ArcCosh[a*x]^3)/(x*Sqrt[1 - a^2*x^2]) - (3*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]*ArcCosh[a*x]^2*Log[1 + E^(2*ArcCosh[a*x])])/Sqrt[1 - a^2*x^2]
- (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[2, -E^(2*ArcCosh[a
*x])])/Sqrt[1 - a^2*x^2] + (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[3, -E^
(2*ArcCosh[a*x])])/(2*Sqrt[1 - a^2*x^2])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
```

```
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5724

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^(m +
 1)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*
f*(m + 1)), x] + Dist[(b*c*n*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*
(d2 + e2*x)^FracPart[p])/(f*(m + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPa
rt[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n
- 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m, p}, x] && EqQ[e1 - c*
d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -
1] && IntegerQ[p + 1/2]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^2\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\text{Subst}\left(\int x^2\tanh(x)\,dx, x, \cosh\right.}{\sqrt{1-a^2x^2}}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{\left(6a\sqrt{-1+ax}\,\sqrt{1+a\right.}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

$$= \frac{a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^3}{\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{x\sqrt{1-a^2x^2}} - \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}}{}$$

Mathematica [A] time = 0.54, size = 137, normalized size = 0.83

$$\frac{a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\left(6\cosh^{-1}(ax)\text{Li}_2\left(-e^{-2\cosh^{-1}(ax)}\right)+3\text{Li}_3\left(-e^{-2\cosh^{-1}(ax)}\right)+2\cosh^{-1}(ax)^2\left(\frac{\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\cosh^{-1}(a\right.}{ax}\right.}{2\sqrt{-((ax-1)(ax+1))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^2*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(2*ArcCosh[a*x]^2*(-ArcCosh[a*x] +
(Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x])/(a*x) - 3*Log[1 + E^(-2
*ArcCosh[a*x])]) + 6*ArcCosh[a*x]*PolyLog[2, -E^(-2*ArcCosh[a*x])] + 3*Poly
Log[3, -E^(-2*ArcCosh[a*x])]))/(2*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)^3}{a^2x^4-x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^4 - x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\text{arcosh}\,(ax)^3}{\sqrt{-a^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.38, size = 313, normalized size = 1.89

$$-\frac{\sqrt{-a^2x^2+1}\,\left(a^2x^2-\sqrt{ax+1}\,\sqrt{ax-1}\,ax-1\right)\operatorname{arccosh}(ax)^3}{x\left(a^2x^2-1\right)}-\frac{2\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{arccosh}(ax)^3\,a}{a^2x^2-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x)
[Out] -(-a^2*x^2+1)^(1/2)*(a^2*x^2-(a*x+1)^(1/2)*(a*x-1)^(1/2)*a*x-1)*arccosh(a*x
)^3/x/(a^2*x^2-1)-2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2
-1)*arccosh(a*x)^3*a+3*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*
x^2-1)*arccosh(a*x)^2*ln(1+(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a+3*(-a^2*x
^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/(a^2*x^2-1)*arccosh(a*x)*polylog(2,
-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)*a-3/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2
)*(a*x+1)^(1/2)/(a^2*x^2-1)*polylog(3,-(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))^2)
*a
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(a^2x^2-1\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^3}{\sqrt{ax+1}\,\sqrt{-ax+1}\,x}-\int\frac{3\left(a^3x^2+\sqrt{ax+1}\,\sqrt{ax-1}\,a^2x-a\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)^2}{\left(\sqrt{ax+1}\,ax^2+(ax+1)\sqrt{ax-1}\,x\right)\sqrt{-ax+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^2/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] (a^2*x^2 - 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^3/(sqrt(a*x + 1)*sqrt(
-a*x + 1)*x) - integrate(3*(a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a
)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))^2/((sqrt(a*x + 1)*a*x^2 + (a*x + 1
)*sqrt(a*x - 1)*x)*sqrt(-a*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\operatorname{acosh}(a\,x)^3}{x^2\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^3/(x^2*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\operatorname{acosh}^3(ax)}{x^2\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**2/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/(x**2*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.260 $\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx$

Optimal. Leaf size=460

$$-\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}(ax)^2\mathrm{Li}_2\left(ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-ax}}+\frac{3ia^2\sqrt{ax-1}\,\cosh^{-1}}{\sqrt{1}}$$

```
[Out] 3/2*a*arccosh(a*x)^2*(a*x-1)^(1/2)/x/(-a*x+1)^(1/2)-6*a^2*arccosh(a*x)*arct
an(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+a^2*arccos
h(a*x)^3*arctan(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2))*(a*x-1)^(1/2)/(-a*x+1)^(1/
2)+3*I*a^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-
a*x+1)^(1/2)-3/2*I*a^2*arccosh(a*x)^2*polylog(2,-I*(a*x+(a*x-1)^(1/2)*(a*x+
1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*polylog(2,I*(a*x+(a*x-1)^(1
/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3/2*I*a^2*arccosh(a*x)^2*p
olylog(2,I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+
3*I*a^2*arccosh(a*x)*polylog(3,-I*(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1
)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*arccosh(a*x)*polylog(3,I*(a*x+(a*x-1)^(1/2)*
(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-3*I*a^2*polylog(4,-I*(a*x+(a*x
-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)+3*I*a^2*polylog(4,I*
(a*x+(a*x-1)^(1/2)*(a*x+1)^(1/2)))*(a*x-1)^(1/2)/(-a*x+1)^(1/2)-1/2*arccosh
(a*x)^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] time = 1.01, antiderivative size = 614, normalized size of antiderivative = 1.33, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5798, 5748, 5761, 4180, 2531, 6609, 2282, 6589, 5662, 2279, 2391}

$$-\frac{3ia^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,-ie^{\cosh^{-1}(ax)}\right)}{2\sqrt{1-a^2x^2}}+\frac{3ia^2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^2\mathrm{PolyLog}\left(2,\right.}{2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] (3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^2)/(2*x*Sqrt[1 - a^2*x^2]) -
 ((1 - a*x)*(1 + a*x)*ArcCosh[a*x]^3)/(2*x^2*Sqrt[1 - a^2*x^2]) - (6*a^2*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*ArcTan[E^ArcCosh[a*x]])/Sqrt[1 - a^
2*x^2] + (a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^3*ArcTan[E^ArcCosh[
a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[
2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - (((3*I)/2)*a^2*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]*ArcCosh[a*x]^2*PolyLog[2, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*
x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*PolyLog[2, I*E^ArcCosh[a*x]]
)/Sqrt[1 - a^2*x^2] + (((3*I)/2)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a
*x]^2*PolyLog[2, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3, (-I)*E^ArcCosh[a*x]])/Sqrt[1 -
 a^2*x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*PolyLog[3,
 I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] - ((3*I)*a^2*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]*PolyLog[4, (-I)*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2] + ((3*I)*a^2*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]*PolyLog[4, I*E^ArcCosh[a*x]])/Sqrt[1 - a^2*x^2]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4180

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(-(I*e) + f*fz*x)/E^(
I*k*Pi)])/(f*fz*I), x] + (-Dist[(d*m)/(f*fz*I), Int[(c + d*x)^(m - 1)*Log[1
 - E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[(d*m)/(f*fz*I), Int[(c +
d*x)^(m - 1)*Log[1 + E^(-(I*e) + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5748

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d1_) + (e
1_.)*(x_))^(p_)*((d2_) + (e2_.)*(x_))^(p_), x_Symbol] :> Simp[((f*x)^(m + 1
)*(d1 + e1*x)^(p + 1)*(d2 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(d1*d2*f*
(m + 1)), x] + (Dist[(c^2*(m + 2*p + 3))/(f^2*(m + 1)), Int[(f*x)^(m + 2)*(
d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^n, x], x] + Dist[(b*c*n*(-(
d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(f*(m +
 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 +
c^2*x^2)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c,
 d1, e1, d2, e2, f, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ
[n, 0] && LtQ[m, -1] && IntegerQ[m] && IntegerQ[p + 1/2]
```

Rule 5761

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/(Sqrt[(d1_) + (e1
_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[1/(c^(m + 1)*Sqrt[-
(d1*d2)]), Subst[Int[(a + b*x)^n*Cosh[x]^m, x], x, ArcCosh[c*x]], x] /; Fre
eQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0]
&& IGtQ[n, 0] && GtQ[d1, 0] && LtQ[d2, 0] && IntegerQ[m]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^3}{x^3\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{1-a^2x^2}}$$

$$= -\frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^2}{x^2}\,dx}{2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\right.}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{\left(a^2\sqrt{-1+ax}\,\sqrt{1+\right.}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} + \frac{a^2\sqrt{-1+ax}\,\sqrt{1+}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\,\sqrt{1+}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\,\sqrt{1+}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\,\sqrt{1+}{}$$

$$= \frac{3a\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^2}{2x\sqrt{1-a^2x^2}} - \frac{(1-ax)(1+ax)\cosh^{-1}(ax)^3}{2x^2\sqrt{1-a^2x^2}} - \frac{6a^2\sqrt{-1+ax}\,\sqrt{1+}{}$$

Mathematica [B] time = 6.26, size = 1051, normalized size = 2.28

$$-\frac{ia^2(ax+1)\left(-16\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)^4 + \frac{64i(ax-1)\cosh^{-1}(ax)^3}{a^2x^2} - 64\sqrt{\frac{ax-1}{ax+1}}\log\left(1+ie^{-\cosh^{-1}(ax)}\right)\cosh^{-1}(ax)^3 + 64\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[ArcCosh[a*x]^3/(x^3*Sqrt[1 - a^2*x^2]),x]
```

```
[Out] ((-1/128*I)*a^2*(1 + a*x)*(7*Pi^4*Sqrt[(-1 + a*x)/(1 + a*x)] + (8*I)*Pi^3*S
qrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x] + 24*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]
*ArcCosh[a*x]^2 + ((192*I)*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2)/(a*x)
 + ((64*I)*(-1 + a*x)*ArcCosh[a*x]^3)/(a^2*x^2) - (32*I)*Pi*Sqrt[(-1 + a*x)
/(1 + a*x)]*ArcCosh[a*x]^3 - 16*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^4 -
 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I/E^ArcCosh[a*x]] + (8
*I)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I/E^ArcCosh[a*x]] + 384*Sqrt[(-
1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] + 48*Pi^2*Sqrt[(
-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 + I/E^ArcCosh[a*x]] - (96*I)*Pi*Sqr
t[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 + I/E^ArcCosh[a*x]] - 64*Sqrt[
(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I/E^ArcCosh[a*x]] - 48*Pi^2*Sq
rt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*Log[1 - I*E^ArcCosh[a*x]] + (96*I)*Pi
*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*Log[1 - I*E^ArcCosh[a*x]] - (8*I
)*Pi^3*Sqrt[(-1 + a*x)/(1 + a*x)]*Log[1 + I*E^ArcCosh[a*x]] + 64*Sqrt[(-1 +
 a*x)/(1 + a*x)]*ArcCosh[a*x]^3*Log[1 + I*E^ArcCosh[a*x]] + (8*I)*Pi^3*Sqrt
[(-1 + a*x)/(1 + a*x)]*Log[Tan[(Pi + (2*I)*ArcCosh[a*x])/4]] - 48*Sqrt[(-1
+ a*x)/(1 + a*x)]*(8 + Pi^2 - (4*I)*Pi*ArcCosh[a*x] - 4*ArcCosh[a*x]^2)*Pol
yLog[2, (-I)/E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I/
E^ArcCosh[a*x]] + 192*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]^2*PolyLog[2,
(-I)*E^ArcCosh[a*x]] - 48*Pi^2*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[2, I*E^Ar
cCosh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[2,
 I*E^ArcCosh[a*x]] + (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, (-I)/
E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3, (-
I)/E^ArcCosh[a*x]] - 384*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*x]*PolyLog[3,
 (-I)*E^ArcCosh[a*x]] - (192*I)*Pi*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[3, I*
E^ArcCosh[a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)/E^ArcCosh[
a*x]] + 384*Sqrt[(-1 + a*x)/(1 + a*x)]*PolyLog[4, (-I)*E^ArcCosh[a*x]]))/Sq
rt[1 - a^2*x^2]
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)^3}{a^2x^5-x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3/(a^2*x^5 - x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^3}{x^3\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] int(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^3}{\sqrt{-a^2x^2+1}\,x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^3/x^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^3/(sqrt(-a^2*x^2 + 1)*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\operatorname{acosh}(a\,x)^3}{x^3\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)),x)
[Out] int(acosh(a*x)^3/(x^3*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^3(ax)}{x^3\sqrt{-(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**3/x**3/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(acosh(a*x)**3/(x**3*sqrt(-(a*x - 1)*(a*x + 1))), x)
```

3.261 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^3)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 3.68, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^3}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^3)/Sqrt[1 - c^2*x^2], x]
```

fricas [A] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b^3\,\text{arcosh}\,(cx)^3+3\,ab^2\,\text{arcosh}\,(cx)^2+3\,a^2b\,\text{arcosh}\,(cx)+a^3\right)\sqrt{-c^2x^2+1}\,\left(fx\right)^m}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-(b^3*arccosh(c*x)^3 + 3*a*b^2*arccosh(c*x)^2 + 3*a^2*b*arccosh(c*
x) + a^3)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(c^2*x^2 - 1), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^3\left(fx\right)^m}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{3}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{3} \left(fx\right)^{m}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^3/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^3*(f*x)^m/sqrt(-c^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{3} \left(f\,x\right)^{m}}{\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^3*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{3}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**3/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**3/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.262 $\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=67

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

```
[Out] 35/64*c^3*Shi(arccosh(a*x))/a-21/64*c^3*Shi(3*arccosh(a*x))/a+7/64*c^3*Shi(
5*arccosh(a*x))/a-1/64*c^3*Shi(7*arccosh(a*x))/a
```

Rubi [A] time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5700, 3312, 3298}

$$\frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-1}(ax)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (35*c^3*SinhIntegral[ArcCosh[a*x]])/(64*a) - (21*c^3*SinhIntegral[3*ArcCosh
[a*x]])/(64*a) + (7*c^3*SinhIntegral[5*ArcCosh[a*x]])/(64*a) - (c^3*SinhInt
egral[7*ArcCosh[a*x]])/(64*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)}\,dx = -\frac{c^3\,\text{Subst}\left(\int \frac{\sinh^7(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{\left(ic^3\right)\text{Subst}\left(\int \left(\frac{35i\sinh(x)}{64x}-\frac{21i\sinh(3x)}{64x}+\frac{7i\sinh(5x)}{64x}-\frac{i\sinh(7x)}{64x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= -\frac{c^3\,\text{Subst}\left(\int \frac{\sinh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a}+\frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\sinh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a}-\frac{\left(21c^3\right.}{}$$

$$= \frac{35c^3\text{Shi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{21c^3\text{Shi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{7c^3\text{Shi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{c^3\text{Shi}\left(7\cosh^{-}\right.}{64a}$$

Mathematica [A] time = 0.28, size = 45, normalized size = 0.67

$$\frac{c^3\left(35\text{Shi}\left(\cosh^{-1}(ax)\right)-21\text{Shi}\left(3\cosh^{-1}(ax)\right)+7\text{Shi}\left(5\cosh^{-1}(ax)\right)-\text{Shi}\left(7\cosh^{-1}(ax)\right)\right)}{64a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x],x]
```

```
[Out] (c^3*(35*SinhIntegral[ArcCosh[a*x]] - 21*SinhIntegral[3*ArcCosh[a*x]] + 7*S
inhIntegral[5*ArcCosh[a*x]] - SinhIntegral[7*ArcCosh[a*x]]))/(64*a)
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^6c^3x^6-3\,a^4c^3x^4+3\,a^2c^3x^2-c^3}{\text{arcosh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2cx^2-c\right)^3}{\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

maple [A] time = 0.15, size = 44, normalized size = 0.66

$$\frac{c^3\,(35\,\text{Shi}\,(\text{arccosh}\,(ax))-21\,\text{Shi}\,(3\,\text{arccosh}\,(ax))+7\,\text{Shi}\,(5\,\text{arccosh}\,(ax))-\text{Shi}\,(7\,\text{arccosh}\,(ax)))}{64a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x),x)
```

```
[Out] 1/64/a*c^3*(35*Shi(arccosh(a*x))-21*Shi(3*arccosh(a*x))+7*Shi(5*arccosh(a*x
))-Shi(7*arccosh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(a^2cx^2-c\right)^3}{\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] -integrate((a^2*c*x^2 - c)^3/arccosh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c-a^2\,c\,x^2\right)^3}{\text{acosh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/acosh(a*x),x)
```

```
[Out] int((c - a^2*c*x^2)^3/acosh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3a^2x^2}{\operatorname{acosh}(ax)}\,dx + \int \left(-\frac{3a^4x^4}{\operatorname{acosh}(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acosh}(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acosh}(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x),x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x), x) + Integral(-3*a**4*x**4/acosh(a*
x), x) + Integral(a**6*x**6/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

3.263 $\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)} dx$

Optimal. Leaf size=50

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

```
[Out] 5/8*c^2*Shi(arccosh(a*x))/a-5/16*c^2*Shi(3*arccosh(a*x))/a+1/16*c^2*Shi(5*a
rccosh(a*x))/a
```

Rubi [A] time = 0.11, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {5700, 3312, 3298}

$$\frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
```

```
[Out] (5*c^2*SinhIntegral[ArcCosh[a*x]])/(8*a) - (5*c^2*SinhIntegral[3*ArcCosh[a*
x]])/(16*a) + (c^2*SinhIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c-a^2cx^2)^2}{\cosh^{-1}(ax)} dx &= \frac{c^2\,\text{Subst}\left(\int \frac{\sinh^5(x)}{x} dx, x, \cosh^{-1}(ax)\right)}{a}\\
&= -\frac{(ic^2)\,\text{Subst}\left(\int \left(\frac{5i\sinh(x)}{8x}-\frac{5i\sinh(3x)}{16x}+\frac{i\sinh(5x)}{16x}\right) dx, x, \cosh^{-1}(ax)\right)}{a}\\
&= \frac{c^2\,\text{Subst}\left(\int \frac{\sinh(5x)}{x} dx, x, \cosh^{-1}(ax)\right)}{16a}-\frac{(5c^2)\,\text{Subst}\left(\int \frac{\sinh(3x)}{x} dx, x, \cosh^{-1}(ax)\right)}{16a}+\frac{(5c
\end{aligned}$$

$$= \frac{5c^2\text{Shi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{5c^2\text{Shi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{c^2\text{Shi}\left(5\cosh^{-1}(ax)\right)}{16a}$$

Mathematica [A]   time = 0.17, size = 34, normalized size = 0.68

$$\frac{c^2\left(10\mathrm{Shi}\left(\cosh^{-1}(ax)\right)-5\mathrm{Shi}\left(3\cosh^{-1}(ax)\right)+\mathrm{Shi}\left(5\cosh^{-1}(ax)\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x],x]
```

```
[Out] (c^2*(10*SinhIntegral[ArcCosh[a*x]] - 5*SinhIntegral[3*ArcCosh[a*x]] + Sinh
Integral[5*ArcCosh[a*x]]))/(16*a)
```

fricas [F]   time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{a^4c^2x^4-2\,a^2c^2x^2+c^2}{\mathrm{arcosh}\left(ax\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x), x)
```

giac [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2-c\right)^2}{\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

maple [A]   time = 0.06, size = 33, normalized size = 0.66

$$\frac{c^2\left(10\,\mathrm{Shi}\left(\mathrm{arccosh}\left(ax\right)\right)-5\,\mathrm{Shi}\left(3\,\mathrm{arccosh}\left(ax\right)\right)+\mathrm{Shi}\left(5\,\mathrm{arccosh}\left(ax\right)\right)\right)}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

```
[Out] 1/16/a*c^2*(10*Shi(arccosh(a*x))-5*Shi(3*arccosh(a*x))+Shi(5*arccosh(a*x)))
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a^2cx^2-c\right)^2}{\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\left(c-a^2\,c\,x^2\right)^2}{\mathrm{acosh}\left(a\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/acosh(a*x),x)
```

```
[Out] int((c - a^2*c*x^2)^2/acosh(a*x), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int\left(-\frac{2a^2x^2}{\operatorname{acosh}(ax)}\right)dx+\int\frac{a^4x^4}{\operatorname{acosh}(ax)}\,dx+\int\frac{1}{\operatorname{acosh}(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x), x) + Integral(a**4*x**4/acosh(a*x),
 x) + Integral(1/acosh(a*x), x))
```

3.264 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=29

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

```
[Out] 3/4*c*Shi(arccosh(a*x))/a-1/4*c*Shi(3*arccosh(a*x))/a
```

Rubi [A] time = 0.08, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5700, 3312, 3298}

$$\frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x],x]
```

```
[Out] (3*c*SinhIntegral[ArcCosh[a*x]])/(4*a) - (c*SinhIntegral[3*ArcCosh[a*x]])/(
4*a)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)}\,dx &= -\frac{c\,\text{Subst}\left(\int \frac{\sinh^3(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{a}\\
&= -\frac{(ic)\,\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4x}-\frac{i\sinh(3x)}{4x}\right)dx,x,\cosh^{-1}(ax)\right)}{a}\\
&= -\frac{c\,\text{Subst}\left(\int \frac{\sinh(3x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{4a}+\frac{(3c)\,\text{Subst}\left(\int \frac{\sinh(x)}{x}\,dx,x,\cosh^{-1}(ax)\right)}{4a}\\
&= \frac{3c\text{Shi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{c\text{Shi}\left(3\cosh^{-1}(ax)\right)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 25, normalized size = 0.86

$$\frac{c\left(3\text{Shi}\left(\cosh^{-1}(ax)\right)-\text{Shi}\left(3\cosh^{-1}(ax)\right)\right)}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x],x]
[Out] (c*(3*SinhIntegral[ArcCosh[a*x]] - SinhIntegral[3*ArcCosh[a*x]]))/(4*a)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2cx^2-c}{\text{arcosh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2cx^2-c}{\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x), x)
```

maple [A] time = 0.06, size = 24, normalized size = 0.83

$$\frac{c\,(3\,\text{Shi}\,(\text{arccosh}\,(ax))-\text{Shi}\,(3\,\text{arccosh}\,(ax)))}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x),x)
[Out] 1/4/a*c*(3*Shi(arccosh(a*x))-Shi(3*arccosh(a*x)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2cx^2-c}{\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
[Out] -integrate((a^2*c*x^2 - c)/arccosh(a*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{c-a^2\,c\,x^2}{\text{acosh}\,(a\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/acosh(a*x),x)
```

```
[Out] int((c - a^2*c*x^2)/acosh(a*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\operatorname{acosh}(ax)}\,dx + \int \left(-\frac{1}{\operatorname{acosh}(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x),x)
```

```
[Out] -c*(Integral(a**2*x**2/acosh(a*x), x) + Integral(-1/acosh(a*x), x))
```

3.265 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
[Out] Defer[Int][1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx=\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Mathematica [A] time = 1.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]),x]
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]), x]
```

fricas [A] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2cx^2-c)\,\text{arcosh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="fricas")
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2cx^2-c)\,\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2}c\,x^{2}+c\right)\operatorname{arccosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\left(a^{2}cx^{2}-c\right)\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x),x, algorithm="maxima")
[Out] -integrate(1/((a^2*c*x^2 - c)*arccosh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)\,\left(c-a^{2}\,c\,x^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(c - a^2*c*x^2)),x)
[Out] int(1/(acosh(a*x)*(c - a^2*c*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2}x^{2}\operatorname{acosh}\left(ax\right)-\operatorname{acosh}\left(ax\right)}\,dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x),x)
[Out] -Integral(1/(a**2*x**2*acosh(a*x) - acosh(a*x)), x)/c
```

3.266 $\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)}, x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
[Out] Defer[Int][1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} \, dx = \int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} \, dx$$

Mathematica [A] time = 6.14, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^2 \cosh^{-1}(ax)} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]),x]
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]), x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(a^4c^2x^4 - 2\,a^2c^2x^2 + c^2)\,\text{arcosh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="fricas")
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a^2cx^2 - c)^2\,\text{arcosh}\,(ax)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^2 \operatorname{arccosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2-c\right)^2 \operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x),x, algorithm="maxima")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)\,\left(c-a^2\,c\,x^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(c - a^2*c*x^2)^2),x)
```

```
[Out] int(1/(acosh(a*x)*(c - a^2*c*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^4x^4\operatorname{acosh}\left(ax\right)-2a^2x^2\operatorname{acosh}\left(ax\right)+\operatorname{acosh}\left(ax\right)}\,dx}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x),x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x) - 2*a**2*x**2*acosh(a*x) + acosh(a*x)), x)
/c**2
```

3.267 $\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$-\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^5\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\right.}{32bc^5\sqrt{cx-}}$$

```
[Out] -1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*x-1)^
(1/2)+1/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*
x-1)^(1/2)+1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^
5/(c*x-1)^(1/2)-1/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^5/(c*x-1)^(1/2
)+1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^5/(c*x-1)
^(1/2)-1/16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^5/(c
*x-1)^(1/2)-1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c
^5/(c*x-1)^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^5\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\text{co}}{32}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(
32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*C
oshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x
]])/(32*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*
ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]
*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b
 + 4*ArcCosh[c*x]])/(16*b*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2
*x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^5*Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^4(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{32c^5\sqrt{-1+cx}\sqrt{1+cx}} +$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x\right.}{32c^5\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cos\right.}{16bc^5\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.51, size = 188, normalized size = 0.55

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(6\left(\frac{a}{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]) +
 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2*a)/b]
*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*Sinh[(4*a)/b]*SinhIntegral[4*(a/b
 + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(3
2*c^5*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^4}{b\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^4}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.66, size = 591, normalized size = 1.74

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\operatorname{Ei}\left(1,6\operatorname{arccosh}(cx)+\frac{6a}{b}\right)\mathrm{e}^{\frac{b\operatorname{arccosh}(cx)+6a}{b}}}{64\,(cx+1)\,c^5\,(cx-1)\,b}+\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/64
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*ar
ccosh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/16*(-
c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^5*ln(a+b*arccosh(c*x))/b+1/3
2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*ar
ccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccos
h(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^5/(c*x-1)/b-1/64*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c
*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^5/(c*x-1)/b+1/32*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)
-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^5/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^4}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^4/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x^4*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{4}\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**4*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

3.268 $\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a\cdot}{}\right.}{16bc^4\sqrt{cx-1}}$$

```
[Out] -1/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
+1/16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^
(1/2)+1/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*
x-1)^(1/2)+1/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*
x-1)^(1/2)-1/16*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^
4/(c*x-1)^(1/2)-1/16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)
/b/c^4/(c*x-1)^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 371, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\right.}{16bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*c^4*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt
[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c
^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegra
l[a/b + ArcCosh[c*x]])/(8*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c
^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^4*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[
(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{8(a+bx)}+\frac{\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cos\right.}{16c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\right)}{16c}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}\right.}{16bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.46, size = 171, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\cos\right.\right.\right.}{16c^4\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[5*
```

```
(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh
[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral
[5*(a/b + ArcCosh[c*x])]))/(16*c^4*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^3}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.42, size = 543, normalized size = 1.83

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,5\,\text{arccosh}\,(cx)+\frac{5a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+5a}{b}}}{32\,(cx+1)\,c^4\,(cx-1)\,b}+\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5
*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arc
cosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccos
h(c*x)-3*a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/32*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c
*x)-5*a/b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a
/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/16*(-c^2*x^2+1)^(1/2)
*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a
+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^3/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{1-c^2\,x^2}}{a+b\,\text{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

3.269 $\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\log\left(a+b\cosh^{-1}(\right.}{8bc^3\sqrt{cx-1}}$$

```
[Out] 1/8*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1
/2)-1/8*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)-1/8*Shi(4*(
a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{4a}{b}\right)\text{Shi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\log\left(\right.}{8bc^3\sqrt{cx-\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8
*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh
[c*x]])/(8*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4
*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c^3*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(-\frac{1}{8(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{4a}{b}+4x\right)}{a+bx}\,dx, x\right.}{8c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{8bc^3\sqrt{-1+cx}\sqrt{1+cx}}-$$

Mathematica [A] time = 0.31, size = 103, normalized size = 0.74

$$-\frac{\sqrt{-((cx-1)(cx+1))}\left(-\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\right.\right.}{8bc^3\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(-(Cosh[(4*a)/b]*CoshIntegral[4*(a/b +
ArcCosh[c*x])]) + Log[a + b*ArcCosh[c*x]] + Sinh[(4*a)/b]*SinhIntegral[4*(a
/b + ArcCosh[c*x])]))/(b*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^2}{b\,\mathrm{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)

maple [A] time = 0.33, size = 227, normalized size = 1.63

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\operatorname{Ei}\left(1,4\operatorname{arccosh}(cx)+\frac{4a}{b}\right)\mathrm{e}^{\frac{b\operatorname{arccosh}(cx)+4a}{b}}}{16\,(cx+1)\,c^3\,(cx-1)\,b}+\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx-\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)

[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^2}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)

[Out] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)

[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)

3.270 $\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=197

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}}{b}\right.}{4bc^2\sqrt{cx-1}}$$

```
[Out] -1/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
+1/4*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(
1/2)+1/4*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(
1/2)-1/4*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-
1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 245, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)}{4bc^2\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(4*b*c^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[
1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/
b + 3*ArcCosh[c*x]])/(4*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx, x,\right.}{4c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right.\right.}{}$$

$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh\right.}{4bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.31, size = 127, normalized size = 0.64

$$\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-}\right.\right.}{4c^2\sqrt{\frac{cx-1}{cx+1}}\,(bcx+b)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Cosh[(3
*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[a/b]*SinhIntegral[a/b +
ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*c^2
*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x}{b\,\text{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

maple [B] time = 0.20, size = 361, normalized size = 1.83

$$\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\operatorname{Ei}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)e^{\frac{b\operatorname{arccosh}(cx)+3a}{b}}}{8(cx+1)c^2(cx-1)b}+\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*
arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/8*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arcc
osh(c*x)-3*a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/8*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp
((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\sqrt{1-c^2x^2}}{a+b\operatorname{acosh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(cx-1)(cx+1)}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

3.271 $\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\log\left(a+b\cosh^{-1}(cx\right.}{2bc\sqrt{cx-1}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2
)-1/2*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-1/2*Shi(2*(a+b*
arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\log\left(a+\right.}{2bc\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2
*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c
*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/
b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c
*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
```

```
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{2c\sqrt{-1+cx}\sqrt{1+cx}} \\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx \right.}{2c\sqrt{-1+cx}\sqrt{1+cx}} \\
&= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}-
\end{aligned}$$

Mathematica [A] time = 0.20, size = 105, normalized size = 0.76

$$\frac{\sqrt{-((cx-1)(cx+1))}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\log\left(a+b\,\text{c}\right.\right.}{2bc\sqrt{\frac{cx-1}{cx+1}}(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[-((-1 + c*x)*(1 + c*x))]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh
[c*x])] - Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + Arc
Cosh[c*x])]))/(2*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b\,\text{arcosh}(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

[Out] integral(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

maple [A] time = 0.18, size = 227, normalized size = 1.63

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{Ei}\left(1,2\,\mathrm{arccosh}\,(cx)+\frac{2a}{b}\right)\mathrm{e}^{\frac{b\,\mathrm{arccosh}(cx)+2a}{b}}}{4\,(cx+1)\,(cx-1)\,cb}+\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)

[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b-1/2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")

[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)),x)

[Out] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)\,(cx+1)}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)

[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)

3.272 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=117

$$\text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}}{b}\right.}{b\sqrt{1-cx}}$$

```
[Out] -Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Shi((a+
b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/
x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 1.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(b*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcC
osh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int]
[1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int\frac{\sqrt{-1+cx}\sqrt{1+cx}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{\sqrt{1-c^2x^2}\int\left(-\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(c^2\sqrt{1-c^2x^2}\right)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\right.}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int\frac{\cosh(x)}{a+bx}\,dx,x,\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\right.}{\sqrt{-1+cx}\sqrt{1}}$$

$$=\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx\operatorname{arcosh}(cx)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x(a+b\operatorname{arccosh}(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))), x)
```

3.273 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=66

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{c\sqrt{cx-1}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{1-cx}}$$

```
[Out] -c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/x^2/(
a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.93, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
+ b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\int \left(\frac{c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\right)dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(c^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\,c\right)}}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx^2\,\text{arcosh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b\operatorname{arccosh}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\operatorname{arcosh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\operatorname{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^2\left(a+b\operatorname{acosh}(cx)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))), x)
```

3.274 $\int \frac{\sqrt{1-c^2x^2}}{x^3(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^3*(a + b*Ar
cCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx^3\operatorname{arcosh}(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^3\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\,\mathrm{arcosh}\left(cx\right)+a)x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^3\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))), x)
```

3.275 $\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^4*(a + b*Ar
cCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{bx^4\,\text{arcosh}\,(cx)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) + a*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\,\text{arcosh}\,(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

maple [A] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{(b\,\mathrm{arcosh}\left(cx\right)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acosh(c*x))), x)
```

3.276 $\int \frac{x^3(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+}{}\right.}{64bc^4\sqrt{cx-1}}$$

```
[Out] -3/64*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2
)+3/64*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)
^(1/2)+1/64*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^4/(c
*x-1)^(1/2)-1/64*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c
^4/(c*x-1)^(1/2)+3/64*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/
c^4/(c*x-1)^(1/2)-3/64*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/
2)/b/c^4/(c*x-1)^(1/2)-1/64*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1
)^(1/2)/b/c^4/(c*x-1)^(1/2)+1/64*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
```

Rubi [A] time = 0.95, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\right.}{64bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(64*b*c^4
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(6
4*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*Co
shIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) + (3*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(64*b
*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*Sin
hIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]]
)/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(7*a)/b
]*SinhIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{3\cosh(x)}{64(a+bx)}-\frac{3\cosh(3x)}{64(a+bx)}-\frac{\cosh(5x)}{64(a+bx)}+\frac{\cosh(7x)}{64(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, c\right.}{64c^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3\sqrt{1-c^2x^2}\cosh\right.}{}$$

$$= -\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^4\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{c}\right.}{64bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.94, size = 215, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-3\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-3*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] + Cosh[(5*a)/b]*CoshIntegral[
5*(a/b + ArcCosh[c*x])] - Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x])
] + 3*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Sinh[(3*a)/b]*SinhInte
gral[3*(a/b + ArcCosh[c*x])] - Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[
c*x])] + Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^4*Sqrt[
(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^5-x^3\right)\sqrt{-c^2x^2+1}}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^5 - x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

maple [B] time = 0.44, size = 725, normalized size = 1.83

$$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,7\,\text{arccosh}\,(cx)+\frac{7a}{b}\right)\mathrm{e}^{\frac{b\,\text{arccosh}(cx)+7a}{b}}}{128\,(cx+1)\,c^4\,(cx-1)\,b}-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1
,7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b-1/
128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7
*arccosh(c*x)-7*a/b)*exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/12
8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*ar
ccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcco
sh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c
*x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b+3/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*
a/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/128*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/
b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/128*(-c^2*x^2+1)^(1/
2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp(
(a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^3}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^3/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(1-c^2\,x^2\right)^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x^3*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3} \left(- \left(cx - 1\right) \left(cx + 1\right)\right)^{\frac{3}{2}}}{a + b \operatorname{acosh}{\left(cx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**3*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

3.277 $\int \frac{x^2(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(\frac{6(a+b}{}\right.}{32bc^3\sqrt{cx-1}}$$

```
[Out] 1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(
1/2)+1/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x
-1)^(1/2)-1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^3
/(c*x-1)^(1/2)-1/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(1/2)
-1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^
(1/2)-1/16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*
x-1)^(1/2)+1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c^
3/(c*x-1)^(1/2)
```

Rubi [A] time = 0.88, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\right.}{32bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(3
2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x]
])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Log[a + b*A
rcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*
Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b
+ 4*ArcCosh[c*x]])/(16*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*
x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int\left(\frac{1}{16(a+bx)}-\frac{\cosh(2x)}{32(a+bx)}-\frac{\cosh(4x)}{16(a+bx)}+\frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx\right.}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A] time = 0.74, size = 188, normalized size = 0.55

$$-\frac{\sqrt{1-c^2x^2}\left(-\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(6\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/32*(Sqrt[1 - c^2*x^2]*(-(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x
])]) - 2*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]
*CoshIntegral[6*(a/b + ArcCosh[c*x])] + 2*Log[a + b*ArcCosh[c*x]] + Sinh[(2
*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 2*Sinh[(4*a)/b]*SinhIntegral[
4*(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])
]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^4-x^2\right)\sqrt{-c^2x^2+1}}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.52, size = 591, normalized size = 1.74

$$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,6\,\text{arccosh}\,(cx)+\frac{6a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+6a}{b}}}{64\,(cx+1)\,c^3\,(cx-1)\,b}-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/6
4*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*a
rccosh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/16*(
-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b+1/
32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*a
rccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(
c*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/32*(-c^2*x^
2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x
)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^2}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

3.278 $\int \frac{x(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=297

$$-\frac{\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a+b}{}\right.}{16bc^2\sqrt{cx-1}}$$

```
[Out] -1/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
+3/16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^
(1/2)-1/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*
x-1)^(1/2)+1/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*
x-1)^(1/2)-3/16*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b/c^
2/(c*x-1)^(1/2)+1/16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)
/b/c^2/(c*x-1)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 371, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16bc^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{5a}{b}\right.}{16bc^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*c^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegr
al[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sq
rt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhInteg
ral[a/b + ArcCosh[c*x]])/(8*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1
 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhInteg
ral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{\cosh(x)}{8(a+bx)}-\frac{3\cosh(3x)}{16(a+bx)}+\frac{\cosh(5x)}{16(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(5x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \right.}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3\sqrt{1-c^2x^2}\cosh\left(\right.\right.}{}$$
$$= -\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cos\right.}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.69, size = 172, normalized size = 0.58

$$\frac{\sqrt{1-c^2x^2}\left(-2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}+\right.\right.\right.}{16}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 3*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - Cosh[(5*a)/b]*CoshIntegral[
```

```
5*(a/b + ArcCosh[c*x])] + 2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*
Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + Sinh[(5*a)/b]*SinhInte
gral[5*(a/b + ArcCosh[c*x])]))/(16*c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*
x))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^3 - x\right)\sqrt{-c^2x^2 + 1}}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{3}{2}} x}{b\,\text{arcosh}\,(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

maple [B] time = 0.23, size = 543, normalized size = 1.83

$$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc + c^2x^2 - 1\right)\text{Ei}\left(1, 5\,\text{arccosh}\,(cx) + \frac{5a}{b}\right)e^{\frac{b\,\text{arccosh}(cx)+5a}{b}}}{32\,(cx+1)\,c^2\,(cx-1)\,b} - \frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/3
2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*a
rccosh(c*x)-5*a/b)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arcco
sh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b+3/32*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/
b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-1/16*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((
a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{3}{2}} x}{b\,\text{arcosh}\,(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-c^2 x^2\right)^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
[Out] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

3.279 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=239

$$\frac{\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\sinh\left(\frac{2a}{b}\right)\operatorname{Shi}\left(\frac{2(a+b\cdots}{}\right.}{2bc\sqrt{cx-1}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2
)-1/8*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)-3/8*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-1/2*Shi(2*(a+
b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)+1/8*Shi(4*(
a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 304, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\operatorname{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\operatorname{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\sinh\left(\frac{2\cdots}{}\right.}{2bc\sqrt{c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2
*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*CoshI
ntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
3*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[
c*x]])/(2*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(4*a)
/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*c*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^4(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(\frac{3}{8(a+bx)}-\frac{\cosh(2x)}{2(a+bx)}+\frac{\cosh(4x)}{8(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{3\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{8c\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= -\frac{3\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\,d\right.}{2c\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\,\text{co}\right.}{8bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.46, size = 147, normalized size = 0.62

$$-\frac{\sqrt{1-c^2x^2}\left(-4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\right.\right.}{8bc\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -1/8*(Sqrt[1 - c^2*x^2]*(-4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x
])] + Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + 3*Log[a + b*ArcC
osh[c*x]] + 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Sinh[(4*
a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(b*c*Sqrt[(-1 + c*x)/(1 + c*x)
]*(1 + c*x))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.21, size = 409, normalized size = 1.71

$$-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\operatorname{Ei}\left(1,4\operatorname{arccosh}\left(cx\right)+\frac{4a}{b}\right)e^{\frac{b\operatorname{arccosh}(cx)+4a}{b}}}{16\left(cx+1\right)\left(cx-1\right)cb}-\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b-1/16*
(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arc
cosh(c*x)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/(c*x-1)/c/b-3/8*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b+1/4*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c
*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+1/4*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*
a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{a+b\operatorname{acosh}(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

3.280 $\int \frac{(1-c^2x^2)^{3/2}}{x(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=216

$$\text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{5\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh}{b}\right.}{4b\sqrt{1-cx}}$$

```
[Out] -5/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/4
*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+5/4
*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/4*Shi
(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Uninteg
rable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 1.75, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(4*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral[(3
*a)/b + 3*ArcCosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c
^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*Arc
Cosh[c*x]])/(4*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Defer[I
nt][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \left(\frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{2c^2x}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{}{\sqrt{-1+cx}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx, x\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)} + \right.\right.}{\sqrt{-1+cx}\,\sqrt{1}}$$

$$= \frac{2\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{5\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\,\mathrm{c}\right.}{4b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{bx\,\mathrm{arcosh}\,(cx)+ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x*arccosh(c*x) + a*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{3}{2}}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{x \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))), x)
```

3.281 $\int \frac{(1-c^2x^2)^{3/2}}{x^2(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=164

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\,(a+b\cosh^{-1}(cx))},x\right)+\frac{c\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{2b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(\cdots\right.}{2b\sqrt{1-cx}}$$

```
[Out] 1/2*c*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2
)-3/2*c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/2*c*Shi(2*(a+
b*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(
1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 1.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-c^2x^2)^{3/2}}{x^2\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -(c*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])
/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcC
osh[c*x]])/(2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (c*Sqrt[1 - c^2*x^2]*Sinh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int\left(-\frac{2c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cdots}{\sqrt{-1+cx}\cdots}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(2c^2\sqrt{1-c^2x^2}\right)\int\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+\cdots}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\sqrt{1-c^2x^2}\int\frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}\cdots\right.}{2b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{bx^2\,\mathrm{arcosh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^2*arccosh(c*x) + a*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\mathrm{arcosh}\,(cx)+a\right)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{3}{2}}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))), x)
```

3.282 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^3*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.45, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{bx^3\,\text{arcosh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{x^{3}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{(b\,\mathrm{arcosh}\left(cx\right)+a)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(1-c^{2}\,x^{2}\right)^{3/2}}{x^{3}\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{3}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acosh(c*x))), x)
```

3.283 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]

[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^4*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])),x]

[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])), x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{bx^4\,\text{arcosh}\,(cx)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] integral((-c^2*x^2 + 1)^(3/2)/(b*x^4*arccosh(c*x) + a*x^4), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\text{arcosh}\,(cx)+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{4} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{3}{2}}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{4}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{x^{4} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acosh(c*x))), x)
```

3.284 $\int \frac{x^3(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{3\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{128bc^4\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\cosh\left(\frac{7a}{b}\right)\operatorname{Chi}\left(\frac{7(a+}{}\right.}{256bc^4\sqrt{cx-1}}$$

```
[Out] -3/128*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/
2)+1/32*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1
)^(1/2)-3/256*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c^4/
(c*x-1)^(1/2)+1/256*Chi(9*(a+b*arccosh(c*x))/b)*cosh(9*a/b)*(-c*x+1)^(1/2)/
b/c^4/(c*x-1)^(1/2)+3/128*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2
)/b/c^4/(c*x-1)^(1/2)-1/32*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)
^(1/2)/b/c^4/(c*x-1)^(1/2)+3/256*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)-1/256*Shi(9*(a+b*arccosh(c*x))/b)*sinh(9*a
/b)*(-c*x+1)^(1/2)/b/c^4/(c*x-1)^(1/2)
```

Rubi [A] time = 1.00, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{128bc^4\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{32bc^4\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\cosh\left(\right.}{256bc^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-3*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(128*b*c^
4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInte
gral[(3*a)/b + 3*ArcCosh[c*x]])/(32*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (
3*Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(
256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(9*a)/b]*
CoshIntegral[(9*a)/b + 9*ArcCosh[c*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(1
28*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*S
inhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(32*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcCosh[c
*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(9
*a)/b]*SinhIntegral[(9*a)/b + 9*ArcCosh[c*x]])/(256*b*c^4*Sqrt[-1 + c*x]*Sq
rt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^3(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{3\cosh(x)}{128(a+bx)}+\frac{\cosh(3x)}{32(a+bx)}-\frac{3\cosh(7x)}{256(a+bx)}+\frac{\cosh(9x)}{256(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(9x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{256c^4\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx,\right.}{256c^4\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{\left(3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{128c^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\right.\right.}{}$$

$$= -\frac{3\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{128bc^4\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cos\right.}{32bc^4\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.33, size = 216, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-6\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+8\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\cosh\left(\frac{7a}{b}\right)\text{Chi}\left(7\left(\frac{a}{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-6*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 8*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Cosh[(7*a)/b]*CoshIntegra
l[7*(a/b + ArcCosh[c*x])] + Cosh[(9*a)/b]*CoshIntegral[9*(a/b + ArcCosh[c*x
])] + 6*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 8*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 3*Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcC
osh[c*x])] - Sinh[(9*a)/b]*SinhIntegral[9*(a/b + ArcCosh[c*x])]))/(256*c^4*
Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^7 - 2\,c^2x^5 + x^3\right)\sqrt{-c^2x^2+1}}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^7 - 2*c^2*x^5 + x^3)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a
), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x^3}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

maple [B] time = 0.45, size = 725, normalized size = 1.83

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc + c^2x^2 - 1\right)\text{Ei}\left(1, 9\,\text{arccosh}\,(cx) + \frac{9a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+9a}{b}}}{512\,(cx+1)\,c^4\,(cx-1)\,b} + \frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/512*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
9*arccosh(c*x)+9*a/b)*exp((b*arccosh(c*x)+9*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/5
12*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-9*
arccosh(c*x)-9*a/b)*exp((b*arccosh(c*x)-9*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/512
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*arc
cosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(-c
^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh
(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^2*
x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x
)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^4/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a/b
)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/512*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*arccosh(c*x)-7*a/b)*
exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^4/(c*x-1)/b-3/256*(-c^2*x^2+1)^(1/2)*
(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+
b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x^3}{b\operatorname{arcosh}(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^3/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\left(1-c^2x^2\right)^{5/2}}{a+b\operatorname{acosh}(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
[Out] int((x^3*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.285 $\int \frac{x^2(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=439

$$\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a+b\cosh^{-1}(cx))}{b}\right)}{32bc^3\sqrt{cx-1}}$$

```
[Out] 1/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x-1)^(
1/2)+1/32*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c^3/(c*x
-1)^(1/2)-1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c^3
/(c*x-1)^(1/2)+1/128*Chi(8*(a+b*arccosh(c*x))/b)*cosh(8*a/b)*(-c*x+1)^(1/2)
/b/c^3/(c*x-1)^(1/2)-5/128*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c^3/(c*x-1
)^(1/2)-1/32*Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c^3/(
c*x-1)^(1/2)-1/32*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/
c^3/(c*x-1)^(1/2)+1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/
2)/b/c^3/(c*x-1)^(1/2)-1/128*Shi(8*(a+b*arccosh(c*x))/b)*sinh(8*a/b)*(-c*x+
1)^(1/2)/b/c^3/(c*x-1)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 556, normalized size of antiderivative = 1.27, number of steps used = 16, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{32bc^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{6a}{b}\right)}{32bc^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(3
2*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x]
])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(8*a)/
b]*CoshIntegral[(8*a)/b + 8*ArcCosh[c*x]])/(128*b*c^3*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(128*b*c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a
)/b + 2*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 -
c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(32*b*c^3*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Sinh[(6*a)/b]*SinhIntegral
[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt
[1 - c^2*x^2]*Sinh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*ArcCosh[c*x]])/(128*b*
c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5}{128(a+bx)}+\frac{\cosh(2x)}{32(a+bx)}+\frac{\cosh(4x)}{32(a+bx)}-\frac{\cosh(6x)}{32(a+bx)}+\frac{\cosh(8x)}{128(a+bx)}\right)dx, x, \cos$$
$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(8x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{128c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{128bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right)}{a+bx}\right.}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh\right.}{32bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.22, size = 233, normalized size = 0.53

$$\sqrt{1-c^2x^2}\left(4\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+4\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\cosh\left(\frac{6a}{b}\right)\text{Chi}\left(6\right.\right.$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]

[Out] (Sqrt[1 - c^2*x^2]*(4*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] + 4*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] - 4*Cosh[(6*a)/b]*CoshIntegral[6*(a/b + ArcCosh[c*x])] + Cosh[(8*a)/b]*CoshIntegral[8*(a/b + ArcCosh[c*x])] - 5*Log[a + b*ArcCosh[c*x]] - 4*Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 4*Sinh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] + 4*Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - Sinh[(8*a)/b]*SinhIntegral[8*(a/b + ArcCosh[c*x])]))/(128*c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^{4}x^{6}-2\,c^{2}x^{4}+x^{2}\right)\sqrt{-c^{2}x^{2}+1}}{b\,\text{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}x^{2}}{b\,\text{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")

[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)

maple [B] time = 0.56, size = 773, normalized size = 1.76

$$\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^{2}x^{2}-1\right)\text{Ei}\left(1,8\,\text{arccosh}\left(cx\right)+\frac{8a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+8a}{b}}}{256\left(cx+1\right)c^{3}\left(cx-1\right)b}+\frac{\sqrt{-c^{2}x^{2}+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)

[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/256*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-8*arccosh(c*x)-8*a/b)*exp((b*arccosh(c*x)-8*a)/b)/(c*x+1)/c^3/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3*ln(a+b*arccosh(c*x))/b-1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/c^3/(c*x-1)/b+1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-4*a/b)*exp((b*arccosh(c*x)-4*a)/b)/(c*x+1)/c^3/(c*x-1)/b-1/64*(-c^2*x^2+1)^

```
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arccosh(c*x)-6*a
/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/c^3/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}x^{2}}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
[Out] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Timed out
```

3.286 $\int \frac{x(1-c^2x^2)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=397

$$-\frac{5\sqrt{1-cx}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64bc^2\sqrt{cx-1}}+\frac{9\sqrt{1-cx}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64bc^2\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\cosh\left(\frac{5a}{b}\right)\operatorname{Chi}\left(\frac{5(a}{}\right.}{64bc^2\sqrt{cx-1}}$$

```
[Out] -5/64*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2
)+9/64*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(-c*x+1)^(1/2)/b/c^2/(c*x-1)
^(1/2)-5/64*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(-c*x+1)^(1/2)/b/c^2/(c
*x-1)^(1/2)+1/64*Chi(7*(a+b*arccosh(c*x))/b)*cosh(7*a/b)*(-c*x+1)^(1/2)/b/c
^2/(c*x-1)^(1/2)+5/64*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b/
c^2/(c*x-1)^(1/2)-9/64*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/
2)/b/c^2/(c*x-1)^(1/2)+5/64*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1
)^(1/2)/b/c^2/(c*x-1)^(1/2)-1/64*Shi(7*(a+b*arccosh(c*x))/b)*sinh(7*a/b)*(-
c*x+1)^(1/2)/b/c^2/(c*x-1)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 497, normalized size of antiderivative = 1.25, number of steps used = 16, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {5798, 5781, 5448, 3303, 3298, 3301}

$$-\frac{5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{9\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64bc^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{5\sqrt{1-c^2x^2}\cosh\left(\right.}{64bc^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(64*b*c^2
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshInt
egral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(5*Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/
(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*
CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (5*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(64
*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (9*Sqrt[1 - c^2*x^2]*Sinh[(3*a)/b]*S
inhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (5*Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c
*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(7*
a)/b]*SinhIntegral[(7*a)/b + 7*ArcCosh[c*x]])/(64*b*c^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64(a+bx)}+\frac{9\cosh(3x)}{64(a+bx)}-\frac{5\cosh(5x)}{64(a+bx)}+\frac{\cosh(7x)}{64(a+bx)}\right) dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\cosh(7x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x\right.}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(5\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(9\sqrt{1-c^2x^2}\,\cosh\right.}{}$$

$$= -\frac{5\sqrt{1-c^2x^2}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64bc^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{9\sqrt{1-c^2x^2}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\,\text{cc}\right.}{64bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.09, size = 216, normalized size = 0.54

$$\frac{\sqrt{1-c^2x^2}\left(-5\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+9\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-5\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(5\left(\frac{a}{b}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-5*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] + 9*Cosh[
(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 5*Cosh[(5*a)/b]*CoshIntegra
l[5*(a/b + ArcCosh[c*x])] + Cosh[(7*a)/b]*CoshIntegral[7*(a/b + ArcCosh[c*x
])] + 5*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 9*Sinh[(3*a)/b]*SinhIn
tegral[3*(a/b + ArcCosh[c*x])] + 5*Sinh[(5*a)/b]*SinhIntegral[5*(a/b + ArcC
osh[c*x])] - Sinh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCosh[c*x])]))/(64*c^2*S
qrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^5-2\,c^2x^3+x\right)\sqrt{-c^2x^2+1}}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

maple [B] time = 0.26, size = 725, normalized size = 1.83

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,7\,\text{arccosh}\,(cx)+\frac{7a}{b}\right)\text{e}^{\frac{b\,\text{arccosh}(cx)+7a}{b}}}{128\,(cx+1)\,c^2\,(cx-1)\,b}+\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,
7*arccosh(c*x)+7*a/b)*exp((b*arccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b+1/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-7*
arccosh(c*x)-7*a/b)*exp((b*arccosh(c*x)-7*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arc
cosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*(-
c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccos
h(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*
x)-a/b)*exp((b*arccosh(c*x)-a)/b)/(c*x+1)/c^2/(c*x-1)/b+9/128*(-c^2*x^2+1)^
(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c*x)-3*a
/b)*exp((b*arccosh(c*x)-3*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1
/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-5*arccosh(c*x)-5*a/b
)*exp((b*arccosh(c*x)-5*a)/b)/(c*x+1)/c^2/(c*x-1)/b-5/128*(-c^2*x^2+1)^(1/2
)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((
a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x}{b\operatorname{arcosh}\left(cx\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-c^2\,x^2\right)^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{a+b\operatorname{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x)), x)
```

3.287 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=339

$$\frac{15\sqrt{1-cx}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32bc\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16bc\sqrt{cx-1}}+\frac{\sqrt{1-cx}\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\ldots\right.}{32bc\sqrt{cx-\ldots}}$$

```
[Out] 15/32*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)-3/16*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)
^(1/2)+1/32*Chi(6*(a+b*arccosh(c*x))/b)*cosh(6*a/b)*(-c*x+1)^(1/2)/b/c/(c*x
-1)^(1/2)-5/16*ln(a+b*arccosh(c*x))*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)-15/32*
Shi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)+3/
16*Shi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1/2)
-1/32*Shi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x+1)^(1/2)/b/c/(c*x-1)^(1
/2)
```

Rubi [A] time = 0.57, antiderivative size = 430, normalized size of antiderivative = 1.27, number of steps used = 13, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {5713, 5701, 3312, 3303, 3298, 3301}

$$\frac{15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{16bc\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\cos\ldots}{32\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]])
/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]
*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*CoshIntegral[(6*a)/b + 6*ArcCosh[c*x
]])/(32*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Log[a + b*
ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (15*Sqrt[1 - c^2*x^2
]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(32*b*c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Sinh[(4*a)/b]*SinhIntegral[(4*a)/
b + 4*ArcCosh[c*x]])/(16*b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*
x^2]*Sinh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(32*b*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(1-c^2x^2\right)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh^6(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \left(\frac{5}{16(a+bx)}-\frac{15\cosh(2x)}{32(a+bx)}+\frac{3\cosh(4x)}{16(a+bx)}-\frac{\cosh(6x)}{32(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\cosh(6x)}{a+bx}\,dx, x, \cosh^{-1}(cx\right.}{32c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{5\sqrt{1-c^2x^2}\log\left(a+b\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cosh\left(\frac{2a}{b}+2x\right.}{a+bx}\right.}{32c\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{15\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{32bc\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+\right.}{16bc\sqrt{-1+cx}\,\sqrt{1+c}}
\end{aligned}$$

Mathematica [A] time = 0.75, size = 191, normalized size = 0.56

$$\frac{\sqrt{1-c^2x^2}\left(15\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-6\cosh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(6\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(15*Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])] -
 6*Cosh[(4*a)/b]*CoshIntegral[4*(a/b + ArcCosh[c*x])] + Cosh[(6*a)/b]*CoshI
ntegral[6*(a/b + ArcCosh[c*x])] - 10*Log[a + b*ArcCosh[c*x]] - 15*Sinh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 6*Sinh[(4*a)/b]*SinhIntegral[4*
(a/b + ArcCosh[c*x])] - Sinh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])])
)/(32*b*c*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4 - 2\,c^2x^2 + 1\right)\sqrt{-c^2x^2 + 1}}{b\operatorname{arcosh}\left(cx\right) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a),
 x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}}{b\operatorname{arcosh}\left(cx\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.24, size = 591, normalized size = 1.74

$$\frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\,\sqrt{cx-1}\,xc + c^2x^2 - 1\right)\operatorname{Ei}\left(1, 6\operatorname{arccosh}\left(cx\right) + \frac{6a}{b}\right)\mathrm{e}^{\frac{b\operatorname{arccosh}(cx)+6a}{b}}}{64\left(cx+1\right)\left(cx-1\right)cb} + \frac{\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6
*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c/b+1/64*(
-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-6*arcc
osh(c*x)-6*a/b)*exp((b*arccosh(c*x)-6*a)/b)/(c*x+1)/(c*x-1)/c/b-5/16*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c*ln(a+b*arccosh(c*x))/b-3/32*(-c^
2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(
c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x^2
+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+
2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b+15/64*(-c^2*x^2+1)^(
1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c*x)-2*a/
b)*exp((b*arccosh(c*x)-2*a)/b)/(c*x+1)/(c*x-1)/c/b-3/32*(-c^2*x^2+1)^(1/2)*
(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-4*arccosh(c*x)-4*a/b)*ex
p((b*arccosh(c*x)-4*a)/b)/(c*x+1)/(c*x-1)/c/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2 + 1\right)^{\frac{5}{2}}}{b\operatorname{arcosh}\left(cx\right) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{a+b\operatorname{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x)), x)
```

3.288 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=310

$$\text{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)-\frac{11\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b\sqrt{1-cx}}+\frac{7\sqrt{cx-1}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\,c}{}\right.}{16b\sqrt{1-cx}}$$

```
[Out] -11/8*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+7/
16*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1
/16*Chi(5*(a+b*arccosh(c*x))/b)*cosh(5*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+
11/8*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-7/1
6*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/
16*Shi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+U
nintegrable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 2.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (11*Sqrt[1 - c^2*x^2]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(8*b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (7*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*CoshIntegral
[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 -
 c^2*x^2]*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (11*Sqrt[1 - c^2*x^2]*Sinh[a/b]*SinhIntegral[a/b
 + ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (7*Sqrt[1 - c^2*x^2]
*Sinh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Sinh[(5*a)/b]*SinhIntegral[(5*a)/b + 5
*ArcCosh[c*x]])/(16*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*De
fer[Int][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])), x])/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int\left(-\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{3c^2x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{}{\sqrt{-1+cx}}\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(3c^2\sqrt{1-c^2x^2}\right)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a\right.}}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int\frac{\cosh^5(x)}{a+bx}dx,x\right.}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int\frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int\left(\frac{5\cosh(x)}{8(a+bx)}+\right.\right.}{\sqrt{-1+}}$$

$$= \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{3\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{3\sqrt{1-c^2x^2}\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{11\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\sqrt{1+cx}}-\frac{7\sqrt{1-c^2x^2}\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\right.}{16b\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)}dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{bx\,\text{arcosh}\,(cx)+ax},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x*arccosh(c*x) + a
*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{5}{2}}}{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}}}{x \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acosh(c*x))), x)
```

3.289 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=255

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)},x\right)+\frac{c\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b\sqrt{1-cx}}-\frac{c\sqrt{cx-1}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\right.}{8b\sqrt{1-cx}}$$

```
[Out] c*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-1/
8*c*Chi(4*(a+b*arccosh(c*x))/b)*cosh(4*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-
15/8*c*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)-c*Shi(2*(a+b*arc
cosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+1/8*c*Shi(4*(a+b*a
rccosh(c*x))/b)*sinh(4*a/b)*(c*x-1)^(1/2)/b/(-c*x+1)^(1/2)+Unintegrable(1/x
^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 2.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((c*Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]
)/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (c*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
(15*c*Sqrt[1 - c^2*x^2]*Log[a + b*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (c*Sqrt[1 - c^2*x^2]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcC
osh[c*x]])/(b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (c*Sqrt[1 - c^2*x^2]*Sinh[(4*
a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(8*b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) - (Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\sqrt{1-c^2x^2}\int\left(\frac{3c^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{}{\sqrt{-1+cx}\,\sqrt{1}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2\sqrt{1-c^2x^2}\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\,(a+b}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15c\sqrt{1-c^2x^2}\,\log\left(a+b\cosh^{-1}(cx)\right)}{8b\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}\right.}{8b\sqrt{-1+cx}\,\sqrt{1+}}$$

<u>Mathematica</u> [A] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])), x]
```

<u>fricas</u> [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{bx^2\,\text{arcosh}\,(cx)+ax^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^2*arccosh(c*x) +
 a*x^2), x)
```

<u>giac</u> [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{\left(b \operatorname{arcosh}\left(c x\right) + a\right) x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))),x)
[Out] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}}}{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x)),x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acosh(c*x))), x)
```

3.290 $\int \frac{(1-c^2x^2)^{5/2}}{x^3(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^3*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3(a+b\cosh^{-1}(cx))}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.42, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^3\,(a+b\cosh^{-1}(cx))}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(c^4x^4-2\,c^2x^2+1)\sqrt{-c^2x^2+1}}{bx^3\,\text{arcosh}\,(cx)+ax^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^3*arccosh(c*x) + a*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}}{(b\,\mathrm{arcosh}\left(cx\right)+a)x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^3), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(1-c^{2}\,x^{2}\right)^{5/2}}{x^{3}\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x^{3}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acosh(c*x))), x)
```

3.291 $\int \frac{(1-c^2x^2)^{5/2}}{x^4(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)

Rubi [A] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]

[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^4*(a + b*ArcCosh[c*x])), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.01, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])),x]

[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])), x]

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{bx^4\,\text{arcosh}\,(cx)+ax^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="fricas")

[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b*x^4*arccosh(c*x) + a*x^4), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{(b\,\text{arcosh}\,(cx)+a)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{x^4\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)*x^4), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**4*(a + b*acosh(c*x))), x)
```

3.292 $\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}+\frac{3\sqrt{ax-1}\,\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-ax}}$$

```
[Out] 1/2*Chi(2*arccosh(a*x))*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)+1/8*Chi(4*arccosh(
a*x))*(a*x-1)^(1/2)/a^5/(-a*x+1)^(1/2)+3/8*ln(arccosh(a*x))*(a*x-1)^(1/2)/a
^5/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 137, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(4\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}+\frac{3\sqrt{ax-1}\sqrt{ax+1}\,\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^5*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[4*ArcCosh[a*x]])/(8
*a^5*Sqrt[1 - a^2*x^2]) + (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]]
)/(8*a^5*Sqrt[1 - a^2*x^2])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^4}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^4(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^5\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{3}{8x}+\frac{\cosh(2x)}{2x}+\frac{\cosh(4x)}{8x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^5\sqrt{1-a^2x^2}}$$
$$= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{8a^5\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(4x}{x}\right.}{8a^5\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^5\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(4\cosh^{-1}(ax\right.}{8a^5\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.70

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(4\text{Chi}\left(2\cosh^{-1}(ax)\right)+\text{Chi}\left(4\cosh^{-1}(ax)\right)+3\log\left(\cosh^{-1}(ax)\right)\right)}{8a^5\sqrt{-((ax-1)(ax+1))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(4*CoshIntegral[2*ArcCosh[a*x]] + Cos
hIntegral[4*ArcCosh[a*x]] + 3*Log[ArcCosh[a*x]]))/(8*a^5*Sqrt[-((-1 + a*x)*
(1 + a*x))])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^4}{\left(a^2x^2-1\right)\operatorname{arcosh}(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^4/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-a^2x^2+1}\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

maple [B] time = 0.71, size = 249, normalized size = 2.54

$$\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,\text{Ei}(1,4\operatorname{arccosh}(ax))}{16a^5\left(a^2x^2-1\right)}+\frac{\sqrt{-a^2x^2+1}\sqrt{ax-1}\sqrt{ax+1}\,\text{Ei}(1,-4\operatorname{arccosh}(ax))}{16a^5\left(a^2x^2-1\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,4*
arccosh(a*x))+1/16*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*
x^2-1)*Ei(1,-4*arccosh(a*x))-3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)/a^5/(a^2*x^2-1)*ln(arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,2*arccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(
a*x-1)^(1/2)*(a*x+1)^(1/2)/a^5/(a^2*x^2-1)*Ei(1,-2*arccosh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(x^4/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.293 $\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{3\sqrt{ax-1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-ax}}$$

```
[Out] 3/4*Chi(arccosh(a*x))*(a*x-1)^(1/2)/a^4/(-a*x+1)^(1/2)+1/4*Chi(3*arccosh(a*
x))*(a*x-1)^(1/2)/a^4/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.45, antiderivative size = 91, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{3\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (3*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(4*a^4*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[3*ArcCosh[a*x]])/(4
*a^4*Sqrt[1 - a^2*x^2])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^3}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^3(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{3\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^4\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\left(3\sqrt{-1+ax}\sqrt{1+ax}\right.}{}$$
$$= \frac{3\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a^4\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 0.92

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\left(3\text{Chi}\left(\cosh^{-1}(ax)\right)+\text{Chi}\left(3\cosh^{-1}(ax)\right)\right)}{4a^4\sqrt{-((ax-1)(ax+1))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*(3*CoshIntegral[ArcCosh[a*x]] + CoshI
ntegral[3*ArcCosh[a*x]]))/(4*a^4*Sqrt[-((-1 + a*x)*(1 + a*x))])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^3}{\left(a^2x^2-1\right)\text{arcosh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^3/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.61, size = 200, normalized size = 3.08

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Ei}\,(1,3\,\text{arccosh}\,(ax))}{8a^4\left(a^2x^2-1\right)}+\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Ei}\,(1,-3\,\text{arccosh}\,(ax))}{8a^4\left(a^2x^2-1\right)}+\frac{3\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,3*a
rccosh(a*x))+1/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^4/(a^2*x^
2-1)*Ei(1,-3*arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a^4/(a^2*x^2-1)*Ei(1,arccosh(a*x))+3/8*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*
(a*x+1)^(1/2)/a^4/(a^2*x^2-1)*Ei(1,-arccosh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-a^2x^2+1}\ \operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(x^3/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(ax-1)(ax+1)}\ \operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.294 $\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}+\frac{\sqrt{ax-1}\,\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-ax}}$$

```
[Out] 1/2*Chi(2*arccosh(a*x))*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)+1/2*ln(arccosh(a*x
))*(a*x-1)^(1/2)/a^3/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 91, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {5798, 5781, 3312, 3301}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}+\frac{\sqrt{ax-1}\sqrt{ax+1}\,\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[2*ArcCosh[a*x]])/(2*a^3*Sqrt[1 -
 a^2*x^2]) + (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]])/(2*a^3*Sqrt[1
 - a^2*x^2])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x^2}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}$$
$$= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \left(\frac{1}{2x}+\frac{\cosh(2x)}{2x}\right)dx, x, \cosh^{-1}(ax)\right)}{a^3\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{x}\right.}{2a^3\sqrt{1-a^2x^2}}$$
$$= \frac{\sqrt{-1+ax}\sqrt{1+ax}\,\text{Chi}\left(2\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}} + \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{2a^3\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.10, size = 60, normalized size = 0.92

$$-\frac{\sqrt{-((ax-1)(ax+1))}\left(\text{Chi}\left(2\cosh^{-1}(ax)\right)+\log\left(\cosh^{-1}(ax)\right)\right)}{2a^3\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -1/2*(Sqrt[-((-1 + a*x)*(1 + a*x))]*(CoshIntegral[2*ArcCosh[a*x]] + Log[Arc
Cosh[a*x]]))/(a^3*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x^2}{\left(a^2x^2-1\right)\text{arcosh}\,(ax)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x^2/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

maple [B] time = 0.47, size = 149, normalized size = 2.29

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{Ei}\left(1, 2\,\text{arccosh}\,(ax)\right)}{4a^3\left(a^2x^2-1\right)} + \frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\operatorname{Ei}\left(1, -2\,\text{arccosh}\,(ax)\right)}{4a^3\left(a^2x^2-1\right)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^2-1)*Ei(1,2*a
rccosh(a*x))+1/4*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^3/(a^2*x^
2-1)*Ei(1,-2*arccosh(a*x))-1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)/a^3/(a^2*x^2-1)*ln(arccosh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(x^2/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.295 $\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{ax-1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-ax}}$$

```
[Out] Chi(arccosh(a*x))*(a*x-1)^(1/2)/a^2/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 41, normalized size of antiderivative = 1.46, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {5798, 5781, 3301}

$$\frac{\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[1 - a^2
*x^2])
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x}{\sqrt{-1+ax}\,\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\
&= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\mathrm{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}\\
&= \frac{\sqrt{-1+ax}\,\sqrt{1+ax}\,\mathrm{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{1-a^2x^2}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 50, normalized size = 1.79

$$-\frac{\sqrt{-((ax-1)(ax+1))}\,\text{Chi}\left(\cosh^{-1}(ax)\right)}{a^2\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] -((Sqrt[-((-1 + a*x)*(1 + a*x))]*CoshIntegral[ArcCosh[a*x]])/(a^2*Sqrt[(-1
+ a*x)/(1 + a*x)]*(1 + a*x)))
```

fricas [F] time = 0.41, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}\,x}{\left(a^2x^2-1\right)\text{arcosh}\,(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)*x/((a^2*x^2 - 1)*arccosh(a*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

maple [B] time = 0.26, size = 100, normalized size = 3.57

$$\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Ei}\,(1,\text{arccosh}\,(ax))}{2a^2\left(a^2x^2-1\right)}+\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Ei}\,(1,-\text{arccosh}\,(ax))}{2a^2\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-1)*Ei(1,arc
cosh(a*x))+1/2*(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a^2/(a^2*x^2-
1)*Ei(1,-arccosh(a*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x}{\sqrt{-a^2x^2+1}\,\text{arcosh}\,(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{x}{\text{acosh}\,(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] int(x/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.296 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=28

$$\frac{\sqrt{ax-1}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-ax}}$$

```
[Out] ln(arccosh(a*x))*(a*x-1)^(1/2)/a/(-a*x+1)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 41, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5674}

$$\frac{\sqrt{ax-1}\sqrt{ax+1}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Log[ArcCosh[a*x]])/(a*Sqrt[1 - a^2*x^2])
```

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}\\ &= \frac{\sqrt{-1+ax}\sqrt{1+ax}\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{1-a^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 47, normalized size = 1.68

$$\frac{\sqrt{\frac{ax-1}{ax+1}}(ax+1)\log\left(\cosh^{-1}(ax)\right)}{a\sqrt{-((ax-1)(ax+1))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Log[ArcCosh[a*x]])/(a*Sqrt[-((-1 + a*
x)*(1 + a*x))])
```

fricas [B] time = 0.56, size = 55, normalized size = 1.96

$$-\frac{\sqrt{a^2x^2-1}\sqrt{-a^2x^2+1}\log\left(\log\left(ax+\sqrt{a^2x^2-1}\right)\right)}{a^3x^2-a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)*log(log(a*x + sqrt(a^2*x^2 - 1)))/(a^
3*x^2 - a)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

maple [A] time = 0.17, size = 48, normalized size = 1.71

$$-\frac{\sqrt{-a^2x^2+1}\,\sqrt{ax-1}\,\sqrt{ax+1}\,\ln(\operatorname{arccosh}(ax))}{a\left(a^2x^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] -(-a^2*x^2+1)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)*ln(arccosh(a*
x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int\frac{1}{\operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(ax-1)(ax+1)}\,\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.297 $\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

[Out] Unintegrable(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)

Rubi [A] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]

[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/(x*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]), x])/Sqrt[1 - a^2*x^2]

Rubi steps

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{x\sqrt{-1+ax}\,\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]

[Out] Integrate[1/(x*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^3-x\right)\operatorname{arcosh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")

[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^3 - x)*arccosh(a*x)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,x\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

maple [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\,\mathrm{arccosh}\left(ax\right)\sqrt{-a^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] int(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^{2}x^{2}+1}\,x\,\mathrm{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x*arccosh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x\,\mathrm{acosh}\left(a\,x\right)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(x*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\mathrm{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.298 $\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=27

$$\operatorname{Int}\left(\frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)},x\right)$$

```
[Out] Unintegrable(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.38, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/(x^2*Sqrt[-1 + a*x]*Sqrt[1 + a*x
]*ArcCosh[a*x]), x])/Sqrt[1 - a^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{x^2\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}\,dx}{\sqrt{1-a^2x^2}}$$

Mathematica [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-a^2x^2}\cosh^{-1}(ax)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-a^2x^2+1}}{\left(a^2x^4-x^2\right)\operatorname{arcosh}(ax)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-a^2*x^2 + 1)/((a^2*x^4 - x^2)*arccosh(a*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,x^2\operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \operatorname{arccosh}(ax)\sqrt{-a^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
[Out] int(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,x^2 \operatorname{arcosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/arccosh(a*x)/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*x^2*arccosh(a*x)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \operatorname{acosh}(a\,x)\,\sqrt{1-a^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)),x)
[Out] int(1/(x^2*acosh(a*x)*(1 - a^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-(ax-1)(ax+1)}\operatorname{acosh}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/acosh(a*x)/(-a**2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)), x)
```

3.299 $\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=197

$$\frac{3\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^4\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh}{b}\right.}{4bc^4\sqrt{1-cx}}$$

```
[Out] 3/4*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1/2)+
1/4*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1
/2)-3/4*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1)^(1
/2)-1/4*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b/c^4/(-c*x+1
)^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 245, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 3312, 3303, 3298, 3301}

$$\frac{3\sqrt{cx-1}\sqrt{cx+1}\,\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\,\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}}-\frac{3\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])
/(4*b*c^4*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(3*a)/b]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^4*Sqrt[1 - c^2*x^2]) - (3*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b
*c^4*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[(3*a)/b]*SinhI
ntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b*c^4*Sqrt[1 - c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{\cosh^3(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \left(\frac{3\cosh(x)}{4(a+bx)}+\frac{\cosh(3x)}{4(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\operatorname{Subst}\left(\int \frac{\cosh(3x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} + \frac{\left(3\sqrt{-1+cx}\right.}{}$$
$$= \frac{\left(3\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}} +$$
$$= \frac{3\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^4\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1+c}}{}$$

Mathematica [A] time = 0.29, size = 130, normalized size = 0.66

$$\frac{\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(3\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+\cosh\left(\frac{3a}{b}\right)\operatorname{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+c\right.\right.}{4bc^4\sqrt{-((cx-1)(cx+1))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(3*Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]] + Cosh[(3*a)/b]*CoshIntegral[3*(a/b + ArcCosh[c*x])] - 3*Sinh[a/b]
*SinhIntegral[a/b + ArcCosh[c*x]] - Sinh[(3*a)/b]*SinhIntegral[3*(a/b + Arc
Cosh[c*x])]))/(4*b*c^4*Sqrt[-((-1 + c*x)*(1 + c*x))])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^3}{ac^2x^2+\left(bc^2x^2-b\right)\operatorname{arcosh}(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.45, size = 349, normalized size = 1.77

$$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\mathrm{Ei}\left(1,3\,\mathrm{arccosh}\,(cx)+\frac{3a}{b}\right)\mathrm{e}^{-\frac{b\,\mathrm{arccosh}(cx)-3a}{b}}}{8b\left(c^2x^2-1\right)c^4}+\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*a
rccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)/b)/b/(c^2*x^2-1)/c^4+1/8*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-3*arccosh(c
*x)-3*a/b)*exp(-(b*arccosh(c*x)+3*a)/b)/b/(c^2*x^2-1)/c^4+3/8*(-c^2*x^2+1)^
(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-arccosh(c*x)-a/b)*e
xp(-(a+b*arccosh(c*x))/b)/b/(c^2*x^2-1)/c^4+3/8*(-c^2*x^2+1)^(1/2)*((c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(
c*x)-a)/b)/b/(c^2*x^2-1)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(cx-1)\,(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.300 $\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc^3\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2bc^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\log\left(a+b\cosh^{-1}(cx\right.}{2bc^3\sqrt{1-cx}}$$

```
[Out] 1/2*Chi(2*(a+b*arccosh(c*x))/b)*cosh(2*a/b)*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1
/2)+1/2*ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1/2)-1/2*Shi(2*(
a+b*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b/c^3/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 178, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {5798, 5781, 3312, 3303, 3298, 3301}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(2*a)/b]*CoshIntegral[(2*a)/b + 2*ArcCos
h[c*x]])/(2*b*c^3*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[a
+ b*ArcCosh[c*x]])/(2*b*c^3*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*Sinh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(2*b*c^3*Sqrt[1 -
 c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh^2(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2(a+bx)}+\frac{\cosh(2x)}{2(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subs}}{2c^3\sqrt{}}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\log\left(a+b\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\right.\right.}{}$$

$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{2bc^3\sqrt{1-c^2x^2}} + \frac{\sqrt{-1+cx}\sqrt{1}}{2}$$

Mathematica [A] time = 0.28, size = 99, normalized size = 0.71

$$-\frac{\sqrt{1-c^2x^2}\left(\cosh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{2a}{b}\right)\text{Shi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\log\left(a+b\cosh^{-1}(cx\right.\right.}{2c^3\sqrt{\frac{cx-1}{cx+1}}(bcx+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -1/2*(Sqrt[1 - c^2*x^2]*(Cosh[(2*a)/b]*CoshIntegral[2*(a/b + ArcCosh[c*x])]
 + Log[a + b*ArcCosh[c*x]] - Sinh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*
x])]))/(c^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(b + b*c*x))
```

fricas [F] time = 1.25, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^2}{ac^2x^2+\left(bc^2x^2-b\right)\text{arcosh}\left(cx\right)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.57, size = 232, normalized size = 1.67

$$\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,2\,\text{arccosh}\,(cx)+\frac{2a}{b}\right)\text{e}^{-\frac{b\,\text{arccosh}(cx)-2a}{b}}}{4b\left(c^2x^2-1\right)c^3}+\frac{\sqrt{-c^2x^2+1}\,\left(\sqrt{cx+1}\,\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*a
rccosh(c*x)+2*a/b)*exp(-(b*arccosh(c*x)-2*a)/b)/b/(c^2*x^2-1)/c^3+1/4*(-c^2
*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-2*arccosh(c
*x)-2*a/b)*exp(-(b*arccosh(c*x)+2*a)/b)/b/(c^2*x^2-1)/c^3-1/2*(-c^2*x^2+1)^
(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3*ln(a+b*arccosh(c*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(cx-1)\,(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.301 $\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{cx-1}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc^2\sqrt{1-cx}}$$

```
[Out] Chi((a+b*arccosh(c*x))/b)*cosh(a/b)*(c*x-1)^(1/2)/b/c^2/(-c*x+1)^(1/2)-Shi(
(a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b/c^2/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 114, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5798, 5781, 3303, 3298, 3301}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\cosh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])/(
b*c^2*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sinh[a/b]*SinhInte
gral[a/b + ArcCosh[c*x]])/(b*c^2*Sqrt[1 - c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
```

```
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}} - \frac{\left(\sqrt{-\right.}{}$$
$$= \frac{\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc^2\sqrt{1-c^2x^2}} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\,\text{si}}{bc^2}$$

Mathematica [A] time = 0.22, size = 81, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)}{c^2\sqrt{\frac{cx-1}{cx+1}}(bcx+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-(Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]]) + Sinh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]]))/(c^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(b +
b*c*x))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x}{ac^2x^2+\left(bc^2x^2-b\right)\text{arcosh}(cx)-a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x) -
a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}(cx)+a)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

maple [B] time = 0.19, size = 173, normalized size = 1.88

$$\frac{\sqrt{-c^2x^2+1}\left(\sqrt{cx+1}\sqrt{cx-1}\,xc+c^2x^2-1\right)\text{Ei}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)e^{-\frac{a+b\,\text{arccosh}(cx)}{b}}}{2b\left(c^2x^2-1\right)c^2}+\frac{\sqrt{-c^2x^2+1}\left(\sqrt{cx+1}\sqrt{cx}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-c^2*x^2+1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,-ar
ccosh(c*x)-a/b)*exp(-(a+b*arccosh(c*x))/b)/b/(c^2*x^2-1)/c^2+1/2*(-c^2*x^2+
1)^(1/2)*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)
*exp(-(b*arccosh(c*x)-a)/b)/b/(c^2*x^2-1)/c^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,\text{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\text{acosh}\,(cx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.302 $\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=35

$$\frac{\sqrt{cx-1}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-cx}}$$

[Out] ln(a+b*arccosh(c*x))*(c*x-1)^(1/2)/b/c/(-c*x+1)^(1/2)

Rubi [A] time = 0.22, antiderivative size = 48, normalized size of antiderivative = 1.37, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5713, 5674}

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[1 - c^2*x^2])

Rule 5674

```
Int[1/(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[Log[a + b*ArcCosh[c*x]]/(b*c*Sqrt[-
(d1*d2)]), x] /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && Eq
Q[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}\\ &= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 54, normalized size = 1.54

$$\frac{\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\log\left(a+b\cosh^{-1}(cx)\right)}{bc\sqrt{-((cx-1)(cx+1))}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Log[a + b*ArcCosh[c*x]])/(b*c*Sqrt[-(
(-1 + c*x)*(1 + c*x))])
```

fricas [B] time = 0.49, size = 65, normalized size = 1.86

$$-\frac{\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}\,\log\left(\frac{b\log\left(cx+\sqrt{c^2x^2-1}\right)+a}{b}\right)}{bc^3x^2-bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*log((b*log(c*x + sqrt(c^2*x^2 - 1)) +
 a)/b)/(b*c^3*x^2 - b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.18, size = 55, normalized size = 1.57

$$-\frac{\sqrt{-c^2x^2+1}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(a+b\,\mathrm{arccosh}\,(cx)\right)}{c\left(c^2x^2-1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)*ln(a+b*arccos
h(c*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{(a+b\,\mathrm{acosh}\,(c\,x))\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.303 $\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{ac^2x^3-ax+\left(bc^2x^3-bx\right)\mathrm{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^3 - a*x + (b*c^2*x^3 - b*x)*arccosh(c
*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^{2}x^{2}+1}\,(b\,\mathrm{arcosh}\left(cx\right)+a)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.304 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{ac^2x^4-ax^2+\left(bc^2x^4-bx^2\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a*c^2*x^4 - a*x^2 + (b*c^2*x^4 - b*x^2)*arcco
sh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right) \sqrt{-c^{2} x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^{2} x^{2} + 1} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^2\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \sqrt{- \left(c x - 1\right) \left(c x + 1\right)} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.305 $\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^2/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 4.43, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^2}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\operatorname{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(x^2/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.306 $\int \frac{x}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 7.16, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c
^2*x^2 + b)*arccosh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{3}{2}} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(x/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}} \left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.307 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\text{arcosh}\left(cx\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b*c^2
*x^2 + b)*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\operatorname{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.308 $\int \frac{1}{x(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 3.56, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{ac^4x^5-2\,ac^2x^3+ax+\left(bc^4x^5-2\,bc^2x^3+bx\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^5 - 2*a*c^2*x^3 + a*x + (b*c^4*x^5 - 2
*b*c^2*x^3 + b*x)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.309 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(3/2)*(1 + c*x
)^(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 2.22, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}}{ac^4x^6-2\,ac^2x^4+ax^2+\left(bc^4x^6-2\,bc^2x^4+bx^2\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a*c^4*x^6 - 2*a*c^2*x^4 + a*x^2 + (b*c^4*x^6 -
 2*b*c^2*x^4 + b*x^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

maple [A] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(-c^{2} x^{2} + 1\right)}^{\frac{3}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)} x^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2} \left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.310 $\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)} dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{(1-c^2x^2)^{3/2}x^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)} dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][(x^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(a +
 b*ArcCosh[c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)} dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^m(-1+cx)^{3/2}(1+cx)^{3/2}}{a+b\cosh^{-1}(cx)} dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{x^m(1-c^2x^2)^{3/2}}{a+b\cosh^{-1}(cx)} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(x^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x]), x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{(c^2x^2-1)\sqrt{-c^2x^2+1}\,x^m}{b\,\mathrm{arcosh}(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.17, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{a+b \,\mathrm{arccosh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

```
[Out] int(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(-c^{2} x^{2} + 1\right)}^{\frac{3}{2}} x^{m}}{b \operatorname{arcosh}\left(c x\right) + a}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^m/(b*arccosh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{a+b\,\mathrm{acosh}\left(c\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)),x)
```

```
[Out] int((x^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m} \left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{a + b \operatorname{acosh}{\left(c x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(x**m*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x)), x)
```

3.311 $\int \frac{x^m \sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}\,x^m}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(a + b*Arc
Cosh[c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^m\sqrt{-1+cx}\,\sqrt{1+cx}}{a+b\cosh^{-1}(cx)}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{1-c^2x^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]),x]
```

```
[Out] Integrate[(x^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x]), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^m}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-c^2x^2+1}}{a+b\,\mathrm{arccosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^m}{b\,\mathrm{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^m/(b*arccosh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m\,\sqrt{1-c^2\,x^2}}{a+b\,\mathrm{acosh}\,(c\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)),x)
[Out] int((x^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m\sqrt{-(cx-1)(cx+1)}}{a+b\,\mathrm{acosh}\,(cx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x)), x)
```

3.312 $\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

[Out] Unintegrable(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^m}{ac^2x^2+\left(bc^2x^2-b\right)\operatorname{arcosh}(cx)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^2*x^2 + (b*c^2*x^2 - b)*arccosh(c*x)
- a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\sqrt{-c^2x^2+1}\,(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{-c^{2}x^{2}+1}\,\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{-c^{2}x^{2}+1}\,(b\,\mathrm{arcosh}\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^{m}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))), x)
```

3.313 $\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^m}{ac^4x^4-2\,ac^2x^2+\left(bc^4x^4-2\,bc^2x^2+b\right)\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^m/(a*c^4*x^4 - 2*a*c^2*x^2 + (b*c^4*x^4 - 2*b
*c^2*x^2 + b)*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\,(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{{\left(-c^{2} x^{2}+1\right)}^{\frac{3}{2}} {\left(b \operatorname{arcosh}\left(c x\right)+a\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{m}}{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**m/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))), x)
```

3.314 $\int \frac{x^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.58, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^m/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^m}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.72, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[x^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^m}{ac^6x^6-3\,ac^4x^4+3\,ac^2x^2+\left(bc^6x^6-3\,bc^4x^4+3\,bc^2x^2-b\right)\text{arcosh}\left(cx\right)-a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^m/(a*c^6*x^6 - 3*a*c^4*x^4 + 3*a*c^2*x^2 + (
b*c^6*x^6 - 3*b*c^4*x^4 + 3*b*c^2*x^2 - b)*arccosh(c*x) - a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{5}{2}} (a+b \operatorname{arccosh}(cx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
[Out] int(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{\left(-c^2x^2+1\right)^{\frac{5}{2}} (b \operatorname{arcosh}(cx)+a)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(x^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^m}{(a+b \operatorname{acosh}(c\,x)) \left(1-c^2\,x^2\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)),x)
[Out] int(x^m/((a + b*acosh(c*x))*(1 - c^2*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m}{(-(cx-1)(cx+1))^{\frac{5}{2}} (a+b \operatorname{acosh}(cx))}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Integral(x**m/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))), x)
```

3.315 $\int \frac{(c-a^2cx^2)^3}{\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=98

$$\frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\text{Chi}\left(7\cosh^{-1}(ax)\right)}{64a}+\frac{c^3(ax-1)}{a\cos}$$

[Out] c^3*(a*x-1)^(7/2)*(a*x+1)^(7/2)/a/arccosh(a*x)+35/64*c^3*Chi(arccosh(a*x))/a-63/64*c^3*Chi(3*arccosh(a*x))/a+35/64*c^3*Chi(5*arccosh(a*x))/a-7/64*c^3*Chi(7*arccosh(a*x))/a

Rubi [A] time = 0.33, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5695, 5781, 5448, 3301}

$$\frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a}-\frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a}+\frac{35c^3\text{Chi}\left(5\cosh^{-1}(ax)\right)}{64a}-\frac{7c^3\text{Chi}\left(7\cosh^{-1}(ax)\right)}{64a}+\frac{c^3(ax-1)}{a\cos}$$

Antiderivative was successfully verified.

[In] Int[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]

[Out] (c^3*(-1 + a*x)^(7/2)*(1 + a*x)^(7/2))/(a*ArcCosh[a*x]) + (35*c^3*CoshIntegral[ArcCosh[a*x]])/(64*a) - (63*c^3*CoshIntegral[3*ArcCosh[a*x]])/(64*a) + (35*c^3*CoshIntegral[5*ArcCosh[a*x]])/(64*a) - (7*c^3*CoshIntegral[7*ArcCosh[a*x]])/(64*a)

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) + (b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a + b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n + 1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^3}{\cosh^{-1}(ax)^2}\,dx = \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \left(7ac^3\right)\int \frac{x(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)}\,dx$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^6(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \left(-\frac{5\cosh(x)}{64x} + \frac{9\cosh(3x)}{64x} - \frac{5\cosh(5x)}{64x} + \frac{\cosh(7x)}{64x}\right) dx, x\right.}{a}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} - \frac{\left(7c^3\right)\text{Subst}\left(\int \frac{\cosh(7x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{64a} + \frac{\left(35c^3\right)\text{Subst}\left(\int \frac{\cos}{}\right.}{6}$$

$$= \frac{c^3(-1+ax)^{7/2}(1+ax)^{7/2}}{a\cosh^{-1}(ax)} + \frac{35c^3\text{Chi}\left(\cosh^{-1}(ax)\right)}{64a} - \frac{63c^3\text{Chi}\left(3\cosh^{-1}(ax)\right)}{64a} + \frac{35c^3\text{Chi}\left(5\right.}{6}$$

Mathematica [B] time = 0.41, size = 257, normalized size = 2.62

$$\frac{c^3\left(64a^7x^7\sqrt{\frac{ax-1}{ax+1}} + 64a^6x^6\sqrt{\frac{ax-1}{ax+1}} - 192a^5x^5\sqrt{\frac{ax-1}{ax+1}} - 192a^4x^4\sqrt{\frac{ax-1}{ax+1}} + 192a^3x^3\sqrt{\frac{ax-1}{ax+1}} + 192a^2x^2\sqrt{\frac{ax-1}{ax+1}} + 35\,\text{c}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^3/ArcCosh[a*x]^2,x]
```

```
[Out] (c^3*(-64*Sqrt[(-1 + a*x)/(1 + a*x)] - 64*a*x*Sqrt[(-1 + a*x)/(1 + a*x)] +
192*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)] + 192*a^3*x^3*Sqrt[(-1 + a*x)/(1 + a
*x)] - 192*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] - 192*a^5*x^5*Sqrt[(-1 + a*x)
/(1 + a*x)] + 64*a^6*x^6*Sqrt[(-1 + a*x)/(1 + a*x)] + 64*a^7*x^7*Sqrt[(-1 +
 a*x)/(1 + a*x)] + 35*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] - 63*ArcCosh[
a*x]*CoshIntegral[3*ArcCosh[a*x]] + 35*ArcCosh[a*x]*CoshIntegral[5*ArcCosh[
a*x]] - 7*ArcCosh[a*x]*CoshIntegral[7*ArcCosh[a*x]]))/(64*a*ArcCosh[a*x])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^6c^3x^6 - 3\,a^4c^3x^4 + 3\,a^2c^3x^2 - c^3}{\text{arcosh}\left(ax\right)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^6*c^3*x^6 - 3*a^4*c^3*x^4 + 3*a^2*c^3*x^2 - c^3)/arccosh(a*x)^
2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(a^2cx^2 - c\right)^3}{\text{arcosh}\left(ax\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)^3/arccosh(a*x)^2, x)
```

maple [A] time = 0.25, size = 107, normalized size = 1.09

$$\frac{c^3 \left(35\mathrm{X}\left(\operatorname{arccosh}(ax)\right)\operatorname{arccosh}(ax) - 63\mathrm{X}\left(3\operatorname{arccosh}(ax)\right)\operatorname{arccosh}(ax) + 35\mathrm{X}\left(5\operatorname{arccosh}(ax)\right)\operatorname{arccosh}(ax)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x)
```

```
[Out] 1/64/a*c^3*(35*Chi(arccosh(a*x))*arccosh(a*x)-63*Chi(3*arccosh(a*x))*arccos
h(a*x)+35*Chi(5*arccosh(a*x))*arccosh(a*x)-7*Chi(7*arccosh(a*x))*arccosh(a*
x)-35*(a*x-1)^(1/2)*(a*x+1)^(1/2)+21*sinh(3*arccosh(a*x))-7*sinh(5*arccosh(
a*x))+sinh(7*arccosh(a*x)))/arccosh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^9c^3x^9 - 4\,a^7c^3x^7 + 6\,a^5c^3x^5 - 4\,a^3c^3x^3 + ac^3x + \left(a^8c^3x^8 - 4\,a^6c^3x^6 + 6\,a^4c^3x^4 - 4\,a^2c^3x^2 + c^3\right)\sqrt{ax+1}\,\sqrt{ax-}}{\left(a^3x^2 + \sqrt{ax+1}\,\sqrt{ax-1}\,a^2x - a\right)\log\left(ax + \sqrt{ax+1}\,\sqrt{ax-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^3/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^9*c^3*x^9 - 4*a^7*c^3*x^7 + 6*a^5*c^3*x^5 - 4*a^3*c^3*x^3 + a*c^3*x + (a
^8*c^3*x^8 - 4*a^6*c^3*x^6 + 6*a^4*c^3*x^4 - 4*a^2*c^3*x^2 + c^3)*sqrt(a*x
+ 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(
a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((7*a^10*c^3*x^10 - 29*a^8*c
^3*x^8 + 46*a^6*c^3*x^6 - 34*a^4*c^3*x^4 + 11*a^2*c^3*x^2 + (7*a^8*c^3*x^8
- 20*a^6*c^3*x^6 + 18*a^4*c^3*x^4 - 4*a^2*c^3*x^2 - c^3)*(a*x + 1)*(a*x - 1
) - c^3 + 7*(2*a^9*c^3*x^9 - 7*a^7*c^3*x^7 + 9*a^5*c^3*x^5 - 5*a^3*c^3*x^3
+ a*c^3*x)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2
*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^3}{{\mathrm{acosh}\left(a\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^3/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)^3/acosh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c^3\left(\int \frac{3a^2x^2}{\operatorname{acosh}^2(ax)}\,dx + \int\left(-\frac{3a^4x^4}{\operatorname{acosh}^2(ax)}\right)dx + \int \frac{a^6x^6}{\operatorname{acosh}^2(ax)}\,dx + \int\left(-\frac{1}{\operatorname{acosh}^2(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**3/acosh(a*x)**2,x)
```

```
[Out] -c**3*(Integral(3*a**2*x**2/acosh(a*x)**2, x) + Integral(-3*a**4*x**4/acosh
(a*x)**2, x) + Integral(a**6*x**6/acosh(a*x)**2, x) + Integral(-1/acosh(a*x
)**2, x))
```

3.316 $\int \frac{\left(c-a^2cx^2\right)^2}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=82

$$\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}-\frac{c^2(ax-1)^{5/2}(ax+1)^{5/2}}{a\cosh^{-1}(ax)}$$

```
[Out] -c^2*(a*x-1)^(5/2)*(a*x+1)^(5/2)/a/arccosh(a*x)+5/8*c^2*Chi(arccosh(a*x))/a
-15/16*c^2*Chi(3*arccosh(a*x))/a+5/16*c^2*Chi(5*arccosh(a*x))/a
```

Rubi [A] time = 0.31, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {5695, 5781, 5448, 3301}

$$\frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a}-\frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a}+\frac{5c^2\text{Chi}\left(5\cosh^{-1}(ax)\right)}{16a}-\frac{c^2(ax-1)^{5/2}(ax+1)^{5/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] -((c^2*(-1 + a*x)^(5/2)*(1 + a*x)^(5/2))/(a*ArcCosh[a*x])) + (5*c^2*CoshInt
egral[ArcCosh[a*x]])/(8*a) - (15*c^2*CoshIntegral[3*ArcCosh[a*x]])/(16*a) +
 (5*c^2*CoshIntegral[5*ArcCosh[a*x]])/(16*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^2}{\cosh^{-1}(ax)^2}\,dx = -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)} + \left(5ac^2\right)\int \frac{x(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)}\,dx$$
$$= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8x}-\frac{3\cosh(3x)}{16x}+\frac{\cosh(5x)}{16x}\right)dx, x, \cosh^{-1}(\right.}{a}$$
$$= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)} + \frac{\left(5c^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{16a} + \frac{\left(5c^2\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{c^2(-1+ax)^{5/2}(1+ax)^{5/2}}{a\cosh^{-1}(ax)} + \frac{5c^2\text{Chi}\left(\cosh^{-1}(ax)\right)}{8a} - \frac{15c^2\text{Chi}\left(3\cosh^{-1}(ax)\right)}{16a} + \frac{5c^2\text{Chi}\left(\right.}{}$$

Mathematica [B] time = 0.32, size = 194, normalized size = 2.37

$$-\frac{c^2\left(16a^5x^5\sqrt{\frac{ax-1}{ax+1}}+16a^4x^4\sqrt{\frac{ax-1}{ax+1}}-32a^3x^3\sqrt{\frac{ax-1}{ax+1}}-32a^2x^2\sqrt{\frac{ax-1}{ax+1}}-10\cosh^{-1}(ax)\text{Chi}\left(\cosh^{-1}(ax)\right)+15\,c\right.}{16a\cosh^{-1}(ax)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^2/ArcCosh[a*x]^2,x]
```

```
[Out] -1/16*(c^2*(16*Sqrt[(-1 + a*x)/(1 + a*x)] + 16*a*x*Sqrt[(-1 + a*x)/(1 + a*x
)] - 32*a^2*x^2*Sqrt[(-1 + a*x)/(1 + a*x)] - 32*a^3*x^3*Sqrt[(-1 + a*x)/(1
+ a*x)] + 16*a^4*x^4*Sqrt[(-1 + a*x)/(1 + a*x)] + 16*a^5*x^5*Sqrt[(-1 + a*x
)/(1 + a*x)] - 10*ArcCosh[a*x]*CoshIntegral[ArcCosh[a*x]] + 15*ArcCosh[a*x]
*CoshIntegral[3*ArcCosh[a*x]] - 5*ArcCosh[a*x]*CoshIntegral[5*ArcCosh[a*x]]
))/(a*ArcCosh[a*x])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{a^4c^2x^4-2\,a^2c^2x^2+c^2}{\text{arcosh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)/arccosh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a^2cx^2-c\right)^2}{\text{arcosh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((a^2*c*x^2 - c)^2/arccosh(a*x)^2, x)
```

maple [A] time = 0.14, size = 87, normalized size = 1.06

$$\frac{c^2\left(10\text{X}\,(\text{arccosh}\,(ax))\,\text{arccosh}\,(ax)-15\text{X}\,(3\,\text{arccosh}\,(ax))\,\text{arccosh}\,(ax)+5\text{X}\,(5\,\text{arccosh}\,(ax))\,\text{arccosh}\,(ax)-\right.}{16a\,\text{arccosh}\,(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

```
[Out] 1/16/a*c^2*(10*Chi(arccosh(a*x))*arccosh(a*x)-15*Chi(3*arccosh(a*x))*arccos
h(a*x)+5*Chi(5*arccosh(a*x))*arccosh(a*x)-10*(a*x-1)^(1/2)*(a*x+1)^(1/2)+5*
sinh(3*arccosh(a*x))-sinh(5*arccosh(a*x)))/arccosh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^7c^2x^7 - 3\,a^5c^2x^5 + 3\,a^3c^2x^3 - ac^2x + \left(a^6c^2x^6 - 3\,a^4c^2x^4 + 3\,a^2c^2x^2 - c^2\right)\sqrt{ax+1}\,\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\,\sqrt{ax-1}\,a^2x - a\right)\log\left(ax + \sqrt{ax+1}\,\sqrt{ax-1}\right)} + \int \frac{5\,a^8c^2x^8 - 16\,a^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a^7*c^2*x^7 - 3*a^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x + (a^6*c^2*x^6 - 3*
a^4*c^2*x^4 + 3*a^2*c^2*x^2 - c^2)*sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*x^2 +
 sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a*x + sqrt(a*x + 1)*sqrt(a*x -
1))) + integrate((5*a^8*c^2*x^8 - 16*a^6*c^2*x^6 + 18*a^4*c^2*x^4 - 8*a^2*c
^2*x^2 + (5*a^6*c^2*x^6 - 9*a^4*c^2*x^4 + 3*a^2*c^2*x^2 + c^2)*(a*x + 1)*(a
*x - 1) + 5*(2*a^7*c^2*x^7 - 5*a^5*c^2*x^5 + 4*a^3*c^2*x^3 - a*c^2*x)*sqrt(
a*x + 1)*sqrt(a*x - 1) + c^2)/((a^4*x^4 + (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a
^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + 1)*log(a*x + sqrt(
a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c - a^2\,c\,x^2\right)^2}{\mathrm{acosh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^2/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)^2/acosh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$c^2\left(\int \left(-\frac{2a^2x^2}{\mathrm{acosh}^2\,(ax)}\right)dx + \int \frac{a^4x^4}{\mathrm{acosh}^2\,(ax)}\,dx + \int \frac{1}{\mathrm{acosh}^2\,(ax)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] c**2*(Integral(-2*a**2*x**2/acosh(a*x)**2, x) + Integral(a**4*x**4/acosh(a*
x)**2, x) + Integral(acosh(a*x)**(-2), x))
```

3.317 $\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=58

$$\frac{3c\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}}{a\cosh^{-1}(ax)}$$

```
[Out] c*(a*x-1)^(3/2)*(a*x+1)^(3/2)/a/arccosh(a*x)+3/4*c*Chi(arccosh(a*x))/a-3/4*
c*Chi(3*arccosh(a*x))/a
```

Rubi [A] time = 0.24, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {5695, 5781, 5448, 3301}

$$\frac{3c\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a}-\frac{3c\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}+\frac{c(ax-1)^{3/2}(ax+1)^{3/2}}{a\cosh^{-1}(ax)}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2))/(a*ArcCosh[a*x]) + (3*c*CoshIntegral[A
rcCosh[a*x]])/(4*a) - (3*c*CoshIntegral[3*ArcCosh[a*x]])/(4*a)
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{c-a^2cx^2}{\cosh^{-1}(ax)^2}\,dx = \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - (3ac)\int \frac{x\sqrt{-1+ax}\,\sqrt{1+ax}}{\cosh^{-1}(ax)}\,dx$$
$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} - \frac{(3c)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4x}+\frac{\cosh(3x)}{4x}\right)dx, x, \cosh^{-1}(ax)\right)}{a}$$
$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(x)}{x}\,dx, x, \cosh^{-1}(ax)\right)}{4a} - \frac{(3c)\,\text{Subst}\left(\int \frac{\cosh(3x)}{x}\,d\right.}{4a}$$
$$= \frac{c(-1+ax)^{3/2}(1+ax)^{3/2}}{a\cosh^{-1}(ax)} + \frac{3c\text{Chi}\left(\cosh^{-1}(ax)\right)}{4a} - \frac{3c\text{Chi}\left(3\cosh^{-1}(ax)\right)}{4a}$$

Mathematica [B] time = 1.78, size = 217, normalized size = 3.74

$$\frac{c\sqrt{ax-1}\left(-3\left(a^2x^2-1\right)\cosh^{-1}(ax)\text{Chi}\left(3\cosh^{-1}(ax)\right)-(ax-1)\cosh^{-1}(ax)\text{Chi}\left(\cosh^{-1}(ax)\right)\left(ax-4\sqrt{ax-1}\,\sqrt{\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)/ArcCosh[a*x]^2,x]
```

```
[Out] (c*Sqrt[-1 + a*x]*(4*((-1 + a*x)/(1 + a*x))^(5/2)*(1 + a*x)^5 - 3*(-1 + a^2
*x^2)*ArcCosh[a*x]*CoshIntegral[3*ArcCosh[a*x]] - (-1 + a*x)*ArcCosh[a*x]*C
oshIntegral[ArcCosh[a*x]]*(1 + a*x - 4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Coth[Ar
cCosh[a*x]/2]) + 4*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]*(Sqrt[(-1 + a*
x)/(1 + a*x)]*(1 + a*x) + (1 - a*x)*Coth[ArcCosh[a*x]/2])*Log[ArcCosh[a*x]]
))/(4*a*((-1 + a*x)/(1 + a*x))^(3/2)*(1 + a*x)^(5/2)*ArcCosh[a*x])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{a^2cx^2-c}{\text{arcosh}\,(ax)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{a^2cx^2-c}{\text{arcosh}\,(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(-(a^2*c*x^2 - c)/arccosh(a*x)^2, x)
```

maple [A] time = 0.12, size = 63, normalized size = 1.09

$$-\frac{c\left(3\sqrt{ax-1}\,\sqrt{ax+1}+3\text{X}\left(3\,\text{arccosh}\,(ax)\right)\text{arccosh}\,(ax)-3\text{X}\left(\text{arccosh}\,(ax)\right)\text{arccosh}\,(ax)-\sinh\left(3\,\text{arccosh}\,(a\right.\right.}{4a\,\text{arccosh}\,(ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

```
[Out] -1/4/a*c*(3*(a*x-1)^(1/2)*(a*x+1)^(1/2)+3*Chi(3*arccosh(a*x))*arccosh(a*x)-
3*Chi(arccosh(a*x))*arccosh(a*x)-sinh(3*arccosh(a*x)))/arccosh(a*x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^5cx^5 - 2\,a^3cx^3 + acx + \left(a^4cx^4 - 2\,a^2cx^2 + c\right)\sqrt{ax+1}\,\sqrt{ax-1}}{\left(a^3x^2 + \sqrt{ax+1}\,\sqrt{ax-1}\,a^2x - a\right)\log\left(ax + \sqrt{ax+1}\,\sqrt{ax-1}\right)} - \int \frac{3\,a^6cx^6 - 7\,a^4cx^4 + 5\,a^2cx^2 + \left(3\,a^4cx^4 - 2\,a\right.}{\left(a^4x^4 + (ax+1)(ax-1)a^2x^2 - 2\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x + (a^4*c*x^4 - 2*a^2*c*x^2 + c)*sqrt(a*x +
 1)*sqrt(a*x - 1))/((a^3*x^2 + sqrt(a*x + 1)*sqrt(a*x - 1)*a^2*x - a)*log(a
*x + sqrt(a*x + 1)*sqrt(a*x - 1))) - integrate((3*a^6*c*x^6 - 7*a^4*c*x^4 +
 5*a^2*c*x^2 + (3*a^4*c*x^4 - 2*a^2*c*x^2 - c)*(a*x + 1)*(a*x - 1) + 3*(2*a
^5*c*x^5 - 3*a^3*c*x^3 + a*c*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - c)/((a^4*x^4
+ (a*x + 1)*(a*x - 1)*a^2*x^2 - 2*a^2*x^2 + 2*(a^3*x^3 - a*x)*sqrt(a*x + 1)
*sqrt(a*x - 1) + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c - a^2\,c\,x^2}{\mathrm{acosh}\left(a\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)/acosh(a*x)^2,x)
```

```
[Out] int((c - a^2*c*x^2)/acosh(a*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-c\left(\int \frac{a^2x^2}{\mathrm{acosh}^2\,(ax)}\,dx + \int \left(-\frac{1}{\mathrm{acosh}^2\,(ax)}\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)/acosh(a*x)**2,x)
```

```
[Out] -c*(Integral(a**2*x**2/acosh(a*x)**2, x) + Integral(-1/acosh(a*x)**2, x))
```

3.318 $\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$

Optimal. Leaf size=66

$$\frac{a\text{Int}\left(\frac{x}{(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)},x\right)}{c}+\frac{1}{ac\sqrt{ax-1}\sqrt{ax+1}\cosh^{-1}(ax)}$$

```
[Out] 1/a/c/arccosh(a*x)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+a*Unintegrable(x/(a*x-1)^(3/
2)/(a*x+1)^(3/2)/arccosh(a*x),x)/c
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] 1/(a*c*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]) + (a*Defer[Int][x/((-1 +
a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x]), x])/c
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx=\frac{1}{ac\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)}+\frac{a\int \frac{x}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}\,dx}{c}$$

Mathematica [A] time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)\cosh^{-1}(ax)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)*ArcCosh[a*x]^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{1}{(a^2cx^2-c)\,\text{arcosh}(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(a^2cx^2-c)\,\text{arcosh}(ax)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="giac")
[Out] integrate(-1/((a^2*c*x^2 - c)*arccosh(a*x)^2), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c \,x^{2}+c \right) \operatorname{arccosh}\left(a x \right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
[Out] int(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ax+\sqrt{ax+1}\,\sqrt{ax-1}}{\left(a^3cx^2+\sqrt{ax+1}\,\sqrt{ax-1}\,a^2cx-ac\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)}+\int \frac{a^4x^4}{\left(a^6cx^6-3\,a^4cx^4+3\,a^2cx^2+\left(a^4cx^4-a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)/arccosh(a*x)^2,x, algorithm="maxima")
[Out] (a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^3*c*x^2 + sqrt(a*x + 1)*sqrt(a*x -
1)*a^2*c*x - a*c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))) + integrate((a^4*
x^4 + (a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + (2*a^3*x^3 - a*x)*sqrt(a*x + 1)*s
qrt(a*x - 1) - 1)/((a^6*c*x^6 - 3*a^4*c*x^4 + 3*a^2*c*x^2 + (a^4*c*x^4 - a^
2*c*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^5*c*x^5 - 2*a^3*c*x^3 + a*c*x)*sqrt(a*x
 + 1)*sqrt(a*x - 1) - c)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^2\,\left(c-a^2\,c\,x^2\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)),x)
[Out] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{1}{a^{2} x^{2} \operatorname{acosh}^{2}{\left(a x \right)} - \operatorname{acosh}^{2}{\left(a x \right)}}\, dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)/acosh(a*x)**2,x)
[Out] -Integral(1/(a**2*x**2*acosh(a*x)**2 - acosh(a*x)**2), x)/c
```

3.319 $\int \frac{1}{\left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2} dx$

Optimal. Leaf size=68

$$-\frac{3a\text{Int}\left(\frac{x}{(ax-1)^{5/2}(ax+1)^{5/2}\cosh^{-1}(ax)},x\right)}{c^2}-\frac{1}{ac^2(ax-1)^{3/2}(ax+1)^{3/2}\cosh^{-1}(ax)}$$

```
[Out] -1/a/c^2/(a*x-1)^(3/2)/(a*x+1)^(3/2)/arccosh(a*x)-3*a*Unintegrable(x/(a*x-1
)^(5/2)/(a*x+1)^(5/2)/arccosh(a*x),x)/c^2
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
```

```
[Out] -(1/(a*c^2*(-1 + a*x)^(3/2)*(1 + a*x)^(3/2)*ArcCosh[a*x])) - (3*a*Defer[Int
][x/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2)*ArcCosh[a*x]), x])/c^2
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2} dx=-\frac{1}{ac^2(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)}-\frac{(3a)\int \frac{x}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)} dx}{c^2}$$

Mathematica [A] time = 13.05, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^2 \cosh^{-1}(ax)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^2*ArcCosh[a*x]^2), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\left(a^4c^2x^4-2\,a^2c^2x^2+c^2\right)\text{arcosh}\,(ax)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(1/((a^4*c^2*x^4 - 2*a^2*c^2*x^2 + c^2)*arccosh(a*x)^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a^2cx^2-c\right)^2 \text{arcosh}\,(ax)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/((a^2*c*x^2 - c)^2*arccosh(a*x)^2), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{2} \operatorname{arccosh}\left(a x\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{ax+\sqrt{ax+1}\,\sqrt{ax-1}}{\left(a^{5}c^{2}x^{4}-2\,a^{3}c^{2}x^{2}+ac^{2}+\left(a^{4}c^{2}x^{3}-a^{2}c^{2}x\right)\sqrt{ax+1}\,\sqrt{ax-1}\right)\log\left(ax+\sqrt{ax+1}\,\sqrt{ax-1}\right)}-\int\frac{}{\left(a^{8}c^{2}x^{8}-4\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^2/arccosh(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))/((a^5*c^2*x^4 - 2*a^3*c^2*x^2 + a*c^2
+ (a^4*c^2*x^3 - a^2*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1))*log(a*x + sqrt(a*x
 + 1)*sqrt(a*x - 1))) - integrate((3*a^4*x^4 - 2*a^2*x^2 + (3*a^2*x^2 - 1)*
(a*x + 1)*(a*x - 1) + 3*(2*a^3*x^3 - a*x)*sqrt(a*x + 1)*sqrt(a*x - 1) - 1)/
((a^8*c^2*x^8 - 4*a^6*c^2*x^6 + 6*a^4*c^2*x^4 - 4*a^2*c^2*x^2 + (a^6*c^2*x^
6 - 2*a^4*c^2*x^4 + a^2*c^2*x^2)*(a*x + 1)*(a*x - 1) + 2*(a^7*c^2*x^7 - 3*a
^5*c^2*x^5 + 3*a^3*c^2*x^3 - a*c^2*x)*sqrt(a*x + 1)*sqrt(a*x - 1) + c^2)*lo
g(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^2\,{\left(c-a^2\,c\,x^2\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)^2),x)
```

```
[Out] int(1/(acosh(a*x)^2*(c - a^2*c*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{a^{4} x^{4} \operatorname{acosh}^{2}{\left(a x \right)} - 2 a^{2} x^{2} \operatorname{acosh}^{2}{\left(a x \right)} + \operatorname{acosh}^{2}{\left(a x \right)}}\, dx}{c^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**2/acosh(a*x)**2,x)
```

```
[Out] Integral(1/(a**4*x**4*acosh(a*x)**2 - 2*a**2*x**2*acosh(a*x)**2 + acosh(a*x
)**2), x)/c**2
```

3.320 $\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=350

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^4\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^4\sqrt{cx-1}}-\frac{5\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^4\sqrt{cx-1}}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^4/(c*x-1)^(1/
2)+3/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^4/(c*x
-1)^(1/2)+5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
^4/(c*x-1)^(1/2)+1/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2
/c^4/(c*x-1)^(1/2)-3/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1
/2)/b^2/c^4/(c*x-1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*
x+1)^(1/2)/b^2/c^4/(c*x-1)^(1/2)-x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+
1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 1.08, antiderivative size = 429, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5778, 5670, 5448, 3303, 3298, 3301}

$$\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^4\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{5\sqrt{1-c^2x^2}\,\sinh\left(\frac{5a}{b}\right)}{16b^2c^4\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^3*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) + (Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a
/b])/(8*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]
]*Sinh[(5*a)/b])/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*
x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^4*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b +
3*ArcCosh[c*x]])/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^
2*x^2]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^4*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^3\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(5c\sqrt{1-c^2x^2}\right)}{b\sqrt{-1+}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx,x,\cosh^{-1}\right.}{bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx,x,\right.}{bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{16bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\right.}{8bc^4\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^3(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^4\sqrt{-1+cx}\sqrt{1+cx}} - \frac{3\sqrt{1-}}{}$$

Mathematica [A] time = 0.77, size = 322, normalized size = 0.92

$$\frac{\sqrt{1-c^2x^2}\left(2\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(16*b*c^3*x^3 - 16*b*c^5*x^5 + 2*(a + b*ArcCosh[c*x])*Co
shIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshInteg
ral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*CoshIntegral[5*(a/b + ArcCo
sh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*
x])]*Sinh[(5*a)/b] - 2*a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 2*b*A
rcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cosh[(3*a)/b]*
SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*a)/b]*SinhI
ntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + A
rcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCos
h[c*x])]))/(16*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^3}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.74, size = 1029, normalized size = 2.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6
+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^4/(c*x-1)/(a+b*arccosh(c*x))/b-5/32*(-c^2
*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c
*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2+1/32*(-c^2*x
^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^4/(c*x-1)/(a+b*arccosh(c*x))/b-
3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3
*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^4/(c*x-1)/b^2-1/
32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(
-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*b-3*x*b*c)/c^4/b^2/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*b*c^4+16*x^5*b*c^
5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^3*b*c^3+5*arccosh(c*x)*exp(
-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*exp(-
5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+5*x*b*c)/c^4/b^2/(a+b*arccosh(c*x))+1/
16*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*E
i(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arcc
osh(c*x)-a/b)*a+x*b*c)/c^4/b^2/(a+b*arccosh(c*x))-1/16*(-c^2*x^2+1)^(1/2)*(
-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^4/(c*x-1)/(a+b*arccos
h(c*x))/b+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^4/(c*x-1)/b^
2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^5 - x^3\right)(cx+1)\sqrt{cx-1} + \left(c^3x^6 - cx^4\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^5 - x^3)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^6 - c*x^4)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
```

```
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((5*c^3*x^5 - 2*c*x^3)*(c*x + 1
)^(3/2)*(c*x - 1) + (10*c^4*x^6 - 11*c^2*x^4 + 3*x^2)*(c*x + 1)*sqrt(c*x -
1) + (5*c^5*x^7 - 9*c^3*x^5 + 4*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c
^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^3*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}{\left(cx\right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

3.321 $\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=154

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^3\sqrt{cx-1}}-\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] 1/2*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^
(1/2)-1/2*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c
*x-1)^(1/2)-x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arc
cosh(c*x))
```

Rubi [A] time = 0.88, antiderivative size = 185, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5798, 5778, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{4a}{b}\right)\mathrm{Shi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{x^2(1-cx)\sqrt{cx}}{bc\sqrt{cx-1}\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*
Sinh[(4*a)/b])/(2*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2
]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c^3*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(4c\sqrt{1-c^2x\right.}}{b\sqrt{-1}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}dx, x, \cosh\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}dx, x, \cosh^{-1}(c\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}dx, x, \cosh^{-1}(cx)\right)}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{4a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{4a}{b}+4x\right)}{a+bx}dx\right.}{2bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)\sinh\left(\frac{4a}{b}\right)}{2b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.49, size = 130, normalized size = 0.84

$$\frac{\sqrt{1-c^2x^2}\left(-\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\cosh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(4\left(\frac{a}{b}\right.\right.\right.}{2b^2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(-2*b*c^2*x^2*(-1 + c^2*x^2) - (a + b*ArcCosh[c*x])*Cosh
Integral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] + (a + b*ArcCosh[c*x])*Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^3*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^2}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) +
a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x^2}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x^2/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.57, size = 422, normalized size = 2.74

$$\frac{\sqrt{-c^2x^2+1}\,\left(-8\sqrt{cx+1}\,\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\,\sqrt{cx+1}\,+4cx\right)}{16\,(cx+1)\,(cx-1)\,c^3b\,(a+b\,\mathrm{arccosh}\,(cx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+4*c*x)/(c*x+1)/(c*x-1)/c^3/b/(a+b*arccosh(c*x))-1/4*(-c^2*x^2+1)^(1/2)*(-(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b
*arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/16*(-c^2*x^2+1)^(1/2)/(c*x-
1)^(1/2)/(c*x+1)^(1/2)*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4
-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei(1,-4*arc
cosh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*a+
b)/c^3/b^2/(a+b*arccosh(c*x))+1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)/c^3/b/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^4-x^2\right)(cx+1)\sqrt{cx-1}\,+\left(c^3x^5-cx^3\right)\sqrt{cx+1}\,\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\,\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^5 - c*x^3)*sqrt(c*x + 1)
)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b
*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate(((4*c^3*x^4 - c*x^2)*(c*x + 1)^
(3/2)*(c*x - 1) + 2*(4*c^4*x^5 - 4*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (
4*c^5*x^6 - 7*c^3*x^4 + 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3
 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x -
 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(
c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \sqrt{- \left(c x - 1\right) \left(c x + 1\right)}}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

3.322 $\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=248

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^2\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}}{b}\right.}{4b^2c^2\sqrt{cx-1}}$$

```
[Out] -1/4*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/
2)+3/4*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-
1)^(1/2)+1/4*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*
x-1)^(1/2)-3/4*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/c
^2/(c*x-1)^(1/2)-x*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*
arccosh(c*x))
```

Rubi [A] time = 0.69, antiderivative size = 418, normalized size of antiderivative = 1.69, number of steps used = 15, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {5798, 5778, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}}{4b^2c^2\sqrt{cx-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*Ar
cCosh[c*x])) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a
/b])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*CoshInte
gral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a
)/b])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[
a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[
c*x]])/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*Cosh[a
/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(3c\sqrt{1-c^2x^2}\right)\int \frac{}{a}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(\right.}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\mathrm{Subst}\left(\int\left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x,\right.}{bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x}}{b}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{\sqrt{1-c^2x}}{b}$$

$$= \frac{x(1-cx)\sqrt{1+cx}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\sqrt{1-}}{}$$

Mathematica [A] time = 0.43, size = 217, normalized size = 0.88

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(4*b*c*x - 4*b*c^3*x^3 + (a + b*ArcCosh[c*x])*CoshIntegr
al[a/b + ArcCosh[c*x]]*Sinh[a/b] - 3*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a
/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - a*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]] - b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*Cos
h[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(3*
a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x}{b^2\,\mathrm{arcosh}\,(cx)^2+2\,ab\,\mathrm{arcosh}\,(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}\,x}{(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)*x/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.32, size = 622, normalized size = 2.51

$$\frac{\sqrt{-c^2x^2+1}\,\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3+4c^4x^4+3\sqrt{cx+1}\,\sqrt{cx-1}\,xc-5c^2x^2+1\right)}{8\,(cx+1)\,c^2\,(cx-1)\,(a+b\operatorname{arccosh}(cx))\,b}-\frac{3\sqrt{-c^2x^2+1}\,\left(-\sqrt{cx+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/(a+b*arcco
sh(c*x))/b-3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x
-1)/b^2-1/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*Ei(1,-3*arccosh(c*x)-3*
a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2
)*(c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))+1/8*(-c^2*x^2+1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/
b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*
c)/c^2/b^2/(a+b*arccosh(c*x))-1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+1/8*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+
a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^3-x\right)(cx+1)\sqrt{cx-1}+\left(c^3x^4-cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^4 - c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c
 + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x - 1)*c^3*
x^3 + (6*c^4*x^4 - 5*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (3*c^5*x^5 - 5*
c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sq
rt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
- 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

3.323 $\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=146

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.34, antiderivative size = 177, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {5713, 5697, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{\sqrt{cx+1}\,\sqrt{1-\cdots}}{bc\sqrt{cx-1}\,\left(a+\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*ArcC
osh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]*Sinh
[(2*a)/b])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(
2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2c\sqrt{1-c^2x^2}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2(a+bx)}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(2x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x,\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}\,\sqrt{1+cx}} + \sqrt{}$$

Mathematica [A] time = 0.22, size = 121, normalized size = 0.83

$$-\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\mathrm{Shi}\left(2\left(\frac{a}{b}+\right.\right.\right.}{b^2c\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*(b*(-1 + c^2*x^2) + (a + b*ArcCosh[c*x])*CoshIntegral[
2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*
SinhIntegral[2*(a/b + ArcCosh[c*x])]))/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(a + b*ArcCosh[c*x])))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{-c^2x^2+1}}{\left(b\operatorname{arcosh}(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.30, size = 361, normalized size = 2.47

$$\frac{\sqrt{-c^2x^2+1}\left(-2\sqrt{cx+1}\sqrt{cx-1}\,x^2c^2+2c^3x^3+\sqrt{cx-1}\sqrt{cx+1}-2cx\right)}{4\,(cx+1)\,(cx-1)\,cb\,(a+b\operatorname{arccosh}(cx))}-\frac{\sqrt{-c^2x^2+1}\left(-\sqrt{cx+1}\sqrt{cx-1}\,x\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c
*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/b/(a+b*arccosh(c*x))-1/2
*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arc
cosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c/b^2-1/4*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2
*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+1/2*
(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c/b/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c +
 (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))) + integrate(((2*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c
*x - 1) + 2*(2*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1) + (2*c^4*x^4 - 3*c^2*
x^2 + 1)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c
*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*
x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{\left(a+b\operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

3.324 $\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=182

$$\frac{\sqrt{1-cx}\,\mathrm{Int}\left(\frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2\sqrt{cx-1}}-$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-Chi((a
+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-(c*x-1)^(1/2
)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/x/(a+b*arccosh(c*x))+(-c*x+1)^(1/2)*
Unintegrable(1/x^2/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a + b*Ar
cCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh
[a/b])/(b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[a/b]*Si
nhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (S
qrt[1 - c^2*x^2]*Defer[Int][1/(x^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(c\sqrt{1-c^2\right.}}{b\sqrt{}}\\
&= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-c^2x^2}\,\mathrm{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\,\mathrm{co}\right.}{b^2\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}+\frac{\sqrt{1-c^2x^2}\int \frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(\sqrt{1-c^2\right.}}{}\\
&= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{\sqrt{1-c^2x^2}\,\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{b^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{1}}{}
\end{aligned}$$

Mathematica [A] time = 33.14, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x\,\text{arcosh}\,(cx)^2+2\,abx\,\text{arcosh}\,(cx)+a^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x) +
a^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x\,(a+b\,\text{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^3+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^2-abcx+\left(b^2c^3x^3+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^2-b^2cx\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c
*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((((c^3*x^3 + 2*c*x)*(c*x + 1
)^(3/2)*(c*x - 1) + (2*c^4*x^4 + c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^
5*x^5 - c^3*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*
x - 1)*a*b*c^3*x^4 - 2*a*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x
^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^
3*x^4 - 2*b^2*c^3*x^4 + b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x
+ 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x*(a + b*acosh(c*x))**2), x)
```

3.325 $\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=98

$$\frac{2\sqrt{1-cx}\,\text{Int}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{1-c^2x^2}}{bcx^2\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)*(-c^2*x^2+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))+
2*(-c*x+1)^(1/2)*Unintegrable(1/x^3/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^(1/2)
```

Rubi [A] time = 0.49, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) + (2*Sqrt[1 - c^2*x^2]*Defer[Int][1/(x^3*(a + b*ArcCosh[c*x]
)), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}\end{aligned}$$

Mathematica [A] time = 10.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^2\,\text{arcosh}\,(cx)^2+2\,abx^2\,\text{arcosh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c*x
) + a^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{\left(b \operatorname{arcosh}(cx)+a\right)^2 x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.36, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^2\left(a+b \operatorname{arccosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c
*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*(c*x + 1)^(3/2)*(c*x
 - 1)*c*x + 2*(2*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqr
t(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^5 -
 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^2*c^3*x^
5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^2\left(a+b \operatorname{acosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**2*(a + b*acosh(c*x))**2), x)
```

3.326 $\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^3*(a + b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 163.26, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-c^2x^2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[Sqrt[1 - c^2*x^2]/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^3\,\text{arcosh}\,(cx)^2+2\,abx^3\,\text{arcosh}\,(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^3\left(a+b\operatorname{arccosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^5+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^4-abcx^3+\left(b^2c^3x^5+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^4-b^2cx^3\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c
*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((c^3*x^3 - 4*c*x)*(c*x
 + 1)^(3/2)*(c*x - 1) + (2*c^4*x^4 - 7*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1)
 + (c^5*x^5 - 3*c^3*x^3 + 2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8
 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4
*x^7 - a*b*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*
(c*x - 1)*b^2*c^3*x^6 - 2*b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^
2*x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(cx-1)(cx+1)}}{x^3\left(a+b\operatorname{acosh}(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**3*(a + b*acosh(c*x))**2), x)
```

3.327 $\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][(Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(x^4*(a + b*Ar
cCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\sqrt{1-c^2x^2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [F] time = 180.01, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[1 - c^2*x^2]/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{b^2x^4\,\text{arcosh}\left(cx\right)^2+2\,abx^4\,\text{arcosh}\left(cx\right)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x
) + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{\left(b\,\text{arcosh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(-c^2*x^2 + 1)/((b*arccosh(c*x) + a)^2*x^4), x)
```

maple [A] time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2x^2+1}}{x^4\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^5-abcx^4+\left(b^2c^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^5-b^2cx^4\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(1/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^3*x^3 - c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c
*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((2*c^3*x^3 - 5*c*x)*(c
*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^4*x^4 - 5*c^2*x^2 + 2)*(c*x + 1)*sqrt(c*x
- 1) + (2*c^5*x^5 - 5*c^3*x^3 + 3*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c
^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a
*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x
 + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 -
b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x
- 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{1-c^2\,x^2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(1/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(1/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(sqrt(-(c*x - 1)*(c*x + 1))/(x**4*(a + b*acosh(c*x))**2), x)
```

3.328 $\int \frac{x^2(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=354

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Chi}\left(\right.}{16b^2c^3\sqrt{cx-}}$$

```
[Out] 1/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)
^(1/2)+1/4*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(
c*x-1)^(1/2)-3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^3/(c*x-1)^(1/2)-1/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(
1/2)/b^2/c^3/(c*x-1)^(1/2)-1/4*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*
x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)+3/16*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/
b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-x^2*(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 1.14, antiderivative size = 439, normalized size of antiderivative = 1.24, number of steps used = 20, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5778, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{4b^2c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\,\text{si}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*
ArcCosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (S
qrt[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*
b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Si
nhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(4*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c
*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(6c\sqrt{1-c^2x\right.}{b\sqrt{-1\right.}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \cos\right.}{bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right)d\right.}{bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{16bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx\right.}{2bc^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}\,\sqrt{1+cx}} -$$

Mathematica [A] time = 1.07, size = 338, normalized size = 0.95

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(-\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-4\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{C}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/16*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(16*b*c^2*x^2 - 32*b*c^4*x^4 + 16*b*c^6
*x^6 - (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)
/b] - 4*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a
)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*ArcCosh
[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh[(2*a)/b]*
SinhIntegral[2*(a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhInt
egral[2*(a/b + ArcCosh[c*x])] + 4*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + Arc
Cosh[c*x])] + 4*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[
c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - 3*b*ArcCo
sh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[
1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^4-x^2\right)\sqrt{-c^2x^2+1}}{b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*x^4 - x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*ar
ccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x^2}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.75, size = 1176, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^
7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1
)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-
1)/c^3/b/(a+b*arccosh(c*x))+3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b
)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/
2)*(32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x-1)^(1/2)*
(c*x+1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+18
*x^2*b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-
6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))+1/16*(-c
^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c^3/b/(a+b*arccosh(c*x))+1/32*(
-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x
)/(c*x+1)/(c*x-1)/c^3/b/(a+b*arccosh(c*x))-1/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arcco
sh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)
^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(
c*x+1)/(c*x-1)/c^3/b/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(
1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh
(c*x)+2*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c
*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)
*exp(-2*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-
4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei(1,-4*arcc
osh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*a+b
)/c^3/b^2/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^4x^6-2\,c^2x^4+x^2\right)(cx+1)\sqrt{cx-1}+\left(c^5x^7-2\,c^3x^5+cx^3\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^6 - 2*c^2*x^4 + x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^7 - 2*c^3*x^5
 + c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c
*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((6*c^5*
x^6 - 7*c^3*x^4 + c*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(6*c^6*x^7 - 11*c^4*
x^5 + 6*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + 3*(2*c^7*x^8 - 5*c^5*x^6 + 4
*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*s
qrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2
 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

3.329 $\int \frac{x\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=348

$$\frac{\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^2\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^2\sqrt{cx-1}}+\frac{5\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\,\mathrm{c}}{}\right.}{16b^2c^2\sqrt{cx-1}}$$

```
[Out] -1/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/
2)+9/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x
-1)^(1/2)-5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
^2/(c*x-1)^(1/2)+1/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2
/c^2/(c*x-1)^(1/2)-9/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1
/2)/b^2/c^2/(c*x-1)^(1/2)+5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*
x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-x*(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 1.06, antiderivative size = 429, normalized size of antiderivative = 1.23, number of steps used = 23, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5798, 5778, 5700, 3312, 3303, 3298, 3301, 5780, 5448}

$$\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{9\sqrt{1-c^2x^2}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{5\sqrt{1-c^2x^2}\,\sinh\left(\frac{5a}{b}\right)}{16b^2c^2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) + (Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh
[a/b])/(8*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (9*Sqrt[1 - c^2*x^2]*Cosh
Integral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*
x]]*Sinh[(5*a)/b])/(16*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b
+ 3*ArcCosh[c*x]])/(16*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 -
c^2*x^2]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(5c\sqrt{1-c^2x^2}\right)\int \frac{}{a}}{b\sqrt{-1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2\sqrt{-1+cx}\,\sqrt{1+cx}} -$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right)dx, x\right.}{bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{4bc^2\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}\,dx, x, c\right.}{8bc^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2c^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{9\sqrt{1-}}{}$$

Mathematica [A] time = 0.95, size = 327, normalized size = 0.94

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(-2\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+9\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(3\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c*x + 32*b*c^3*x^3 - 16*b*c^5*x^5 - 2*
(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] + 9*(a + b*
ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] - 5*a*Cosh
Integral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 5*b*ArcCosh[c*x]*CoshInteg
ral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 2*a*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x
]] - 9*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 9*b*ArcCosh[c
*x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*ArcCosh[c*x]*Cosh[(5*a)/b]*SinhI
ntegral[5*(a/b + ArcCosh[c*x])]))/(16*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^3-x\right)\sqrt{-c^2x^2+1}}{b^2\operatorname{arcosh}(cx)^2+2ab\operatorname{arcosh}(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

[Out] integral(-(c^2*x^3 - x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")

[Out] integrate((-c^2*x^2 + 1)^(3/2)*x/(b*arccosh(c*x) + a)^2, x)

maple [B] time = 0.41, size = 1029, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)

[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+5/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*arccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+1/32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^3*b*c^3+5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+5*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))+3/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b-9/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+1/16*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-3/32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arccosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^4x^5-2\,c^2x^3+x\right)(cx+1)\sqrt{cx-1}+\left(c^5x^6-2\,c^3x^4+cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")

[Out] ((c^4*x^5 - 2*c^2*x^3 + x)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^6 - 2*c^3*x^4 + c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate((5*(c^5*x^

```
5 - c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + (10*c^6*x^6 - 17*c^4*x^4 + 8*c^2*x
^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (5*c^7*x^7 - 12*c^5*x^5 + 9*c^3*x^3 - 2*c
*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^
3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*s
qrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x
^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-c^2\,x^2\right)^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x*(-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

3.330 $\int \frac{(1-c^2x^2)^{3/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=246

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{2b^2c\sqrt{cx-1}}+\frac{\sqrt{1-cx}\,\cosh\left(\frac{2a}{b}\right)\mathrm{Shi}\left(\frac{2}{}\right.}{b^2c\sqrt{cx-1}}$$

```
[Out] cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-
1/2*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1
/2)-Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^(1
/2)+1/2*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1
)^(1/2)-(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x
))
```

Rubi [A] time = 0.52, antiderivative size = 305, normalized size of antiderivative = 1.24, number of steps used = 11, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5713, 5697, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{\sqrt{1-c^2x^2}\,\mathrm{cos}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]*
Sinh[(2*a)/b])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Co
shIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2
*ArcCosh[c*x]])/(b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^2*x^2]*C
osh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4c\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{a+bx}\,dx, x, \cos\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(4\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4(a+bx)}+\frac{\sinh(4x)}{8(a+bx)}\right) d\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(4x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{2bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx\right.}{bc\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{b^2c\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 0.62, size = 232, normalized size = 0.94

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-2*b + 4*b*c^2*x^2 - 2*b*c^4*x^4 + 2*(a + b*
ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*A
rcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sinh[(4*a)/b] - 2*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 2*b*ArcCosh[c*x]*Cosh[(2*a)
/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + a*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] + b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + A
rcCosh[c*x])]))/(2*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\,\text{arcosh}\,(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.34, size = 737, normalized size = 3.00

$$-\frac{\sqrt{-c^2x^2+1}\,\left(-8\sqrt{cx+1}\,\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\,\sqrt{cx+1}\,+4cx\right)}{16\,(cx+1)\,(cx-1)\,cb\,(a+b\,\text{arccosh}\,(cx))}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c*x)/(c*x+1)/(c*x-1)/c/b/(a+b*arccosh(c*x))+1/4*(-c^2*x^2+1)^(1/2)*(-(c
*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*
arccosh(c*x)+4*a)/b)/(c*x+1)/(c*x-1)/c/b^2+1/16*(-c^2*x^2+1)^(1/2)/(c*x-1)^
(1/2)/(c*x+1)^(1/2)*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei(1,-4*arccos
h(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/
c/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
/c/b/(a+b*arccosh(c*x))+1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-1)/c/
b/(a+b*arccosh(c*x))-1/2*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x
*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)/(c*x+1
)/(c*x-1)/c/b^2-1/4*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(2*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x
)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/c/b^2
/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^4x^4-2\,c^2x^2+1\right)(cx+1)\sqrt{cx-1}\,+\left(c^5x^5-2\,c^3x^3+cx\right)\sqrt{cx+1}\,\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\,\right)}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*
x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((4*c^4*x^4
- 3*c^2*x^2 - 1)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^5*x^5 - 3*c^3*x^3 + c*x
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^6*x^6 - 9*c^4*x^4 + 6*c^2*x^2 - 1)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a
*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b +
(b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3
*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2 x^2\right)^{3/2}}{(a+b \operatorname{acosh}(c x))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2,x)
[Out] int((1 - c^2*x^2)^(3/2)/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{(a+b \operatorname{acosh}(cx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(a + b*acosh(c*x))**2, x)
```

3.331 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{1-cx}\,\mathrm{Int}\left(\frac{c^2x^2-1}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{9\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx\right.}{b}\right.}{4b^2\sqrt{cx-1}}$$

```
[Out] 9/4*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-3/
4*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-
9/4*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)+3/
4*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-
(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x/(a+b*arccosh(c*x))-(-c
*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^2/(a+b*arccosh(c*x)),x)/b/c/(c*x-1)^
(1/2)
```

Rubi [A] time = 0.87, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) - (9*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Si
nh[a/b])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIn
tegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + (9*Sqrt[1 - c^2*x^2]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
)/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]
*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]) - (Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^2*(a + b*ArcCosh[c*x])
), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(3c\sqrt{1-c\right.}{b\sqrt{}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh^3(x)}{a+bx}\,dx, x, \cosh^{-1}\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{3i\sinh(x)}{4(a+bx)} - \frac{i\sinh(3x)}{4(a+bx)}\right.\right.}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(3x)}{a+bx}\,dx, x, \cosh^{-1}\right.}{4b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int \frac{-1+c^2x^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(9\sqrt{1-c^2\right.}{}$$

$$= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{9\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4b^2\sqrt{-1+cx}\,\sqrt{1+cx}} -$$

Mathematica [A] time = 34.30, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x\,\text{arcosh}\,(cx)^2 + 2\,abx\,\text{arcosh}\,(cx) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x*arccosh(c*x)^2 + 2*a*b*x*arccosh(c*x)
+ a^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{x\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^{4}x^{4}-2\,c^{2}x^{2}+1\right)(cx+1)\sqrt{cx-1}+\left(c^{5}x^{5}-2\,c^{3}x^{3}+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{3}+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^{2}x^{2}-abcx+\left(b^{2}c^{3}x^{3}+\sqrt{cx+1}\,\sqrt{cx-1}\,b^{2}c^{2}x^{2}-b^{2}cx\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*
c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(((3*
c^5*x^5 - c^3*x^3 - 2*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (6*c^6*x^6 - 7*c^4*x
^4 + 1)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*sqrt(c*
x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a
*b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 +
b^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x*(a + b*acosh(c*x))**2), x)
```

3.332 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=158

$$-\frac{2c\sqrt{1-cx}\,\text{Int}\left(\frac{c^2x^2-1}{x\left(a+b\cosh^{-1}(cx)\right)},x\right)}{b\sqrt{cx-1}}-\frac{2\sqrt{1-cx}\,\text{Int}\left(\frac{c^2x^2-1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\sqrt{cx+1}\left(1-c^2x^2\right)^{3/2}}{bcx^2\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))-
2*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^3/(a+b*arccosh(c*x)),x)/b/c/(c*
x-1)^(1/2)-2*c*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x/(a+b*arccosh(c*x))
,x)/b/(c*x-1)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (2*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^3*(a
 + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*c*Sqrt[1 -
 c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^3\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}}-\frac{(2c\cdots}{\cdots}$$

Mathematica [A] time = 64.94, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^2\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^2\,\text{arcosh}\,(cx)^2+2\,abx^2\,\text{arcosh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^2*arccosh(c*x)^2 + 2*a*b*x^2*arccosh(c
*x) + a^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{x^2\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^4x^4-2\,c^2x^2+1\right)(cx+1)\sqrt{cx-1}+\left(c^5x^5-2\,c^3x^3+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((2*c^5*x^5 + c^3*x^3 - 3*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(2*c^6*x^6 - c
^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (2*c^7*x^7 - 3*c^5*x^5 +
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b^
2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{3}{2}}}{x^2(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**2*(a + b*acosh(c*x))**2), x)
```

3.333 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(x^3*(a
+ b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 160.49, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(3/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^3\operatorname{arcosh}(cx)^2+2\,abx^3\operatorname{arcosh}(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^3*arccosh(c*x)^2 + 2*a*b*x^3*arccosh(c
*x) + a^2*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}}}{x^{3} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^{4} x^{4}-2 c^{2} x^{2}+1\right)(cx+1)\sqrt{cx-1}+\left(c^{5} x^{5}-2 c^{3} x^{3}+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3} x^{5}+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^{2} x^{4}-abcx^{3}+\left(b^{2} c^{3} x^{5}+\sqrt{cx+1}\,\sqrt{cx-1}\,b^{2} c^{2} x^{4}-b^{2} c x^{3}\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
((c^5*x^5 + 3*c^3*x^3 - 4*c*x)*(c*x + 1)^(3/2)*(c*x - 1) + (2*c^6*x^6 + 3*c
^4*x^4 - 8*c^2*x^2 + 3)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7 - 3*c^3*x^3 + 2*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*b^
2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}{x^{3} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**3*(a + b*acosh(c*x))**2), x)
```

3.334 $\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{4\sqrt{1-cx}\,\mathrm{Int}\left(\frac{c^2x^2-1}{x^5\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1-c^2x^2\right)^{3/2}}{bcx^4\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(-c^2*x^2+1)^(3/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^4/(a+b*arccosh(c*x))-
4*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)/x^5/(a+b*arccosh(c*x)),x)/b/c/(c*
x-1)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2])/(b*c*x^4*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (4*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)/(x^5*(a
 + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1-c^2x^2\right)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= -\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\ &= \frac{(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}}{bcx^4\sqrt{-1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4\sqrt{1-c^2x^2}\right)\int \frac{-1+c^2x^2}{x^5\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}\end{aligned}$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(3/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 1.01, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{b^2x^4\,\mathrm{arcosh}\,(cx)^2+2\,abx^4\,\mathrm{arcosh}\,(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((-c^2*x^2 + 1)^(3/2)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c
*x) + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(3/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{3}{2}}}{x^4\left(a+b\operatorname{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^4x^4-2\,c^2x^2+1\right)(cx+1)\sqrt{cx-1}+\left(c^5x^5-2\,c^3x^3+cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^5-abcx^4+\left(b^2c^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^5-b^2cx^4\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(3/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] ((c^4*x^4 - 2*c^2*x^2 + 1)*(c*x + 1)*sqrt(c*x - 1) + (c^5*x^5 - 2*c^3*x^3 +
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x -
 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^
2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) - integrate(
(5*(c^3*x^3 - c*x)*(c*x + 1)^(3/2)*(c*x - 1) + 4*(2*c^4*x^4 - 3*c^2*x^2 + 1
)*(c*x + 1)*sqrt(c*x - 1) + 3*(c^5*x^5 - 2*c^3*x^3 + c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^7 - 2*a*b*c^3*x^7
 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1) +
(b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^2*c^3*x^7 + b^2*c*x^5
+ 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{3/2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(3/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}}{x^{4}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(3/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(3/2)/(x**4*(a + b*acosh(c*x))**2), x)
```

3.335 $\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=454

$$-\frac{\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^3\sqrt{cx-1}}-\frac{\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8b^2c^3\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\mathrm{Chi}\left(\frac{6(a}{}\right.}{16b^2c^3\sqrt{cx-1}}$$

```
[Out] 1/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)
^(1/2)+1/8*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^3/(
c*x-1)^(1/2)-3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^3/(c*x-1)^(1/2)+1/16*cosh(8*a/b)*Shi(8*(a+b*arccosh(c*x))/b)*(-c*x+1)^(
1/2)/b^2/c^3/(c*x-1)^(1/2)-1/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c
*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-1/8*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/
b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)+3/16*Chi(6*(a+b*arccosh(c*x))/b)*si
nh(6*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-1/16*Chi(8*(a+b*arccosh(c*x)
)/b)*sinh(8*a/b)*(-c*x+1)^(1/2)/b^2/c^3/(c*x-1)^(1/2)-x^2*(-c^2*x^2+1)^(5/2
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 1.51, antiderivative size = 565, normalized size of antiderivative = 1.24, number of steps used = 29, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5778, 5780, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{2a}{b}\right)\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{\sqrt{1-c^2x^2}\,\sinh\left(\frac{4a}{b}\right)\mathrm{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{8b^2c^3\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\,\sinh}{16b^2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x^2*(1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) - (Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[1 - c^
2*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(8*b^2*c^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*
ArcCosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (S
qrt[1 - c^2*x^2]*CoshIntegral[(8*a)/b + 8*ArcCosh[c*x]]*Sinh[(8*a)/b])/(16*
b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(2*a)/b]*Si
nhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*
x]])/(8*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(
6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Cosh[(8*a)/b]*SinhIntegral[(8*a)/b + 8*
ArcCosh[c*x]])/(16*b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{x^2(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(8c\sqrt{1-c^2x^2}\right)}{b\sqrt{-1+}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx, x, \cosh^{-}\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(2\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh}{32(a}\right.\right.}{bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}} +$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(5\sqrt{1-c^2x^2}\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx, x\right.}{16bc^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x^2(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c^3\sqrt{-1+cx}\sqrt{1+cx}} - \cdots$$

Mathematica [A] time = 1.74, size = 446, normalized size = 0.98

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+2\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(4\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b*c^2*x^2 + 48*b*c^4*x^4 - 48*b*c^6*x^6
+ 16*b*c^8*x^8 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*
Sinh[(2*a)/b] + 2*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]
*Sinh[(4*a)/b] - 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 3
*b*ArcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + a*Cosh
Integral[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] + b*ArcCosh[c*x]*CoshIntegra
l[8*(a/b + ArcCosh[c*x])]*Sinh[(8*a)/b] - a*Cosh[(2*a)/b]*SinhIntegral[2*(a
/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + Ar
cCosh[c*x])] - 2*a*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] - 2*b
*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])] + 3*a*Cosh
[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] + 3*b*ArcCosh[c*x]*Cosh[(6*a
)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])] - a*Cosh[(8*a)/b]*SinhIntegral[8*
(a/b + ArcCosh[c*x])] - b*ArcCosh[c*x]*Cosh[(8*a)/b]*SinhIntegral[8*(a/b +
ArcCosh[c*x])]))/(16*b^2*c^3*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^6-2\,c^2x^4+x^2\right)\sqrt{-c^2x^2+1}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^6 - 2*c^2*x^4 + x^2)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2
 + 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x^2}{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x^2/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.87, size = 1676, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/256*(-c^2*x^2+1)^(1/2)*(-128*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^8*c^8+128*x^9*
c^9+256*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6-320*c^7*x^7-160*(c*x+1)^(1/2)*(
c*x-1)^(1/2)*x^4*c^4+272*c^5*x^5+32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-88*
c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+8*c*x)/(c*x+1)/(c*x-1)/c^3/b/(a+b*arcco
sh(c*x))-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-
1)*Ei(1,8*arccosh(c*x)+8*a/b)*exp((b*arccosh(c*x)+8*a)/b)/(c*x+1)/(c*x-1)/c
^3/b^2-1/256*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(128*(c*x-1)^(1
/2)*(c*x+1)^(1/2)*x^7*b*c^7+128*x^8*b*c^8-192*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x
^5*b*c^5-256*x^6*b*c^6+80*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+160*x^4*b*c
^4-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c-32*x^2*b*c^2+8*arccosh(c*x)*Ei(1,-8*
arccosh(c*x)-8*a/b)*exp(-8*a/b)*b+8*Ei(1,-8*arccosh(c*x)-8*a/b)*exp(-8*a/b)
*a+b)/c^3/b^2/(a+b*arccosh(c*x))+5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)/c^3/b/(a+b*arccosh(c*x))-1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1
/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4
-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)
*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1)/c^3/b/(a+b*arccosh(c*x))+3/32*(-c^2*x
^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x
)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2
+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)
*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1
)/(c*x-1)/c^3/b/(a+b*arccosh(c*x))-1/16*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)
+4*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2+1/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/
(c*x-1)/c^3/b/(a+b*arccosh(c*x))-1/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2
*a)/b)/(c*x+1)/(c*x-1)/c^3/b^2-1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(
1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2
*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))-1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*(c*x+
1)^(1/2)*(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei(1,-4*arccosh(c*x
)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/c^3/b
^2/(a+b*arccosh(c*x))+1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(
```

```
32*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+18*x^2
*b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-6*a/
b)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c^3/b^2/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^6x^8-3\,c^4x^6+3\,c^2x^4-x^2\right)(cx+1)\sqrt{cx-1}+\left(c^7x^9-3\,c^5x^7+3\,c^3x^5-cx^3\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^8 - 3*c^4*x^6 + 3*c^2*x^4 - x^2)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^
9 - 3*c^5*x^7 + 3*c^3*x^5 - c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))) + integrate(((8*c^7*x^8 - 17*c^5*x^6 + 10*c^3*x^4 - c*x^2)*(c*x + 1)^(
3/2)*(c*x - 1) + 2*(8*c^8*x^9 - 22*c^6*x^7 + 21*c^4*x^5 - 8*c^2*x^3 + x)*(c
*x + 1)*sqrt(c*x - 1) + (8*c^9*x^10 - 27*c^7*x^8 + 33*c^5*x^6 - 17*c^3*x^4
+ 3*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)
*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^
2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x^2*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.336 $\int \frac{x\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=448

$$\frac{5\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{64b^2c^2\sqrt{cx-1}}-\frac{27\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{64b^2c^2\sqrt{cx-1}}+\frac{25\sqrt{1-cx}\,\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\right.}{64b^2c^2\sqrt{cx-}}$$

```
[Out] -5/64*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1
/2)+27/64*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c^2/(c
*x-1)^(1/2)-25/64*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^
2/c^2/(c*x-1)^(1/2)+7/64*cosh(7*a/b)*Shi(7*(a+b*arccosh(c*x))/b)*(-c*x+1)^(
1/2)/b^2/c^2/(c*x-1)^(1/2)+5/64*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1
)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-27/64*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)
*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)+25/64*Chi(5*(a+b*arccosh(c*x))/b)*sin
h(5*a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-7/64*Chi(7*(a+b*arccosh(c*x))
/b)*sinh(7*a/b)*(-c*x+1)^(1/2)/b^2/c^2/(c*x-1)^(1/2)-x*(-c^2*x^2+1)^(5/2)*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 1.33, antiderivative size = 555, normalized size of antiderivative = 1.24, number of steps used = 29, number of rules used = 9, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.346, Rules used = {5798, 5778, 5700, 3312, 3303, 3298, 3301, 5780, 5448}

$$\frac{5\sqrt{1-c^2x^2}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{64b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{27\sqrt{1-c^2x^2}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{64b^2c^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{25\sqrt{1-c^2x^2}\,\text{si}}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (x*(1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) + (5*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*Si
nh[a/b])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (27*Sqrt[1 - c^2*x^2]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(64*b^2*c^2*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCo
sh[c*x]]*Sinh[(5*a)/b])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (7*Sqrt
[1 - c^2*x^2]*CoshIntegral[(7*a)/b + 7*ArcCosh[c*x]]*Sinh[(7*a)/b])/(64*b^2
*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*Cosh[a/b]*SinhInt
egral[a/b + ArcCosh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (27*
Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(64
*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (25*Sqrt[1 - c^2*x^2]*Cosh[(5*a)/b
]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + (7*Sqrt[1 - c^2*x^2]*Cosh[(7*a)/b]*SinhIntegral[(7*a)/b + 7*ArcC
osh[c*x]])/(64*b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5700

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Dist[(-d)^p/c, Subst[Int[(a + b*x)^n*Sinh[x]^(2*p + 1), x], x,
 ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e, 0] &&
IGtQ[p, 0]
```

Rule 5778

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d1_) + (e
1_.)*(x_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((f*x)^m*Sq
rt[1 + c*x]*Sqrt[-1 + c*x]*(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])
^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(-(d1*d2))^IntPart[p]*(d1 + e1*x)^
FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*c*(n + 1)*(1 + c*x)^FracPart[p]*(-1
 + c*x)^FracPart[p]), Int[(f*x)^(m - 1)*(-1 + c^2*x^2)^(p - 1/2)*(a + b*Arc
Cosh[c*x])^(n + 1), x], x] - Dist[(c*(m + 2*p + 1)*(-(d1*d2))^IntPart[p]*(d
1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(b*f*(n + 1)*(1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^(m + 1)*(-1 + c^2*x^2)^(p - 1/2)*
(a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f}
, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && LtQ[n, -1] && IGtQ[m, -3]
 && IGtQ[p + 1/2, 0]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int\frac{x(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\int\frac{\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(7c\sqrt{1-c^2x^2}\right)}{b\sqrt{-1+c}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int\frac{\sinh^5(x)}{a+bx}dx,x,\cosh^{-1}(cx)\right)}{bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{5i\sinh(x)}{8(a+bx)}-\frac{5i\sinh(3x)}{16(a+bx)}+\right.\right.}{bc^2\sqrt{-1+cx}\sqrt{1+c}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{1-c^2x^2}\,\text{Subst}\left(\int\frac{\sinh(5x)}{a+bx}dx,x,\cosh^{-1}(cx)\right)}{16bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(35\sqrt{1-c^2x^2}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sinh\left(\frac{a}{b}+x\right)}{a+bx}dx,\right.}{64bc^2\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{x(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{5\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{64b^2c^2\sqrt{-1+cx}\sqrt{1+cx}} - 2$$

Mathematica [A] time = 1.43, size = 436, normalized size = 0.97

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-5\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+27\sinh\left(\frac{3a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(1 - c^2*x^2)^(5/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-64*b*c*x + 192*b*c^3*x^3 - 192*b*c^5*x^5 +
64*b*c^7*x^7 - 5*(a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh
[a/b] + 27*(a + b*ArcCosh[c*x])*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(
3*a)/b] - 25*a*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 25*b*Ar
cCosh[c*x]*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 7*a*CoshInt
egral[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 7*b*ArcCosh[c*x]*CoshIntegral
[7*(a/b + ArcCosh[c*x])]*Sinh[(7*a)/b] + 5*a*Cosh[a/b]*SinhIntegral[a/b + A
rcCosh[c*x]] + 5*b*ArcCosh[c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]]
- 27*a*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] - 27*b*ArcCosh[c*
x]*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 25*a*Cosh[(5*a)/b]*
SinhIntegral[5*(a/b + ArcCosh[c*x])] + 25*b*ArcCosh[c*x]*Cosh[(5*a)/b]*Sinh
Integral[5*(a/b + ArcCosh[c*x])] - 7*a*Cosh[(7*a)/b]*SinhIntegral[7*(a/b +
ArcCosh[c*x])] - 7*b*ArcCosh[c*x]*Cosh[(7*a)/b]*SinhIntegral[7*(a/b + ArcCo
sh[c*x])]))/(64*b^2*c^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^5-2\,c^2x^3+x\right)\sqrt{-c^2x^2+1}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^5 - 2*c^2*x^3 + x)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)*x/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.56, size = 1499, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/128*(-c^2*x^2+1)^(1/2)*(-64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^7*c^7+64*c^8*x^
8+112*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5-144*c^6*x^6-56*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3+104*c^4*x^4+7*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-25*c^2*x^2+
1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b-7/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,7*arccosh(c*x)+7*a/b)*exp((b*ar
ccosh(c*x)+7*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2-1/128*(-c^2*x^2+1)^(1/2)/(c*x-1)
^(1/2)/(c*x+1)^(1/2)*(64*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*b*c^6+64*x^7*b*c^7
-80*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*b*c^4-112*x^5*b*c^5+24*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x^2*b*c^2+56*x^3*b*c^3+7*arccosh(c*x)*Ei(1,-7*arccosh(c*x)-7*a/
b)*exp(-7*a/b)*b+7*Ei(1,-7*arccosh(c*x)-7*a/b)*exp(-7*a/b)*a-(c*x+1)^(1/2)*
(c*x-1)^(1/2)*b-7*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-5/128*(-c^2*x^2+1)^(1/2
)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^6+20*(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+13*c^2*x^2-1
)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+25/128*(-c^2*x^2+1)^(1/2)*(-(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,5*arccosh(c*x)+5*a/b)*exp((b*ar
ccosh(c*x)+5*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+9/128*(-c^2*x^2+1)^(1/2)*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*
c-5*c^2*x^2+1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b-27/128*(-c^2*x^2+1)
^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,3*arccosh(c*x)+3*a
/b)*exp((b*arccosh(c*x)+3*a)/b)/(c*x+1)/c^2/(c*x-1)/b^2+5/128*(-c^2*x^2+1)^
(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x
)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+
x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-9/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(
c*x)*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b
)*exp(-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/c^2/b^2/(a+b*arccosh
(c*x))+5/128*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(16*(c*x+1)^(1/
2)*(c*x-1)^(1/2)*x^4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*
b*c^2-20*x^3*b*c^3+5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b
+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+5*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+
5*x*b*c)/c^2/b^2/(a+b*arccosh(c*x))-5/128*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2
)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c*x+1)/c^2/(c*x-1)/(a+b*arccosh(c*x))/b+5/1
28*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,arc
cosh(c*x)+a/b)*exp((a+b*arccosh(c*x))/b)/(c*x+1)/c^2/(c*x-1)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^{6}x^{7}-3\,c^{4}x^{5}+3\,c^{2}x^{3}-x\right)(cx+1)\sqrt{cx-1}+\left(c^{7}x^{8}-3\,c^{5}x^{6}+3\,c^{3}x^{4}-cx^{2}\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^{2}x-abc+\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,b^{2}c^{2}x-b^{2}c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^7 - 3*c^4*x^5 + 3*c^2*x^3 - x)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^8
- 3*c^5*x^6 + 3*c^3*x^4 - c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2
 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x
+ 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
)) + integrate((7*(c^7*x^7 - 2*c^5*x^5 + c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1)
 + (14*c^8*x^8 - 37*c^6*x^6 + 33*c^4*x^4 - 11*c^2*x^2 + 1)*(c*x + 1)*sqrt(c
*x - 1) + (7*c^9*x^9 - 23*c^7*x^7 + 27*c^5*x^5 - 13*c^3*x^3 + 2*c*x)*sqrt(c
*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^2 - 2*
a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*c^3*x^2 + b^2*c
 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((x*(1 - c^2*x^2)^(5/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.337 $\int \frac{(1-c^2x^2)^{5/2}}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=351

$$-\frac{15\sqrt{1-cx}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{16b^2c\sqrt{cx-1}}+\frac{3\sqrt{1-cx}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4(a+b\cosh^{-1}(cx))}{b}\right)}{4b^2c\sqrt{cx-1}}-\frac{3\sqrt{1-cx}\,\sinh\left(\frac{6a}{b}\right)\text{Chi}}{16b^2c\sqrt{cx-}}$$

```
[Out] 15/16*cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x-1)^
(1/2)-3/4*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c/(c*x
-1)^(1/2)+3/16*cosh(6*a/b)*Shi(6*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/c
/(c*x-1)^(1/2)-15/16*Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(-c*x+1)^(1/2)
/b^2/c/(c*x-1)^(1/2)+3/4*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(-c*x+1)^(
1/2)/b^2/c/(c*x-1)^(1/2)-3/16*Chi(6*(a+b*arccosh(c*x))/b)*sinh(6*a/b)*(-c*x
+1)^(1/2)/b^2/c/(c*x-1)^(1/2)-(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.65, antiderivative size = 436, normalized size of antiderivative = 1.24, number of steps used = 14, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {5713, 5697, 5780, 5448, 3303, 3298, 3301}

$$-\frac{15\sqrt{1-c^2x^2}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{16b^2c\sqrt{cx-1}\sqrt{cx+1}}+\frac{3\sqrt{1-c^2x^2}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{4b^2c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3\sqrt{1-c^2x^2}\,\text{si}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[-1 + c*x]*(a + b*
ArcCosh[c*x])) - (15*Sqrt[1 - c^2*x^2]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x
]]*Sinh[(2*a)/b])/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*Sqrt[1 - c^2
*x^2]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(4*b^2*c*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*CoshIntegral[(6*a)/b + 6*Arc
Cosh[c*x]]*Sinh[(6*a)/b])/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (15*Sqr
t[1 - c^2*x^2]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(16*b^
2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*Sqrt[1 - c^2*x^2]*Cosh[(4*a)/b]*Sinh
Integral[(4*a)/b + 4*ArcCosh[c*x]])/(4*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
+ (3*Sqrt[1 - c^2*x^2]*Cosh[(6*a)/b]*SinhIntegral[(6*a)/b + 6*ArcCosh[c*x]]
)/(16*b^2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6c\sqrt{1-c^2x^2}\right)\int \frac{x\left(-1+c^2x^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx}{b\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{a+bx}\,dx,x,\cosh^{-}\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(6\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32(a+bx)} - \frac{\sinh(4x)}{8(a+bx)} + \frac{\sinh}{32(a}\right.\right.}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(15\sqrt{1-c^2x^2}\,\cosh\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{2a}{b}+2x\right)}{a+bx}\,dx,\right.}{16bc\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bc\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{15\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)\sinh\left(\frac{2a}{b}\right)}{16b^2c\sqrt{-1+cx}\,\sqrt{1+cx}} +$$

Mathematica [A] time = 1.04, size = 343, normalized size = 0.98

$$\frac{\sqrt{cx-1}\,\sqrt{cx+1}\left(15\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-12\sinh\left(\frac{4a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Ch}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-16*b + 48*b*c^2*x^2 - 48*b*c^4*x^4 + 16*b*c
^6*x^6 + 15*(a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b + ArcCosh[c*x])]*Sinh[
(2*a)/b] - 12*(a + b*ArcCosh[c*x])*CoshIntegral[4*(a/b + ArcCosh[c*x])]*Sin
h[(4*a)/b] + 3*a*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] + 3*b*A
rcCosh[c*x]*CoshIntegral[6*(a/b + ArcCosh[c*x])]*Sinh[(6*a)/b] - 15*a*Cosh[
(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - 15*b*ArcCosh[c*x]*Cosh[(2*a
)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] + 12*a*Cosh[(4*a)/b]*SinhIntegral
[4*(a/b + ArcCosh[c*x])] + 12*b*ArcCosh[c*x]*Cosh[(4*a)/b]*SinhIntegral[4*(
a/b + ArcCosh[c*x])] - 3*a*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x]
)] - 3*b*ArcCosh[c*x]*Cosh[(6*a)/b]*SinhIntegral[6*(a/b + ArcCosh[c*x])]))/
(16*b^2*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*arccosh(c*x)^2 +
 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.43, size = 1176, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/64*(-c^2*x^2+1)^(1/2)*(-32*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^6*c^6+32*c^7*x^7
+48*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4-64*c^5*x^5-18*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2+38*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-6*c*x)/(c*x+1)/(c*x-1
)/c/b/(a+b*arccosh(c*x))-3/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*Ei(1,6*arccosh(c*x)+6*a/b)*exp((b*arccosh(c*x)+6*a)/b)/(
c*x+1)/(c*x-1)/c/b^2-1/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)*(3
2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*b*c^5+32*x^6*b*c^6-32*(c*x-1)^(1/2)*(c*x+
1)^(1/2)*x^3*b*c^3-48*x^4*b*c^4+6*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+18*x^2*
b*c^2+6*arccosh(c*x)*exp(-6*a/b)*Ei(1,-6*arccosh(c*x)-6*a/b)*b+6*exp(-6*a/b
)*Ei(1,-6*arccosh(c*x)-6*a/b)*a-b)/c/b^2/(a+b*arccosh(c*x))+5/16*(-c^2*x^2+
1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)/c/b/(a+b*arccosh(c*x))-3/32*(-c^2*x^2+
1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+8*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c*x)/(c*x+1)
/(c*x-1)/c/b/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x
-1)^(1/2)*x*c+c^2*x^2-1)*Ei(1,4*arccosh(c*x)+4*a/b)*exp((b*arccosh(c*x)+4*a
)/b)/(c*x+1)/(c*x-1)/c/b^2+15/64*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-
1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c*x+1)/(c*x-
1)/c/b/(a+b*arccosh(c*x))-15/32*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x*c+c^2*x^2-1)*Ei(1,2*arccosh(c*x)+2*a/b)*exp((b*arccosh(c*x)+2*a)/b)
/(c*x+1)/(c*x-1)/c/b^2-15/64*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*ar
ccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a
-b)/c/b^2/(a+b*arccosh(c*x))+3/32*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*x^4*b*c^4-4*(c*x+1)^(1/2)*
(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei(1,-4*arccosh(c*x)-4*a/b)*
exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*a+b)/c/b^2/(a+b*arc
cosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^2 +
```

```
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate(((6*c^6*x^6 - 11*c^4*x^4 + 4*c^2*x^2 + 1)*(c*x + 1)^(3/2)*(c*x
 - 1) + 6*(2*c^7*x^7 - 5*c^5*x^5 + 4*c^3*x^3 - c*x)*(c*x + 1)*sqrt(c*x - 1)
 + (6*c^8*x^8 - 19*c^6*x^6 + 21*c^4*x^4 - 9*c^2*x^2 + 1)*sqrt(c*x + 1))*sqr
t(-c*x + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2
+ 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^
4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*
c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(1-c^2\,x^2\right)^{5/2}}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2,x)
[Out] int((1 - c^2*x^2)^(5/2)/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(a + b*acosh(c*x))**2, x)
```

3.338 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=386

$$\frac{\sqrt{1-cx}\,\mathrm{Int}\left(\frac{\left(c^2x^2-1\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{cx-1}}-\frac{25\sqrt{1-cx}\,\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2\sqrt{cx-1}}+\frac{25\sqrt{1-cx}\,\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh\right.}{b}\right.}{16b^2\sqrt{cx-1}}$$

```
[Out] 25/8*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/2)-2
5/16*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(1/
2)+5/16*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^
(1/2)-25/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(-c*x+1)^(1/2)/b^2/(c*x-1)^(
1/2)+25/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(-c*x+1)^(1/2)/b^2/(c*x-
1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(-c*x+1)^(1/2)/b^2/(c
*x-1)^(1/2)-(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x/(a+b*arcco
sh(c*x))+(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x^2/(a+b*arccosh(c*x)),x
)/b/c/(c*x-1)^(1/2)
```

Rubi [A] time = 0.98, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*x*Sqrt[-1 + c*x]*(a +
b*ArcCosh[c*x])) - (25*Sqrt[1 - c^2*x^2]*CoshIntegral[a/b + ArcCosh[c*x]]*S
inh[a/b])/(8*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2]*Cosh
Integral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (5*Sqrt[1 - c^2*x^2]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]*
Sinh[(5*a)/b])/(16*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (25*Sqrt[1 - c^2*x^2
]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (25*Sqrt[1 - c^2*x^2]*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcC
osh[c*x]])/(16*b^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (5*Sqrt[1 - c^2*x^2]*Cos
h[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) + (Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x^2*(a + b*
ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(5c\sqrt{1-c^2x\right.}{b\sqrt{-1}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh^5(x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(5i\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int\left(\frac{5i\sinh(x)}{8(a+bx)} - \frac{5i\sinh(3x)}{16(a+bx)} + \right.\right.}{b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{1-c^2x^2}\right)\text{Subst}\left(\int \frac{\sinh(5x)}{a+bx}\,dx, x, \cosh^{-1}(c\right.}{16b\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\sqrt{1-c^2x^2}\int \frac{\left(-1+c^2x^2\right)^2}{x^2\left(a+b\cosh^{-1}(cx)\right)}\,dx}{bc\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(25\sqrt{1-c^2x\right.}{}$$

$$= \frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx\sqrt{-1+cx}\left(a+b\cosh^{-1}(cx)\right)} - \frac{25\sqrt{1-c^2x^2}\,\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8b^2\sqrt{-1+cx}\sqrt{1+cx}} +$$

Mathematica [A] time = 9.00, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4 - 2\,c^2x^2 + 1\right)\sqrt{-c^2x^2+1}}{b^2x\,\text{arcosh}\,(cx)^2 + 2\,abx\,\text{arcosh}\,(cx) + a^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x*arccosh(c*x)^2
 + 2*a*b*x*arccosh(c*x) + a^2*x), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}}}{x \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^{6} x^{6}-3 \, c^{4} x^{4}+3 \, c^{2} x^{2}-1\right) (c x+1) \sqrt{c x-1}+\left(c^{7} x^{7}-3 \, c^{5} x^{5}+3 \, c^{3} x^{3}-c x\right) \sqrt{c x+1}\right) \sqrt{-c x+1}}{a b c^{3} x^{3}+\sqrt{c x+1} \sqrt{c x-1} \, a b c^{2} x^{2}-a b c x+\left(b^{2} c^{3} x^{3}+\sqrt{c x+1} \sqrt{c x-1} \, b^{2} c^{2} x^{2}-b^{2} c x\right) \log \left(c x+\sqrt{c x+1} \sqrt{c x}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^3 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 - a*b*c*x + (b^2*c^3*x^3 + sqrt(c*
x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 - b^2*c*x)*log(c*x + sqrt(c*x + 1)*sqrt(c*
x - 1))) + integrate(((5*c^7*x^7 - 8*c^5*x^5 + c^3*x^3 + 2*c*x)*(c*x + 1)^(
3/2)*(c*x - 1) + (10*c^8*x^8 - 23*c^6*x^6 + 15*c^4*x^4 - c^2*x^2 - 1)*(c*x
+ 1)*sqrt(c*x - 1) + 5*(c^9*x^9 - 3*c^7*x^7 + 3*c^5*x^5 - c^3*x^3)*sqrt(c*x
 + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^6 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^4 - 2*a*
b*c^3*x^4 + a*b*c*x^2 + 2*(a*b*c^4*x^5 - a*b*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^6 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^4 - 2*b^2*c^3*x^4 + b
^2*c*x^2 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{5}{2}}}{x \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x*(a + b*acosh(c*x))**2), x)
```

3.339 $\int \frac{(1-c^2x^2)^{5/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{4c\sqrt{1-cx}\,\mathrm{Int}\left(\frac{(c^2x^2-1)^2}{x(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{cx-1}}+\frac{2\sqrt{1-cx}\,\mathrm{Int}\left(\frac{(c^2x^2-1)^2}{x^3(a+b\cosh^{-1}(cx))},x\right)}{bc\sqrt{cx-1}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(1-c^2x^2)^{5/2}}{bcx^2\,(a+b\cosh^{-1}(cx))}$$

```
[Out] -(-c^2*x^2+1)^(5/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))+
2*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x^3/(a+b*arccosh(c*x)),x)/b/c/(
c*x-1)^(1/2)+4*c*(-c*x+1)^(1/2)*Unintegrable((c^2*x^2-1)^2/x/(a+b*arccosh(c
*x)),x)/b/(c*x-1)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] ((1 - c*x)^3*(1 + c*x)^(5/2)*Sqrt[1 - c^2*x^2])/(b*c*x^2*Sqrt[-1 + c*x]*(a
+ b*ArcCosh[c*x])) + (2*Sqrt[1 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x^3*
(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (4*c*Sqrt[1
 - c^2*x^2]*Defer[Int][(-1 + c^2*x^2)^2/(x*(a + b*ArcCosh[c*x])), x])/(b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^2(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{(1-cx)^3(1+cx)^{5/2}\sqrt{1-c^2x^2}}{bcx^2\sqrt{-1+cx}\,(a+b\cosh^{-1}(cx))}+\frac{\left(2\sqrt{1-c^2x^2}\right)\int \frac{(-1+c^2x^2)^2}{x^3(a+b\cosh^{-1}(cx))}\,dx}{bc\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(4c\sqrt{1\right.}{}$$

Mathematica [A] time = 17.84, size = 0, normalized size = 0.00

$$\int \frac{(1-c^2x^2)^{5/2}}{x^2\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^2*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(c^4x^4-2\,c^2x^2+1)\sqrt{-c^2x^2+1}}{b^2x^2\,\mathrm{arcosh}\,(cx)^2+2\,abx^2\,\mathrm{arcosh}\,(cx)+a^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^2*arccosh(c*x)
^2 + 2*a*b*x^2*arccosh(c*x) + a^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{x^2\left(a+b\operatorname{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^3-abcx^2+\left(b^2c^3x^4+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^3-b^2cx^2\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^4 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 - a*b*c*x^2 + (b^2*c^3*x^4 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 - b^2*c*x^2)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((4*c^7*x^7 - 5*c^5*x^5 - 2*c^3*x^3 + 3*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(4*c^8*x^8 - 8*c^6*x^6 + 3*c^4*x^4 + 2*c^2*x^2 - 1
)*(c*x + 1)*sqrt(c*x - 1) + (4*c^9*x^9 - 11*c^7*x^7 + 9*c^5*x^5 - c^3*x^3 -
 c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^7 + (c*x + 1)*(c*x - 1)*a*b*
c^3*x^5 - 2*a*b*c^3*x^5 + a*b*c*x^3 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*sqrt(c*
x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^7 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^5 - 2*b
^2*c^3*x^5 + b^2*c*x^3 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*sqrt(c*x + 1)*sqrt(c
*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^2*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-(cx-1)(cx+1))^{\frac{5}{2}}}{x^2 (a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**2*(a + b*acosh(c*x))**2), x)
```

3.340 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^3*(a +
b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 21.18, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^3\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^3*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b^2x^3\,\text{arcosh}\,(cx)^2+2\,abx^3\,\text{arcosh}\,(cx)+a^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^3*arccosh(c*x)
^2 + 2*a*b*x^3*arccosh(c*x) + a^2*x^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{x^3\left(a+b\,\mathrm{arccosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^5+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^4-abcx^3+\left(b^2c^3x^5+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^4-b^2cx^3\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^3/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^5 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^4 - a*b*c*x^3 + (b^2*c^3*x^5 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^4 - b^2*c*x^3)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((3*c^7*x^7 - 2*c^5*x^5 - 5*c^3*x^3 + 4*c*x)*(c*x
+ 1)^(3/2)*(c*x - 1) + 3*(2*c^8*x^8 - 3*c^6*x^6 - c^4*x^4 + 3*c^2*x^2 - 1)*
(c*x + 1)*sqrt(c*x - 1) + (3*c^9*x^9 - 7*c^7*x^7 + 3*c^5*x^5 + 3*c^3*x^3 -
2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^8 + (c*x + 1)*(c*x - 1)*a*b
*c^3*x^6 - 2*a*b*c^3*x^6 + a*b*c*x^4 + 2*(a*b*c^4*x^7 - a*b*c^2*x^5)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^8 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^6 - 2*
b^2*c^3*x^6 + b^2*c*x^4 + 2*(b^2*c^4*x^7 - b^2*c^2*x^5)*sqrt(c*x + 1)*sqrt(
c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^3*(a + b*acosh(c*x))^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}}{x^3\left(a+b\,\mathrm{acosh}\,(cx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**3/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((-(c*x - 1)*(c*x + 1))**(5/2)/(x**3*(a + b*acosh(c*x))**2), x)
```

3.341 $\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\mathrm{Int}\left(\frac{\left(1-c^2x^2\right)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(x^4*(a + b*ArcCosh[c*x])^2), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\sqrt{1-c^2x^2}\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 179.70, size = 0, normalized size = 0.00

$$\int \frac{\left(1-c^2x^2\right)^{5/2}}{x^4\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(1 - c^2*x^2)^(5/2)/(x^4*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^4x^4-2\,c^2x^2+1\right)\sqrt{-c^2x^2+1}}{b^2x^4\,\mathrm{arcosh}\,(cx)^2+2\,abx^4\,\mathrm{arcosh}\,(cx)+a^2x^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((c^4*x^4 - 2*c^2*x^2 + 1)*sqrt(-c^2*x^2 + 1)/(b^2*x^4*arccosh(c*x)^2 + 2*a*b*x^4*arccosh(c*x) + a^2*x^4), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((-c^2*x^2 + 1)^(5/2)/((b*arccosh(c*x) + a)^2*x^4), x)
```

maple [A] time = 1.25, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2x^2+1\right)^{\frac{5}{2}}}{x^4\left(a+b\operatorname{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^6x^6-3\,c^4x^4+3\,c^2x^2-1\right)(cx+1)\sqrt{cx-1}+\left(c^7x^7-3\,c^5x^5+3\,c^3x^3-cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}}{abc^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x^5-abcx^4+\left(b^2c^3x^6+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x^5-b^2cx^4\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*x^2+1)^(5/2)/x^4/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -((c^6*x^6 - 3*c^4*x^4 + 3*c^2*x^2 - 1)*(c*x + 1)*sqrt(c*x - 1) + (c^7*x^7
- 3*c^5*x^5 + 3*c^3*x^3 - c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^3*x^6 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^5 - a*b*c*x^4 + (b^2*c^3*x^6 + sqrt(
c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^5 - b^2*c*x^4)*log(c*x + sqrt(c*x + 1)*sqr
t(c*x - 1))) + integrate(((2*c^7*x^7 + c^5*x^5 - 8*c^3*x^3 + 5*c*x)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(2*c^8*x^8 - c^6*x^6 - 6*c^4*x^4 + 7*c^2*x^2 - 2)*(c
*x + 1)*sqrt(c*x - 1) + (2*c^9*x^9 - 3*c^7*x^7 - 3*c^5*x^5 + 7*c^3*x^3 - 3*
c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)/(a*b*c^5*x^9 + (c*x + 1)*(c*x - 1)*a*b*c
^3*x^7 - 2*a*b*c^3*x^7 + a*b*c*x^5 + 2*(a*b*c^4*x^8 - a*b*c^2*x^6)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^9 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^7 - 2*b^
2*c^3*x^7 + b^2*c*x^5 + 2*(b^2*c^4*x^8 - b^2*c^2*x^6)*sqrt(c*x + 1)*sqrt(c*
x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(1-c^2\,x^2\right)}^{5/2}}{x^4\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))^2),x)
```

```
[Out] int((1 - c^2*x^2)^(5/2)/(x^4*(a + b*acosh(c*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*x**2+1)**(5/2)/x**4/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.342 $\int \frac{x^5}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=337

$$-\frac{5\sqrt{cx-1}\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^6\sqrt{1-cx}}-\frac{15\sqrt{cx-1}\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^6\sqrt{1-cx}}-\frac{5\sqrt{cx-1}\sinh\left(\frac{5a}{b}\right)\mathrm{Chi}\left(\frac{5(a}{}\right.}{16b^2c^6\sqrt{1-cx}}$$

```
[Out] -x^5*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+5/8*cosh(a/b)*Shi(
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)+15/16*cosh(3*a/b
)*Shi(3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)+5/16*cos
h(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)-5
/8*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+1)^(1/2)
-15/16*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b^2/c^6/(-c*x+
1)^(1/2)-5/16*Chi(5*(a+b*arccosh(c*x))/b)*sinh(5*a/b)*(c*x-1)^(1/2)/b^2/c^6
/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.86, antiderivative size = 424, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{5\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^6\sqrt{1-c^2x^2}}-\frac{15\sqrt{cx-1}\sqrt{cx+1}\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{16b^2c^6\sqrt{1-c^2x^2}}-5\sqrt{c}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[a/b + ArcCosh[c*x]]*
Sinh[a/b])/(8*b^2*c^6*Sqrt[1 - c^2*x^2]) - (15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(16*b^2*c^6*Sqrt[1 -
 c^2*x^2]) - (5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(5*a)/b + 5*ArcCo
sh[c*x]]*Sinh[(5*a)/b])/(16*b^2*c^6*Sqrt[1 - c^2*x^2]) + (5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^6*Sqrt[1
 - c^2*x^2]) + (15*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(3*a)/b]*SinhIntegral[
(3*a)/b + 3*ArcCosh[c*x]])/(16*b^2*c^6*Sqrt[1 - c^2*x^2]) + (5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*Cosh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*
b^2*c^6*Sqrt[1 - c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^5}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4}{a+b\cosh^{-1}(cx)}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\cosh^4}{}\right.}{bc^6\sqrt{1-c^2x}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\left(\frac{\sinh}{8(a+}\right.\right.}{bc^6}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\sinh(5}{a+bx}\right.}{16bc^6\sqrt{1-c^2x^2}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(5\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subs}}{8bc^6\sqrt{1-}}$$

$$= -\frac{x^5\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)} - \frac{5\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(\right.}{8b^2c^6\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.70, size = 190, normalized size = 0.56

$$\frac{\sqrt{1-c^2x^2}\,\left(\frac{16bc^5x^5}{a+b\cosh^{-1}(cx)} + 5\left(2\sinh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right) + 3\sinh\left(\frac{3a}{b}\right)\mathrm{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right) + \sinh\left(\frac{5a}{b}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((16*b*c^5*x^5)/(a + b*ArcCosh[c*x]) + 5*(2*CoshIntegral
[a/b + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sin
h[(3*a)/b] + CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] - 2*Cosh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*SinhIntegral[3*(a/b +
 ArcCosh[c*x])] - Cosh[(5*a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])])))/(16
*b^2*c^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^5}{a^2c^2x^2 + \left(b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2 - a^2 + 2\left(abc^2x^2-ab\right)\operatorname{arcosh}(cx)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^5/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.84, size = 1046, normalized size = 3.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/32*(-c^2*x^2+1)^(1/2)*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^5*c^5+16*c^6*x^
6+20*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3-28*c^4*x^4-5*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x*c+13*c^2*x^2-1)/(c^2*x^2-1)/c^6/b/(a+b*arccosh(c*x))-5/32*((c*x+1)
^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,5*arccosh(c*x)+
5*a/b)*exp(-(b*arccosh(c*x)-5*a)/b)/b^2/(c^2*x^2-1)/c^6+1/32*(-c^2*x^2+1)^(
1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^6*(16*(c*x+1)^(1/2)*(c*x-1)^
(1/2)*x^4*b*c^4+16*x^5*b*c^5-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2-20*x^
3*b*c^3+5*arccosh(c*x)*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*b+(c*x+1)^(1
/2)*(c*x-1)^(1/2)*b+5*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)*a+5*x*b*c)/b^
2/(a+b*arccosh(c*x))-5/32*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2
)*x^3*c^3+4*c^4*x^4+3*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2
-1)/c^6/b/(a+b*arccosh(c*x))-15/32*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2
-1)*(-c^2*x^2+1)^(1/2)*Ei(1,3*arccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)
/b)/b^2/(c^2*x^2-1)/c^6+5/16*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/(c^2*x^2-1)/c^6*(arccosh(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^
(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*a
rccosh(c*x))+5/32*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1
)/c^6*(4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*E
i(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(
-3*a/b)*a-(c*x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/b^2/(a+b*arccosh(c*x))-5/1
6*(-c^2*x^2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c^2*x^2-
1)/c^6/b/(a+b*arccosh(c*x))-5/16*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1
)*(-c^2*x^2+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/b^2/
(c^2*x^2-1)/c^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^8 - cx^6 + \left(c^2x^7 - x^5\right)\sqrt{cx+1}\,\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^8 - c*x^6 + (c^2*x^7 - x^5)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((5
*c^5*x^9 - 11*c^3*x^7 + 6*c*x^5 + (5*c^3*x^7 - 4*c*x^5)*(c*x + 1)*(c*x - 1)
 + 5*(2*c^4*x^8 - 3*c^2*x^6 + x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)
^(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c
*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*
```

```
c^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^
4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^5/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**5/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.343 $\int \frac{x^4}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^5\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{2b^2c^5\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2}{}\right.}{b^2c^5\sqrt{1-c}}$$

[Out] -x^4*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(2*a/b)*Shi(2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)+1/2*cosh(4*a/b)*Shi(4*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)-Chi(2*(a+b*arccosh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)-1/2*Chi(4*(a+b*arccosh(c*x))/b)*sinh(4*a/b)*(c*x-1)^(1/2)/b^2/c^5/(-c*x+1)^(1/2)

Rubi [A] time = 0.79, antiderivative size = 301, normalized size of antiderivative = 1.28, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^5\sqrt{1-c^2x^2}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(\frac{4a}{b}+4\cosh^{-1}(cx)\right)}{2b^2c^5\sqrt{1-c^2x^2}}+\cdots$$

Antiderivative was successfully verified.

[In] Int[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]

[Out] -((x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*x]]*Sinh[(2*a)/b])/(b^2*c^5*Sqrt[1 - c^2*x^2]) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(4*a)/b + 4*ArcCosh[c*x]]*Sinh[(4*a)/b])/(2*b^2*c^5*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c^5*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[(4*a)/b]*SinhIntegral[(4*a)/b + 4*ArcCosh[c*x]])/(2*b^2*c^5*Sqrt[1 - c^2*x^2])

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^4}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{a+b\cosh^{-1}(cx)}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\cosh^3}{}\right.}{bc^5\sqrt{1-c^2x}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(4\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\left(\frac{\sinh}{4(a+}\right.\right.}{bc^5\sqrt{1-}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\sinh(4x}{a+bx}\right.}{2bc^5\sqrt{1-c^2x^2}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\,\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subs}}{bc^5\sqrt{1-}}$$

$$= -\frac{x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-}\right.}{b^2c^5\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.51, size = 149, normalized size = 0.63

$$\frac{\sqrt{1-c^2x^2}\left(\frac{2bc^4x^4}{a+b\cosh^{-1}(cx)}+2\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+\sinh\left(\frac{4a}{b}\right)\text{Chi}\left(4\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-2\cosh}{2b^2c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((2*b*c^4*x^4)/(a + b*ArcCosh[c*x]) + 2*CoshIntegral[2*(
a/b + ArcCosh[c*x])]*Sinh[(2*a)/b] + CoshIntegral[4*(a/b + ArcCosh[c*x])]*S
inh[(4*a)/b] - 2*Cosh[(2*a)/b]*SinhIntegral[2*(a/b + ArcCosh[c*x])] - Cosh[
(4*a)/b]*SinhIntegral[4*(a/b + ArcCosh[c*x])]))/(2*b^2*c^5*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^4}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4}{\sqrt{-c^2x^2+1}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

maple [B] time = 0.87, size = 758, normalized size = 3.21

$$-\frac{\sqrt{-c^2x^2+1}\left(-8\sqrt{cx+1}\sqrt{cx-1}\,x^4c^4+8c^5x^5+8\sqrt{cx+1}\sqrt{cx-1}\,x^2c^2-12c^3x^3-\sqrt{cx-1}\sqrt{cx+1}+4cx\right.}{16\left(c^2x^2-1\right)c^5\left(a+b\,\text{arccosh}\left(cx\right)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/16*(-c^2*x^2+1)^(1/2)*(-8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+8*c^5*x^5+
8*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-12*c^3*x^3-(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*((c*x+1)^(1/2)*(c*x-1)^(1
/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,4*arccosh(c*x)+4*a/b)*exp(-(b*ar
ccosh(c*x)-4*a)/b)/b^2/(c^2*x^2-1)/c^5+1/16*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^5*(8*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*b*c^3+8*
x^4*b*c^4-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c-8*x^2*b*c^2+4*arccosh(c*x)*Ei
(1,-4*arccosh(c*x)-4*a/b)*exp(-4*a/b)*b+4*Ei(1,-4*arccosh(c*x)-4*a/b)*exp(-
4*a/b)*a+b)/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/4*(-c^2*x^2+1)^(1/2)*(-2*
(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(c*x-1)^(1/2)*(c*x+1)^(1/2)-2
```

```
*c*x)/(c^2*x^2-1)/c^5/(a+b*arccosh(c*x))/b-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)
*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arccosh(c*x)+2*a/b)*exp(-(b*arcco
sh(c*x)-2*a)/b)/b^2/(c^2*x^2-1)/c^5+1/4*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/(c^2*x^2-1)/c^5*(2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*b*c+2*x^2*b*c^
2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*b+2*Ei(1,-2*arccos
h(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh(c*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^7 - cx^5 + \left(c^2x^6 - x^4\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,abc^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^7 - c*x^5 + (c^2*x^6 - x^4)*sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4
*c^5*x^8 - 9*c^3*x^6 + 5*c*x^4 + (4*c^3*x^6 - 3*c*x^4)*(c*x + 1)*(c*x - 1)
+ 4*(2*c^4*x^7 - 3*c^2*x^5 + x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c
^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4
 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(a + b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{-(cx-1)(cx+1)}\,(a + b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.344 $\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^4\sqrt{1-cx}}-\frac{3\sqrt{cx-1}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^4\sqrt{1-cx}}+\frac{3\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+}{}\right.}{4b^2c^4\sqrt{1-cx}}$$

```
[Out] -x^3*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+3/4*cosh(a/b)*Shi(
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)+3/4*cosh(3*a/b)*
Shi(3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)-3/4*Chi((a
+b*arccosh(c*x))/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)-3/4*Chi(
3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)*(c*x-1)^(1/2)/b^2/c^4/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.77, antiderivative size = 298, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {5798, 5775, 5670, 5448, 3303, 3298, 3301}

$$-\frac{3\sqrt{cx-1}\,\sqrt{cx+1}\,\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^4\sqrt{1-c^2x^2}}-\frac{3\sqrt{cx-1}\,\sqrt{cx+1}\,\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b^2c^4\sqrt{1-c^2x^2}}+\frac{3\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[a/b + ArcCosh[c*x]]*
Sinh[a/b])/(4*b^2*c^4*Sqrt[1 - c^2*x^2]) - (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]*Sinh[(3*a)/b])/(4*b^2*c^4*Sqrt[1 - c
^2*x^2]) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh[a/b]*SinhIntegral[a/b + Arc
Cosh[c*x]])/(4*b^2*c^4*Sqrt[1 - c^2*x^2]) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^4*Sqrt[1 -
c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{a+b\cosh^{-1}(cx)}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\cosh^2}{}\right.}{bc^4\sqrt{1-c^2x}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int\left(\frac{\sinh}{4(a+}\right.\right.}{bc^4\sqrt{1-}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\sinh(x}{a+bx}\right.}{4bc^4\sqrt{1-c^2x^2}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(3\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{a}{b}\right)\right)\mathrm{Subs}}{4bc^4\sqrt{1-}}$$

$$= -\frac{x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{3\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(\right.}{4b^2c^4\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.48, size = 144, normalized size = 0.61

$$\frac{\sqrt{1-c^2x^2}\left(\frac{4bc^3x^3}{a+b\cosh^{-1}(cx)}+3\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+3\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-3\cosh\left(\frac{a}{b}\right)\right.}{4b^2c^4\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*((4*b*c^3*x^3)/(a + b*ArcCosh[c*x]) + 3*CoshIntegral[a/b
 + ArcCosh[c*x]]*Sinh[a/b] + 3*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3
*a)/b] - 3*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] - 3*Cosh[(3*a)/b]*Sin
hIntegral[3*(a/b + ArcCosh[c*x])]))/(4*b^2*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^3}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.65, size = 634, normalized size = 2.68

$$-\frac{\sqrt{-c^2x^2+1}\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^3c^3+4c^4x^4+3\sqrt{cx+1}\,\sqrt{cx-1}\,xc-5c^2x^2+1\right)}{8\left(c^2x^2-1\right)c^4b\left(a+b\,\text{arccosh}\left(cx\right)\right)}-\frac{3\left(\sqrt{cx+1}\,\sqrt{cx-1}\,xc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/8*(-c^2*x^2+1)^(1/2)*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^3*c^3+4*c^4*x^4+3
*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c-5*c^2*x^2+1)/(c^2*x^2-1)/c^4/b/(a+b*arccos
h(c*x))-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*
Ei(1,3*arccosh(c*x)+3*a/b)*exp(-(b*arccosh(c*x)-3*a)/b)/b^2/(c^2*x^2-1)/c^4
+1/8*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)*x^2*b*c^2+4*x^3*b*c^3+3*arccosh(c*x)*Ei(1,-3*arccos
h(c*x)-3*a/b)*exp(-3*a/b)*b+3*Ei(1,-3*arccosh(c*x)-3*a/b)*exp(-3*a/b)*a-(c*
x+1)^(1/2)*(c*x-1)^(1/2)*b-3*x*b*c)/b^2/(a+b*arccosh(c*x))+3/8*(-c^2*x^2+1)
^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^4*(arccosh(c*x)*exp(-a/b)*
Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp(-a/b)*Ei(1,-arc
cosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*arccosh(c*x))-3/8*(-c^2*x^2+1)^(1/2)*(-(c*
```

```
x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/(c^2*x^2-1)/c^4/b/(a+b*arccosh(c*x)
)-3/8*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,a
rccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/b^2/(c^2*x^2-1)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^6 - cx^4 + \left(c^2x^5 - x^3\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,abc^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^6 - c*x^4 + (c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3
*c^5*x^7 - 7*c^3*x^5 + 4*c*x^3 + (3*c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1)
+ 3*(2*c^4*x^6 - 3*c^2*x^4 + x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*
x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1
)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c
^3*x^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4
 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(a + b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\sqrt{1 - c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{-(cx-1)(cx+1)}\,(a + b\,\mathrm{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.345 $\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt{cx-1}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{b^2c^3\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}}{bc\sqrt{1-cx}\,(a+b\cosh^{-1}(}$$

```
[Out] -x^2*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(2*a/b)*Shi(2*
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^3/(-c*x+1)^(1/2)-Chi(2*(a+b*arcco
sh(c*x))/b)*sinh(2*a/b)*(c*x-1)^(1/2)/b^2/c^3/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 175, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {5798, 5775, 5670, 5448, 12, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}\,\sinh\left(\frac{2a}{b}\right)\text{Chi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^3\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\sqrt{cx+1}\,\cosh\left(\frac{2a}{b}\right)\text{Shi}\left(\frac{2a}{b}+2\cosh^{-1}(cx)\right)}{b^2c^3\sqrt{1-c^2x^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(2*a)/b + 2*ArcCosh[c*
x]]*Sinh[(2*a)/b])/(b^2*c^3*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*Cosh[(2*a)/b]*SinhIntegral[(2*a)/b + 2*ArcCosh[c*x]])/(b^2*c^3*Sqrt[1 -
 c^2*x^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
```

```
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{a+b\cosh^{-1}(cx)}}{bc\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\cosh(\cdots}{a\cdots}\right.}{bc^3\sqrt{1-c^2x\cdots}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\sinh(2\cdots}{2(a+b\cdots}\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\mathrm{Subst}\left(\int \frac{\sinh(2x\cdots}{a+bx}\right.}{bc^3\sqrt{1-c^2x^2}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\,\cosh\left(\frac{2a}{b}\right)\right)\mathrm{Subs}\cdots}{bc^3\sqrt{1-\cdots}}$$

$$= -\frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\mathrm{Chi}\left(\frac{2a}{b}+2\cosh^{-}\cdots\right.}{b^2c^3\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.29, size = 117, normalized size = 0.86

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(2\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)-\cosh\left(\frac{2a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(2\left(\frac{a}{b}+\right.\right.\right.}{b^2c^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(b*c^2*x^2 + (a + b*ArcCosh[c*x])*CoshIntegral[2*(a/b +
ArcCosh[c*x])]*Sinh[(2*a)/b] - (a + b*ArcCosh[c*x])*Cosh[(2*a)/b]*SinhInteg
ral[2*(a/b + ArcCosh[c*x])]))/(b^2*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*
ArcCosh[c*x]))
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^2}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh
(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{\sqrt{-c^2x^2+1}\,(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

maple [B] time = 0.54, size = 377, normalized size = 2.77

$$-\frac{\sqrt{-c^2x^2+1}\left(-2\sqrt{cx+1}\sqrt{cx-1}\,x^2c^2+2c^3x^3+\sqrt{cx-1}\sqrt{cx+1}-2cx\right)}{4\left(c^2x^2-1\right)c^3\left(a+b\operatorname{arccosh}(cx)\right)b}-\frac{\left(\sqrt{cx+1}\sqrt{cx-1}\,xc+c^2x^2-1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] -1/4*(-c^2*x^2+1)^(1/2)*(-2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+2*c^3*x^3+(
c*x-1)^(1/2)*(c*x+1)^(1/2)-2*c*x)/(c^2*x^2-1)/c^3/(a+b*arccosh(c*x))/b-1/2*
((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2+1)^(1/2)*Ei(1,2*arcco
sh(c*x)+2*a/b)*exp(-(b*arccosh(c*x)-2*a)/b)/b^2/(c^2*x^2-1)/c^3+1/4*(-c^2*x
^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3*(2*(c*x+1)^(1/2)*(c
*x-1)^(1/2)*x*b*c+2*x^2*b*c^2+2*arccosh(c*x)*Ei(1,-2*arccosh(c*x)-2*a/b)*ex
p(-2*a/b)*b+2*Ei(1,-2*arccosh(c*x)-2*a/b)*exp(-2*a/b)*a-b)/b^2/(a+b*arccosh
(c*x))+1/2*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)/c^3/(
a+b*arccosh(c*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^5-cx^3+\left(c^2x^4-x^2\right)\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^5 - c*x^3 + (c^2*x^4 - x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1
)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2
*c^5*x^6 - 5*c^3*x^4 + (2*c^3*x^4 - c*x^2)*(c*x + 1)*(c*x - 1) + 3*c*x^2 +
2*(2*c^4*x^5 - 3*c^2*x^3 + x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)^(3/2
)*(c*x - 1)*b^2*c^3*x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x -
1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*lo
g(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x
^2 + 2*(a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2
*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(cx-1)(cx+1)}\,(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.346 $\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=130

$$-\frac{\sqrt{cx-1}\,\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}+\frac{\sqrt{cx-1}\,\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-cx}}-\frac{x\sqrt{cx-1}}{bc\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -x*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+cosh(a/b)*Shi((a+b*a
rccosh(c*x))/b)*(c*x-1)^(1/2)/b^2/c^2/(-c*x+1)^(1/2)-Chi((a+b*arccosh(c*x))
/b)*sinh(a/b)*(c*x-1)^(1/2)/b^2/c^2/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 169, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {5798, 5775, 5658, 3303, 3298, 3301}

$$-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\sinh\left(\frac{a}{b}\right)\operatorname{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-c^2x^2}}+\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\cosh\left(\frac{a}{b}\right)\operatorname{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-c^2x^2}}-\frac{x\sqrt{cx-}}{bc\sqrt{1-c^2x^2}\,(a}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*
x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*CoshIntegral[(a + b*ArcCosh[c*x])/b]*
Sinh[a/b])/(b^2*c^2*Sqrt[1 - c^2*x^2]) + (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Cosh
[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b^2*c^2*Sqrt[1 - c^2*x^2])
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5775

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[((f*x)^m*(a
 + b*ArcCosh[c*x])^(n + 1))/(b*c*Sqrt[-(d1*d2)]*(n + 1)), x] - Dist[(f*m)/(
```

```
b*c*Sqrt[-(d1*d2)]*(n + 1)), Int[(f*x)^(m - 1)*(a + b*ArcCosh[c*x])^(n + 1)
, x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, f, m}, x] && EqQ[e1 - c*d1, 0]
&& EqQ[e2 + c*d2, 0] && LtQ[n, -1] && GtQ[d1, 0] && LtQ[d2, 0]
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{a+b\cosh^{-1}(cx)}\,d$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\right.}{x}\right.}{b^2c^2\sqrt{1-c^2}}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\cosh\left(\frac{a}{b}\right)\right)\text{Subst}}{b^2c^2\sqrt{1-}}$$

$$= -\frac{x\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)} - \frac{\sqrt{-1+cx}\sqrt{1+cx}\,\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c^2\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.23, size = 107, normalized size = 0.82

$$\frac{\sqrt{1-c^2x^2}\left(\sinh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\left(a+b\cosh^{-1}(cx)\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right.}{b^2c^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*(b*c*x + (a + b*ArcCosh[c*x])*CoshIntegral[a/b + ArcCosh
[c*x]]*Sinh[a/b] - (a + b*ArcCosh[c*x])*Cosh[a/b]*SinhIntegral[a/b + ArcCos
h[c*x]]))/(b^2*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arccosh(c
*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

maple [B] time = 0.28, size = 283, normalized size = 2.18

$$\frac{\sqrt{-c^2x^2+1}\,\sqrt{cx-1}\,\sqrt{cx+1}\,\left(\mathrm{arccosh}\,(cx)\,\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}\left(1,-\mathrm{arccosh}\,(cx)-\frac{a}{b}\right)b+\sqrt{cx+1}\,\sqrt{cx-1}\,b+\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}\left(1,\right.\right.}{2c^2\left(c^2x^2-1\right)b^2\,(a+b\,\mathrm{arccosh}\,(cx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-c^2*x^2+1)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^2/(c^2*x^2-1)*(arccosh
(c*x)*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*b+(c*x+1)^(1/2)*(c*x-1)^(1/2)*b+exp
(-a/b)*Ei(1,-arccosh(c*x)-a/b)*a+x*b*c)/b^2/(a+b*arccosh(c*x))-1/2*(-c^2*x^
2+1)^(1/2)*(-(c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)/c^2/(c^2*x^2-1)/b/(
a+b*arccosh(c*x))-1/2*((c*x+1)^(1/2)*(c*x-1)^(1/2)*x*c+c^2*x^2-1)*(-c^2*x^2
+1)^(1/2)*Ei(1,arccosh(c*x)+a/b)*exp(-(b*arccosh(c*x)-a)/b)/b^2/c^2/(c^2*x^
2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^4-cx^2+\left(c^2x^3-x\right)\sqrt{cx+1}\,\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^4 - c*x^2 + (c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*
sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x +
1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^
2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5
*x^5 + (c*x + 1)*(c*x - 1)*c^3*x^3 - 3*c^3*x^3 + (2*c^4*x^4 - 3*c^2*x^2 + 1
)*sqrt(c*x + 1)*sqrt(c*x - 1) + 2*c*x)/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^3*
x^2 + 2*(b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 -
2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^2 + 2*(a*b*c^4*x^3
- a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c
)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(a+b\,\mathrm{acosh}\,(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\,\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.347 $\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{cx-1}}{bc\sqrt{1-cx}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 50, normalized size of antiderivative = 1.35, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5713, 5676}

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
)))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 50, normalized size = 1.35

$$-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x]
)))
```

fricas [B] time = 0.61, size = 75, normalized size = 2.03

$$\frac{\sqrt{c^2x^2-1}\,\sqrt{-c^2x^2+1}}{abc^3x^2 - abc + \left(b^2c^3x^2 - b^2c\right)\log\left(cx + \sqrt{c^2x^2-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 -
b^2*c)*log(c*x + sqrt(c^2*x^2 - 1)))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2x^2+1}\,(b\,\mathrm{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.06, size = 57, normalized size = 1.54

$$\frac{\sqrt{-(cx-1)(cx+1)}\,\sqrt{cx-1}\,\sqrt{cx+1}}{c\left(c^2x^2-1\right)(a+b\,\mathrm{arccosh}\,(cx))\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] (-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/(c^2*x^2-1)/(a+b*arc
cosh(c*x))/b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3 + \left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1} - cx}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x + \left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,abc^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*
x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(-(c^2*
x^2 - (c*x + 1)*(c*x - 1) - 1)/(((c*x + 1)^(3/2)*(c*x - 1)*b^2*c^2*x^2 + 2*
(b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^4*x^4 - 2*b^2*c^2*
x^2 + b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*(
c*x + 1)*sqrt(c*x - 1) + (a*b*c^4*x^4 - 2*a*b*c^2*x^2 + a*b)*sqrt(c*x + 1))
*sqrt(-c*x + 1)), x)
```

mupad [B] time = 0.45, size = 59, normalized size = 1.59

$$-\frac{b\sqrt{1-c^2x^2}\ \sqrt{c\,x-1}\ \sqrt{c\,x+1}}{c\ (a+b\,\mathrm{acosh}\,(c\,x))\ \left(b^2-b^2\,c^2\,x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] -(b*(1 - c^2*x^2)^(1/2)*(c*x - 1)^(1/2)*(c*x + 1)^(1/2))/(c*(a + b*acosh(c*
x))*(b^2 - b^2*c^2*x^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-(cx-1)(cx+1)}\ (a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.348 $\int \frac{1}{x\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{\sqrt{cx-1}\,\mathrm{Int}\left(\frac{1}{x^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)/b/c/x/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)-(c*x-1)^(1/2)*Uninte
grable(1/x^2/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*
x]))) - (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(a + b*ArcCosh[c*x]
)), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bcx\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\left(a+b\cosh^{-1}\right.}}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 4.82, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^2x^3-a^2x+\left(b^2c^2x^3-b^2x\right)\operatorname{arcosh}(cx)^2+2\left(abc^2x^3-abx\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^3 - a^2*x + (b^2*c^2*x^3 - b^2*x)*a
rccosh(c*x)^2 + 2*(a*b*c^2*x^3 - a*b*x)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(a + b \operatorname{arccosh}\left(cx\right)\right)^{2} \sqrt{-c^{2} x^{2} + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{3} + \left(c^{2}x^{2} - 1\right)\sqrt{cx+1}\,\sqrt{cx-1} - cx}{\left(\left(cx+1\right)\sqrt{cx-1}\,b^{2}c^{2}x^{2} + \left(b^{2}c^{3}x^{3} - b^{2}cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right) + \left(\left(cx+1\right)\sqrt{cx-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^2 + (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*
c^2*x^2 + (a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrat
e((c^5*x^5 - c^3*x^3 + (c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (2*c^4*x^4 -
 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)^(3/2)*(c*x - 1)*b^
2*c^3*x^4 + 2*(b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^
5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)*a*b*c^3*x^4 + 2*(
a*b*c^4*x^5 - a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c
^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{- \left(cx - 1\right) \left(cx + 1\right)} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.349 $\int \frac{1}{x^2\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2\sqrt{cx-1}\,\mathrm{Int}\left(\frac{1}{x^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}}{bcx^2\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)/b/c/x^2/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)-2*(c*x-1)^(1/2)*Un
integrable(1/x^3/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[
c*x]))) - (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^3*(a + b*ArcCosh[
c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bcx^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(2\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^3\left(a+b\right.}}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 1.59, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^2x^4-a^2x^2+\left(b^2c^2x^4-b^2x^2\right)\mathrm{arcosh}\,(cx)^2+2\left(abc^2x^4-abx^2\right)\mathrm{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^2*x^4 - a^2*x^2 + (b^2*c^2*x^4 - b^2*x^
2)*arccosh(c*x)^2 + 2*(a*b*c^2*x^4 - a*b*x^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c^2x^2+1}\,(b\operatorname{arcosh}(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\,(a+b\operatorname{arccosh}(cx))^2\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1}-cx}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x^3+\left(b^2c^3x^4-b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*arccosh(c*x))^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((((c*x + 1)*sq
rt(c*x - 1)*b^2*c^2*x^3 + (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*
x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x^3 + (a*b*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - inte
grate((2*c^5*x^5 - 3*c^3*x^3 + (2*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*
(2*c^4*x^4 - 3*c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((c*x + 1)^
(3/2)*(c*x - 1)*b^2*c^3*x^5 + 2*(b^2*c^4*x^6 - b^2*c^2*x^4)*(c*x + 1)*sqrt(
c*x - 1) + (b^2*c^5*x^7 - 2*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c
*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x - 1)
*a*b*c^3*x^5 + 2*(a*b*c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b
*c^5*x^7 - 2*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2\,(a+b\operatorname{acosh}(c\,x))^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{-(cx-1)(cx+1)}\,(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*acosh(c*x))**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.350 $\int \frac{x^3}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.57, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 29.99, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^3}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^{4}+\sqrt{cx+1}\,\sqrt{cx-1}\,x^{3}}{\left((cx+1)\sqrt{cx-1}\,b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}-b^{2}c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 - 5*c^3*x^5 + 4*
c*x^3 + (c^3*x^5 - 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 - 7*c^2*x^4 +
3*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)
^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sq
rt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*
x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x
^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^
3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a
*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.351 $\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=107

$$\frac{2\sqrt{cx-1}\,\text{Int}\left(\frac{x}{\left(c^2x^2-1\right)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{x^2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))+
2*(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^2/(a+b*arccosh(c*x)),x)/b/c/(-c*
x+1)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^2*Sqrt[-1 + c*x])/(b*c*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(a +
b*ArcCosh[c*x]))) + (2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2
*x^2)^2*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{x^2\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2\sqrt{-1+cx}\,\sqrt{1\right.}{}$$

Mathematica [A] time = 5.99, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x^2}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4
 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x
^2 + a*b)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^2/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^3+\sqrt{cx+1}\sqrt{cx-1}\,x^2}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,abc^2x\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2
*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 -
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^3*x^4 + (c*x + 1)*(c
*x - 1)*c*x^2 - 3*c*x^2 + 2*(2*c^2*x^3 - x)*sqrt(c*x + 1)*sqrt(c*x - 1))/((
(b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^5 - 2*
b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^6 - 3*b^2*c^5
*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(
c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x
 - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
 + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqr
t(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(a+b\,\mathrm{acosh}(c\,x))^2\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.352 $\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(3/2)*(1 + c*x)^(3
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 22.71, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,x}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*x/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 -
 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2
 + a*b)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-c^{2} x^{2}+1\right)^{\frac{3}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx^{2} + \sqrt{cx+1}\,\sqrt{cx-1}\,x}{\left((cx+1)\sqrt{cx-1}\,b^{2}c^{2}x + \left(b^{2}c^{3}x^{2} - b^{2}c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*
b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((c^5*x^5 + (c*x + 1)*(c*x -
 1)*c^3*x^3 + c^3*x^3 + (2*c^4*x^4 + c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x -
1) - 2*c*x)/(((b^2*c^5*x^4 - b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^
2*c^6*x^5 - 2*b^2*c^4*x^3 + b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x
^6 - 3*b^2*c^5*x^4 + 3*b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^4 - a*b*c^3*x^2)*(c*x +
 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^5 - 2*a*b*c^4*x^3 + a*b*c^2*x)*(c*x + 1)
*sqrt(c*x - 1) + (a*b*c^7*x^6 - 3*a*b*c^5*x^4 + 3*a*b*c^3*x^2 - a*b*c)*sqrt
(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(- \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.353 $\int \frac{1}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=102

$$\frac{2c\sqrt{cx-1}\,\text{Int}\left(\frac{x}{(c^2x^2-1)^2(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))+2*c*
(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^2/(a+b*arccosh(c*x)),x)/b/(-c*x+1)
^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*(1 - c*x)*Sqrt[1 + c*x]*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))) + (2*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2*x^2
)^2*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$= -\frac{\sqrt{-1+cx}}{bc(1-cx)\sqrt{1+cx}\,\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(2c\sqrt{-1+cx}\,\sqrt{1+\right.}{}$$

Mathematica [A] time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\operatorname{arcosh}(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\operatorname{arcosh}(cx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4*x^4 - 2
*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c^2*x^2 +
 a*b)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.26, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x+\left(b^2c^3x^2-b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (
b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)
*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((2*c^4*x^4 - c^2*x^2 + (2*c^2*x
^2 - 1)*(c*x + 1)*(c*x - 1) + 2*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x -
1) - 1)/(((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^6*x^6 - 3
*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x +
 sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)^(3/2
)*(c*x - 1) + 2*(a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*(c*x + 1)*sqrt(c*x
- 1) + (a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*sqrt(c*x + 1))*s
qrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{acosh}(c\,x))^2\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.354 $\int \frac{1}{x(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(3/2)*(1 + c*x)^
(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 23.25, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^5-2\,a^2c^2x^3+a^2x+\left(b^2c^4x^5-2\,b^2c^2x^3+b^2x\right)\text{arcosh}\,(cx)^2+2\,\left(abc^4x^5-2\,abc^2x^3+abx\right)\text{arcc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^5 - 2*a^2*c^2*x^3 + a^2*x + (b^2*c^4
*x^5 - 2*b^2*c^2*x^3 + b^2*x)*arccosh(c*x)^2 + 2*(a*b*c^4*x^5 - 2*a*b*c^2*x
^3 + a*b*x)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx+\sqrt{cx+1}\,\sqrt{cx-1}}{\left(\left(cx+1\right)\sqrt{cx-1}\,b^{2}c^{2}x^{2}+\left(b^{2}c^{3}x^{3}-b^{2}cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(\left(cx+1\right)\sqrt{cx-1}\,abc\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/((((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^2 +
 (b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^2 + (a*b*c^3*x^3 -
a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((3*c^5*x^5 - 3*c^3*x^3
+ (3*c^3*x^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (6*c^4*x^4 - 5*c^2*x^2 + 1)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^5*x^6 - b^2*c^3*x^4)*(c*x + 1)^(3/2)*(c*
x - 1) + 2*(b^2*c^6*x^7 - 2*b^2*c^4*x^5 + b^2*c^2*x^3)*(c*x + 1)*sqrt(c*x -
 1) + (b^2*c^7*x^8 - 3*b^2*c^5*x^6 + 3*b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x +
1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^5*x^6 -
 a*b*c^3*x^4)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^7 - 2*a*b*c^4*x^5 +
a*b*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^8 - 3*a*b*c^5*x^6 + 3*a*b
*c^3*x^4 - a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{3}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.355 $\int \frac{1}{x^2(1-c^2x^2)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(3/2)*(1 + c*x
)^(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 21.04, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}}{a^2c^4x^6-2\,a^2c^2x^4+a^2x^2+\left(b^2c^4x^6-2\,b^2c^2x^4+b^2x^2\right)\operatorname{arcosh}(cx)^2+2\left(abc^4x^6-2\,abc^2x^4+abx^2\right)\operatorname{arco}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)/(a^2*c^4*x^6 - 2*a^2*c^2*x^4 + a^2*x^2 + (b^2*c
^4*x^6 - 2*b^2*c^2*x^4 + b^2*x^2)*arccosh(c*x)^2 + 2*(a*b*c^4*x^6 - 2*a*b*c
^2*x^4 + a*b*x^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-c^2x^2+1\right)^{\frac{3}{2}}(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x^3+\left(b^2c^3x^4-b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] (c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x^3 +
 (b^2*c^3*x^4 - b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x^3 + (a*b*c^3*x^4
- a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate((4*c^5*x^5 - 5*c^3*
x^3 + (4*c^3*x^3 - 3*c*x)*(c*x + 1)*(c*x - 1) + 2*(4*c^4*x^4 - 4*c^2*x^2 +
1)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^5*x^7 - b^2*c^3*x^5)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^8 - 2*b^2*c^4*x^6 + b^2*c^2*x^4)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^7*x^9 - 3*b^2*c^5*x^7 + 3*b^2*c^3*x^5 - b^2*c*x^3)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^5*x^7 - a*b*c^3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^6*x^8 - 2*a*
b*c^4*x^6 + a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^7*x^9 - 3*a*b*c^5
*x^7 + 3*a*b*c^3*x^5 - a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-(cx-1)(cx+1)\right)^{\frac{3}{2}}(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**2), x)
```

3.356 $\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=109

$$-\frac{4\sqrt{cx-1}\,\mathrm{Int}\left(\frac{x^3}{\left(c^2x^2-1\right)^3\left(a+b\cosh^{-1}(cx)\right)},x\right)}{bc\sqrt{1-cx}}-\frac{x^4\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))-
4*(c*x-1)^(1/2)*Unintegrable(x^3/(c^2*x^2-1)^3/(a+b*arccosh(c*x)),x)/b/c/(-
c*x+1)^(1/2)
```

Rubi [A] time = 0.65, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((x^4*Sqrt[-1 + c*x])/(b*c*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2]*(
a + b*ArcCosh[c*x]))) - (4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1
 + c^2*x^2)^3*(a + b*ArcCosh[c*x])), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^4}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{x^4\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4\sqrt{-1+cx}\right.}{}$$

Mathematica [A] time = 5.14, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^4/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^4}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\mathrm{arcosh}\,(cx)^2-a^2+2\left(abc^6x^6-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^4/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(x^4/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^5+\sqrt{cx+1}\sqrt{cx-1}\,x^4}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^5 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^4)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^3*x^6 + 3*(c*x + 1)*(c*x - 1)*
c*x^4 - 5*c*x^4 + 4*(2*c^2*x^5 - x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c
^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^
8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c
^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^
8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1)
+ (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4}{(a+b\operatorname{acosh}(c\,x))^2\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x^4/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\operatorname{acosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**4/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.357 $\int \frac{x^3}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^3/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{x^3}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 50.72, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^3/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^3}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^6x^6-3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^3/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\left(-c^{2} x^{2}+1\right)^{\frac{5}{2}} \left(a+b \,\mathrm{arccosh}\left(c x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^4 + \sqrt{cx+1}\,\sqrt{cx-1}\,x}{\left(\left(b^2c^4x^3 - b^2c^2x\right)(cx+1)\sqrt{cx-1} + \left(b^2c^5x^4 - 2\,b^2c^3x^2 + b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x^4 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^3)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((c^5*x^7 + 3*c^3*x^5 - 4*c*x^3 + (c
^3*x^5 + 2*c*x^3)*(c*x + 1)*(c*x - 1) + (2*c^4*x^6 + 5*c^2*x^4 - 3*x^2)*sqr
t(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b
^2*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^
5*x^4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*
x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a
*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^
5*x^4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^3}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x^3/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(-(cx-1)(cx+1))^{\frac{5}{2}}(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**3/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.358 $\int \frac{x^2}{(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x^2/((-1 + c*x)^(5/2)*(1 + c*x)^(5
/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 7.83, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x^2/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x^2}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^6x^6-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x^2/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x
^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a
^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(x^2/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.89, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
[Out] int(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^3+\sqrt{cx+1}\sqrt{cx-1}\,x^2}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
[Out] -(c*x^3 + sqrt(c*x + 1)*sqrt(c*x - 1)*x^2)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b
*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqr
t(c*x + 1))*sqrt(-c*x + 1)) - integrate((2*c^5*x^6 + c^3*x^4 + (2*c^3*x^4 +
 c*x^2)*(c*x + 1)*(c*x - 1) - 3*c*x^2 + 2*(2*c^4*x^5 + c^2*x^3 - x)*sqrt(c*
x + 1)*sqrt(c*x - 1))/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^
4 - 4*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x +
1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^
4 - 4*a*b*c^3*x^2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2}{(a+b\operatorname{acosh}(c\,x))^2\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
[Out] int(x^2/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x**2/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.359 $\int \frac{x}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{x}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.39, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c*x)^(5/2)*(1 + c*x)^(5/2
)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x}{(-1+cx)^{5/2}(1+cx)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 45.93, size = 0, normalized size = 0.00

$$\int \frac{x}{(1-c^2x^2)^{5/2}\,(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[x/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,x}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^6x^6-3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*x/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2
 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2
 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-c^2x^2+1\right)^{\frac{5}{2}}\left(a+b\operatorname{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx^2+\sqrt{cx+1}\,\sqrt{cx-1}\,}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2\,b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*x)/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)) - integrate((3*c^5*x^5 + 3*(c*x + 1)*(c*x - 1)*c^
3*x^3 - c^3*x^3 + (6*c^4*x^4 - c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - 2
*c*x)/(((b^2*c^7*x^6 - 2*b^2*c^5*x^4 + b^2*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(b^2*c^8*x^7 - 3*b^2*c^6*x^5 + 3*b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^9*x^8 - 4*b^2*c^7*x^6 + 6*b^2*c^5*x^4 - 4*b^2*c^3*x^
2 + b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1)) + ((a*b*c^7*x^6 - 2*a*b*c^5*x^4 + a*b*c^3*x^2)*(c*x + 1)^(3/2)*(c*x -
1) + 2*(a*b*c^8*x^7 - 3*a*b*c^6*x^5 + 3*a*b*c^4*x^3 - a*b*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (a*b*c^9*x^8 - 4*a*b*c^7*x^6 + 6*a*b*c^5*x^4 - 4*a*b*c^3*x^
2 + a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(1-c^2\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(x/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(x/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.360 $\int \frac{1}{(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=102

$$-\frac{4c\sqrt{cx-1}\,\text{Int}\left(\frac{x}{(c^2x^2-1)^3(a+b\cosh^{-1}(cx))},x\right)}{b\sqrt{1-cx}}-\frac{\sqrt{cx-1}\sqrt{cx+1}}{bc\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))-4*c*
(c*x-1)^(1/2)*Unintegrable(x/(c^2*x^2-1)^3/(a+b*arccosh(c*x)),x)/b/(-c*x+1)
^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(Sqrt[-1 + c*x]/(b*c*(1 - c*x)^2*(1 + c*x)^(3/2)*Sqrt[1 - c^2*x^2]*(a + b*
ArcCosh[c*x]))) - (4*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][x/((-1 + c^2
*x^2)^3*(a + b*ArcCosh[c*x])), x])/(b*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

$$=-\frac{\sqrt{-1+cx}}{bc(1-cx)^2(1+cx)^{3/2}\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}-\frac{\left(4c\sqrt{-1+cx}\sqrt{1\right.}{}$$

Mathematica [A] time = 3.84, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\text{arcosh}\left(cx\right)^2-a^2+2\left(abc^6x^6-3\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c^2*x^2 +
 (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2 - a^2 +
 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{cx+1}\sqrt{cx-1}}{\left(\left(b^2c^4x^3-b^2c^2x\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^4-2b^2c^3x^2+b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^3 - b^2*c^2*x)*(c*x + 1)*
sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(c*x + 1))*sqrt(-
c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^3 - a*b*c^2*x
)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 + a*b*c)*sqrt(c*x
+ 1))*sqrt(-c*x + 1)) - integrate((4*c^4*x^4 - 3*c^2*x^2 + (4*c^2*x^2 - 1)*
(c*x + 1)*(c*x - 1) + 4*(2*c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - 1)/
(((b^2*c^6*x^6 - 2*b^2*c^4*x^4 + b^2*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2
*(b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b^2*c^3*x^3 - b^2*c*x)*(c*x + 1)*sqrt(c*x
 - 1) + (b^2*c^8*x^8 - 4*b^2*c^6*x^6 + 6*b^2*c^4*x^4 - 4*b^2*c^2*x^2 + b^2)
*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a
*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*(c*x + 1)*sqrt(c*x - 1
) + (a*b*c^8*x^8 - 4*a*b*c^6*x^6 + 6*a*b*c^4*x^4 - 4*a*b*c^2*x^2 + a*b)*sqr
t(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a+b\operatorname{acosh}(c\,x))^2\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral(1/((-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.361 $\int \frac{1}{x(1-c^2x^2)^{5/2}(a+b\cosh^{-1}(cx))^2} dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)

Rubi [A] time = 0.54, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Int[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]

Rubi steps

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2} dx = \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{1}{x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2} dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 41.02, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]

[Out] Integrate[1/(x*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^7-3\,a^2c^4x^5+3\,a^2c^2x^3-a^2x+\left(b^2c^6x^7-3\,b^2c^4x^5+3\,b^2c^2x^3-b^2x\right)\text{arcosh}\left(cx\right)^2+2\left(abc^6x^7\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")

[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^7 - 3*a^2*c^4*x^5 + 3*a^2*c^2*x^3 - a^2*x + (b^2*c^6*x^7 - 3*b^2*c^4*x^5 + 3*b^2*c^2*x^3 - b^2*x)*arccosh(c*x)^2 + 2*(a*b*c^6*x^7 - 3*a*b*c^4*x^5 + 3*a*b*c^2*x^3 - a*b*x)*arccosh(c*x)), x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-c^{2}x^{2}+1\right)^{\frac{5}{2}}\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{cx+1}\,\sqrt{cx-1}}{\left(\left(b^{2}c^{4}x^{4}-b^{2}c^{2}x^{2}\right)(cx+1)\sqrt{cx-1}+\left(b^{2}c^{5}x^{5}-2\,b^{2}c^{3}x^{3}+b^{2}cx\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima"
)
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^4 - a*b*c
^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*s
qrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((5*c^5*x^5 - 5*c^3*x^3 + (5*c^3*x
^3 - 2*c*x)*(c*x + 1)*(c*x - 1) + (10*c^4*x^4 - 7*c^2*x^2 + 1)*sqrt(c*x + 1
)*sqrt(c*x - 1))/(((b^2*c^7*x^8 - 2*b^2*c^5*x^6 + b^2*c^3*x^4)*(c*x + 1)^(3
/2)*(c*x - 1) + 2*(b^2*c^8*x^9 - 3*b^2*c^6*x^7 + 3*b^2*c^4*x^5 - b^2*c^2*x^
3)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^10 - 4*b^2*c^7*x^8 + 6*b^2*c^5*x^6
- 4*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c
*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^8 - 2*a*b*c^5*x^6 + a*b*c^3*x^4)*(c*x
+ 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^9 - 3*a*b*c^6*x^7 + 3*a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^10 - 4*a*b*c^7*x^8 + 6*a*b*c
^5*x^6 - 4*a*b*c^3*x^4 + a*b*c*x^2)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^2\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(x*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.362 $\int \frac{1}{x^2(1-c^2x^2)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][1/(x^2*(-1 + c*x)^(5/2)*(1 + c*x)^
(5/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{1}{x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 15.68, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(x^2*(1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}}{a^2c^6x^8-3\,a^2c^4x^6+3\,a^2c^2x^4-a^2x^2+\left(b^2c^6x^8-3\,b^2c^4x^6+3\,b^2c^2x^4-b^2x^2\right)\text{arcosh}\left(cx\right)^2+2\left(abc^6x^8\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)/(a^2*c^6*x^8 - 3*a^2*c^4*x^6 + 3*a^2*c^2*x^4 -
 a^2*x^2 + (b^2*c^6*x^8 - 3*b^2*c^4*x^6 + 3*b^2*c^2*x^4 - b^2*x^2)*arccosh(
c*x)^2 + 2*(a*b*c^6*x^8 - 3*a*b*c^4*x^6 + 3*a*b*c^2*x^4 - a*b*x^2)*arccosh(
c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c^2x^2+1\right)^{\frac{5}{2}}(b\operatorname{arcosh}(cx)+a)^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac"
)
```

```
[Out] integrate(1/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2*x^2), x)
```

maple [A] time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(-c^2x^2+1\right)^{\frac{5}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cx+\sqrt{cx+1}\sqrt{cx-}}{\left(\left(b^2c^4x^5-b^2c^2x^3\right)(cx+1)\sqrt{cx-1}+\left(b^2c^5x^6-2\,b^2c^3x^4+b^2cx^2\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\sqrt{cx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxim
a")
```

```
[Out] -(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(((b^2*c^4*x^5 - b^2*c^2*x^3)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^6 - 2*b^2*c^3*x^4 + b^2*c*x^2)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4*x^5 - a*b
*c^2*x^3)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^6 - 2*a*b*c^3*x^4 + a*b*c*x^
2)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate((6*c^5*x^5 - 7*c^3*x^3 + 3*(2
*c^3*x^3 - c*x)*(c*x + 1)*(c*x - 1) + 2*(6*c^4*x^4 - 5*c^2*x^2 + 1)*sqrt(c*
x + 1)*sqrt(c*x - 1) + c*x)/(((b^2*c^7*x^9 - 2*b^2*c^5*x^7 + b^2*c^3*x^5)*(
c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^10 - 3*b^2*c^6*x^8 + 3*b^2*c^4*x^6
- b^2*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^11 - 4*b^2*c^7*x^9 + 6*
b^2*c^5*x^7 - 4*b^2*c^3*x^5 + b^2*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(
c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^9 - 2*a*b*c^5*x^7 + a*b*c^
3*x^5)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^10 - 3*a*b*c^6*x^8 + 3*a*b*
c^4*x^6 - a*b*c^2*x^4)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9*x^11 - 4*a*b*c^7*
x^9 + 6*a*b*c^5*x^7 - 4*a*b*c^3*x^5 + a*b*c*x^3)*sqrt(c*x + 1))*sqrt(-c*x +
 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x^2\,(a+b\,\mathrm{acosh}\,(c\,x))^2\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int(1/(x^2*(a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^{2}\left(-\left(cx-1\right)\left(cx+1\right)\right)^{\frac{5}{2}}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/(x**2*(-(c*x - 1)*(c*x + 1))**(5/2)*(a + b*acosh(c*x))**2), x)
```

3.363 $\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{\left(1-c^2x^2\right)^{3/2}(fx)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((Sqrt[1 - c^2*x^2]*Defer[Int][((f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/
(a + b*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\sqrt{1-c^2x^2}\int \frac{(fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(1-c^2x^2\right)^{3/2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(c^2x^2-1\right)\sqrt{-c^2x^2+1}\,(fx)^m}{b^2\operatorname{arcosh}(cx)^2+2ab\operatorname{arcosh}(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*
a*b*arccosh(c*x) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.33, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(-c^{2}x^{2}+1\right)^{\frac{3}{2}}}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\left(c^{4}f^{m}x^{4}-2\,c^{2}f^{m}x^{2}+f^{m}\right)(cx+1)\sqrt{cx-1}\,x^{m}+\left(c^{5}f^{m}x^{5}-2\,c^{3}f^{m}x^{3}+cf^{m}x\right)\sqrt{cx+1}\,x^{m}\right)\sqrt{-cx+1}}{abc^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^{2}x-abc+\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,b^{2}c^{2}x-b^{2}c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}-\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] ((c^4*f^m*x^4 - 2*c^2*f^m*x^2 + f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*x^5 - 2*c^3*f^m*x^3 + c*f^m*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*
x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c
*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))) - integrate(((c^5*f^m*(m + 4)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m
- 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^6*f^m*(m + 4)*x^6 - c^4*f^m*(5
*m + 12)*x^4 + 4*c^2*f^m*(m + 1)*x^2 - f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m +
 (c^7*f^m*(m + 4)*x^7 - 3*c^5*f^m*(m + 3)*x^5 + 3*c^3*f^m*(m + 2)*x^3 - c*f
^m*(m + 1)*x)*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c
*x - 1)*a*b*c^3*x^3 - 2*a*b*c^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^
2)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3
*x^3 - 2*b^2*c^3*x^3 + b^2*c*x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1
)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(f\,x\right)}^m\,{\left(1-c^2\,x^2\right)}^{3/2}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int(((f*x)^m*(1 - c^2*x^2)^(3/2))/(a + b*acosh(c*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.364 $\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{\sqrt{1-c^2x^2}\,(fx)^m}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (Sqrt[1 - c^2*x^2]*Defer[Int][((f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(a + b
*ArcCosh[c*x])^2, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \frac{\sqrt{1-c^2x^2}\int \frac{(fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\sqrt{1-c^2x^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[((f*x)^m*Sqrt[1 - c^2*x^2])/(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,\left(fx\right)^m}{b^2\operatorname{arcosh}\left(cx\right)^2+2\,ab\operatorname{arcosh}\left(cx\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x
) + a^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \sqrt{-c^2x^2+1}}{(a+b\,\mathrm{arccosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(\left(c^2f^mx^2-f^m\right)(cx+1)\sqrt{cx-1}\,x^m+\left(c^3f^mx^3-cf^mx\right)\sqrt{cx+1}\,x^m\right)\sqrt{-cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}+\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -((c^2*f^m*x^2 - f^m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^m*x)
*sqrt(c*x + 1)*x^m)*sqrt(-c*x + 1)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x -
1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x
 - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((((c^3*f^m*(m
+ 2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)^(3/2)*(c*x - 1)*x^m + (2*c^4*f^m*(m +
 2)*x^4 - c^2*f^m*(3*m + 2)*x^2 + f^m*m)*(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5
*f^m*(m + 2)*x^5 - c^3*f^m*(2*m + 3)*x^3 + c*f^m*(m + 1)*x)*sqrt(c*x + 1)*x
^m)*sqrt(-c*x + 1)/(a*b*c^5*x^5 + (c*x + 1)*(c*x - 1)*a*b*c^3*x^3 - 2*a*b*c
^3*x^3 + a*b*c*x + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1
) + (b^2*c^5*x^5 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^3 - 2*b^2*c^3*x^3 + b^2*c*
x + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m\,\sqrt{1-c^2\,x^2}}{(a+b\,\mathrm{acosh}\,(c\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2,x)
```

```
[Out] int(((f*x)^m*(1 - c^2*x^2)^(1/2))/(a + b*acosh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \sqrt{-(cx-1)(cx+1)}}{(a+b\,\mathrm{acosh}\,(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m*sqrt(-(c*x - 1)*(c*x + 1))/(a + b*acosh(c*x))**2, x)
```

3.365 $\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=92

$$\frac{fm\sqrt{cx-1}\,\mathrm{Int}\left(\frac{(fx)^{m-1}}{a+b\cosh^{-1}(cx)},x\right)}{bc\sqrt{1-cx}}-\frac{\sqrt{cx-1}\,(fx)^m}{bc\sqrt{1-cx}\,\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] -(f*x)^m*(c*x-1)^(1/2)/b/c/(a+b*arccosh(c*x))/(-c*x+1)^(1/2)+f*m*(c*x-1)^(1
/2)*Unintegrable((f*x)^(-1+m)/(a+b*arccosh(c*x)),x)/b/c/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -(((f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[1 - c^2*x^2]*(a + b*ArcC
osh[c*x]))) + (f*m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^(-1 + m)/(
a + b*ArcCosh[c*x]), x])/(b*c*Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx &= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}\\ &= -\frac{(fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)}+\frac{\left(fm\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^{-1}}{a+b\cosh}}{bc\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\sqrt{1-c^2x^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/(Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,\left(fx\right)^m}{a^2c^2x^2+\left(b^2c^2x^2-b^2\right)\operatorname{arcosh}\left(cx\right)^2-a^2+2\left(abc^2x^2-ab\right)\operatorname{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^2*x^2 + (b^2*c^2*x^2 - b^2)*arc
cosh(c*x)^2 - a^2 + 2*(a*b*c^2*x^2 - a*b)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}}{\sqrt{-c^{2}x^{2}+1}\,(b\,\mathrm{arcosh}\,(cx)+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/(sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}}{\sqrt{-c^{2}x^{2}+1}\;\left(a+b\,\mathrm{arccosh}\,(cx)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^{2}f^{m}x^{2}-f^{m}\right)\sqrt{cx+1}\,\sqrt{cx-1}\,x^{m}+\left(c^{3}f^{m}x^{3}-cf^{m}x\right)x^{m}}{\left((cx+1)\sqrt{cx-1}\,b^{2}c^{2}x+\left(b^{2}c^{3}x^{2}-b^{2}c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left((cx+1)\sqrt{cx-1}\,abc^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -((c^2*f^m*x^2 - f^m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^3*f^m*x^3 - c*f^
m*x)*x^m)/(((c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1
)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x
+ 1)) + integrate(((c^3*f^m*m*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^
m + (2*c^4*f^m*m*x^4 - 3*c^2*f^m*m*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)
*x^m + (c^5*f^m*m*x^5 - c^3*f^m*(2*m + 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/((((c*
x + 1)^(3/2)*(c*x - 1)*b^2*c^3*x^3 + 2*(b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1
)*sqrt(c*x - 1) + (b^2*c^5*x^5 - 2*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*sq
rt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)^(3/2)*(c*x
 - 1)*a*b*c^3*x^3 + 2*(a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) +
 (a*b*c^5*x^5 - 2*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(f\,x\right)}^m}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}\left(a+b\operatorname{acosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(1/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((f*x)**m/(sqrt(-(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**2), x)
```

3.366 $\int \frac{(fx)^m}{(1-c^2x^2)^{3/2}(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.56, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^m/((-1 + c*x)^(3/2)*(1 + c
*x)^(3/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2])
```

Rubi steps

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2x^2+1}\,(fx)^m}{a^2c^4x^4-2\,a^2c^2x^2+\left(b^2c^4x^4-2\,b^2c^2x^2+b^2\right)\text{arcosh}\,(cx)^2+a^2+2\left(abc^4x^4-2\,abc^2x^2+ab\right)\text{arcosh}\,(cx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^4*x^4 - 2*a^2*c^2*x^2 + (b^2*c^4
*x^4 - 2*b^2*c^2*x^2 + b^2)*arccosh(c*x)^2 + a^2 + 2*(a*b*c^4*x^4 - 2*a*b*c
^2*x^2 + a*b)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{3}{2}}(a+b\operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{cf^m x x^m + \sqrt{cx+1}\sqrt{cx-1}\,f^m x^m}{\left((cx+1)\sqrt{cx-1}\,b^2c^2x + \left(b^2c^3x^2 - b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + \left((cx+1)\sqrt{cx-1}\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] (c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/((((c*x + 1)*sqrt(c*x -
1)*b^2*c^2*x + (b^2*c^3*x^2 - b^2*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + ((c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x + (a*b*
c^3*x^2 - a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) - integrate(((c^3*f^m*(m -
2)*x^3 - c*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 2)*x^4
- c^2*f^m*(3*m - 2)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m
*(m - 2)*x^5 - c^3*f^m*(2*m - 1)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^5*x^5
 - b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^6*x^6 - 2*b^2*c^4*x^4
+ b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^7*x^7 - 3*b^2*c^5*x^5 + 3*b
^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1)) + ((a*b*c^5*x^5 - a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) +
2*(a*b*c^6*x^6 - 2*a*b*c^4*x^4 + a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*
b*c^7*x^7 - 3*a*b*c^5*x^5 + 3*a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x + 1))*sqrt(-c
*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m}{(a+b\operatorname{acosh}(c\,x))^2\,\left(1-c^2\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)),x)
```

```
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.367 $\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(f*x)^m/((-1 + c*x)^(5/2)*(1 + c*x
)^(5/2)*(a + b*ArcCosh[c*x])^2), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m}{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{\left(1-c^2x^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[(f*x)^m/((1 - c^2*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(fx)^m}{a^2c^6x^6-3\,a^2c^4x^4+3\,a^2c^2x^2+\left(b^2c^6x^6-3\,b^2c^4x^4+3\,b^2c^2x^2-b^2\right)\operatorname{arcosh}(cx)^2-a^2+2\left(abc^6x^6-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m/(a^2*c^6*x^6 - 3*a^2*c^4*x^4 + 3*a^2*c
^2*x^2 + (b^2*c^6*x^6 - 3*b^2*c^4*x^4 + 3*b^2*c^2*x^2 - b^2)*arccosh(c*x)^2
 - a^2 + 2*(a*b*c^6*x^6 - 3*a*b*c^4*x^4 + 3*a*b*c^2*x^2 - a*b)*arccosh(c*x)
), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(b \operatorname{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m/((-c^2*x^2 + 1)^(5/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m}{(-c^2x^2+1)^{\frac{5}{2}}(a+b \operatorname{arccosh}(cx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{cf^m xx^m + \sqrt{cx+1}\,\sqrt{cx-1}\,f^m}{\left(\left(b^2c^4x^3 - b^2c^2x\right)(cx+1)\sqrt{cx-1} + \left(b^2c^5x^4 - 2\,b^2c^3x^2 + b^2c\right)\sqrt{cx+1}\right)\sqrt{-cx+1}\,\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right) +}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m/(-c^2*x^2+1)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="max
ima")
```

```
[Out] -(c*f^m*x*x^m + sqrt(c*x + 1)*sqrt(c*x - 1)*f^m*x^m)/(((b^2*c^4*x^3 - b^2*c
^2*x)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 - 2*b^2*c^3*x^2 + b^2*c)*sqrt(
c*x + 1))*sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^4
*x^3 - a*b*c^2*x)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^5*x^4 - 2*a*b*c^3*x^2 +
a*b*c)*sqrt(c*x + 1))*sqrt(-c*x + 1)) + integrate(((c^3*f^m*(m - 4)*x^3 - c
*f^m*(m - 1)*x)*(c*x + 1)*(c*x - 1)*x^m + (2*c^4*f^m*(m - 4)*x^4 - c^2*f^m*
(3*m - 4)*x^2 + f^m*m)*sqrt(c*x + 1)*sqrt(c*x - 1)*x^m + (c^5*f^m*(m - 4)*x
^5 - c^3*f^m*(2*m - 3)*x^3 + c*f^m*(m + 1)*x)*x^m)/(((b^2*c^7*x^7 - 2*b^2*c
^5*x^5 + b^2*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(b^2*c^8*x^8 - 3*b^2*c^
6*x^6 + 3*b^2*c^4*x^4 - b^2*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (b^2*c^9*x^9
 - 4*b^2*c^7*x^7 + 6*b^2*c^5*x^5 - 4*b^2*c^3*x^3 + b^2*c*x)*sqrt(c*x + 1))*
sqrt(-c*x + 1)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + ((a*b*c^7*x^7 - 2*a
*b*c^5*x^5 + a*b*c^3*x^3)*(c*x + 1)^(3/2)*(c*x - 1) + 2*(a*b*c^8*x^8 - 3*a*
b*c^6*x^6 + 3*a*b*c^4*x^4 - a*b*c^2*x^2)*(c*x + 1)*sqrt(c*x - 1) + (a*b*c^9
*x^9 - 4*a*b*c^7*x^7 + 6*a*b*c^5*x^5 - 4*a*b*c^3*x^3 + a*b*c*x)*sqrt(c*x +
1))*sqrt(-c*x + 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(f\,x)^m}{(a+b\,\mathrm{acosh}(c\,x))^2\,\left(1-c^2\,x^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)),x)
```

```
[Out] int((f*x)^m/((a + b*acosh(c*x))^2*(1 - c^2*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m/(-c**2*x**2+1)**(5/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.368 $\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}\,dx$

Optimal. Leaf size=32

$$-\frac{\sqrt{ax-1}}{2a\sqrt{1-ax}\cosh^{-1}(ax)^2}$$

[Out] -1/2*(a*x-1)^(1/2)/a/arccosh(a*x)^2/(-a*x+1)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 45, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {5713, 5676}

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]

[Out] -(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(2*a*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1-a^2x^2}\cosh^{-1}(ax)^3}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^3}\,dx}{\sqrt{1-a^2x^2}}\\ &= -\frac{\sqrt{-1+ax}\sqrt{1+ax}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.41

$$-\frac{\sqrt{ax-1}\sqrt{ax+1}}{2a\sqrt{1-a^2x^2}\cosh^{-1}(ax)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^3),x]

[Out] -1/2*(Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[1 - a^2*x^2]*ArcCosh[a*x]^2)

fricas [B] time = 0.60, size = 56, normalized size = 1.75

$$\frac{\sqrt{a^2x^2-1}\,\sqrt{-a^2x^2+1}}{2\left(a^3x^2-a\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(a^2*x^2 - 1)*sqrt(-a^2*x^2 + 1)/((a^3*x^2 - a)*log(a*x + sqrt(a^2*
x^2 - 1))^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2x^2+1}\,\operatorname{arcosh}\left(ax\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*x^2 + 1)*arccosh(a*x)^3), x)
```

maple [A] time = 0.07, size = 51, normalized size = 1.59

$$\frac{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\sqrt{ax-1}\,\sqrt{ax+1}}{2a\left(a^2x^2-1\right)\operatorname{arccosh}\left(ax\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x)
```

```
[Out] 1/2*(-(a*x-1)*(a*x+1))^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(a^2*x^2-1)/arcc
osh(a*x)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{a^7x^7-3\,a^5x^5+3\,a^3x^3+\left(a^4x^4-a^2x^2\right)(ax+1)^{\frac{3}{2}}(ax-1)^{\frac{3}{2}}+\left(3\,a^5x^5-5\,a^3x^3+2\,ax\right)(ax+1)(ax-1)+\left(3\,a^6x\right.}{2\left((ax+1)^2(ax-1)^{\frac{3}{2}}a^4x^3+3\left(a^5x^4-a^3x^2\right)(ax\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/arccosh(a*x)^3/(-a^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 + (a^4*x^4 - a^2*x^2)*(a*x + 1)^(3/2)
*(a*x - 1)^(3/2) + (3*a^5*x^5 - 5*a^3*x^3 + 2*a*x)*(a*x + 1)*(a*x - 1) + (3
*a^6*x^6 - 7*a^4*x^4 + 5*a^2*x^2 - 1)*sqrt(a*x + 1)*sqrt(a*x - 1) - a*x - (
a^5*x^5 - 2*a^3*x^3 - (a^2*x^2 - 1)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2) - (a^3*
x^3 - a*x)*(a*x + 1)*(a*x - 1) + (a^4*x^4 - 2*a^2*x^2 + 1)*sqrt(a*x + 1)*sq
rt(a*x - 1) + a*x)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1)))/(((a*x + 1)^2*(a
*x - 1)^(3/2)*a^4*x^3 + 3*(a^5*x^4 - a^3*x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 3
*(a^6*x^5 - 2*a^4*x^3 + a^2*x)*(a*x + 1)*sqrt(a*x - 1) + (a^7*x^6 - 3*a^5*x
^4 + 3*a^3*x^2 - a)*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*s
qrt(a*x - 1))^2) - integrate(-1/2*(2*a^6*x^6 - 3*a^4*x^4 - (2*a^2*x^2 - 3)*
(a*x + 1)^2*(a*x - 1)^2 - 4*(a^3*x^3 - a*x)*(a*x + 1)^(3/2)*(a*x - 1)^(3/2)
 - 4*(a^2*x^2 - 1)*(a*x + 1)*(a*x - 1) + 4*(a^5*x^5 - 2*a^3*x^3 + a*x)*sqrt
(a*x + 1)*sqrt(a*x - 1) + 1)/(((a*x + 1)^(5/2)*(a*x - 1)^2*a^4*x^4 + 4*(a^5
*x^5 - a^3*x^3)*(a*x + 1)^2*(a*x - 1)^(3/2) + 6*(a^6*x^6 - 2*a^4*x^4 + a^2*
x^2)*(a*x + 1)^(3/2)*(a*x - 1) + 4*(a^7*x^7 - 3*a^5*x^5 + 3*a^3*x^3 - a*x)*
```

```
(a*x + 1)*sqrt(a*x - 1) + (a^8*x^8 - 4*a^6*x^6 + 6*a^4*x^4 - 4*a^2*x^2 + 1)
*sqrt(a*x + 1))*sqrt(-a*x + 1)*log(a*x + sqrt(a*x + 1)*sqrt(a*x - 1))), x)
```

mupad [B] time = 0.41, size = 48, normalized size = 1.50

$$\frac{\sqrt{1-a^2\,x^2}\,\sqrt{a\,x-1}\,\sqrt{a\,x+1}}{a\,\mathrm{acosh}\left(a\,x\right)^2\,\left(2\,a^2\,x^2-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^3*(1 - a^2*x^2)^(1/2)),x)
```

```
[Out] ((1 - a^2*x^2)^(1/2)*(a*x - 1)^(1/2)*(a*x + 1)^(1/2))/(a*acosh(a*x)^2*(2*a^
2*x^2 - 2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\left(ax-1\right)\left(ax+1\right)}\,\mathrm{acosh}^{3}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/acosh(a*x)**3/(-a**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*acosh(a*x)**3), x)
```

3.369 $\int \frac{x^3(d-c^2dx^2)}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=259

$$\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\operatorname{erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}\,c}{}$$

```
[Out] 3/32*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)/c^4+3/32*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)-1/32*d*exp(6*a/b)*erf(6^(1/2)*(a+b*arc
cosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4-1/32*d*erfi(6^(1/2)*
(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)+2
*d*x^3*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 1.75, antiderivative size = 269, normalized size of antiderivative = 1.04, number of steps used = 27, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}\,de^{\frac{6a}{b}}\operatorname{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4}-\frac{\sqrt{\frac{3\pi}{2}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (3*d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^4) - (d*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]
*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4) + (3*d*Sqrt[Pi/2]*Erf
i[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^4*E^((2*a)/b))
 - (d*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*
b^(3/2)*c^4*E^((6*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^3\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\int \frac{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(12cd)\int \frac{x^4\sqrt{-}}{\sqrt{a+b}}}{b}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\operatorname{Subst}\left(\int \frac{\cosh^2(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(6d)\operatorname{Subst}\left(\int\left(-\frac{1}{8\sqrt{a+bx}}+\frac{\cosh(4x)}{8\sqrt{a+bx}}\right)dx, x,\right.}{bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(3d)\operatorname{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\operatorname{Subst}\left(\int \frac{e^{-6x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^4} +$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\operatorname{Subst}\left(\int e^{\frac{6a}{b}-\frac{6x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-}}\right.}{8b^2c^4}$$

$$= -\frac{2dx^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{3de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^4} - \frac{de^{\frac{6a}{b}}\sqrt{}}{}$$

Mathematica [A] time = 2.65, size = 300, normalized size = 1.16

$$\frac{de^{-\frac{6a}{b}}\left(e^{\frac{6a}{b}}\left(-3\sqrt{2}\,e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)+\sqrt{6}\,e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-(Sqrt[6]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh
[c*x]))/b]) + 3*Sqrt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1
/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((6*a)/b)*(-64*c^3*x^3*Sqrt[(-1 + c*x)
/(1 + c*x)] - 64*c^4*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - 3*Sqrt[2]*E^((2*a)/b)
*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*
E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b]
 + 10*Sinh[2*ArcCosh[c*x]] + 8*Sinh[4*ArcCosh[c*x]] + 2*Sinh[6*ArcCosh[c*x]
])))/(32*b*c^4*E^((6*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(c^2dx^2-d\right)x^3}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\left(-c^2d\,x^2+d\right)}{\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(c^2dx^2-d\right)x^3}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\left(d-c^2\,d\,x^2\right)}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{x^3}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^2x^5}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

3.370 $\int \frac{x^2\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=340

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{erf}}{8}$$

```
[Out] 1/8*d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+1
/8*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)+1
/16*d*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c^3+1/16*d*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-1/16*d*exp(5*a/b)*erf(5^(1/2)*(a+b*arcc
osh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3-1/16*d*erfi(5^(1/2)*(
a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(5*a/b)+2*
d*x^2*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 1.77, antiderivative size = 350, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.259, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,de^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3}+\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}-\frac{\sqrt{5\pi}\,de^{\frac{5a}{b}}\mathrm{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^3}+\frac{\sqrt{\pi}\,de^{-\frac{a}{b}}\mathrm{Er}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCo
sh[c*x]]) + (d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b
^(3/2)*c^3) + (d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(16*b^(3/2)*c^3) - (d*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a
 + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c^3*E^(a/b)) + (d*Sqrt[3*Pi]*Erfi[(
Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^3*E^((3*a)/b)) -
(d*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)
*c^3*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^2\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\int \frac{x\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(10cd)\int \frac{x^3\sqrt{-1}}{\sqrt{a+b}}}{b}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(\right.}{bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right)dx, x,\right.}{bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\text{Subst}\left(\int \frac{\cosh(3x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(5d)\,\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-}}\right.}{8b^2c^3}$$

$$= -\frac{2dx^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c^3} + \frac{de^{\frac{3a}{b}}\sqrt{3\pi}}{}$$

Mathematica [A] time = 1.56, size = 384, normalized size = 1.13

$$de^{-\frac{5a}{b}}\left(-4cxe^{\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}} - 4e^{\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}} - 2e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(cx)\right) + 2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(cx)\right) - 2e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(-4*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] - 4*c*E^((5*a)/b)*x*Sqrt[(-1
+ c*x)/(1 + c*x)] - 2*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b +
 ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-5*(a
+ b*ArcCosh[c*x]))/b] + Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]
*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 2*E^((4*a)/b)*Sqrt[-((a + b*ArcC
osh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - Sqrt[3]*E^((8*a)/b)*S
qrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + Sqrt[5]*E^
((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b]
- 2*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5*ArcCosh[c*x]]))
/(16*b*c^3*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(c^2dx^2-d\right)x^2}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(-c^2d\,x^2+d\right)}{(a+b\operatorname{arccosh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(c^2dx^2-d\right)x^2}{(b\operatorname{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(d-c^2\,d\,x^2\right)}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{x^2}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(cx)}\operatorname{acosh}(cx)}\right)dx+\int\frac{c^2x^4}{a\sqrt{a+b\operatorname{acosh}(cx)}+b\sqrt{a+b\operatorname{acosh}(c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*aco
sh(c*x)), x) + Integral(c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*
acosh(c*x))*acosh(c*x)), x))
```

3.371 $\int \frac{x(d-c^2dx^2)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\operatorname{erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\operatorname{er}}{}$$

```
[Out] 1/4*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(
1/2)/b^(3/2)/c^2+1/4*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/
2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)-1/4*d*exp(4*a/b)*erf(2*(a+b*arccosh(c*x)
)^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2-1/4*d*erfi(2*(a+b*arccosh(c*x))^(1/2)
/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+2*d*x*(c*x-1)^(3/2)*(c*x+1)^(3/2)
/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 251, normalized size of antiderivative = 1.04, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{\sqrt{\pi}\,de^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}c^2}-\frac{\sqrt{\pi}\,de^{-\frac{4a}{b}}\operatorname{Erfi}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{\sqrt{\frac{\pi}{2}}\,de^{-\frac{2a}{b}}\,\mathrm{E}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*Sqrt[a + b*ArcCosh
[c*x]]) - (d*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]
)/(4*b^(3/2)*c^2) + (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2) - (d*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCos
h[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^((4*a)/b)) + (d*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*c^2*E^((2*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x\left(d-c^2dx^2\right)}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(2d)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc}-\frac{(8cd)\int \frac{x^2\sqrt{-1+c}}{\sqrt{a+b\,\mathrm{co}}}}{b}$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{(2d)\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{bc^2}-$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{(2d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}}-\frac{\cosh(2x)}{2\sqrt{a+bx}}\right)dx,x,\mathrm{co}\right.}{bc^2}$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{d\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{bc^2}-\frac{d\,\mathrm{S}}{}$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{d\,\mathrm{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{2bc^2}+\frac{d\,\mathrm{S}}{}$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{d\,\mathrm{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right.}{b^2c^2}$$

$$=-\frac{2dx\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bc\sqrt{a+b\cosh^{-1}(cx)}}-\frac{de^{\frac{4a}{b}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{er}}{}$$

Mathematica [A] time = 4.27, size = 331, normalized size = 1.37

$$\frac{de^{-\frac{4a}{b}}\left(2\sqrt{2\pi}\,e^{\frac{6a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+2\sqrt{2\pi}\,e^{\frac{2a}{b}}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)+\frac{\sqrt{b}\left(-\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{4\left(a+b\cosh^{-1}(c\right.}{b}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2))/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d*(2*E^((6*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
] + 2*E^((2*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]] + (Sqrt[b]*(-(Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcC
osh[c*x]))/b]) - Sqrt[2]*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[
1/2, (-2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*(8*c*x*((-1 + c*x)/(1 + c*x
))^(3/2)*(1 + c*x)^3 + Sqrt[2]*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1
/2, (2*(a + b*ArcCosh[c*x]))/b] + E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b])))/Sqrt[a + b*ArcCosh[c*x]]))/(4*b^(3/2)
*c^2*E^((4*a)/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(c^2dx^2-d\right)x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)*x/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-c^2d\,x^2+d\right)}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(c^2dx^2-d\right)x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)*x/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(d-c^2\,d\,x^2\right)}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x*(d - c^2*d*x^2))/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int\left(-\frac{x}{a\sqrt{a+b\operatorname{acosh}\left(cx\right)}+b\sqrt{a+b\operatorname{acosh}\left(cx\right)}\operatorname{acosh}\left(cx\right)}\right)dx+\int\frac{c^{2}x^{3}}{a\sqrt{a+b\operatorname{acosh}\left(cx\right)}+b\sqrt{a+b\operatorname{acosh}\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(-x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh(
c*x)), x) + Integral(c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*aco
sh(c*x))*acosh(c*x)), x))
```

3.372 $\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=233

$$\frac{3\sqrt{\pi}\,de^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\text{er}$$

```
[Out] 3/4*d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+3/4
*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-1/4*d
*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/
b^(3/2)/c-1/4*d*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(
1/2)/b^(3/2)/c/exp(3*a/b)+2*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/(a+b*arccosh(
c*x))^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5695, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\,de^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}+\frac{3\sqrt{\pi}\,de^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c}-\frac{\sqrt{3\pi}\,de^{-\frac{3a}{b}}\text{E}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (2*d*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (3*
d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c) - (
d*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(
4*b^(3/2)*c) + (3*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(
3/2)*c*E^(a/b)) - (d*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(4*b^(3/2)*c*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{d-c^2dx^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6cd)\int \frac{x\sqrt{-1+cx}\sqrt{1+cx}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\,\text{Subst}\left(\int \frac{\cosh(x)\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(6d)\,\text{Subst}\left(\int \left(-\frac{\cosh(x)}{4\sqrt{a+bx}} + \frac{\cosh(3x)}{4\sqrt{a+bx}}\right) dx, x, \cosh^{-1}(cx\right.}{bc}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(3d)\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2bc} - \frac{(3d)\,\text{Subs}}{}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4bc} + \frac{(3d)\,\text{Subs}}{}$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(3d)\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2b^2c} + \cdots$$

$$= \frac{2d(-1+cx)^{3/2}(1+cx)^{3/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{3de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c} - \frac{de^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{a}}{}\right.}{4b^{3/2}c}$$

Mathematica [A] time = 1.70, size = 246, normalized size = 1.06

$$\frac{e^{-\frac{3a}{b}}\left(-3de^{\frac{4a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right) - \sqrt{3}\,d\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right) + de^{\frac{2a}{b}}\left(3\sqrt{\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*d*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] -
 Sqrt[3]*d*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*
x]))/b] + d*E^((2*a)/b)*(3*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a
+ b*ArcCosh[c*x])/b)] + E^(a/b)*(-6*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) +
Sqrt[3]*E^((3*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c
*x]))/b] + 2*Sinh[3*ArcCosh[c*x]])))/(4*b*c*E^((3*a)/b)*Sqrt[a + b*ArcCosh[
c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{c^2dx^2 - d}{(b\,\mathrm{arcosh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(-(c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [**F(-2)**] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{-c^2d\,x^2 + d}{(a + b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c^2dx^2 - d}{(b\,\mathrm{arcosh}\,(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{d - c^2\,d\,x^2}{(a + b\,\mathrm{acosh}\,(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((d - c^2*d*x^2)/(a + b*acosh(c*x))^(3/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \frac{c^2x^2}{a\sqrt{a + b\,\mathrm{acosh}\,(cx)} + b\sqrt{a + b\,\mathrm{acosh}\,(cx)}\,\mathrm{acosh}\,(cx)}\,dx + \int \left(-\frac{1}{a\sqrt{a + b\,\mathrm{acosh}\,(cx)} + b\sqrt{a + b\,\mathrm{acosh}\,(cx

3.373 $\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{2d\text{Int}\left(\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}},x\right)}{bc}-\frac{\sqrt{\frac{\pi}{2}}de^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}de^{-\frac{2a}{b}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}}+\frac{2d(}{bcx}$$

```
[Out] -1/2*d*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^
(1/2)/b^(3/2)-1/2*d*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*
Pi^(1/2)/b^(3/2)/exp(2*a/b)+2*d*(c*x-1)^(3/2)*(c*x+1)^(3/2)/b/c/x/(a+b*arcc
osh(c*x))^(1/2)+2*d*Unintegrable(1/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(a+b*arc
cosh(c*x))^(1/2),x)/b/c
```

Rubi [A] time = 1.66, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2))/(b*c*x*Sqrt[a + b*ArcCosh
[c*x]]) - (d*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/
Sqrt[b]])/b^(3/2) - (d*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/S
qrt[b]])/(b^(3/2)*E^((2*a)/b)) + (2*d*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} - \frac{(4cd)\int \frac{\sqrt{-1+cx}\,\sqrt{}}{\sqrt{a+b\cosh}}}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(4d)\,\mathrm{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} - \frac{(2}{}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(4d)\,\mathrm{Subst}\left(\int \left(\frac{1}{2\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}}\right) dx, x, \cosh}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} + \frac{(2}{}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d\,\mathrm{Subst}\left(\int \frac{e^{-2x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} - \frac{d\,\mathrm{Sub}}{}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{(2d)\,\mathrm{Subst}\left(\int e^{\frac{2a}{b}-\frac{2x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c}}{b^2}$$

$$= -\frac{2d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{de^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{erf}\left(\frac{\sqrt{2}\,\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{de^{-\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\,\mathrm{e}}{}$$

Mathematica [A] time = 4.46, size = 0, normalized size = 0.00

$$\int \frac{d-c^2dx^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[(d - c^2*d*x^2)/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{-c^2 d\,x^2 + d}{x\,(a + b\operatorname{arccosh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{c^2dx^2 - d}{(b\operatorname{arcosh}(cx) + a)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] -integrate((c^2*d*x^2 - d)/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{d - c^2\,d\,x^2}{x\,(a + b\operatorname{acosh}(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)),x)
```

```
[Out] int((d - c^2*d*x^2)/(x*(a + b*acosh(c*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-d\left(\int \frac{c^2x^2}{ax\sqrt{a + b\operatorname{acosh}(cx)} + bx\sqrt{a + b\operatorname{acosh}(cx)}\,\operatorname{acosh}(cx)}\,dx + \int \left(-\frac{1}{ax\sqrt{a + b\operatorname{acosh}(cx)} + bx\sqrt{a + b\operatorname{ac}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)/x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] -d*(Integral(c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*acosh(c
*x))*acosh(c*x)), x) + Integral(-1/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a
 + b*acosh(c*x))*acosh(c*x)), x))
```

3.374 $\int \frac{x^3(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=479

$$-\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\operatorname{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{8a}{b}}\operatorname{erf}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\sqrt{\frac{3\pi}{2}}\,d^2e^{6}\ldots$$

```
[Out] 3/64*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^4+1/64*d^2*exp(8*a/b)*erf(2*2^(1/2)*(a+b*arccosh(c*x))^(1
/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4+3/64*d^2*erfi(2^(1/2)*(a+b*arccos
h(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(2*a/b)+1/64*d^2*erf
i(2*2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/b^(3/2)/c^4/
exp(8*a/b)-1/32*d^2*exp(4*a/b)*erf(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(
1/2)/b^(3/2)/c^4-1/32*d^2*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)
/b^(3/2)/c^4/exp(4*a/b)-1/64*d^2*exp(6*a/b)*erf(6^(1/2)*(a+b*arccosh(c*x))^
(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4-1/64*d^2*erfi(6^(1/2)*(a+b*arcc
osh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^4/exp(6*a/b)-2*d^2*x^3*
(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 2.13, antiderivative size = 491, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{3\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}+\frac{\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{8a}{b}}\operatorname{Erf}\left(\frac{2\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4}-\sqrt{\frac{3\pi}{2}}\,d^2e\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*A
rcCosh[c*x]]) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/
Sqrt[b]])/(32*b^(3/2)*c^4) + (3*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) + (d^2*E^((8*a)/b)*Sqrt[P
i/2]*Erf[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4) -
(d^2*E^((6*a)/b)*Sqrt[(3*Pi)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]])/(32*b^(3/2)*c^4) - (d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(32*b^(3/2)*c^4*E^((4*a)/b)) + (3*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt
[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^((2*a)/b)) + (d^2*Sqrt[Pi
/2]*Erfi[(2*Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*b^(3/2)*c^4*E^(
(8*a)/b)) - (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqr
t[b]])/(32*b^(3/2)*c^4*E^((6*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^3\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\int \frac{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(16cd^2\right)\int}{}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\text{Subst}\left(\int \frac{\cosh^2(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}\right.}{bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(6d^2\right)\text{Subst}\left(\int\left(\frac{1}{16\sqrt{a+bx}} - \frac{\cosh(2x)}{32\sqrt{a+bx}} - \frac{\cosh}{16\sqrt{a}}\right.\right.}{bc^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(8x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^4} + \frac{(}{}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-8x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{16bc^4} + \frac{d^2}{}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{8a}{b}-\frac{8x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c}\right.}{8b^2c^4}$$

$$= -\frac{2d^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32b^{3/2}c^4} + \frac{3d^2e^{\frac{2a}{b}}\sqrt{}}{}$$

Mathematica [A] time = 3.78, size = 527, normalized size = 1.10

$$-\frac{d^2e^{-\frac{8a}{b}}\left(128c^4x^4e^{\frac{8a}{b}}\sqrt{\frac{cx-1}{cx+1}} + 128c^3x^3e^{\frac{8a}{b}}\sqrt{\frac{cx-1}{cx+1}} - 26e^{\frac{8a}{b}}\sinh\left(2\cosh^{-1}(cx)\right) - 18e^{\frac{8a}{b}}\sinh\left(4\cosh^{-1}(cx)\right) - 2e^{\frac{8a}{b}}\text{s}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(128*c^3*E^((8*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128*c^4*E^
((8*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] - Sqrt[2]*Sqrt[-((a + b*ArcCosh[c*
x])/b)]*Gamma[1/2, (-8*(a + b*ArcCosh[c*x]))/b] + Sqrt[6]*E^((2*a)/b)*Sqrt[
-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] + 2*E^((
4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x])
)/b] - 3*Sqrt[2]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2
*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[2]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]
*Gamma[1/2, (2*(a + b*ArcCosh[c*x]))/b] - 2*E^((12*a)/b)*Sqrt[a/b + ArcCosh
[c*x]]*Gamma[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((14*a)/b)*Sqrt[a
/b + ArcCosh[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] + Sqrt[2]*E^((16*
a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (8*(a + b*ArcCosh[c*x]))/b] - 26*
E^((8*a)/b)*Sinh[2*ArcCosh[c*x]] - 18*E^((8*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*
E^((8*a)/b)*Sinh[6*ArcCosh[c*x]] + E^((8*a)/b)*Sinh[8*ArcCosh[c*x]]))/(b*c^
4*E^((8*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2-d\right)^2x^3}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}\left(-c^{2}d\,x^{2}+d\right)^{2}}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2-d\right)^2x^3}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^3/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^3*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{2}\left(\int \frac{x^{3}}{a\sqrt{a+b\operatorname{acosh}\left(cx\right)}+b\sqrt{a+b\operatorname{acosh}\left(cx\right)}\operatorname{acosh}\left(cx\right)}\,dx+\int \left(-\frac{2c^{2}x^{5}}{a\sqrt{a+b\operatorname{acosh}\left(cx\right)}+b\sqrt{a+b\operatorname{acosh}\left(\right.}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**5/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**7/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

3.375 $\int \frac{x^2\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=462

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\operatorname{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\sqrt{7\pi}\,d$$

```
[Out] 5/64*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^
3+5/64*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(
a/b)+1/64*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(
1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(3*a/b)-3/64*d^2*exp(5*a/b)*erf(5^(1/
2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3-3/64*d^2*
erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c^3
/exp(5*a/b)+1/64*d^2*exp(7*a/b)*erf(7^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2
))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3+1/64*d^2*erfi(7^(1/2)*(a+b*arccosh(c*x))^(1
/2)/b^(1/2))*7^(1/2)*Pi^(1/2)/b^(3/2)/c^3/exp(7*a/b)-2*d^2*x^2*(c*x-1)^(5/2
)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 2.27, antiderivative size = 474, normalized size of antiderivative = 1.03, number of steps used = 42, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {5776, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\frac{\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}-\frac{3\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\operatorname{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3}+\sqrt{7\pi}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*A
rcCosh[c*x]]) + (5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b
]])/(64*b^(3/2)*c^3) + (d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) - (3*d^2*E^((5*a)/b)*Sqrt[5*Pi]*E
rf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3) + (d^2*E^(
(7*a)/b)*Sqrt[7*Pi]*Erf[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^
(3/2)*c^3) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(64*b^
(3/2)*c^3*E^(a/b)) + (d^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]]
)/Sqrt[b]])/(64*b^(3/2)*c^3*E^((3*a)/b)) - (3*d^2*Sqrt[5*Pi]*Erfi[(Sqrt[5]*
Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E^((5*a)/b)) + (d^2*Sqr
t[7*Pi]*Erfi[(Sqrt[7]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(64*b^(3/2)*c^3*E
^((7*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x^2\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(14cd^2\right)\ldots}{\ldots}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh\ldots\right.}{bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(4d^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}} - \frac{3\cosh(3x)}{16\sqrt{a+bx}} + \ldots\right.\right.}{bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx\ldots\right.}{32bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int \frac{e^{-7x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\ldots\right.}{64bc^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(7d^2\right)\text{Subst}\left(\int e^{\frac{7a}{b}-\frac{7x^2}{b}}\,dx, x, \sqrt{a+b\,\text{co}\ldots}\right.}{32b^2c^3}$$

$$= -\frac{2d^2x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64b^{3/2}c^3} + \frac{d^2e^{\frac{3a}{b}}\ldots}{\ldots}$$

Mathematica [A] time = 2.99, size = 498, normalized size = 1.08

$$-\frac{d^2e^{-\frac{7a}{b}}\left(10cxe^{\frac{7a}{b}}\sqrt{\frac{cx-1}{cx+1}} + 10e^{\frac{7a}{b}}\sqrt{\frac{cx-1}{cx+1}} + 2e^{\frac{7a}{b}}\sinh\left(3\cosh^{-1}(cx)\right) - 6e^{\frac{7a}{b}}\sinh\left(5\cosh^{-1}(cx)\right) + 2e^{\frac{7a}{b}}\sinh\left(7\,\text{c}\ldots\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/64*(d^2*(10*E^((7*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 10*c*E^((7*a)/b)*x*
Sqrt[(-1 + c*x)/(1 + c*x)] + 5*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1
/2, a/b + ArcCosh[c*x]] - Sqrt[7]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2
, (-7*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*E^((2*a)/b)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] - Sqrt[3]*E^((4*a)/b)*S
qrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 5*
E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x
])/b)] + Sqrt[3]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b
*ArcCosh[c*x]))/b] - 3*Sqrt[5]*E^((12*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[
1/2, (5*(a + b*ArcCosh[c*x]))/b] + Sqrt[7]*E^((14*a)/b)*Sqrt[a/b + ArcCosh[
c*x]]*Gamma[1/2, (7*(a + b*ArcCosh[c*x]))/b] + 2*E^((7*a)/b)*Sinh[3*ArcCosh
[c*x]] - 6*E^((7*a)/b)*Sinh[5*ArcCosh[c*x]] + 2*E^((7*a)/b)*Sinh[7*ArcCosh[
c*x]]))/(b*c^3*E^((7*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="frica
s")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 - d\right)^2 x^2}{\left(b \operatorname{arcosh}(cx) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(-c^2 d\, x^2 + d\right)^2}{\left(a + b \operatorname{arccosh}(cx)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 - d\right)^2 x^2}{\left(b \operatorname{arcosh}(cx) + a\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x^2/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 \left(d - c^2\, d\, x^2\right)^2}{\left(a + b \operatorname{acosh}(c\, x)\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x^2*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \left(\int \frac{x^2}{a\sqrt{a + b \operatorname{acosh}(cx)} + b\sqrt{a + b \operatorname{acosh}(cx)} \operatorname{acosh}(cx)}\, dx + \int \left(-\frac{2c^2x^4}{a\sqrt{a + b \operatorname{acosh}(cx)} + b\sqrt{a + b \operatorname{acosh}(cx)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*ac
osh(c*x)), x) + Integral(-2*c**2*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a
+ b*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**6/(a*sqrt(a + b*acosh(c*
x)) + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

3.376 $\int \frac{x(d-c^2dx^2)^2}{(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=363

$$-\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\,\mathrm{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{\frac{6a}{b}}\,\mathrm{erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{\sqrt{\pi}\,d^2e^{-\frac{4}{b}}}{}$$

```
[Out] 5/32*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)/c^2+5/32*d^2*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))
*2^(1/2)*Pi^(1/2)/b^(3/2)/c^2/exp(2*a/b)-1/4*d^2*exp(4*a/b)*erf(2*(a+b*arcc
osh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2-1/4*d^2*erfi(2*(a+b*arccosh(c
*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^2/exp(4*a/b)+1/32*d^2*exp(6*a/b)*erf
(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2)/c^2+1/3
2*d^2*erfi(6^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/
2)/c^2/exp(6*a/b)-2*d^2*x*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x)
)^(1/2)
```

Rubi [A] time = 1.78, antiderivative size = 375, normalized size of antiderivative = 1.03, number of steps used = 32, number of rules used = 9, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{\sqrt{\pi}\,d^2e^{\frac{4a}{b}}\,\mathrm{Erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2}+\frac{5\sqrt{\frac{\pi}{2}}\,d^2e^{\frac{2a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}+\frac{\sqrt{\frac{3\pi}{2}}\,d^2e^{\frac{6a}{b}}\,\mathrm{Erf}\left(\frac{\sqrt{6}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c^2}-\frac{\sqrt{\pi}\,d^2e^{-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*Sqrt[a + b*Arc
Cosh[c*x]]) - (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)*c^2) + (5*d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a
 + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) + (d^2*E^((6*a)/b)*Sqrt[(3*P
i)/2]*Erf[(Sqrt[6]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2) - (
d^2*Sqrt[Pi]*Erfi[(2*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*c^2*E^(
(4*a)/b)) + (5*d^2*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(16*b^(3/2)*c^2*E^((2*a)/b)) + (d^2*Sqrt[(3*Pi)/2]*Erfi[(Sqrt[6]*Sqrt[
a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c^2*E^((6*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{x\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(12cd^2\right)\int \frac{x^2(\cdots}{\cdots}}{\cdots}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc^2} + \cdots$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{\left(2d^2\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4x)}{8\sqrt{a+bx}}\cdots\right.\right.}{bc^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4bc^2} - \frac{(3\cdots}{\cdots}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{8bc^2} - \frac{d^2\,\text{S}\cdots}{\cdots}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\cdots\right.}{4b^2c^2}$$

$$= -\frac{2d^2x\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^2} + \frac{5d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\cdots}{\cdots}$$

Mathematica [A] time = 7.29, size = 508, normalized size = 1.40

$$d^2e^{-\frac{6a}{b}}\left(\frac{\sqrt{b}\left(128c^4x^4e^{\frac{6a}{b}}\sqrt{\frac{cx-1}{cx+1}}+128c^3x^3e^{\frac{6a}{b}}\sqrt{\frac{cx-1}{cx+1}}-42e^{\frac{6a}{b}}\sinh\left(2\cosh^{-1}(cx)\right)-8e^{\frac{6a}{b}}\sinh\left(4\cosh^{-1}(cx)\right)-2e^{\frac{6a}{b}}\sinh\left(6\cosh^{-1}(cx)\right)+\sqrt{6}\sqrt{-\frac{a+b\cos\cdots}{b}}\cdots\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(d - c^2*d*x^2)^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (d^2*(16*E^((8*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt
[b]] + 16*E^((4*a)/b)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]] + (Sqrt[b]*(128*c^3*E^((6*a)/b)*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 128
*c^4*E^((6*a)/b)*x^4*Sqrt[(-1 + c*x)/(1 + c*x)] + Sqrt[6]*Sqrt[-((a + b*Arc
Cosh[c*x])/b)]*Gamma[1/2, (-6*(a + b*ArcCosh[c*x]))/b] - 8*E^((2*a)/b)*Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-4*(a + b*ArcCosh[c*x]))/b] - 11*Sq
rt[2]*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-2*(a + b*Arc
Cosh[c*x]))/b] + 11*Sqrt[2]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2,
 (2*(a + b*ArcCosh[c*x]))/b] + 8*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, (4*(a + b*ArcCosh[c*x]))/b] - Sqrt[6]*E^((12*a)/b)*Sqrt[a/b + ArcCos
h[c*x]]*Gamma[1/2, (6*(a + b*ArcCosh[c*x]))/b] - 42*E^((6*a)/b)*Sinh[2*ArcC
osh[c*x]] - 8*E^((6*a)/b)*Sinh[4*ArcCosh[c*x]] - 2*E^((6*a)/b)*Sinh[6*ArcCo
sh[c*x]]))/Sqrt[a + b*ArcCosh[c*x]]))/(32*b^(3/2)*c^2*E^((6*a)/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2-d\right)^2x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(-c^2d\,x^2+d\right)^2}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2-d\right)^2x}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2*x/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(d-c^2\,d\,x^2\right)}^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((x*(d - c^2*d*x^2)^2)/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \frac{x}{a \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\, dx + \int \left(- \frac{2 c^{2} x^{3}}{a \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b \sqrt{a + b \operatorname{acosh}{\left(</$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(x/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(c*x))*acosh
(c*x)), x) + Integral(-2*c**2*x**3/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b
*acosh(c*x))*acosh(c*x)), x) + Integral(c**4*x**5/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

3.377 $\int \frac{\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=351

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}\,d}{}$$

```
[Out] 5/8*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+5
/8*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-5
/16*d^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/b^(3/2)/c-5/16*d^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/b^(3/2)/c/exp(3*a/b)+1/16*d^2*exp(5*a/b)*erf(5^(1/2)*(a+b*ar
ccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c+1/16*d^2*erfi(5^(1/2)
*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)*Pi^(1/2)/b^(3/2)/c/exp(5*a/b)-2*
d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 0.92, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {5695, 5781, 5448, 3307, 2180, 2204, 2205}

$$\frac{5\sqrt{\pi}\,d^2e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5\sqrt{3\pi}\,d^2e^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{\sqrt{5\pi}\,d^2e^{\frac{5a}{b}}\text{Erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16b^{3/2}c}+\frac{5\sqrt{\pi}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d^2*(-1 + c*x)^(5/2)*(1 + c*x)^(5/2))/(b*c*Sqrt[a + b*ArcCosh[c*x]]) +
(5*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c
) - (5*d^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(16*b^(3/2)*c) + (d^2*E^((5*a)/b)*Sqrt[5*Pi]*Erf[(Sqrt[5]*Sqrt[a +
b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c) + (5*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(8*b^(3/2)*c*E^(a/b)) - (5*d^2*Sqrt[3*Pi]*Erfi[(Sq
rt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c*E^((3*a)/b)) + (d^2
*Sqrt[5*Pi]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(16*b^(3/2)*c
*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5695

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((-d)^p*(-1 + c*x)^(p + 1/2)*(1 + c*x)^(p + 1/2)*(a + b*Ar
cCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[(c*(-d)^p*(2*p + 1))/(b*(n +
1)), Int[x*(-1 + c*x)^(p - 1/2)*(1 + c*x)^(p - 1/2)*(a + b*ArcCosh[c*x])^(n
 + 1), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && LtQ[n
, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(10cd^2\right)\int \frac{x(-1+cx)^{3/2}(1+cx)^{3/2}}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(10d^2\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^4(x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(10d^2\right)\text{Subst}\left(\int \left(\frac{\cosh(x)}{8\sqrt{a+bx}}-\frac{3\cosh(3x)}{16\sqrt{a+bx}}+\frac{\cosh(5x)}{16\sqrt{a+bx}}\right)d\right.}{bc}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(5d^2\right)\text{Subst}\left(\int \frac{\cosh(5x)}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{8bc}+\frac{\left(5d^2\right.}{}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(5d^2\right)\text{Subst}\left(\int \frac{e^{-5x}}{\sqrt{a+bx}}\,dx,x,\cosh^{-1}(cx)\right)}{16bc}+\frac{\left(5d^2\right)}{}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{\left(5d^2\right)\text{Subst}\left(\int e^{\frac{5a}{b}-\frac{5x^2}{b}}\,dx,x,\sqrt{a+b\cosh^{-1}(cx)}\right)}{8b^2c}$$

$$= -\frac{2d^2(-1+cx)^{5/2}(1+cx)^{5/2}}{bc\sqrt{a+b\cosh^{-1}(cx)}}+\frac{5d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8b^{3/2}c}-\frac{5d^2e^{\frac{3a}{b}}\sqrt{3\pi}\,\text{erf}\left(\right.}{16}$$

Mathematica [A] time = 1.90, size = 387, normalized size = 1.10

$$-\frac{d^2e^{-\frac{5a}{b}}\left(20cxe^{\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}}+20e^{\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}}-10e^{\frac{5a}{b}}\sinh\left(3\cosh^{-1}(cx)\right)+2e^{\frac{5a}{b}}\sinh\left(5\cosh^{-1}(cx)\right)+10e^{\frac{6a}{b}}\sqrt{\frac{a}{b}+\cdots}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^2/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] -1/16*(d^2*(20*E^((5*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)] + 20*c*E^((5*a)/b)*x*
Sqrt[(-1 + c*x)/(1 + c*x)] + 10*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[
1/2, a/b + ArcCosh[c*x]] - Sqrt[5]*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/
2, (-5*(a + b*ArcCosh[c*x]))/b] + 5*Sqrt[3]*E^((2*a)/b)*Sqrt[-((a + b*ArcCo
sh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] - 10*E^((4*a)/b)*Sqrt[
-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - 5*Sqrt[3
]*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/
b] + Sqrt[5]*E^((10*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*Arc
Cosh[c*x]))/b] - 10*E^((5*a)/b)*Sinh[3*ArcCosh[c*x]] + 2*E^((5*a)/b)*Sinh[5
*ArcCosh[c*x]]))/(b*c*E^((5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 - d\right)^2}{\left(b\,\mathrm{arcosh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2d\,x^2 + d\right)^2}{\left(a + b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^2dx^2 - d\right)^2}{\left(b\,\mathrm{arcosh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c^2*d*x^2 - d)^2/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(d - c^2\,d\,x^2\right)^2}{\left(a + b\,\mathrm{acosh}\left(c\,x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((d - c^2*d*x^2)^2/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2\left(\int \left(-\frac{2c^2x^2}{a\sqrt{a + b\,\mathrm{acosh}\left(cx\right)} + b\sqrt{a + b\,\mathrm{acosh}\left(cx\right)}\,\mathrm{acosh}\left(cx\right)}\right)dx + \int \frac{c^4x^4}{a\sqrt{a + b\,\mathrm{acosh}\left(cx\right)} + b\sqrt{a + b\,\mathrm{acosh}\left(c\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*sqrt(a + b*acosh(c*x)) + b*sqrt(a + b*acosh(
c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*sqrt(a + b*acosh(c*x)) + b*sq
rt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*sqrt(a + b*acosh(c*x))
 + b*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

3.378 $\int \frac{(d-c^2dx^2)^2}{x(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=289

$$\frac{2d^2\text{Int}\left(\frac{1}{x^2\sqrt{cx-1}\sqrt{cx+1}\sqrt{a+b\cosh^{-1}(cx)}},x\right)}{bc}+\frac{\sqrt{\pi}d^2e^{\frac{4a}{b}}\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}}-\frac{3\sqrt{\frac{\pi}{2}}d^2e^{\frac{2a}{b}}\text{erf}\left(\frac{\sqrt{2}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{\pi}}{}$$

```
[Out] -3/4*d^2*exp(2*a/b)*erf(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1/2)*P
i^(1/2)/b^(3/2)-3/4*d^2*erfi(2^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/b^(3/2)/exp(2*a/b)+1/4*d^2*exp(4*a/b)*erf(2*(a+b*arccosh(c*x))
^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)+1/4*d^2*erfi(2*(a+b*arccosh(c*x))^(1/2)/b^
(1/2))*Pi^(1/2)/b^(3/2)/exp(4*a/b)-2*d^2*(c*x-1)^(5/2)*(c*x+1)^(5/2)/b/c/x/
(a+b*arccosh(c*x))^(1/2)+2*d^2*Unintegrable(1/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)/(a+b*arccosh(c*x))^(1/2),x)/b/c
```

Rubi [A] time = 2.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
```

```
[Out] (-2*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(1 - c^2*x^2)^2)/(b*c*x*Sqrt[a + b*Arc
Cosh[c*x]]) + (d^2*E^((4*a)/b)*Sqrt[Pi]*Erf[(2*Sqrt[a + b*ArcCosh[c*x]])/Sq
rt[b]])/(4*b^(3/2)) + (d^2*E^((2*a)/b)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[a + b*A
rcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)) - (d^2*E^((2*a)/b)*Sqrt[2*Pi]*Erf[(Sqrt
[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/b^(3/2) + (d^2*Sqrt[Pi]*Erfi[(2*Sqr
t[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2)*E^((4*a)/b)) + (d^2*Sqrt[Pi/2]*
Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(2*b^(3/2)*E^((2*a)/b)) -
 (d^2*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(b^(3/2)
*E^((2*a)/b)) + (2*d^2*Defer[Int][1/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[
a + b*ArcCosh[c*x]]), x])/(b*c)
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(2d^2\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}}{x^2\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{bc} + \frac{\left(8cd^2\right)\int \frac{(-1\cdots}{\sqrt{a\cdots}}}{\cdots}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{b} + \cdots$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\left(8d^2\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{a+bx}} - \frac{\cosh(2x)}{2\sqrt{a+bx}} + \frac{\cosh(4x)}{8\sqrt{a+bx}}\cdots\right.\right.}{b}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{\cosh(\cdots}{\sqrt{a+\cdots}}\right.}{\cdots}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2d^2\sqrt{a+b\cosh^{-1}(cx)}}{b^2} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{-4x}\cdots}{\sqrt{a+b\cdots}}\right.}{\cdots}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2\,\text{Subst}\left(\int e^{\frac{4a}{b}-\frac{4x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(c\cdots}\right.}{b^2}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi\cdots}}{\cdots}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} - \frac{d^2e^{\frac{2a}{b}}\sqrt{2\pi\cdots}}{\cdots}$$

$$= -\frac{2d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(1-c^2x^2\right)^2}{bcx\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d^2e^{\frac{4a}{b}}\sqrt{\pi}\,\text{erf}\left(\frac{2\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}} + \frac{d^2e^{\frac{2a}{b}}\sqrt{\frac{\pi}{2}}\cdots}{\cdots}$$

Mathematica [A] time = 3.42, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^2}{x\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[(d - c^2*d*x^2)^2/(x*(a + b*ArcCosh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas"
)
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{2}}{x \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c^{2} d x^{2}-d\right)^{2}}{\left(b\,\mathrm{arcosh}\left(c x\right)+a\right)^{\frac{3}{2}} x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^2/x/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c^2*d*x^2 - d)^2/((b*arccosh(c*x) + a)^(3/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(d-c^2\,d\,x^2\right)}^2}{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)),x)
```

```
[Out] int((d - c^2*d*x^2)^2/(x*(a + b*acosh(c*x))^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$d^{2} \left(\int \left(- \frac{2 c^{2} x^{2}}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} \operatorname{acosh}{\left(c x \right)}}\right)\, dx + \int \frac{c^{4} x^{4}}{a x \sqrt{a + b \operatorname{acosh}{\left(c x \right)}} + b x \sqrt{a + b a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**2/x/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] d**2*(Integral(-2*c**2*x**2/(a*x*sqrt(a + b*acosh(c*x)) + b*x*sqrt(a + b*ac
osh(c*x))*acosh(c*x)), x) + Integral(c**4*x**4/(a*x*sqrt(a + b*acosh(c*x))
+ b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x) + Integral(1/(a*x*sqrt(a + b*a
cosh(c*x)) + b*x*sqrt(a + b*acosh(c*x))*acosh(c*x)), x))
```

3.379 $\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=351

$$-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(2\sqrt{\cosh\right.}}{256a\sqrt{ax-1}\,\sqrt{ax+1}}$$

```
[Out] -1/4*c*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2
)+1/32*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1
/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2
^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/256*c*
erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*
x+1)^(1/2)+1/256*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)
/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2
)+3/8*c*x*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.72, antiderivative size = 363, normalized size of antiderivative = 1.03, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5713, 5685, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5780}

$$-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh\right.}}{256a\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (3*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/8 + (c*x*(1 - a*x)*(1 + a*x)
*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/4 - (c*Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*
c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c
*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[Arc
Cosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c -
a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
```

```
!IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+ax}}{4\sqrt{-1+ax}}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

$$= \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}$$

Mathematica [A] time = 0.26, size = 154, normalized size = 0.44

$$-\frac{c\sqrt{c-a^2cx^2}\left(-\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-4\cosh^{-1}(ax)\right)+8\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)+\sqrt{\cosh^{-1}(ax}\right.}{128a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/128*(c*Sqrt[c - a^2*c*x^2]*(-(Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -4*ArcCosh[
a*x]]) + 8*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + Sqrt[A
rcCosh[a*x]]*(32*ArcCosh[a*x]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[a*x]]
```

```
- Gamma[3/2, 4*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqr
t[ArcCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.80, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(a x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\mathrm{acosh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{acosh}{\left(a x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(1/2),x)
```

```
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acosh(a*x)), x)
```

3.380 $\int \sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}\,dx$

Optimal. Leaf size=205

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x$$

```
[Out] -1/3*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+
1/32*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)
*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*x*(-a^2*c*
x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\operatorname{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{1}{2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh
[a*x]^(3/2))/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*
c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
 - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(a\sqrt{c-a^2cx^2}\right)}{4\sqrt{-1+a}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\right.}{4a\sqrt{}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\right.}{4a\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\right.}{8a\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\right.}{16a\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\mathrm{Subst}\left(\right.}{8a\sqrt{-}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{3a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erf}}{16a\sqrt{-1+}}$$

Mathematica [A] time = 0.14, size = 117, normalized size = 0.57

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(16\cosh^{-1}(ax)^2+3\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}(ax)\right)+3\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}(ax)\right)\right)}{48a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/48*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(16*ArcCosh[a*x]^2 + 3*Sqrt[2]*Sqrt[
-ArcCosh[a*x]]*Gamma[3/2, -2*ArcCosh[a*x]] + 3*Sqrt[2]*Sqrt[ArcCosh[a*x]]*G
amma[3/2, 2*ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[Ar
cCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2}c\,x^{2}+c}\,\sqrt{\operatorname{arccosh}\left(a x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2}cx^{2}+c}\sqrt{\operatorname{arcosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}\left(a\,x\right)}\,\sqrt{c-a^2\,c\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c\left(ax-1\right)\left(ax+1\right)}\sqrt{\operatorname{acosh}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x)), x)
```

3.381 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/3*arccosh(a*x)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{3/2}}{3a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(3/2))/(3*a*Sqrt[c - a^2*c*x^2
])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\sqrt{-a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(a*x))/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.85

$$\frac{2\operatorname{arccosh}(ax)^{\frac{3}{2}}\sqrt{ax-1}\,\sqrt{ax+1}}{3a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/3*arccosh(a*x)^(3/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\sqrt{-a^2cx^2 + c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{acosh}(a\,x)}}{\sqrt{c - a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.382 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx$

Optimal. Leaf size=95

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}\,\text{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(1/2)+1/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x/(-a^2*x^2+1)/arccosh(a*x)^(1/2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[a*x]])/(c*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 +
 a*x]*Defer[Int][x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(2*c*Sqrt[c - a^
2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx &= -\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}} dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\sqrt{\cosh^{-1}(ax)}}{c\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}} dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 1.67, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{3/2}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}(ax)}}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{acosh}(a\,x)}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(sqrt(acosh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.383 $\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{a\sqrt{ax-1}\sqrt{ax+1}\,\text{Int}\left(\frac{x}{(1-a^2x^2)\sqrt{\cosh^{-1}(ax)}},x\right)}{3c^2\sqrt{c-a^2cx^2}}+\frac{a\sqrt{ax-1}\sqrt{ax+1}\,\text{Int}\left(\frac{x}{(a^2x^2-1)^2\sqrt{\cosh^{-1}(ax)}},x\right)}{6c^2\sqrt{c-a^2cx^2}}+\frac{2x\sqrt{\cosh^{-1}(ax)}}{3c^2\sqrt{c-a^2cx^2}}$$

```
[Out] 1/3*x*arccosh(a*x)^(1/2)/c/(-a^2*c*x^2+c)^(3/2)+2/3*x*arccosh(a*x)^(1/2)/c^
2/(-a^2*c*x^2+c)^(1/2)+1/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(-a
^2*x^2+1)/arccosh(a*x)^(1/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)+1/6*a*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)^(1/2),x)/c^2/(-a^
2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcCosh[a*x]])/(3*c^2*Sqrt[c - a^2*c*x^2]) + (x*Sqrt[ArcCosh[a*x]
])/(3*c^2*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt
[1 + a*x]*Defer[Int][x/((1 - a^2*x^2)*Sqrt[ArcCosh[a*x]]), x])/(3*c^2*Sqrt[
c - a^2*c*x^2]) + (a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x
^2)^2*Sqrt[ArcCosh[a*x]]), x])/(6*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{5/2}(1+ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}\\
&= \frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}}-\frac{\left(2\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{\sqrt{\cosh^{-1}(ax)}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{3c^2\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{-1\right.}}{}\\
&= \frac{2x\sqrt{\cosh^{-1}(ax)}}{3c^2\sqrt{c-a^2cx^2}}+\frac{x\sqrt{\cosh^{-1}(ax)}}{3c^2(1-ax)(1+ax)\sqrt{c-a^2cx^2}}+\frac{\left(a\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{}{(-1+a^2x^2)}}{6c^2\sqrt{c-a^2cx^2}}
\end{aligned}$$

Mathematica [A] time = 2.27, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}(ax)}}{(c-a^2cx^2)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[a*x]]/(c - a^2*c*x^2)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}(ax)}}{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

```
[Out] int(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}(ax)}}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(1/2)/(-a^2*c*x^2+c)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(a*x))/(-a^2*c*x^2 + c)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{acosh}(a\,x)}}{{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2),x)
```

```
[Out] int(acosh(a*x)^(1/2)/(c - a^2*c*x^2)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}(ax)}}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(1/2)/(-a**2*c*x**2+c)**(5/2),x)
```

```
[Out] Integral(sqrt(acosh(a*x))/(-c*(a*x - 1)*(a*x + 1))**(5/2), x)
```

3.384 $\int \left(c-a^2cx^2\right)^{3/2} \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=511

$$-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\operatorname{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\operatorname{erfi}\left(2\sqrt{c\right.}}{2048a\sqrt{ax-1}\,\sqrt{ax}}$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2)+3/8*c*x*arccosh(a*x)^(3/2)*(-
a^2*c*x^2+c)^(1/2)-3/20*c*arccosh(a*x)^(5/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)
^(1/2)/(a*x+1)^(1/2)+3/128*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/
2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*c*erfi(2^(1/2)*
arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(
a*x+1)^(1/2)-3/2048*c*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/2048*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/
2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+27/256*c*(-a^2*c*x^2+
c)^(1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-9/32*a*c*x^2*(-a^
2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/32*c*(-a^
2*x^2+1)^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)
```

Rubi [A] time = 1.10, antiderivative size = 523, normalized size of antiderivative = 1.02, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5713, 5685, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205, 5716, 5701}

$$-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\operatorname{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{2048a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\operatorname{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{3\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\operatorname{Erfi}\left(2\sqrt{\right.}}{2048a\sqrt{ax-1}\,\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (27*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (9*a*c*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(32*Sqrt[-1 +
a*x]*Sqrt[1 + a*x]) + (3*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh
[a*x]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*Ar
cCosh[a*x]^(3/2))/8 + (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[
a*x]^(3/2))/4 - (3*c*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/(20*a*Sqrt[-1
+ a*x]*Sqrt[1 + a*x]) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCos
h[a*x]]])/(2048*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c -
a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) - (3*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(2048*
a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
```

```
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+}}{4\sqrt{-1+}}$$

$$= \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(a}}{32a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(a}}{32a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{9c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

$$= \frac{27c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{256a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{9acx^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{32\sqrt{-1+ax}\sqrt{1+ax}} + \frac{3c\left(1-\right.}{}$$

Mathematica [A] time = 0.48, size = 198, normalized size = 0.39

$$\frac{c\sqrt{c-a^2cx^2}\left(60\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-3\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-384*ArcCosh[a*x]^3 - 480*ArcCosh[a*x]*Cosh[2*ArcCo
sh[a*x]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
 + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 5*Sq
rt[-ArcCosh[a*x]]*Gamma[5/2, -4*ArcCosh[a*x]] + 5*Sqrt[ArcCosh[a*x]]*Gamma[
5/2, 4*ArcCosh[a*x]] + 640*ArcCosh[a*x]^2*Sinh[2*ArcCosh[a*x]]))/(2560*a*Sq
rt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)]   time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F]   time = 0.62, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c\, x^{2}+c\right)^{\frac{3}{2}} \operatorname{arccosh}\left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \operatorname{arcosh}\left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2), x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.385 $\int \sqrt{c-a^2cx^2} \cosh^{-1}(ax)^{3/2}\,dx$

Optimal. Leaf size=302

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] 1/2*x*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)-1/5*arccosh(a*x)^(5/2)*(-a^2*
c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/128*erf(2^(1/2)*arccosh(a*x)
^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
+3/128*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/16*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/
2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/8*a*x^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x
)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}+\frac{3\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}{5a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (3*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (3*a*x^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*Sqrt[-1 + a*x]*Sq
rt[1 + a*x]) + (x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/2 - (Sqrt[c - a^2
*c*x^2]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi
/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (3*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int \sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx^2}\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(3a\sqrt{c-a^2\right.}{4\sqrt{}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx}}{5a\sqrt{-1}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx}}{5a\sqrt{-1}}$$

$$= -\frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2} - \frac{\sqrt{c-a^2cx}}{5a\sqrt{-1}}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx}$$

$$= \frac{3\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{3ax^2\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx}$$

Mathematica [A] time = 0.35, size = 136, normalized size = 0.45

$$\frac{\sqrt{c-a^2cx^2}\left(15\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+15\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)-8\sqrt{\cosh^{-1}(ax)}\left(16\cosh^{-1}(ax\right.\right.}{640a\sqrt{\frac{ax-1}{ax+1}}(ax+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 15*Sq
rt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 8*Sqrt[ArcCosh[a*x]]*(16*ArcCos
h[a*x]^2 + 15*Cosh[2*ArcCosh[a*x]] - 20*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]]))
)/(640*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F**] time = 0.70, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c\,x^{2}+c}\, \operatorname{arccosh}\left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c x^{2}+c}\, \operatorname{arcosh}\left(a x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}\left(a\,x\right)^{3/2}\,\sqrt{c-a^2\,c\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{- c \left(a x - 1\right) \left(a x + 1\right)} \operatorname{acosh}^{\frac{3}{2}}{\left(a x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(3/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(3/2), x)
```

3.386 $\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/5*arccosh(a*x)^(5/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{5/2}}{5a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(5/2))/(5*a*Sqrt[c - a^2*c*x^2
])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.85

$$\frac{2\operatorname{arccosh}(ax)^{\frac{5}{2}}\sqrt{ax-1}\,\sqrt{ax+1}}{5a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/5*arccosh(a*x)^(5/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(3/2)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a\,x)^{3/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{\sqrt{-c(ax-1)(ax+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Integral(acosh(a*x)**(3/2)/sqrt(-c*(a*x - 1)*(a*x + 1)), x)
```

3.387 $\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{3a\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Int}\left(\frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(3/2)/c/(-a^2*c*x^2+c)^(1/2)+3/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x*arccosh(a*x)^(1/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(3/2))/(c*Sqrt[c - a^2*c*x^2]) + (3*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*Sqrt[ArcCosh[a*x]])/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{3/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$
$$=\frac{x\cosh^{-1}(ax)^{3/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(3a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\sqrt{\cosh^{-1}(ax)}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.75, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{3/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(3/2)/(c - a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^{\frac{3}{2}}}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{3}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^(3/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^{3/2}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(acosh(a*x)^(3/2)/(c - a^2*c*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}(ax)}{\left(-c\,(ax-1)\,(ax+1)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(3/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Integral(acosh(a*x)**(3/2)/(-c*(a*x - 1)*(a*x + 1))**(3/2), x)
```

3.388 $\int \left(c-a^2cx^2\right)^{3/2} \cosh^{-1}(ax)^{5/2}\,dx$

Optimal. Leaf size=580

$$-\frac{15\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(2\right.}{16384a\sqrt{ax-1}\,\sqrt{}}$$

```
[Out] 1/4*x*(-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2)+3/8*c*x*arccosh(a*x)^(5/2)*(-
a^2*c*x^2+c)^(1/2)+45/256*c*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)-15/32*a*c*x^2*arccosh(a*x)^(3/2)*(-a^2*c*x^2+c)^(1/2
)/(a*x-1)^(1/2)/(a*x+1)^(1/2)+5/32*c*(-a^2*x^2+1)^2*arccosh(a*x)^(3/2)*(-a^
2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/28*c*arccosh(a*x)^(7/2)*(-
a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/512*c*erf(2^(1/2)*arcco
sh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1
)^(1/2)-15/512*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*
x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-15/16384*c*erf(2*arccosh(a*x)^(1
/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/16384*c
*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(
a*x+1)^(1/2)+225/512*c*x*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)+15/256*c*x
*(-a*x+1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 592, normalized size of antiderivative = 1.02, number of steps used = 40, number of rules used = 15, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {5713, 5685, 5683, 5676, 5664, 5759, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5716, 5780}

$$-\frac{15\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{16384a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\right.}{16384a\sqrt{ax-1}\,}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (225*c*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/512 + (15*c*x*(1 - a*x)*(1
 + a*x)*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/256 + (45*c*Sqrt[c - a^2*c*
x^2]*ArcCosh[a*x]^(3/2))/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*a*c*x^2
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
+ (5*c*(1 - a^2*x^2)^2*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(32*a*Sqrt[-
1 + a*x]*Sqrt[1 + a*x]) + (3*c*x*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/8
+ (c*x*(1 - a*x)*(1 + a*x)*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/4 - (3*c
*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/2))/(28*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]
) - (15*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(16384*a*
Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sq
rt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c*Sqr
t[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(16384*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) - (15*c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[
ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{1}{4}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2} + \frac{\left(3c\sqrt{c-a^2cx^2}\right)\int\sqrt{-1+}}{4\sqrt{-1+}}$$

$$= \frac{5c\left(1-a^2x^2\right)^2\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{3}{8}cx\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{5/2}$$

$$= \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} - \frac{15acx^2\sqrt{c-a^2cx^2}\,\cosh}{32\sqrt{-1+ax}\,\sqrt{1+}}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

$$= \frac{225}{512}cx\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)} + \frac{15}{256}cx(1-ax)(1+ax)\sqrt{c-a^2cx^2}\,\sqrt{\cos}$$

Mathematica [A] time = 0.51, size = 213, normalized size = 0.37

$$\frac{c\sqrt{c-a^2cx^2}\left(420\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right) - 420\sqrt{2\pi}\,\sqrt{\cosh^{-1}(ax)}\,\mathrm{erfi}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (c*Sqrt[c - a^2*c*x^2]*(-1536*ArcCosh[a*x]^4 - 4480*ArcCosh[a*x]^2*Cosh[2*A
rcCosh[a*x]] + 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a
*x]]] - 420*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]]
+ 7*Sqrt[-ArcCosh[a*x]]*Gamma[7/2, -4*ArcCosh[a*x]] + 7*Sqrt[ArcCosh[a*x]]*
Gamma[7/2, 4*ArcCosh[a*x]] + 3360*ArcCosh[a*x]*Sinh[2*ArcCosh[a*x]] + 3584*
ArcCosh[a*x]^3*Sinh[2*ArcCosh[a*x]]))/(14336*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(
1 + a*x)*Sqrt[ArcCosh[a*x]])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**F**] time = 0.63, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \operatorname{arccosh}\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \operatorname{arcosh}\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2), x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \mathrm{acosh}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.389 $\int \sqrt{c-a^2cx^2} \cosh^{-1}(ax)^{5/2} \, dx$

Optimal. Leaf size=330

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)}{7a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] 1/2*x*arccosh(a*x)^(5/2)*(-a^2*c*x^2+c)^(1/2)+5/16*arccosh(a*x)^(3/2)*(-a^2
*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/8*a*x^2*arccosh(a*x)^(3/2)*
(-a^2*c*x^2+c)^(1/2)/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/7*arccosh(a*x)^(7/2)*(-a
^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/512*erf(2^(1/2)*arccosh(
a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(
1/2)-15/512*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/32*x*(-a^2*c*x^2+c)^(1/2)*arccosh(
a*x)^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5713, 5683, 5676, 5664, 5759, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{256a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax}{7a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```

```
[Out] (15*x*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/32 + (5*Sqrt[c - a^2*c*x^2]*A
rcCosh[a*x]^(3/2))/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (5*a*x^2*Sqrt[c -
a^2*c*x^2]*ArcCosh[a*x]^(3/2))/(8*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (x*Sqrt[c
 - a^2*c*x^2]*ArcCosh[a*x]^(5/2))/2 - (Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(7/
2))/(7*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]
*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (1
5*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(256*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
```

```
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}\,dx = \frac{\sqrt{c-a^2cx^2}\int\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} - \frac{\sqrt{c-a^2cx^2}\int\frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\sqrt{1+ax}}\,dx}{2\sqrt{-1+ax}\sqrt{1+ax}} - \frac{\left(5a\sqrt{c-a^2\right.}{4\sqrt{}}$$

$$= -\frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2} - \frac{\sqrt{c-a^2c}}{7a\sqrt{-1}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} - \frac{5ax^2\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{8\sqrt{-1+ax}\sqrt{1+ax}} + \frac{1}{2}x\sqrt{c-a}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

$$= \frac{15}{32}x\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)} + \frac{5\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}{16a\sqrt{-1+ax}\sqrt{1+ax}} - \frac{5ax^2\sqrt{c-a^2c}}{8\sqrt{-1+}}$$

Mathematica [A] time = 0.52, size = 148, normalized size = 0.45

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(-105\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+105\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+8\sqrt{\cosh^{-1}(ax)}\right.}{3584a\sqrt{\frac{ax-1}{ax+1}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2),x]
```

```
[Out] -1/3584*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(-105*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[
ArcCosh[a*x]]] + 105*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 8*Sqrt[A
```

```
rcCosh[a*x]]*(64*ArcCosh[a*x]^3 + 140*ArcCosh[a*x]*Cosh[2*ArcCosh[a*x]] - 7
*(15 + 16*ArcCosh[a*x]^2)*Sinh[2*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a
*x)]*(1 + a*x))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c\,x^{2}+c}\, \operatorname{arccosh}\left(a x\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a^{2} c x^{2} + c} \operatorname{arcosh}\left(a x\right)^{\frac{5}{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\mathrm{acosh}\left(a\,x\right)}^{5/2}\,\sqrt{c-a^2\,c\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)*acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.390 $\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx$

Optimal. Leaf size=48

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

```
[Out] 2/7*arccosh(a*x)^(7/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{c-a^2cx^2}}\,dx &= \frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{\sqrt{-1+ax}\,\sqrt{1+ax}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\,\sqrt{1+ax}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$\frac{2\sqrt{ax-1}\,\sqrt{ax+1}\,\cosh^{-1}(ax)^{7/2}}{7a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/Sqrt[c - a^2*c*x^2],x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*ArcCosh[a*x]^(7/2))/(7*a*Sqrt[c - a^2*c*x^2
])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/sqrt(-a^2*c*x^2 + c), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.85

$$\frac{2\operatorname{arccosh}(ax)^{\frac{7}{2}}\sqrt{ax-1}\,\sqrt{ax+1}}{7a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x)
```

```
[Out] 2/7*arccosh(a*x)^(7/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\sqrt{-a^2cx^2+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(a*x)^(5/2)/sqrt(-a^2*c*x^2 + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}(a\,x)^{5/2}}{\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2),x)
```

```
[Out] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(1/2),x)
```

```
[Out] Timed out
```

3.391 $\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=95

$$\frac{5a\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Int}\left(\frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2},x\right)}{2c\sqrt{c-a^2cx^2}}+\frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}$$

```
[Out] x*arccosh(a*x)^(5/2)/c/(-a^2*c*x^2+c)^(1/2)+5/2*a*(a*x-1)^(1/2)*(a*x+1)^(1/
2)*Unintegrable(x*arccosh(a*x)^(3/2)/(-a^2*x^2+1),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[a*x]^(5/2))/(c*Sqrt[c - a^2*c*x^2]) + (5*a*Sqrt[-1 + a*x]*Sqrt[1
 + a*x]*Defer[Int][(x*ArcCosh[a*x]^(3/2))/(1 - a^2*x^2), x])/(2*c*Sqrt[c -
a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{\cosh^{-1}(ax)^{5/2}}{(-1+ax)^{3/2}(1+ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= \frac{x\cosh^{-1}(ax)^{5/2}}{c\sqrt{c-a^2cx^2}}+\frac{\left(5a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{x\cosh^{-1}(ax)^{3/2}}{1-a^2x^2}\,dx}{2c\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 1.64, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}(ax)^{5/2}}{\left(c-a^2cx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[a*x]^(5/2)/(c - a^2*c*x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

maple [A] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}(ax)^{\frac{5}{2}}}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
[Out] int(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}(ax)^{\frac{5}{2}}}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(arccosh(a*x)^(5/2)/(-a^2*c*x^2 + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}(a\,x)^{5/2}}{\left(c-a^2\,c\,x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2),x)
[Out] int(acosh(a*x)^(5/2)/(c - a^2*c*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(a*x)**(5/2)/(-a**2*c*x**2+c)**(3/2),x)
[Out] Timed out
```

3.392 $\int \left(a^2-x^2\right)^{3/2} \sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} \, dx$

Optimal. Leaf size=368

$$\frac{3}{8}a^2x\sqrt{a^2-x^2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}-\frac{\sqrt{\pi}\,a^3\sqrt{a^2-x^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\frac{\pi}{2}}\,a^3\sqrt{a^2-}}{16}$$

```
[Out] -1/4*a^3*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/
32*a^3*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1
+x/a)^(1/2)/(1+x/a)^(1/2)-1/32*a^3*erfi(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)
*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)-1/256*a^3*erf(2*arcc
osh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/256
*a^3*erfi(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+
x/a)^(1/2)+1/4*x*(a^2-x^2)^(3/2)*arccosh(x/a)^(1/2)+3/8*a^2*x*(a^2-x^2)^(1/
2)*arccosh(x/a)^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 376, normalized size of antiderivative = 1.02, number of steps used = 25, number of rules used = 12, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5713, 5685, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205, 5780}

$$-\frac{\sqrt{\pi}\,a^3\sqrt{a^2-x^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{256\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\frac{\pi}{2}}\,a^3\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}+\frac{\sqrt{\pi}\,a^3\sqrt{a^2-x^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}}\right.}{256\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] (3*a^2*x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/8 + ((a - x)*x*(a + x)*Sqrt[a^
2 - x^2]*Sqrt[ArcCosh[x/a]])/4 - (a^3*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/(
4*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erf[2*Sqrt[
ArcCosh[x/a]]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a^3*Sqrt[Pi/2]*Sqrt[a
^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]
) + (a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[ArcCosh[x/a]]])/(256*Sqrt[-1
+ x/a]*Sqrt[1 + x/a]) - (a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[A
rcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
```

```
!IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2} \sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\, dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int \left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\, dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{\left(a\sqrt{a^2-x^2}\right)\int \frac{x\left(-1+\frac{x^2}{a^2}\right)}{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\, dx}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{(3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{(3a}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

$$= \frac{3}{8}a^2x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} + \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a^3}{}$$

Mathematica [A] time = 0.29, size = 165, normalized size = 0.45

$$-\frac{a^4\sqrt{a^2-x^2}\left(-\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-4\cosh^{-1}\left(\frac{x}{a}\right)\right)+8\sqrt{2}\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}\left(\frac{x}{a}\right)\right)+\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,\left(\right.}{128\sqrt{\frac{x-a}{a+x}}\,(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -1/128*(a^4*Sqrt[a^2 - x^2]*(-(Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -4*ArcCosh[x/
a]]) + 8*Sqrt[2]*Sqrt[-ArcCosh[x/a]]*Gamma[3/2, -2*ArcCosh[x/a]] + Sqrt[Arc
Cosh[x/a]]*(32*ArcCosh[x/a]^(3/2) + 8*Sqrt[2]*Gamma[3/2, 2*ArcCosh[x/a]] -
Gamma[3/2, 4*ArcCosh[x/a]])))/(Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[
x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*sqrt(arccosh(x/a)), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

```
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a^2 - x^2)^(3/2)*sqrt(arccosh(x/a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\, \left(a^2 - x^2\right)^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(3/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(1/2),x)
```

```
[Out] Integral((-(-a + x)*(a + x))**(3/2)*sqrt(acosh(x/a)), x)
```

3.393 $\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x$$

```
[Out] -1/3*a*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/32
*a*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)-1/32*a*erfi(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1
/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+1/2*x*(a^2-x^2)^(1/2)*arcc
osh(x/a)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5713, 5683, 5676, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{\sqrt{\frac{\pi}{2}}\,a\sqrt{a^2-x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{16\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] (x*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/2 - (a*Sqrt[a^2 - x^2]*ArcCosh[x/a]^
(3/2))/(3*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf
[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (a*Sqrt[P
i/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(16*Sqrt[-1 + x/a]*S
qrt[1 + x/a])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx = \frac{\sqrt{a^2-x^2}\int \sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{\sqrt{a^2-x^2}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx}{2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\sqrt{a^2-x^2}\int \frac{x}{\sqrt{\cosh}}}{4a\sqrt{-1+\frac{x}{a}}\sqrt{1}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\right.}{4\sqrt{}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\right.}{4\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\right.}{8\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\right.}{16\sqrt{-1}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{\left(a\sqrt{a^2-x^2}\right)\text{Subst}\left(\right.}{8\sqrt{-}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{a\sqrt{\frac{\pi}{2}}\sqrt{a^2-x^2}\,\text{erf}\left(\right.}{16\sqrt{-1+\frac{x}{a}}}$$

Mathematica [A] time = 0.15, size = 121, normalized size = 0.57

$$-\frac{a^2\sqrt{a^2-x^2}\left(16\cosh^{-1}\left(\frac{x}{a}\right)^2+3\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},2\cosh^{-1}\left(\frac{x}{a}\right)\right)+3\sqrt{2}\sqrt{-\cosh^{-1}\left(\frac{x}{a}\right)}\,\Gamma\left(\frac{3}{2},-2\cosh^{-1}\left(\right.\right.\right.}{48\sqrt{\frac{x-a}{a+x}}\,(a+x)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]],x]
```

```
[Out] -1/48*(a^2*Sqrt[a^2 - x^2]*(16*ArcCosh[x/a]^2 + 3*Sqrt[2]*Sqrt[-ArcCosh[x/a
]]*Gamma[3/2, -2*ArcCosh[x/a]] + 3*Sqrt[2]*Sqrt[ArcCosh[x/a]]*Gamma[3/2, 2*
ArcCosh[x/a]]))/(Sqrt[(-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 - x^2} \sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(a^2 - x^2)*sqrt(arccosh(x/a)), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 - x^2} \, \sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
[Out] int((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 - x^2} \sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(1/2)*arccosh(x/a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(a^2 - x^2)*sqrt(arccosh(x/a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)} \, \sqrt{a^2 - x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(1/2),x)
[Out] int(acosh(x/a)^(1/2)*(a^2 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{- \left(-a + x\right) \left(a + x\right)} \sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(1/2)*acosh(x/a)**(1/2),x)
[Out] Integral(sqrt(-(-a + x)*(a + x))*sqrt(acosh(x/a)), x)
```

3.394 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

```
[Out] 2/3*a*arccosh(x/a)^(3/2)*(-1+x/a)^(1/2)*(1+x/a)^(1/2)/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2a\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{a^2-x^2}}\,dx &= \frac{\left(\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}\\ &= \frac{2a\sqrt{-1+\frac{x}{a}}\,\sqrt{1+\frac{x}{a}}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 1.00

$$\frac{2a\sqrt{\frac{x}{a}-1}\,\sqrt{\frac{x}{a}+1}\,\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{3\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(3/2))/(3*Sqrt[a^2 - x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(x/a))/sqrt(a^2 - x^2), x)
```

maple [A] time = 0.09, size = 44, normalized size = 0.88

$$\frac{2\operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} a\sqrt{\frac{-a+x}{a}}\,\sqrt{\frac{a+x}{a}}}{3\sqrt{-\left(-a+x\right)\left(a+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/3*arccosh(x/a)^(3/2)*a/(-(-a+x)*(a+x))^(1/2)*((-a+x)/a)^(1/2)*((a+x)/a)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/sqrt(a^2 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(1/2),x)
[Out] Integral(sqrt(acosh(x/a))/sqrt(-(-a + x)*(a + x)), x)
```

3.395 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\,\mathrm{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{2a^3\sqrt{a^2-x^2}}+\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}$$

```
[Out] x*arccosh(x/a)^(1/2)/a^2/(a^2-x^2)^(1/2)+1/2*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*U
nintegrable(x/(1-x^2/a^2)/arccosh(x/a)^(1/2),x)/a^3/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*Sqrt[ArcCosh[x/a]])/(a^2*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a
]*Defer[Int][x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(2*a^3*Sqrt[a^2 - x^
2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}\\ &= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{a^2\sqrt{a^2-x^2}}+\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{2a^3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.96, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(3/2), x)
```

maple [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(3/2),x)
```

```
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/(-(-a + x)*(a + x))**(3/2), x)
```

3.396 $\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$

Optimal. Leaf size=200

$$\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\,\text{Int}\left(\frac{x}{\left(1-\frac{x^2}{a^2}\right)\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{3a^5\sqrt{a^2-x^2}}+\frac{\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\,\text{Int}\left(\frac{x}{\left(\frac{x^2}{a^2}-1\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}},x\right)}{6a^5\sqrt{a^2-x^2}}+\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2\left(a^2-x^2\right)^{3/2}}+\frac{2x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}}$$

```
[Out] 1/3*x*arccosh(x/a)^(1/2)/a^2/(a^2-x^2)^(3/2)+2/3*x*arccosh(x/a)^(1/2)/a^4/(
a^2-x^2)^(1/2)+1/3*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*Unintegrable(x/(1-x^2/a^2)/
arccosh(x/a)^(1/2),x)/a^5/(a^2-x^2)^(1/2)+1/6*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*
Unintegrable(x/(-1+x^2/a^2)^2/arccosh(x/a)^(1/2),x)/a^5/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
```

```
[Out] (2*x*Sqrt[ArcCosh[x/a]])/(3*a^4*Sqrt[a^2 - x^2]) + (x*Sqrt[ArcCosh[x/a]])/(
3*a^2*(a - x)*(a + x)*Sqrt[a^2 - x^2]) + (Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Defe
r[Int][x/((1 - x^2/a^2)*Sqrt[ArcCosh[x/a]]), x])/(3*a^5*Sqrt[a^2 - x^2]) +
(Sqrt[-1 + x/a]*Sqrt[1 + x/a]*Defer[Int][x/((-1 + x^2/a^2)^2*Sqrt[ArcCosh[x
/a]]), x])/(6*a^5*Sqrt[a^2 - x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(-1+\frac{x}{a}\right)^{5/2}\left(1+\frac{x}{a}\right)^{5/2}}\,dx}{a^4\sqrt{a^2-x^2}}\\ &= \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}}+\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(-1+\frac{x^2}{a^2}\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}\,dx}{6a^5\sqrt{a^2-x^2}}-\frac{\left(2\sqrt{-1+\frac{x}{a}}\sqrt{\right.}}{}\\ &= \frac{2x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^4\sqrt{a^2-x^2}}+\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{3a^2(a-x)(a+x)\sqrt{a^2-x^2}}+\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x}{\left(-1+\frac{x^2}{a^2}\right)^2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}}{6a^5\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 2.32, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{\left(a^2-x^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2),x]
[Out] Integrate[Sqrt[ArcCosh[x/a]]/(a^2 - x^2)^(5/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2 - x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(5/2), x)
```

maple [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arccosh}\left(\frac{x}{a}\right)}}{\left(a^2 - x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
[Out] int(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{arcosh}\left(\frac{x}{a}\right)}}{\left(a^2 - x^2\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(1/2)/(a^2-x^2)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(arccosh(x/a))/(a^2 - x^2)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(a^2 - x^2\right)^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(5/2),x)
[Out] int(acosh(x/a)^(1/2)/(a^2 - x^2)^(5/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\operatorname{acosh}\left(\frac{x}{a}\right)}}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(1/2)/(a**2-x**2)**(5/2),x)
```

```
[Out] Integral(sqrt(acosh(x/a))/(-(-a + x)*(a + x))**(5/2), x)
```

3.397 $\int \left(a^2-x^2\right)^{3/2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=525

$$\frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}-\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{4}x\left(a^2-x^2\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}+\frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cos}}{32a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{}}}$$

```
[Out] 1/4*x*(a^2-x^2)^(3/2)*arccosh(x/a)^(3/2)+3/8*a^2*x*arccosh(x/a)^(3/2)*(a^2-
x^2)^(1/2)-3/20*a^3*arccosh(x/a)^(5/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/
a)^(1/2)+3/128*a^3*erf(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^
2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a^3*erfi(2^(1/2)*arccosh(x/a)^(
1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)-3/2048*
a^3*erf(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/
a)^(1/2)-3/2048*a^3*erfi(2*arccosh(x/a)^(1/2))*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1
+x/a)^(1/2)/(1+x/a)^(1/2)+3/32*(a^2-x^2)^(5/2)*arccosh(x/a)^(1/2)/a/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)+27/256*a^3*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)-9/32*a*x^2*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a
)^(1/2)/(1+x/a)^(1/2)
```

Rubi [A] time = 1.28, antiderivative size = 533, normalized size of antiderivative = 1.02, number of steps used = 27, number of rules used = 13, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.542, Rules used = {5713, 5685, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205, 5716, 5701}

$$-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}a^3\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{3\sqrt{\pi}a^3\sqrt{a^2-x^2}\,\mathrm{Erfi}\left(2\sqrt{c}\right.}{2048\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (27*a^3*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(256*Sqrt[-1 + x/a]*Sqrt[1 + x/
a]) - (9*a*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(32*Sqrt[-1 + x/a]*Sqrt[
1 + x/a]) + (3*(a^2 - x^2)^(5/2)*Sqrt[ArcCosh[x/a]])/(32*a*Sqrt[-1 + x/a]*S
qrt[1 + x/a]) + (3*a^2*x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/8 + ((a - x)*x
*(a + x)*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/4 - (3*a^3*Sqrt[a^2 - x^2]*Arc
Cosh[x/a]^(5/2))/(20*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a
^2 - x^2]*Erf[2*Sqrt[ArcCosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a^3*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt
[-1 + x/a]*Sqrt[1 + x/a]) - (3*a^3*Sqrt[Pi]*Sqrt[a^2 - x^2]*Erfi[2*Sqrt[Arc
Cosh[x/a]]])/(2048*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a^3*Sqrt[Pi/2]*Sqrt[a
^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a
])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5685

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(x*(d1 + e1*x)^p*(d2 + e2*x)^
p*(a + b*ArcCosh[c*x])^n)/(2*p + 1), x] + (Dist[(2*d1*d2*p)/(2*p + 1), Int[
(d1 + e1*x)^(p - 1)*(d2 + e2*x)^(p - 1)*(a + b*ArcCosh[c*x])^n, x], x] - Di
st[(b*c*n*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/((2*p + 1)*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a + b*ArcCos
h[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1,
c*d1] && EqQ[e2, -(c*d2)] && GtQ[n, 0] && GtQ[p, 0] && IntegerQ[p - 1/2]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
```

```
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5716

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e*(p
 + 1)), x] - Dist[(b*n*(-d)^p)/(2*c*(p + 1)), Int[(1 + c*x)^(p + 1/2)*(-1 +
 c*x)^(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d,
e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1] && IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \left(a^2-x^2\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}dx = -\frac{\left(a^2\sqrt{a^2-x^2}\right)\int\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{4}(a-x)x(a+x)\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{\left(3a\sqrt{a^2-x^2}\right)\int x\left(-1+\frac{x^2}{a^2}\right)\sqrt{\cosh}}{8\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} + \frac{1}{4}(a-x)x(a+x$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}$$

$$= -\frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3}{8}a^2x\sqrt{a^2-x^2}$$

$$= -\frac{9a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{}}{32a\sqrt{-1+\frac{x}{}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{}}{32a\sqrt{-1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{}}{32a\sqrt{-1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{}}{32a\sqrt{-1+\frac{x}{a}}}$$

$$= \frac{27a^3\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{256\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{9ax^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{32\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{3\left(a^2-x^2\right)^{5/2}\sqrt{}}{32a\sqrt{-1+\frac{x}{a}}}$$

Mathematica [A] time = 0.48, size = 219, normalized size = 0.42

$$\frac{a^4\sqrt{a^2-x^2}\left(60\sqrt{2\pi}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\operatorname{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)+60\sqrt{2\pi}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\operatorname{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)-384\,\mathrm{co}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a^2 - x^2)^(3/2)*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^4*Sqrt[a^2 - x^2]*(-384*ArcCosh[x/a]^3 - 480*ArcCosh[x/a]*Cosh[2*ArcCosh
[x/a]] + 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] +
 60*Sqrt[2*Pi]*Sqrt[ArcCosh[x/a]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 5*Sqrt
[-ArcCosh[x/a]]*Gamma[5/2, -4*ArcCosh[x/a]] + 5*Sqrt[ArcCosh[x/a]]*Gamma[5/
2, 4*ArcCosh[x/a]] + 640*ArcCosh[x/a]^2*Sinh[2*ArcCosh[x/a]]))/(2560*Sqrt[(
-a + x)/(a + x)]*(a + x)*Sqrt[ArcCosh[x/a]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="giac")
[Out] integrate((a^2 - x^2)^(3/2)*arccosh(x/a)^(3/2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
[Out] int((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a^2 - x^2\right)^{\frac{3}{2}} \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2-x^2)^(3/2)*arccosh(x/a)^(3/2),x, algorithm="maxima")
[Out] integrate((a^2 - x^2)^(3/2)*arccosh(x/a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}\left(\frac{x}{a}\right)^{3/2} \left(a^2 - x^2\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(3/2),x)
[Out] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2-x**2)**(3/2)*acosh(x/a)**(3/2),x)
[Out] Timed out
```

3.398 $\int \sqrt{a^2-x^2} \cosh^{-1}\left(\frac{x}{a}\right)^{3/2} dx$

Optimal. Leaf size=316

$$\frac{3\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2-x^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2-x^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x\sqrt{}$$

```
[Out] 1/2*x*arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2)-1/5*a*arccosh(x/a)^(5/2)*(a^2-x^2)
^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a*erf(2^(1/2)*arccosh(x/a)^(1/2))
*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2)/(1+x/a)^(1/2)+3/128*a*erfi
(2^(1/2)*arccosh(x/a)^(1/2))*2^(1/2)*Pi^(1/2)*(a^2-x^2)^(1/2)/(-1+x/a)^(1/2
)/(1+x/a)^(1/2)+3/16*a*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/(-1+x/a)^(1/2)/(1
+x/a)^(1/2)-3/8*x^2*(a^2-x^2)^(1/2)*arccosh(x/a)^(1/2)/a/(-1+x/a)^(1/2)/(1+
x/a)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5713, 5683, 5676, 5664, 5781, 3312, 3307, 2180, 2204, 2205}

$$\frac{3\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2-x^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{3\sqrt{\frac{\pi}{2}}\, a\sqrt{a^2-x^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)}{64\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}-\frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}}+\frac{1}{2}x$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (3*a*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(16*Sqrt[-1 + x/a]*Sqrt[1 + x/a])
- (3*x^2*Sqrt[a^2 - x^2]*Sqrt[ArcCosh[x/a]])/(8*a*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]) + (x*Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2))/2 - (a*Sqrt[a^2 - x^2]*ArcCos
h[x/a]^(5/2))/(5*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) + (3*a*Sqrt[Pi/2]*Sqrt[a^2 -
 x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[-1 + x/a]*Sqrt[1 + x/a]) +
(3*a*Sqrt[Pi/2]*Sqrt[a^2 - x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]])/(64*Sqrt[
-1 + x/a]*Sqrt[1 + x/a])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5683

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*Sqrt[(d1_) + (e1_.)*(x_)]*Sqr
t[(d2_) + (e2_.)*(x_)], x_Symbol] :> Simp[(x*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x
]*(a + b*ArcCosh[c*x])^n)/2, x] + (-Dist[(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(2*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(a + b*ArcCosh[c*x])^n/(Sqrt[1 + c*x]
*Sqrt[-1 + c*x]), x], x] - Dist[(b*c*n*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/(2*
Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(a + b*ArcCosh[c*x])^(n - 1), x], x])
/; FreeQ[{a, b, c, d1, e1, d2, e2}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)]
 && GtQ[n, 0]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}dx = \frac{\sqrt{a^2-x^2}\int \sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}dx}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}$$

$$= \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{\sqrt{a^2-x^2}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}dx}{2\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{\left(3\sqrt{a^2-x^2}\right)\int x\sqrt{c}}{4a\sqrt{-1+\frac{x}{a}}\sqrt{}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\right.}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\right.}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= -\frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\frac{x}{a}\right)^{3/2} - \frac{a\sqrt{a^2-x^2}\cosh^{-1}\left(\right.}{5\sqrt{-1+\frac{x}{a}}\sqrt{1+}}$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\right.$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\right.$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\right.$$

$$= \frac{3a\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{16\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} - \frac{3x^2\sqrt{a^2-x^2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{8a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}} + \frac{1}{2}x\sqrt{a^2-x^2}\cosh^{-1}\left(\right.$$

Mathematica [A] time = 0.39, size = 144, normalized size = 0.46

$$\frac{a^2\sqrt{a^2-x^2}\left(15\sqrt{2\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)+15\sqrt{2\pi}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\right)-8\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}\left(16\cosh^{-1}\left(\frac{x}{a}\right)^2+\right.\right.}{640\sqrt{\frac{x-a}{a+x}}(a+x)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a^2 - x^2]*ArcCosh[x/a]^(3/2),x]
```

```
[Out] (a^2*Sqrt[a^2 - x^2]*(15*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[ArcCosh[x/a]]] + 15*Sq
rt[2*Pi]*Erfi[Sqrt[2]*Sqrt[ArcCosh[x/a]]] - 8*Sqrt[ArcCosh[x/a]]*(16*ArcCos
h[x/a]^2 + 15*Cosh[2*ArcCosh[x/a]] - 20*ArcCosh[x/a]*Sinh[2*ArcCosh[x/a]]))
)/(640*Sqrt[(-a + x)/(a + x)]*(a + x))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 - x^2} \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a^2 - x^2)*arccosh(x/a)^(3/2), x)
```

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} \sqrt{a^2 - x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a^2 - x^2} \operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)*(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a^2 - x^2)*arccosh(x/a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \operatorname{acosh}\left(\frac{x}{a}\right)^{3/2} \sqrt{a^2 - x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(3/2)*(a^2 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\left(-a + x\right)\left(a + x\right)} \operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)*(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(-a + x)*(a + x))*acosh(x/a)**(3/2), x)
```

3.399 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx$

Optimal. Leaf size=50

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

```
[Out] 2/5*a*arccosh(x/a)^(5/2)*(-1+x/a)^(1/2)*(1+x/a)^(1/2)/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {5713, 5676}

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2-x^2}}\,dx &= \frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}}\,dx}{\sqrt{a^2-x^2}}\\ &= \frac{2a\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 50, normalized size = 1.00

$$\frac{2a\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\cosh^{-1}\left(\frac{x}{a}\right)^{5/2}}{5\sqrt{a^2-x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/Sqrt[a^2 - x^2],x]
```

```
[Out] (2*a*Sqrt[-1 + x/a]*Sqrt[1 + x/a]*ArcCosh[x/a]^(5/2))/(5*Sqrt[a^2 - x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

maple [A] time = 0.08, size = 44, normalized size = 0.88

$$\frac{2 \operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{5}{2}} a \sqrt{\frac{-a+x}{a}}\, \sqrt{\frac{a+x}{a}}}{5 \sqrt{-\left(-a+x\right)\left(a+x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x)
```

```
[Out] 2/5*arccosh(x/a)^(5/2)*a/(-(-a+x)*(a+x))^(1/2)*((-a+x)/a)^(1/2)*((a+x)/a)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/sqrt(a^2 - x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\operatorname{acosh}\left(\frac{x}{a}\right)^{3/2}}{\sqrt{a^2 - x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(1/2),x)
```

```
[Out] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\sqrt{-\left(-a+x\right)\left(a+x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(1/2),x)
```

```
[Out] Integral(acosh(x/a)**(3/2)/sqrt(-(-a + x)*(a + x)), x)
```

3.400 $\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$

Optimal. Leaf size=98

$$\frac{3\sqrt{\frac{x}{a}-1}\sqrt{\frac{x}{a}+1}\,\mathrm{Int}\left(\frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}},x\right)}{2a^3\sqrt{a^2-x^2}}+\frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}$$

```
[Out] x*arccosh(x/a)^(3/2)/a^2/(a^2-x^2)^(1/2)+3/2*(-1+x/a)^(1/2)*(1+x/a)^(1/2)*U
nintegrable(x*arccosh(x/a)^(1/2)/(1-x^2/a^2),x)/a^3/(a^2-x^2)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] (x*ArcCosh[x/a]^(3/2))/(a^2*Sqrt[a^2 - x^2]) + (3*Sqrt[-1 + x/a]*Sqrt[1 + x
/a]*Defer[Int][(x*Sqrt[ArcCosh[x/a]])/(1 - x^2/a^2), x])/(2*a^3*Sqrt[a^2 -
x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(-1+\frac{x}{a}\right)^{3/2}\left(1+\frac{x}{a}\right)^{3/2}}\,dx}{a^2\sqrt{a^2-x^2}}\\ &= \frac{x\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{a^2\sqrt{a^2-x^2}}+\frac{\left(3\sqrt{-1+\frac{x}{a}}\sqrt{1+\frac{x}{a}}\right)\int \frac{x\sqrt{\cosh^{-1}\left(\frac{x}{a}\right)}}{1-\frac{x^2}{a^2}}\,dx}{2a^3\sqrt{a^2-x^2}}\end{aligned}$$

Mathematica [A] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{-1}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2),x]
```

```
[Out] Integrate[ArcCosh[x/a]^(3/2)/(a^2 - x^2)^(3/2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arccosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

```
[Out] int(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{arcosh}\left(\frac{x}{a}\right)^{\frac{3}{2}}}{\left(a^2-x^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(arccosh(x/a)^(3/2)/(a^2-x^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(arccosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\operatorname{acosh}\left(\frac{x}{a}\right)^{3/2}}{\left(a^2-x^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(3/2),x)
```

```
[Out] int(acosh(x/a)^(3/2)/(a^2 - x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{acosh}^{\frac{3}{2}}\left(\frac{x}{a}\right)}{\left(-\left(-a+x\right)\left(a+x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(acosh(x/a)**(3/2)/(a**2-x**2)**(3/2),x)
```

```
[Out] Integral(acosh(x/a)**(3/2)/(-(-a + x)*(a + x))**(3/2), x)
```

3.401 $\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx$

Optimal. Leaf size=65

$$\frac{\sqrt{\pi}\sqrt{x-1}\,\text{erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}+\frac{\sqrt{\pi}\sqrt{x-1}\,\text{erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x}}$$

```
[Out] 1/2*erf(arccosh(x)^(1/2))*Pi^(1/2)*(-1+x)^(1/2)/(1-x)^(1/2)+1/2*erfi(arccos
h(x)^(1/2))*Pi^(1/2)*(-1+x)^(1/2)/(1-x)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 83, normalized size of antiderivative = 1.28, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5798, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\sqrt{x-1}\sqrt{x+1}\,\text{Erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}+\frac{\sqrt{\pi}\sqrt{x-1}\sqrt{x+1}\,\text{Erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] (Sqrt[Pi]*Sqrt[-1 + x]*Sqrt[1 + x]*Erf[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x^2])
 + (Sqrt[Pi]*Sqrt[-1 + x]*Sqrt[1 + x]*Erfi[Sqrt[ArcCosh[x]]])/(2*Sqrt[1 - x
^2])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{\sqrt{1-x^2}\sqrt{\cosh^{-1}(x)}}\,dx &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\int \frac{x}{\sqrt{-1+x}\sqrt{1+x}\sqrt{\cosh^{-1}(x)}}\,dx}{\sqrt{1-x^2}}\\ &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{\cosh(x)}{\sqrt{x}}\,dx,x,\cosh^{-1}(x)\right)}{\sqrt{1-x^2}}\\ &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{e^{-x}}{\sqrt{x}}\,dx,x,\cosh^{-1}(x)\right)}{2\sqrt{1-x^2}}+\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\operatorname{Subst}\left(\int \frac{e}{\sqrt{}}\right.}{2\sqrt{1-x^2}}\\ &= \frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\operatorname{Subst}\left(\int e^{-x^2}\,dx,x,\sqrt{\cosh^{-1}(x)}\right)}{\sqrt{1-x^2}}+\frac{\left(\sqrt{-1+x}\sqrt{1+x}\right)\operatorname{Subst}\left(\right.}{\sqrt{1-}}\\ &= \frac{\sqrt{\pi}\sqrt{-1+x}\sqrt{1+x}\operatorname{erf}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}+\frac{\sqrt{\pi}\sqrt{-1+x}\sqrt{1+x}\operatorname{erfi}\left(\sqrt{\cosh^{-1}(x)}\right)}{2\sqrt{1-x^2}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 1.11

$$-\frac{\sqrt{-((x-1)(x+1))}\left(\sqrt{-\cosh^{-1}(x)}\,\Gamma\left(\frac{1}{2},-\cosh^{-1}(x)\right)-\sqrt{\cosh^{-1}(x)}\,\Gamma\left(\frac{1}{2},\cosh^{-1}(x)\right)\right)}{2\sqrt{\frac{x-1}{x+1}}\,(x+1)\sqrt{\cosh^{-1}(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/(Sqrt[1 - x^2]*Sqrt[ArcCosh[x]]),x]
```

```
[Out] -1/2*(Sqrt[-((-1 + x)*(1 + x))]*(Sqrt[-ArcCosh[x]]*Gamma[1/2, -ArcCosh[x]]
- Sqrt[ArcCosh[x]]*Gamma[1/2, ArcCosh[x]]))/(Sqrt[(-1 + x)/(1 + x)]*(1 + x)
*Sqrt[ArcCosh[x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^2+1}\sqrt{\operatorname{arcosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="giac")
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

maple [F] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^{2}+1}\,\sqrt{\operatorname{arccosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
[Out] int(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-x^{2}+1}\,\sqrt{\operatorname{arcosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x^2+1)^(1/2)/arccosh(x)^(1/2),x, algorithm="maxima")
[Out] integrate(x/(sqrt(-x^2 + 1)*sqrt(arccosh(x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{\sqrt{\operatorname{acosh}(x)}\,\sqrt{1-x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)),x)
[Out] int(x/(acosh(x)^(1/2)*(1 - x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-(x-1)(x+1)}\,\sqrt{\operatorname{acosh}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(-x**2+1)**(1/2)/acosh(x)**(1/2),x)
[Out] Integral(x/(sqrt(-(x - 1)*(x + 1))*sqrt(acosh(x))), x)
```

3.402 $\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=438

$$-\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{\right.}}{64a\sqrt{ax-1}\,\sqrt{a}}$$

```
[Out] 1/384*c^2*erf(6^(1/2)*arccosh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/384*c^2*erfi(6^(1/2)*arccosh(a*x)^(1/2
))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/1
28*c^2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/128*c^2*erfi(2^(1/2)*arccosh(a*x)^(1/2))
*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-3/64*c
^2*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/
(a*x+1)^(1/2)-3/64*c^2*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-5/8*c^2*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x
)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.44, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$-\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{6}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\right.}{64a\sqrt{ax-1}\,\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (-5*c^2*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1
+ a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(6
4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*
Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (c^2
*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt
[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt
[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2*Sqrt[Pi/2]*S
qrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(64*a*Sqrt[-1 + a*x]*S
qrt[1 + a*x]) + (c^2*Sqrt[Pi/6]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[6]*Sqrt[ArcCo
sh[a*x]]])/(64*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^6(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5}{16\sqrt{x}} - \frac{15\cosh(2x)}{32\sqrt{x}} + \frac{3\cosh(4x)}{16\sqrt{x}} - \frac{\cosh(6x)}{32\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(6x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} -$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{64a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{(c}{}$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\,\sqrt{1+ax}} -$$

$$= -\frac{5c^2\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{64a\sqrt{-1+ax}\,\sqrt{1+ax}} + \frac{15c^2\sqrt{\frac{\pi}{2}}\,\sqrt{}}{6}$$

Mathematica [A] time = 0.44, size = 209, normalized size = 0.48

$$-\frac{c^2\sqrt{c-a^2cx^2}\left(240\cosh^{-1}(ax)+45\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)-18\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},4\cosh^{-1}(ax)\right)\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/384*(c^2*Sqrt[c - a^2*c*x^2]*(240*ArcCosh[a*x] - Sqrt[6]*Sqrt[-ArcCosh[a
*x]]*Gamma[1/2, -6*ArcCosh[a*x]] + 18*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*Arc
Cosh[a*x]] - 45*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + 4
5*Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 18*Sqrt[ArcCosh[a
*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*A
rcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt(arccosh(a*x)), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arccosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/sqrt(arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c-a^2\,c\,x^2\right)^{5/2}}{\sqrt{\mathrm{acosh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(1/2),x)
[Out] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
[Out] Timed out
```

3.403 $\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=294

$$-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(2\sqrt{\cosh\right.}}{32a\sqrt{ax-1}\,\sqrt{ax+1}}$$

```
[Out] 1/8*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)
/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/8*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/
2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c*erf(2
*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)-1/32*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*
x-1)^(1/2)/(a*x+1)^(1/2)-3/4*c*(-a^2*c*x^2+c)^(1/2)*arccosh(a*x)^(1/2)/a/(a
*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh\right.}}{32a\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] (-3*c*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 +
a*x]) - (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sq
rt[-1 + a*x]*Sqrt[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi]*Sqrt
[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(32*a*Sqrt[-1 + a*x]*Sqrt[1 + a
*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/
(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
```

```
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx &= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh^4(x)}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}\\
&= -\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}-\frac{\cosh(2x)}{2\sqrt{x}}+\frac{\cosh(4x)}{8\sqrt{x}}\right)dx,x,\cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}\\
&= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(4x)}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}+\cdots\\
&= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx,x,\cosh^{-1}(ax)\right)}{16a\sqrt{-1+ax}\,\sqrt{1+ax}}-\cdots\\
&= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx,x,\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}-\cdots\\
&= -\frac{3c\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{4a\sqrt{-1+ax}\,\sqrt{1+ax}}-\frac{c\sqrt{\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{32a\sqrt{-1+ax}\,\sqrt{1+ax}}+\frac{c\sqrt{\frac{\pi}{2}}\sqrt{c-a^2c\cdots}}{4a\sqrt{\cdots}}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 153, normalized size = 0.52

$$-\frac{c\sqrt{c-a^2cx^2}\left(\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-4\cosh^{-1}(ax)\right)-4\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{\cosh^{-1}(a\cdots}\right.}{32a\sqrt{\frac{ax-1}{ax+1}}\,(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/32*(c*Sqrt[c - a^2*c*x^2]*(Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x
]] - 4*Sqrt[2]*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[ArcCo
sh[a*x]]*(24*Sqrt[ArcCosh[a*x]] + 4*Sqrt[2]*Gamma[1/2, 2*ArcCosh[a*x]] - Ga
mma[1/2, 4*ArcCosh[a*x]])))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[Ar
cCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arcosh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt(arccosh(a*x)), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arccosh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\sqrt{\operatorname{arcosh}\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/sqrt(arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{\sqrt{\mathrm{acosh}\left(a\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(1/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-c\,(ax-1)\,(ax+1))^{\frac{3}{2}}}{\sqrt{\operatorname{acosh}\,(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/sqrt(acosh(a*x)), x)
```

3.404 $\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=175

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{ax-1}\sqrt{ax+1}}$$

```
[Out] 1/8*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/8*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-(-a^2*c*x^2+c)^(
1/2)*arccosh(a*x)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5701, 3312, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\sqrt{ax+1}}-\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{ax-1}\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -((Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(4*a*S
qrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]
*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{\sinh^2(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}}-\frac{\cosh(2x)}{2\sqrt{x}}\right)dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx}}{}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{2a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-}}{}$$

$$= -\frac{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx}}{4a\sqrt{-}}$$

Mathematica [A] time = 0.15, size = 114, normalized size = 0.65

$$-\frac{\sqrt{-c(ax-1)(ax+1)}\left(8\cosh^{-1}(ax)+\sqrt{2}\sqrt{\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},2\cosh^{-1}(ax)\right)-\sqrt{2}\sqrt{-\cosh^{-1}(ax)}\,\Gamma\left(\frac{1}{2},-2\cos\right.\right.}{8a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/Sqrt[ArcCosh[a*x]],x]
```

```
[Out] -1/8*(Sqrt[-(c*(-1 + a*x)*(1 + a*x))]*(8*ArcCosh[a*x] - Sqrt[2]*Sqrt[-ArcCo
sh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] + Sqrt[2]*Sqrt[ArcCosh[a*x]]*Gamma[1/2
, 2*ArcCosh[a*x]]))/(a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*
x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}}{\sqrt{\operatorname{arcosh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt(arccosh(a*x)), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2c\,x^2 + c}}{\sqrt{\operatorname{arccosh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}}{\sqrt{\operatorname{arcosh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/sqrt(arccosh(a*x)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - a^2\,c\,x^2}}{\sqrt{\operatorname{acosh}(a\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(1/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax - 1)\,(ax + 1)}}{\sqrt{\operatorname{acosh}(ax)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/sqrt(acosh(a*x)), x)
```

3.405 $\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=46

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

```
[Out] 2*(a*x-1)^(1/2)*(a*x+1)^(1/2)*arccosh(a*x)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= \frac{2\sqrt{-1+ax}\sqrt{1+ax}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{\cosh^{-1}(ax)}}{a\sqrt{c-a^2cx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]),x]
[Out] (2*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Sqrt[ArcCosh[a*x]])/(a*Sqrt[c - a^2*c*x^2])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2 + c}\,\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x))), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.89

$$\frac{2\sqrt{\operatorname{arccosh}(ax)}\,\sqrt{ax-1}\,\sqrt{ax+1}}{a\sqrt{-(ax-1)(ax+1)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x)
[Out] 2*arccosh(a*x)^(1/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1/
2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2 + c}\,\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*sqrt(arccosh(a*x))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{\operatorname{acosh}(a\,x)}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)),x)
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c(ax-1)(ax+1)}\,\sqrt{\operatorname{acosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*sqrt(acosh(a*x))), x)
```

3.406 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] -((Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/((-1 + a*x)^(3/2)*(1 + a*x)^(3
/2)*Sqrt[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2]))
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx = -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*Sqrt[ArcCosh[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x))), x)
```

maple [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arccosh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c x^{2}+c\right)^{\frac{3}{2}} \sqrt{\operatorname{arcosh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*sqrt(arccosh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\operatorname{acosh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c\left(a x-1\right)\left(a x+1\right)\right)^{\frac{3}{2}} \sqrt{\operatorname{acosh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(1/2),x)
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*sqrt(acosh(a*x))), x)
```

3.407 $\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}},x\right)$$

```
[Out] Unintegrable(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] (Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][1/((-1 + a*x)^(5/2)*(1 + a*x)^(5/2
)*Sqrt[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx = \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

Mathematica [A] time = 2.48, size = 0, normalized size = 0.00

$$\int \frac{1}{(c-a^2cx^2)^{5/2}\sqrt{\cosh^{-1}(ax)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*Sqrt[ArcCosh[a*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-a^2cx^2+c)^{\frac{5}{2}}\sqrt{\operatorname{arcosh}(ax)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccosh(a*x))), x)
```

maple [A] time = 0.74, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c\,x^{2}+c\right)^{\frac{5}{2}} \sqrt{\operatorname{arccosh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

```
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^{2} c x^{2}+c\right)^{\frac{5}{2}} \sqrt{\operatorname{arcosh}\left(a x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*sqrt(arccosh(a*x))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{\mathrm{acosh}\left(a\,x\right)}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(1/2)*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(- c \left(a x - 1\right) \left(a x + 1\right)\right)^{\frac{5}{2}} \sqrt{\operatorname{acosh}{\left(a x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(1/2),x)
```

```
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(5/2)*sqrt(acosh(a*x))), x)
```

3.408 $\int \frac{(c-a^2cx^2)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=433

$$\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\right.}{16a\sqrt{ax-1}\,}$$

```
[Out] -15/32*c^2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^
(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+15/32*c^2*erfi(2^(1/2)*arccosh(a*x)^(1/
2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+3/8
*c^2*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2
)/(a*x+1)^(1/2)-3/8*c^2*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^
(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-1/32*c^2*erf(6^(1/2)*arccosh(a*x)^(1/2)
)*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/32*
c^2*erfi(6^(1/2)*arccosh(a*x)^(1/2))*6^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2*(-a^2*c*x^2+c)^(5/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)/a/arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 444, normalized size of antiderivative = 1.03, number of steps used = 20, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5713, 5697, 5780, 5448, 3308, 2180, 2204, 2205}

$$\frac{3\sqrt{\pi}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{15\sqrt{\frac{\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{16a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{3\pi}{2}}\,c^2\sqrt{c-a^2cx^2}\,\mathrm{Erf}}{16a\sqrt{ax-1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*c^2*(1 - a*x)^3*(1 + a*x)^(5/2)*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*S
qrt[ArcCosh[a*x]]) + (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh
[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (15*c^2*Sqrt[Pi/2]*Sqrt[c - a
^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*
x]) - (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[6]*Sqrt[ArcCosh[a*x]
]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (3*c^2*Sqrt[Pi]*Sqrt[c - a^2*c*x^
2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (15*c^2
*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqr
t[-1 + a*x]*Sqrt[1 + a*x]) + (c^2*Sqrt[(3*Pi)/2]*Sqrt[c - a^2*c*x^2]*Erfi[S
qrt[6]*Sqrt[ArcCosh[a*x]]])/(16*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\left(c^2\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{5/2}(1+ax)^{5/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12ac^2\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)^2}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^5(x)}{\sqrt{x}}\,dx, x, \text{cc}\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(12c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(\frac{5\sinh(2x)}{32\sqrt{x}} - \frac{\sinh(4x)}{8\sqrt{x}} + \right.\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(6x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(a\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-6x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{16a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(3c^2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-6x^2}\,dx, x, \sqrt{\cosh^{-1}(a\right.\right.}{8a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c^2(1-ax)^3(1+ax)^{5/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{3c^2\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{8a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{15c^2\sqrt{}}{}$$

Mathematica [A] time = 1.27, size = 411, normalized size = 0.95

$$\frac{c^2\sqrt{c-a^2cx^2}\,e^{-6\cosh^{-1}(ax)}\left(-64a^2x^2e^{6\cosh^{-1}(ax)} - 16\sqrt{2\pi}\,e^{6\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right) + 16\sqrt{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(5/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (c^2*Sqrt[c - a^2*c*x^2]*(-1 + 6*E^(2*ArcCosh[a*x]) + E^(4*ArcCosh[a*x]) +
52*E^(6*ArcCosh[a*x]) + E^(8*ArcCosh[a*x]) + 6*E^(10*ArcCosh[a*x]) - E^(12*
ArcCosh[a*x]) - 64*a^2*E^(6*ArcCosh[a*x])*x^2 - 16*E^(6*ArcCosh[a*x])*Sqrt[
2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 16*E^(6*ArcCosh[
a*x])*Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + Sqrt
[6]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -6*ArcCosh[a*x]] - 12
*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] - Sqrt[
2]*E^(6*ArcCosh[a*x])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -2*ArcCosh[a*x]] - Sqr
t[2]*E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 2*ArcCosh[a*x]] - 12*
E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]] + Sqrt[6]*
E^(6*ArcCosh[a*x])*Sqrt[ArcCosh[a*x]]*Gamma[1/2, 6*ArcCosh[a*x]]))/(32*a*E^
(6*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/arccosh(a*x)^(3/2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}}{\operatorname{arccosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{5}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(5/2)/arccosh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{5/2}}{{\mathrm{acosh}\left(a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(3/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(5/2)/acosh(a*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
```

```
[Out] Timed out
```

3.409 $\int \frac{(c-a^2cx^2)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=286

$$\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}$$

```
[Out] -1/2*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2
)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*c*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1
/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/4*c*erf(2
*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^
(1/2)-1/4*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x
-1)^(1/2)/(a*x+1)^(1/2)-2*(-a^2*c*x^2+c)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/
a/arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 295, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 8, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5713, 5697, 5780, 5448, 3308, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\frac{\pi}{2}}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{ax-1}\,\sqrt{ax+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*c*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*Sqr
t[ArcCosh[a*x]]) + (c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]
])/(4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*E
rf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (c*Sqrt[
Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(4*a*Sqrt[-1 + a*x]*Sqr
t[1 + a*x]) + (c*Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a
*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5780

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)
^2)^(p_.), x_Symbol] :> Dist[(-d)^p/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x
]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d, e, n}
, x] && EqQ[c^2*d + e, 0] && IGtQ[p, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh^3(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \left(-\frac{\sinh(2x)}{4\sqrt{x}}+\frac{\sinh(4x)}{8\sqrt{x}}\right)dx,\right.}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} - \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(4x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-4x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{2a\sqrt{-1+ax}\,\sqrt{1+ax}} -$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{\left(c\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-4x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\,\sqrt{\cosh^{-1}(ax)}} + \frac{c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{4a\sqrt{-1+ax}\,\sqrt{1+ax}} - \frac{c\sqrt{\frac{\pi}{2}}\,\sqrt{c-}}{}$$

Mathematica [A] time = 0.48, size = 239, normalized size = 0.84

$$-\frac{c\sqrt{c-a^2cx^2}\,e^{-4\cosh^{-1}(ax)}\left(16a^2x^2e^{4\cosh^{-1}(ax)}+4\sqrt{2\pi}\,e^{4\cosh^{-1}(ax)}\sqrt{\cosh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)-4\sqrt{2\pi}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(3/2),x]
```

```
[Out] -1/8*(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x]
) + 16*a^2*E^(4*ArcCosh[a*x])*x^2 + 4*E^(4*ArcCosh[a*x])*Sqrt[2*Pi]*Sqrt[Ar
cCosh[a*x]]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]] - 4*E^(4*ArcCosh[a*x])*Sqrt[2*P
i]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[ArcCosh[a*x]]] + 2*E^(4*ArcCosh[a*x
])*Sqrt[-ArcCosh[a*x]]*Gamma[1/2, -4*ArcCosh[a*x]] + 2*E^(4*ArcCosh[a*x])*S
qrt[ArcCosh[a*x]]*Gamma[1/2, 4*ArcCosh[a*x]]))/(a*E^(4*ArcCosh[a*x])*Sqrt[(
-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(3/2), x)
```

maple [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{\operatorname{arccosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{{\mathrm{acosh}\left(a\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(3/2),x)
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}}{\operatorname{acosh}^{\frac{3}{2}}{\left(ax\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
[Out] Integral((-c*(a*x - 1)*(a*x + 1))**(3/2)/acosh(a*x)**(3/2), x)
```

3.410 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}\sqrt{c-a^2}}{a\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] -1/2*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/
a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+1/2*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*
Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2*(a*x-1)^(1/2)
*(a*x+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 176, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {5713, 5697, 5670, 5448, 12, 3308, 2180, 2204, 2205}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{ax+1}(1-ax)\sqrt{c-a^2}}{a\sqrt{ax-1}\sqrt{\cosh^{-1}(ax}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (2*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(a*Sqrt[-1 + a*x]*Sqrt[ArcC
osh[a*x]]) - (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]
])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (Sqrt[Pi/2]*Sqrt[c - a^2*c*x^2]*Erfi[
Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{3/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\sqrt{\cosh^{-1}(ax)}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(x)\sinh(x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{2\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} + \frac{\left(2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\sinh(2x)}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{c-a^2cx^2}\,\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx, x, \cosh^{-1}(ax)\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{c-a^2 \cdots}}{}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\left(2\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx, x, \sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \cdots$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{a\sqrt{-1+ax}\sqrt{\cosh^{-1}(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{a\sqrt{-1+ax}\sqrt{1+ax}} + \frac{\sqrt{\frac{\pi}{2}}\sqrt{c-a^2 \cdots}}{a\sqrt{\cdots}}$$

Mathematica [A] time = 0.27, size = 127, normalized size = 0.75

$$\frac{\sqrt{c-a^2cx^2}\left(-4a^2x^2-\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)+\sqrt{2\pi}\sqrt{\cosh^{-1}(ax)}\,\text{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right) \cdots\right)}{2a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\sqrt{\cosh^{-1}(ax)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(3/2),x]
```

```
[Out] (Sqrt[c - a^2*c*x^2]*(4 - 4*a^2*x^2 - Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erf[Sqr
t[2]*Sqrt[ArcCosh[a*x]]] + Sqrt[2*Pi]*Sqrt[ArcCosh[a*x]]*Erfi[Sqrt[2]*Sqrt[
ArcCosh[a*x]]]))/(2*a*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*Sqrt[ArcCosh[a*x
]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(3/2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2c\,x^2 + c}}{\operatorname{arccosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2 + c}}{\operatorname{arcosh}(ax)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c - a^2\,c\,x^2}}{\operatorname{acosh}(a\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(3/2),x)
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\,(ax-1)\,(ax+1)}}{\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acosh(a*x)**(3/2), x)
```

3.411 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=46

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{3/2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{a\sqrt{c-a^2cx^2}\sqrt{\cosh^{-1}(ax)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(a*Sqrt[c - a^2*c*x^2]*Sqrt[ArcCosh[a*x]]
)
```

fricas [A] time = 0.50, size = 59, normalized size = 1.28

$$\frac{2\sqrt{-a^2cx^2+c}\,\sqrt{a^2x^2-1}}{\left(a^3cx^2-ac\right)\sqrt{\log\left(ax+\sqrt{a^2x^2-1}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*sqrt(log(a*x +
sqrt(a^2*x^2 - 1))))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2)), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.89

$$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{\sqrt{\operatorname{arccosh}\left(ax\right)}\,a\sqrt{-\left(ax-1\right)\left(ax+1\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x)
```

```
[Out] -2/arccosh(a*x)^(1/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^(1
/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a^2cx^2+c}\,\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(3/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)^{3/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,\operatorname{acosh}^{\frac{3}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(3/2),x)
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(3/2)), x)
```

3.412 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{4a\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2-1\right)^2\sqrt{\cosh^{-1}(ax)}},x\right)}{c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{a\left(c-a^2cx^2\right)^{3/2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(1/2)+4*
a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)^(1/
2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(a*c*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2]*Sqrt[A
rcCosh[a*x]]) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2*x
^2)^2*Sqrt[ArcCosh[a*x]]), x])/(c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx &= -\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}}{ac(1-ax)\sqrt{1+ax}\,\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}+\frac{\left(4a\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int -}{c\sqrt{c-a^2}}\end{aligned}$$

Mathematica [A] time = 1.79, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2)), x)
```

maple [A]   time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}\operatorname{arccosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x)
```

maxima [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}}\operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(3/2)), x)
```

mupad [A]   time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [A]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-c\left(ax-1\right)\left(ax+1\right)\right)^{\frac{3}{2}}\operatorname{acosh}^{\frac{3}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(3/2),x)
[Out] Integral(1/((-c*(a*x - 1)*(a*x + 1))**(3/2)*acosh(a*x)**(3/2)), x)
```

3.413 $\int \frac{1}{(c-a^2cx^2)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$

Optimal. Leaf size=110

$$-\frac{8a\sqrt{ax-1}\,\sqrt{ax+1}\,\mathrm{Int}\left(\frac{x}{(a^2x^2-1)^3\sqrt{\cosh^{-1}(ax)}},x\right)}{c^2\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{a\left(c-a^2cx^2\right)^{5/2}\sqrt{\cosh^{-1}(ax)}}$$

```
[Out] -2*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(1/2)-8*
a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^3/arccosh(a*x)^(1/
2),x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(a*c^2*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2]*
Sqrt[ArcCosh[a*x]]) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 +
 a^2*x^2)^3*Sqrt[ArcCosh[a*x]]), x])/(c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx=\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\,\sqrt{\cosh^{-1}(ax)}}-\frac{\left(8a\sqrt{-1+ax}\,\sqrt{1+ax}\right)}{c^2\sqrt{c}}$$

Mathematica [A] time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="fricas")
```

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}} \operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(3/2)), x)
```

maple [A] time = 0.73, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}} \operatorname{arccosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}} \operatorname{arcosh}\left(ax\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)^{3/2}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(1/(acosh(a*x)^(3/2)*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(3/2),x)
[Out] Timed out
```

3.414 $\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(2\sqrt{c\right.}}{3a\sqrt{ax-1}\,\sqrt{ax+}}$$

```
[Out] -2/3*(-a^2*c*x^2+c)^(3/2)*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(3/2)-
2/3*c*erf(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/
2)/(a*x+1)^(1/2)-2/3*c*erfi(2*arccosh(a*x)^(1/2))*Pi^(1/2)*(-a^2*c*x^2+c)^(
1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*c*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*c*erf
i(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-
1)^(1/2)/(a*x+1)^(1/2)-16/3*c*x*(-a*x+1)*(a*x+1)*(-a^2*c*x^2+c)^(1/2)/arcco
sh(a*x)^(1/2)
```

Rubi [A] time = 0.75, antiderivative size = 337, normalized size of antiderivative = 1.02, number of steps used = 19, number of rules used = 11, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.458, Rules used = {5713, 5697, 5776, 5701, 3312, 3307, 2180, 2204, 2205, 5781, 5448}

$$-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(2\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\,\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\,\sqrt{ax+1}}-\frac{2\sqrt{\pi}\,c\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(2\sqrt{\right.}}{3a\sqrt{ax-1}\,\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] Int[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (2*c*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[-1 + a*x]*A
rcCosh[a*x]^(3/2)) - (16*c*x*(1 - a^2*x^2)*Sqrt[c - a^2*c*x^2])/(3*Sqrt[Arc
Cosh[a*x]]) - (2*c*Sqrt[Pi]*Sqrt[c - a^2*c*x^2]*Erf[2*Sqrt[ArcCosh[a*x]]])/
(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Er
f[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]) - (2*c*Sq
rt[Pi]*Sqrt[c - a^2*c*x^2]*Erfi[2*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*
Sqrt[1 + a*x]) + (2*c*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[ArcC
osh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rule 5776

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.)*((d_) + (e_
.)*(x_)^2)^(p_.), x_Symbol] :> Simp[((f*x)^m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]*(
d + e*x^2)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] + (Dist[(f*m*(
-d)^p)/(b*c*(n + 1)), Int[(f*x)^(m - 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p -
 1/2)*(a + b*ArcCosh[c*x])^(n + 1), x], x] - Dist[(c*(-d)^p*(m + 2*p + 1))/
(b*f*(n + 1)), Int[(f*x)^(m + 1)*(1 + c*x)^(p - 1/2)*(-1 + c*x)^(p - 1/2)*(
a + b*ArcCosh[c*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ
[c^2*d + e, 0] && LtQ[n, -1] && IGtQ[m, -3] && IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{\left(c-a^2cx^2\right)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx = -\frac{\left(c\sqrt{c-a^2cx^2}\right)\int \frac{(-1+ax)^{3/2}(1+ax)^{3/2}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{\left(8ac\sqrt{c-a^2cx^2}\right)\int \frac{x\left(-1+a^2x^2\right)}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\,\sqrt{1+ax}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2cx^2}\right)\int \frac{\sqrt{-1}}{\sqrt{c}}}{3\sqrt{-1+ax}\,\sqrt{1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(16c\sqrt{c-a^2cx^2}\right)\text{Subst}}{3a\sqrt{-1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(16c\sqrt{c-a^2cx^2}\right)\text{Subst}}{3a}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}}{3a\sqrt{-1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(4c\sqrt{c-a^2cx^2}\right)\text{Subst}}{3a\sqrt{-1+}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{\left(8c\sqrt{c-a^2cx^2}\right)\text{Subst}}{3a\sqrt{-1}}$$

$$= \frac{2c(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\,\cosh^{-1}(ax)^{3/2}} - \frac{16cx\left(1-a^2x^2\right)\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} - \frac{2c\sqrt{\pi}\,\sqrt{c-a^2cx^2}\,\text{erf}\left(2\right.}{3a\sqrt{-1+ax}\,\sqrt{}}$$

Mathematica [A] time = 0.60, size = 317, normalized size = 0.96

$$-\frac{c\sqrt{c-a^2cx^2}\,e^{-4\cosh^{-1}(ax)}\left(16a^2x^2e^{4\cosh^{-1}(ax)} + 64a^2x^2\sqrt{\frac{ax-1}{ax+1}}\,e^{4\cosh^{-1}(ax)}\cosh^{-1}(ax) + 64ax\sqrt{\frac{ax-1}{ax+1}}\,e^{4\cosh^{-1}(ax)}\cos\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c - a^2*c*x^2)^(3/2)/ArcCosh[a*x]^(5/2),x]
```

```
[Out] -1/24*(c*Sqrt[c - a^2*c*x^2]*(-1 - 14*E^(4*ArcCosh[a*x]) - E^(8*ArcCosh[a*x
]) + 16*a^2*E^(4*ArcCosh[a*x])*x^2 + 8*ArcCosh[a*x] - 8*E^(8*ArcCosh[a*x])*
```

```
ArcCosh[a*x] + 64*a*E^(4*ArcCosh[a*x])*x*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh
[a*x] + 64*a^2*E^(4*ArcCosh[a*x])*x^2*Sqrt[(-1 + a*x)/(1 + a*x)]*ArcCosh[a*
x] - 16*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -4*ArcCosh[a*x]
] + 16*Sqrt[2]*E^(4*ArcCosh[a*x])*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCo
sh[a*x]] + 16*Sqrt[2]*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*Ar
cCosh[a*x]] - 16*E^(4*ArcCosh[a*x])*ArcCosh[a*x]^(3/2)*Gamma[1/2, 4*ArcCosh
[a*x]]))/(a*E^(4*ArcCosh[a*x])*Sqrt[(-1 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh
[a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(5/2), x)
```

maple [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}}}{\operatorname{arccosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-a^2cx^2+c\right)^{\frac{3}{2}}}{\operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((-a^2*c*x^2 + c)^(3/2)/arccosh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c-a^2\,c\,x^2\right)}^{3/2}}{{\mathrm{acosh}\left(a\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(5/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(3/2)/acosh(a*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Timed out
```

3.415 $\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=201

$$\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\mathrm{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\mathrm{erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}-\frac{2\sqrt{ax}}{}$$

```
[Out] 2/3*erf(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*Pi^(1/2)*(-a^2*c*x^2+c)^(1/2)/a
/(a*x-1)^(1/2)/(a*x+1)^(1/2)+2/3*erfi(2^(1/2)*arccosh(a*x)^(1/2))*2^(1/2)*P
i^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/(a*x-1)^(1/2)/(a*x+1)^(1/2)-2/3*(a*x-1)^(1/2
)*(a*x+1)^(1/2)*(-a^2*c*x^2+c)^(1/2)/a/arccosh(a*x)^(3/2)-8/3*x*(-a^2*c*x^2
+c)^(1/2)/arccosh(a*x)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 207, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {5713, 5697, 5666, 3307, 2180, 2204, 2205}

$$\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\mathrm{Erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}+\frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\mathrm{Erfi}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right)}{3a\sqrt{ax-1}\sqrt{ax+1}}-\frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}}+\frac{2(1-}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (2*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2])/(3*a*Sqrt[-1 + a*x]*ArcCosh
[a*x]^(3/2)) - (8*x*Sqrt[c - a^2*c*x^2])/(3*Sqrt[ArcCosh[a*x]]) + (2*Sqrt[2
*Pi]*Sqrt[c - a^2*c*x^2]*Erf[Sqrt[2]*Sqrt[ArcCosh[a*x]]])/(3*a*Sqrt[-1 + a*
x]*Sqrt[1 + a*x]) + (2*Sqrt[2*Pi]*Sqrt[c - a^2*c*x^2]*Erfi[Sqrt[2]*Sqrt[Arc
Cosh[a*x]]])/(3*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x])
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5697

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*((d1_) + (e1_.)*(x_))^(p_.)*((
d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[(Sqrt[1 + c*x]*Sqrt[-1 + c*x]*
(d1 + e1*x)^p*(d2 + e2*x)^p*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x]
 - Dist[(c*(2*p + 1)*(-(d1*d2))^(p - 1/2)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x])/
(b*(n + 1)*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[x*(-1 + c^2*x^2)^(p - 1/2)*(a
 + b*ArcCosh[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, p}, x
] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && LtQ[n, -1] && IntegerQ[p - 1/2]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c-a^2cx^2}}{\cosh^{-1}(ax)^{5/2}}\,dx = \frac{\sqrt{c-a^2cx^2}\int \frac{\sqrt{-1+ax}\sqrt{1+ax}}{\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} + \frac{\left(4a\sqrt{c-a^2cx^2}\right)\int \frac{x}{\cosh^{-1}(ax)^{3/2}}\,dx}{3\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{\cosh(2x)}{\sqrt{x}}\,dx,x,c\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(4\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int \frac{e^{-2x}}{\sqrt{x}}\,dx,x,\cosh\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{\left(8\sqrt{c-a^2cx^2}\right)\text{Subst}\left(\int e^{-2x^2}\,dx,x,\sqrt{c}\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

$$= \frac{2(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}}{3a\sqrt{-1+ax}\cosh^{-1}(ax)^{3/2}} - \frac{8x\sqrt{c-a^2cx^2}}{3\sqrt{\cosh^{-1}(ax)}} + \frac{2\sqrt{2\pi}\sqrt{c-a^2cx^2}\,\text{erf}\left(\sqrt{2}\sqrt{\cosh^{-1}(ax)}\right.}{3a\sqrt{-1+ax}\sqrt{1+ax}}$$

Mathematica [A] time = 0.32, size = 141, normalized size = 0.70

$$-\frac{2\sqrt{c-a^2cx^2}\left((ax+1)\left(ax+4ax\sqrt{\frac{ax-1}{ax+1}}\cosh^{-1}(ax)-1\right)+\sqrt{2}\left(-\cosh^{-1}(ax)\right)^{3/2}\Gamma\left(\frac{1}{2},-2\cosh^{-1}(ax)\right)+\sqrt{2}\cos\right.}{3a\sqrt{\frac{ax-1}{ax+1}}(ax+1)\cosh^{-1}(ax)^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[c - a^2*c*x^2]/ArcCosh[a*x]^(5/2),x]
```

```
[Out] (-2*Sqrt[c - a^2*c*x^2]*((1 + a*x)*(-1 + a*x + 4*a*x*Sqrt[(-1 + a*x)/(1 + a
*x)]*ArcCosh[a*x]) + Sqrt[2]*(-ArcCosh[a*x])^(3/2)*Gamma[1/2, -2*ArcCosh[a*
x]] + Sqrt[2]*ArcCosh[a*x]^(3/2)*Gamma[1/2, 2*ArcCosh[a*x]]))/(3*a*Sqrt[(-1
 + a*x)/(1 + a*x)]*(1 + a*x)*ArcCosh[a*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(5/2), x)
```

maple [F] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2c\,x^2+c}}{\operatorname{arccosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] int((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a^2cx^2+c}}{\operatorname{arcosh}(ax)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a^2*c*x^2 + c)/arccosh(a*x)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c-a^2\,c\,x^2}}{\operatorname{acosh}(a\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(5/2),x)
```

```
[Out] int((c - a^2*c*x^2)^(1/2)/acosh(a*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c\left(ax-1\right)\left(ax+1\right)}}{\operatorname{acosh}^{\frac{5}{2}}\left(ax\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
[Out] Integral(sqrt(-c*(a*x - 1)*(a*x + 1))/acosh(a*x)**(5/2), x)
```

3.416 $\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=48

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/arccosh(a*x)^(3/2)/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {5713, 5676}

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{5/2}}\,dx &= \frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{\sqrt{-1+ax}\sqrt{1+ax}\cosh^{-1}(ax)^{5/2}}\,dx}{\sqrt{c-a^2cx^2}}\\ &= -\frac{2\sqrt{-1+ax}\sqrt{1+ax}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 48, normalized size = 1.00

$$-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x]*Sqrt[1 + a*x])/(3*a*Sqrt[c - a^2*c*x^2]*ArcCosh[a*x]^(3/
2))
```

fricas [A] time = 0.66, size = 59, normalized size = 1.23

$$\frac{2\sqrt{-a^2cx^2+c}\sqrt{a^2x^2-1}}{3\left(a^3cx^2-ac\right)\log\left(ax+\sqrt{a^2x^2-1}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(-a^2*c*x^2 + c)*sqrt(a^2*x^2 - 1)/((a^3*c*x^2 - a*c)*log(a*x + sqr
t(a^2*x^2 - 1))^(3/2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a^2cx^2+c}\ \operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2)), x)
```

maple [A] time = 0.08, size = 41, normalized size = 0.85

$$-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{3\mathrm{arccosh}\left(ax\right)^{\frac{3}{2}}a\sqrt{-\left(ax-1\right)\left(ax+1\right)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x)
```

```
[Out] -2/3/arccosh(a*x)^(3/2)/a/(-(a*x-1)*(a*x+1)*c)^(1/2)*(a*x-1)^(1/2)*(a*x+1)^
(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{-a^2cx^2+c}\ \operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(1/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(-a^2*c*x^2 + c)*arccosh(a*x)^(5/2)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{1}{\mathrm{acosh}\left(a\,x\right)^{5/2}\,\sqrt{c-a^2\,c\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)),x)
```

```
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-c\,(ax-1)\,(ax+1)}\,\operatorname{acosh}^{\frac{5}{2}}(ax)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(1/2)/acosh(a*x)**(5/2),x)
```

```
[Out] Integral(1/(sqrt(-c*(a*x - 1)*(a*x + 1))*acosh(a*x)**(5/2)), x)
```

3.417 $\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=114

$$\frac{4a\sqrt{ax-1}\sqrt{ax+1}\,\mathrm{Int}\left(\frac{x}{\left(a^2x^2-1\right)^2\cosh^{-1}(ax)^{3/2}},x\right)}{3c\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\sqrt{ax+1}}{3a\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(3/2)+
4/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^2/arccosh(a*x)
^(3/2),x)/c/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(3*a*c*(1 - a*x)*Sqrt[1 + a*x]*Sqrt[c - a^2*c*x^2]*ArcC
osh[a*x]^(3/2)) + (4*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1 + a^2
*x^2)^2*ArcCosh[a*x]^(3/2)), x])/(3*c*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx=-\frac{\left(\sqrt{-1+ax}\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{3/2}(1+ax)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{3ac(1-ax)\sqrt{1+ax}\sqrt{c-a^2cx^2}\cosh^{-1}(ax)^{3/2}}+\frac{\left(4a\sqrt{-1+ax}\sqrt{1+ax}\right)\int}{3c\sqrt{c-}}$$

Mathematica [A] time = 1.82, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{3/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(3/2)*ArcCosh[a*x]^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2)), x)
```

maple [**A**] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^{\frac{3}{2}} \operatorname{arccosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{3}{2}} \operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(3/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(3/2)*arccosh(a*x)^(5/2)), x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\operatorname{acosh}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)),x)
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(3/2)), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(3/2)/acosh(a*x)**(5/2),x)
[Out] Timed out
```

3.418 $\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{8a\sqrt{ax-1}\,\sqrt{ax+1}\,\text{Int}\left(\frac{x}{\left(a^2x^2-1\right)^3\cosh^{-1}(ax)^{3/2}},x\right)}{3c^2\sqrt{c-a^2cx^2}}-\frac{2\sqrt{ax-1}\,\sqrt{ax+1}}{3a\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{3/2}}$$

```
[Out] -2/3*(a*x-1)^(1/2)*(a*x+1)^(1/2)/a/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(3/2)-
8/3*a*(a*x-1)^(1/2)*(a*x+1)^(1/2)*Unintegrable(x/(a^2*x^2-1)^3/arccosh(a*x)
^(3/2),x)/c^2/(-a^2*c*x^2+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] (-2*Sqrt[-1 + a*x])/(3*a*c^2*(1 - a*x)^2*(1 + a*x)^(3/2)*Sqrt[c - a^2*c*x^2
]*ArcCosh[a*x]^(3/2)) - (8*a*Sqrt[-1 + a*x]*Sqrt[1 + a*x]*Defer[Int][x/((-1
 + a^2*x^2)^3*ArcCosh[a*x]^(3/2)), x])/(3*c^2*Sqrt[c - a^2*c*x^2])
```

Rubi steps

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx=\frac{\left(\sqrt{-1+ax}\,\sqrt{1+ax}\right)\int \frac{1}{(-1+ax)^{5/2}(1+ax)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx}{c^2\sqrt{c-a^2cx^2}}$$

$$=-\frac{2\sqrt{-1+ax}}{3ac^2(1-ax)^2(1+ax)^{3/2}\sqrt{c-a^2cx^2}\,\cosh^{-1}(ax)^{3/2}}-\frac{\left(8a\sqrt{-1+ax}\,\sqrt{1+ax}\right.}{3c^2\sqrt{}}$$

Mathematica [A] time = 2.36, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c-a^2cx^2\right)^{5/2}\cosh^{-1}(ax)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)),x]
```

```
[Out] Integrate[1/((c - a^2*c*x^2)^(5/2)*ArcCosh[a*x]^(5/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(5/2)), x)
```

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2c\,x^2+c\right)^{\frac{5}{2}}\operatorname{arccosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
[Out] int(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a^2cx^2+c\right)^{\frac{5}{2}}\operatorname{arcosh}\left(ax\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a^2*c*x^2+c)^(5/2)/arccosh(a*x)^(5/2),x, algorithm="maxima")
[Out] integrate(1/((-a^2*c*x^2 + c)^(5/2)*arccosh(a*x)^(5/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\mathrm{acosh}\left(a\,x\right)^{5/2}\,{\left(c-a^2\,c\,x^2\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)),x)
[Out] int(1/(acosh(a*x)^(5/2)*(c - a^2*c*x^2)^(5/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-a**2*c*x**2+c)**(5/2)/acosh(a*x)**(5/2),x)
[Out] Timed out
```

3.419 $\int x^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{8bc^3(n+1)\sqrt{cx-1}\sqrt{cx+1}}+\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{4(}{}\right.}{c^3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/8*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)+(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c
^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*
arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c^3/(((a+b*arccosh(c*x))/
b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-2(n+3)}e^{\frac{4a}{b}}\sqrt{d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c^3*(1 + n)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]) + (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma
[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c^3*E^((4*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(3 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d-c^2dx^2}\,\int x^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int\left(-\frac{1}{8}(a+bx)^n+\frac{1}{8}(a+bx)^n\cosh(4x)\right)\,dx,x,c\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int(a+b\right.}{8c^3\sqrt{-1+}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\mathrm{Subst}\left(\int e^{-4x}(a\right.}{16c^3\sqrt{-1+c}}$$

$$=-\frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{4^{-3-n}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\right.}{}$$

Mathematica [A] time = 1.07, size = 181, normalized size = 0.72

$$-\frac{d\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(4^{-n}e^{-\frac{4a}{b}}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{4\left(a+b\cosh^{-1}(c\right.}{b}\right.\right.\right.}{64c^3\sqrt{-d(cx-1)(cx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/64*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((-8*(
a + b*ArcCosh[c*x]))/(b + b*n) + ((a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(
a + b*ArcCosh[c*x]))/b] - E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1
 + n, (4*(a + b*ArcCosh[c*x]))/b])/(4^n*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])
^2/b^2))^n)))/(c^3*Sqrt[-(d*(-1 + c*x)*(1 + c*x))])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^nx^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n} \sqrt{-c^{2} d \,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^{2} d x^{2} + d} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^{2} \sqrt{- d \left(c x - 1\right) \left(c x + 1\right)} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

3.420 $\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n dx$

Optimal. Leaf size=379

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\left(a+b\right.}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)-1/8*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)+1/8*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*
(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(
c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (3^(-1 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a
+ b*ArcCosh[c*x]))/b])/(8*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) - (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a +
b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^2*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt
[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x])
)/b])/(8*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
```

```
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d-c^2dx^2}\int x\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}$$
$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(-\frac{1}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx\right.}{c^2\sqrt{-1+cx}\sqrt{1+cx}}$$
$$=-\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\cosh(x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^2\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{d-c^2d}}{}$$
$$=\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\,\text{Su}}{}$$
$$=\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{8c^2\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 1.28, size = 241, normalized size = 0.64

$$-\frac{de^{-\frac{3a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\left(-3^{-n}e^{\frac{6a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{2n}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\Gamma\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/24*(d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*((3*E^
((4*a)/b)*Gamma[1 + n, a/b + ArcCosh[c*x]])/(a/b + ArcCosh[c*x])^n + (Gamma
[1 + n, (-3*(a + b*ArcCosh[c*x]))/b]/3^n - 3*E^((2*a)/b)*Gamma[1 + n, -((a
+ b*ArcCosh[c*x])/b)] - (E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamm
a[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/(3^n*(-((a + b*ArcCosh[c*x])^2/b^2))^
n))/(-((a + b*ArcCosh[c*x])/b))^n))/(c^2*E^((3*a)/b)*Sqrt[-(d*(-1 + c*x)*(1
 + c*x))])
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.34, size = 0, normalized size = 0.00

$$\int x\,(a+b\,\text{arccosh}\,(cx))^n\,\sqrt{-c^2d\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,(a+b\,\text{acosh}\,(c\,x))^n\,\sqrt{d-c^2\,d\,x^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{-d\,(cx-1)\,(cx+1)}\;(a+b\,\text{acosh}\,(cx))^n\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

3.421 $\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n dx$

Optimal. Leaf size=253

$$-\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{2bc(n+1)\sqrt{cx-1}\sqrt{cx+1}}+\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+2^(-3-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))
/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2^(-3-n)*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+
b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2(a+b\cosh^{-1}(cx))}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{d-c^2d}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c*(1 + n)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) + (2^(-3 - n)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sq
rt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x
]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
```

```
e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
 !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n dx &= \frac{\sqrt{d-c^2dx^2}\int \sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\ &= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a+bx)^n\sinh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\sqrt{1+cx}}\\ &= -\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int\left(\frac{1}{2}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx,x,\cos\ldots\right.}{c\sqrt{-1+cx}\sqrt{1+cx}}\\ &= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int (a+bx)^{\ldots}\right.}{2c\sqrt{-1+c\ldots}}\\ &= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-2x}(a+\ldots\right.}{4c\sqrt{-1+cx}\ldots}\\ &= -\frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc(1+n)\sqrt{-1+cx}\sqrt{1+cx}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\left(a+b\,c\ldots\right.}{\ldots}\end{aligned}$$

Mathematica [A] time = 0.70, size = 214, normalized size = 0.85

$$\frac{d2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{n+2}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\ldots\right.}{b^2}\right.\right.}{bc(n+\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-3 - n)*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(
2^(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^
n - b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))
/b] + b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(
a + b*ArcCosh[c*x]))/b]))/(b*c*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a + b \operatorname{arccosh}\left(c x\right)\right)^{n} \sqrt{-c^{2} d \,x^{2}+d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^{2} d x^{2} + d} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{- d \left(c x - 1\right) \left(c x + 1\right)} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n, x)
```

3.422 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=212

$$d\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}},x\right)-\frac{de^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2\sqrt{d-c^2dx^2}}$$

```
[Out] -1/2*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+1/2
*d*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1
/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d*Uninteg
rable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 1.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] (Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c
*x])/b)])/(2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b
))^n) - (E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a
 + b*ArcCosh[c*x])/b])/(2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x]
)/b)^n) - (Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx &= \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{\sqrt{d-c^2dx^2}\int \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\right)dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= -\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int(a+bx)^n\,c\right.}{\sqrt{-1+cx}\,\sqrt{1}}\\
&= \frac{\sqrt{d-c^2dx^2}\,\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\sqrt{d-c^2dx^2}\,S}{}\\
&= \frac{e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b}{}\right.}{2\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\text{arccosh}\,(cx))^n\sqrt{-c^2d\,x^2+d}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x,x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\text{acosh}\,(c\,x))^n\sqrt{d-c^2\,d\,x^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\operatorname{acosh}\left(cx\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x, x)
```

3.423 $\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=92

$$d\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)-\frac{cd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{b(n+1)\sqrt{d-c^2dx^2}}$$

[Out] -c*d*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-c^2*d*x^2+d)^(1/2)+d*Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)

Rubi [A] time = 0.90, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]

[Out] (c*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(b*(1 + n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx &= \frac{\sqrt{d-c^2dx^2}\int \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{\sqrt{d-c^2dx^2}\int \left(\frac{c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\right)dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= -\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(c^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}\\
&= \frac{c\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\sqrt{1+cx}}-\frac{\sqrt{d-c^2dx^2}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\sqrt{1+cx}}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2,x]

[Out] Integrate[(Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n)/x^2, x]

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}(cx)+a)^n}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}\sqrt{-c^{2}d\,x^{2}+d}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-c^{2}dx^{2}+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n*(-c^2*d*x^2+d)^(1/2)/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}\sqrt{d-c^{2}\,d\,x^{2}}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2))/x^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-d\,(cx-1)\,(cx+1)}\,\left(a+b\,\mathrm{acosh}\,(cx)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n*(-c**2*d*x**2+d)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(-d*(c*x - 1)*(c*x + 1))*(a + b*acosh(c*x))**n/x**2, x)
```

3.424 $\int x^2 \left(d - c^2dx^2\right)^{3/2} \left(a + b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=658

$$-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{16bc^3(n+1)\sqrt{cx-1}\sqrt{cx+1}}-\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+\right.}{c^3\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -1/16*d*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2^(-7-n)*3^(-1-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-6*(a
+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/(((-a-b*arccosh(c*x
))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-2*n)*d*(a+b*arccosh(c*x))^n*GAMM
A(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(4*a/b)/(((-a-b*
arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-n)*d*(a+b*arccosh(c*x
))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(2*a/b)
/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-n)*d*exp(2*a
/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(
1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-2*n)*
d*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)*(-c^2*d
*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^
(-7-n)*3^(-1-n)*d*exp(6*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(a+b*arccosh(
c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 658, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 5448, 3307, 2181}

$$-\frac{d2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-2n-7}e}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c^3*(1 + n)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]) - (2^(-7 - n)*3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
 - 2*n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) + (2^(-7 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d*E^((2*a
)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcC
osh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
) - (2^(-7 - 2*n)*d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*
Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*((a + b*ArcCosh[c*x])/b)^n) + (2^(-7 - n)*3^(-1 - n)*d*E^((6*a)/b)*Sqrt[d
- c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b
])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
```

```
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^4(x)\,dx,x,\cos\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{16}(a+bx)^n-\frac{1}{32}(a+bx)^n\cosh(2x)\right.\right.}{c^3\sqrt{-1+}}$$

$$=-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{32c^3}$$

$$=-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{64c^3\sqrt{-}}$$

$$=-\frac{d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{2^{-7-n}3^{-1-n}de^{-\frac{6a}{b}}\sqrt{d-c^2}}{}$$

Mathematica [A] time = 3.37, size = 438, normalized size = 0.67

$$d^2 2^{-2n-7} 3^{-n-1} e^{-\frac{6a}{b}} \sqrt{\frac{cx-1}{cx+1}} (cx+1) \left(a+b\cosh^{-1}(cx)\right)^n \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n} \left(2^n e^{\frac{6a}{b}} \left(2^{n+3} 3^{n+1} \left(a+b\cosh^{-1}(cx)\right)\left(-\right.\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b
*ArcCosh[c*x]))/b] - 3^(1 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n
*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^n*3^(1 + n)*b*E^((4*a)/b)*(1
 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + 2^
n*3^(1 + n)*b*E^((8*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n
, (2*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((10*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] + 2^n*E^((6*a)
/b)*(2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2
))^n - b*E^((6*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*
(a + b*ArcCosh[c*x]))/b])))/(b*c^3*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*
(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2dx^4 - dx^2\right)\sqrt{-c^2dx^2 + d}\,(b\,\text{arcosh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral(-(c^2*d*x^4 - d*x^2)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n,
x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^2 \left(-c^2 d\,x^2 + d\right)^{\frac{3}{2}} (a + b\,\text{arccosh}\,(cx))^n \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^2dx^2 + d\right)^{\frac{3}{2}} (b\,\text{arcosh}\,(cx) + a)^n x^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 (a + b\,\mathrm{acosh}\,(c\,x))^n \left(d - c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.425 $\int x \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=578

$$-\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\,(}{}$$

```
[Out] -1/32*5^(-1-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c*x))/b)*(-
c^2*d*x^2+d)^(1/2)/c^2/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)+1/32*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/
b)*(-c^2*d*x^2+d)^(1/2)/(3^n)/c^2/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/16*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arcco
sh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*d*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(
a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/32*d*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,
3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(3^n)/c^2/(((a+b*arccosh(c*x))
/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/32*5^(-1-n)*d*exp(5*a/b)*(a+b*arccosh(
c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*a
rccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 578, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5781, 5448, 3307, 2181}

$$-\frac{d5^{-n-1}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{32c^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{d3^{-n}e^{-\frac{3a}{b}}\sqrt{d-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -(5^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5*
(a + b*ArcCosh[c*x]))/b])/(32*c^2*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(-((a + b*ArcCosh[c*x])/b))^n) + (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*E^((3*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d*Sqrt[d - c^2*d
*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(16*c
^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (d
*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*Arc
Cosh[c*x])/b])/(16*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) - (d*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 +
n, (3*(a + b*ArcCosh[c*x]))/b])/(32*3^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((
a + b*ArcCosh[c*x])/b)^n) + (5^(-1 - n)*d*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(
a + b*ArcCosh[c*x])^n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,d}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^4(x)\,dx,x,\cosh^{-}\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{1}{8}(a+bx)^n\cosh(x)-\frac{3}{16}(a+bx)^n\cos\right.\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(5x)\,dx,x,\cosh^{-1}(cx)\right)}{16c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-5x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{32c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{\right.}}{}$$

$$=-\frac{5^{-1-n}de^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma}{32c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 2.19, size = 500, normalized size = 0.87

$$-\frac{d^2 15^{-n-1}e^{-\frac{5a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-3n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(-3^{n+1}\left(-\frac{\left(a+b\cosh^{-}\right.}{b^2}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -1/32*(15^(-1 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[
c*x])^n*(2*15^(1 + n)*E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*A
rcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]] + (a/b + ArcCos
h[c*x])^n*(-(3^(1 + n)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (
-5*(a + b*ArcCosh[c*x]))/b]) + 3*5^(1 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*
x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] - 2*15^(1 + n)*
E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, -((a + b*Arc
Cosh[c*x])/b)] + 5^(1 + n)*E^((8*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*Arc
Cosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] - 4*5^(1 + n)
*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^
2))^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*E^((10*a)/b)*(a/
b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a +
b*ArcCosh[c*x]))/b])))/(c^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCo
sh[c*x])^2/b^2))^(3*n))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2dx^3-dx\right)\sqrt{-c^2dx^2+d}\,(b\operatorname{arcosh}(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral(-(c^2*d*x^3 - d*x)*sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x\left(-c^2d\,x^2+d\right)^{\frac{3}{2}}(a+b\,\mathrm{arccosh}(cx))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\operatorname{arcosh}(cx)+a)^nx\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n*x, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^n\,{\left(d-c^2\,d\,x^2\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
[Out] Timed out
```

3.426 $\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=450

$$-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{8bc(n+1)\sqrt{cx-1}\sqrt{cx+1}}-\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,\right.}{c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -3/8*d*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(1/2
)/(c*x+1)^(1/2)-d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(
-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-3-n)*d*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b
*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2*a/b)/(((-a-b*arccosh(c*x))/b
)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-3-n)*d*exp(2*a/b)*(a+b*arccosh(c*x))^n
*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*
x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAM
MA(1+n,4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(6+2*n))/c/(((a+b*ar
ccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$-\frac{d2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{4\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}+\frac{d2^{-n-3}e^{-\frac{2a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-3*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*c*(1 + n)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*G
amma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*E^((4*a)/b)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*d*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x
]))/b])/(c*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])
/b))^n) - (2^(-3 - n)*d*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x]
)^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]*((a + b*ArcCosh[c*x])/b)^n) + (d*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh^4(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{3}{8}(a+bx)^n-\frac{1}{2}(a+bx)^n\cosh(2x)+\frac{1}{8}(\right.\right.}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(\right.}{8c\sqrt{-}}$$

$$=-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(d\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\right.}{16c\sqrt{-1}}$$

$$=-\frac{3d\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{4^{-3-n}de^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\left(a\right.}{}$$

Mathematica [A] time = 2.21, size = 384, normalized size = 0.85

$$\frac{d^24^{-n-3}e^{-\frac{4a}{b}}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3\,2^{2n+3}e^{\frac{4a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{(a+b\cosh}{b^2}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (4^(-3 - n)*d^2*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n
*(3*2^(3 + 2*n)*E^((4*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/
b^2))^(2*n) + b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/
```

```
b))^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*b*E^((2*a)/b)*(
1 + n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n
, (-2*(a + b*ArcCosh[c*x]))/b] + 2^(3 + n)*b*E^((6*a)/b)*(1 + n)*(-((a + b*
ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a +
b*ArcCosh[c*x]))/b] - b*E^((8*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*(-((a +
b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b]))/(b*c*E
^((4*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n
))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(-c^2d\,x^2+d\right)^{\frac{3}{2}}\left(a+b\,\text{arccosh}\,(cx)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+b\,\text{acosh}\,(c\,x)\right)^n\left(d-c^2\,d\,x^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.427 $\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx$

Optimal. Leaf size=415

$$d^2\text{Int}\left(\frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{d-c^2dx^2}},x\right)+\frac{d^2 3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3(a}{}\right.}{8\sqrt{d-c^2dx^2}}$$

```
[Out] 1/8*3^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^
2+d)^(1/2)-5/8*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+
d)^(1/2)+5/8*d^2*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))
/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(
1/2)-1/8*3^(-1-n)*d^2*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arcc
osh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d
*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),
x)
```

Rubi [A] time = 1.84, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(d-c^2dx^2)^{3/2}(a+b\cosh^{-1}(cx))^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] -(3^(-1 - n)*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*
(a + b*ArcCosh[c*x]))/b])/(8*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a
 + b*ArcCosh[c*x])/b))^n) + (5*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n
*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (5*d*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (3^(-1 - n)*d*E^((3*a)/b)*Sqr
t[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]
))/b])/(8*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d*Sqr
t[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{c^4x^3\left(a+b\,\mathrm{c}\right.}{\sqrt{-1+c}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+b\,\mathrm{c}\right.}{\sqrt{-1+}}}{\sqrt{-1+cx}\,\sqrt{1+c}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int (a\right.}{\sqrt{-1}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int e\right.}{\sqrt{-1+c}}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{de^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{3^{-1-n}de^{-\frac{3a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1\right.}{8\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x,x]
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\mathrm{arcosh}\,(cx)+a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcosh}\left(c x\right)+a\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(d-c^2\,d\,x^2\right)}^{3/2}}{x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Integral((-d*(c*x - 1)*(c*x + 1))**(3/2)*(a + b*acosh(c*x))**n/x, x)
```

3.428 $\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=292

$$d^2\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)-\frac{3cd^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{2b(n+1)\sqrt{d-c^2dx^2}}+\frac{cd^2 2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a\right.}{}$$

```
[Out] -3/2*c*d^2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-c
^2*d*x^2+d)^(1/2)+2^(-3-n)*c*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arc
cosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(2*a/b)/(((-a-b*arccosh(c*x))/
b)^n)/(-c^2*d*x^2+d)^(1/2)-2^(-3-n)*c*d^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*G
AMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh
(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^2*Unintegrable((a+b*arccosh(c*x))^n/x^2
/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 1.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (3*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*(1 + n)*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (2^(-3 - n)*c*d*Sqrt[d - c^2*d*x^2]*(a + b*ArcCo
sh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-3 - n)*c*d*E^((2
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*Ar
cCosh[c*x]))/b])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
- (d*Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\left(-\frac{2c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{c^4x^2\left(a+b\cosh\right.}{\sqrt{-1+cx}\,\sqrt{}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(2c^2d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh\right.}{\sqrt{-1+cx}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(c\right.}{x^2\sqrt{-1+cx}\,\sqrt{1}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{2cd\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(c\right.}{x^2\sqrt{-1+cx}\,\sqrt{1}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(c\right.}{x^2\sqrt{-1+cx}\,\sqrt{1}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d\sqrt{d-c^2dx^2}\right)\int\frac{\left(a+b\cosh^{-1}(c\right.}{x^2\sqrt{-1+cx}\,\sqrt{1}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3cd\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2^{-3-n}cde^{-\frac{2a}{b}}\sqrt{d-c^2dx^2}\,\left(a+\right.}{}$$

Mathematica [A] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
[Out] Integrate[((d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(-c^2dx^2+d\right)^{\frac{3}{2}}(b\,\text{arcosh}\,(cx)+a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
[Out] integral((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d x^{2}+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcosh}\left(c x\right)+a\right)^{n}}{x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^n\,\left(d-c^2\,d\,x^2\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

3.429 $\int x^2 \left(d-c^2dx^2\right)^{5/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=870

$$\frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-n-7}3^{-n-1}d^2e^{-\frac{6a}{b}}\sqrt{d}}{}$$

```
[Out] -5/128*d^2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c^3/(1+n)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+2^(-11-3*n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-8*(a+
b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(8*a/b)/(((-a-b*arccosh(c*x)
)/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*(a+b*arccosh(c*x)
)^n*GAMMA(1+n,-6*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/exp(6*a/b)/
(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+d^2*(a+b*arccosh(c*
x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c
^3/exp(4*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7
-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2
+d)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^
(1/2)-2^(-7-n)*d^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh
(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)
/(c*x+1)^(1/2)-d^2*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh
(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/(2^(8+2*n))/c^3/(((a+b*arccosh(c*x))/b)^n)/(
c*x-1)^(1/2)/(c*x+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,6*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^3/(((a+b*arc
cosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2^(-11-3*n)*d^2*exp(8*a/b)*(a+
b*arccosh(c*x))^n*GAMMA(1+n,8*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^
3/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.24, antiderivative size = 870, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{2^{-3n-11}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\text{Gamma}\left(n+1,-\frac{8\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{2^{-n-7}3^{-n-1}d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(128*b*c^3*(1 + n
)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-11 - 3*n)*d^2*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (-8*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((8*
a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-7
 - n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n,
 (-6*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) + (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(4 + n))*c^3*E^((4
*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (2^(-7
 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a +
b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (2^(-7 - n)*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a
 + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*E^((4*a)/b)*Sqrt[d
 - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/
b])/(2^(2*(4 + n))*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b
)^n) + (2^(-7 - n)*3^(-1 - n)*d^2*E^((6*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b])/(c^3*Sqrt[-1 + c*x]
*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-11 - 3*n)*d^2*E^((8*a)/b)
```

```
*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (8*(a + b*ArcCosh[
c*x]))/b])/(c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x^2(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int(a+bx)^n\cosh^2(x)\sinh^6(x)\,dx,x,\cosh^{-1}\right.}{c^3\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int\left(-\frac{5}{128}(a+bx)^n+\frac{1}{32}(a+bx)^n\cosh(2x)+\right.\right.}{}$$
$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\right.}{128c^3}$$
$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\right.}{256c^3\sqrt{}}$$
$$= -\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{128bc^3(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2^{-11-3n}d^2e^{-\frac{8a}{b}}\sqrt{d-c^2dx^2}}{}$$

Mathematica [A] time = 7.41, size = 677, normalized size = 0.78

$$\frac{d^3 2^{-3n-11}3^{-n-1}e^{-\frac{8a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(e^{\frac{8a}{b}}\left(5a2^{3n+4}3^{n+1}\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-11 - 3*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*A
rcCosh[c*x])^n*(-(3^(1 + n)*b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (
-8*(a + b*ArcCosh[c*x]))/b]) + 4^(2 + n)*b*E^((2*a)/b)*(1 + n)*(a/b + ArcCo
sh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b] - 2^(3 + n)*3^(1 + n)*
b*E^((4*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-4*(a + b*ArcCos
h[c*x]))/b] - 3^(1 + n)*4^(2 + n)*b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x]
)^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b] + E^((8*a)/b)*(5*2^(4 + 3*n)*
3^(1 + n)*a*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 5*2^(4 + 3*n)*3^(1 + n)*b*A
rcCosh[c*x]*(-((a + b*ArcCosh[c*x])^2/b^2))^n + 3^(1 + n)*4^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a + b*ArcCosh
[c*x]))/b] + 2^(3 + n)*3^(1 + n)*b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*
x])/b))^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b] - 4^(2 + n)*b*E^((6*a)/b
)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/b] -
4^(2 + n)*b*E^((6*a)/b)*n*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (6*(a
+ b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))
^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)*b*E^((8*a)/b)*n*(-(
(a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (8*(a + b*ArcCosh[c*x]))/b])))/(b*c
^3*E^((8*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^
n)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(c^4d^2x^6-2\,c^2d^2x^4+d^2x^2\right)\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
```

```
[Out] integral((c^4*d^2*x^6 - 2*c^2*d^2*x^4 + d^2*x^2)*sqrt(-c^2*d*x^2 + d)*(b*ar
ccosh(c*x) + a)^n, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int x^{2} \left(-c^{2} d \,x^{2}+d \right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(-c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{2}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(d-c^2\,d\,x^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.430 $\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=793

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}}{}$$

```
[Out] 1/128*7^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-7*(a+b*arccosh(c*x))/b)*
(-c^2*d*x^2+d)^(1/2)/c^2/exp(7*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-1/128*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c
*x))/b)*(-c^2*d*x^2+d)^(1/2)/(5^n)/c^2/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^
n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/128*3^(1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA
(1+n,-3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(3*a/b)/(((-a-b*a
rccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-5/128*d^2*(a+b*arccosh(c*x))
^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/exp(a/b)/(((-a
-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/128*d^2*exp(a/b)*(a+b*
arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/((
(a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/128*3^(1-n)*d^2*exp(
3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d
)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/128*d^
2*exp(5*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(-c^2*d
*x^2+d)^(1/2)/(5^n)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1
/2)-1/128*7^(-1-n)*d^2*exp(7*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,7*(a+b*arc
cosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 793, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {5798, 5781, 5448, 3307, 2181}

$$\frac{d^2 7^{-n-1}e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{7\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{128c^2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d^2 5^{-n}e^{-\frac{5a}{b}}\sqrt{d-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (7^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-7
*(a + b*ArcCosh[c*x]))/b])/(128*c^2*E^((7*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[
c*x])^n*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*E^((5*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (3^(1 - n)*d
^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCo
sh[c*x]))/b])/(128*c^2*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*A
rcCosh[c*x])/b))^n) - (5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gam
ma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(128*c^2*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) + (5*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2
]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(128*c^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (3^(1 - n)*d^2*E^((
3*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*A
rcCosh[c*x]))/b])/(128*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x
])/b)^n) + (d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(128*5^n*c^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (7^(-1 - n)*d^2*E^((7*a)/b)*Sqrt[d - c^2
*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b])/(1
28*c^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int x(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(x)\sinh^6(x)\,dx,x,\cosh^{-1}(c\right.}{c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$
$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(-\frac{5}{64}(a+bx)^n\cosh(x)+\frac{9}{64}(a+bx)^n\cosh\right.\right.}{c^2\sqrt{-1+}}$$
$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\cosh(7x)\,dx,x,\cosh^{-1}(cx)\right)}{64c^2\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{(5}{}$$
$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int e^{-7x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{128c^2\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d\right.}}{}$$
$$=\frac{7^{-1-n}d^2e^{-\frac{7a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+\right.}{128c^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 3.98, size = 633, normalized size = 0.80

$$\frac{d^3 5^{-n}21^{-n-1}e^{-\frac{7a}{b}}\sqrt{\frac{cx-1}{cx+1}}(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-3n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\left(e^{\frac{2a}{b}}\left(21^{n+1}\left(-\frac{(a+b\,\mathrm{co}}{}\right.\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (21^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(-(105^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCos
h[c*x])^2/b^2))^(2*n)*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + ArcCosh[c*
x])^n*(-(3^(1 + n)*5^n*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (
-7*(a + b*ArcCosh[c*x]))/b]) + E^((2*a)/b)*(21^(1 + n)*(-((a + b*ArcCosh[c*
x])^2/b^2))^(2*n)*Gamma[1 + n, (-5*(a + b*ArcCosh[c*x]))/b] - 9*5^n*7^(1 +
n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n)*Gamma[1 + n, (-3*(a +
b*ArcCosh[c*x]))/b] + 105^(1 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2
))^(2*n)*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] - 5^n*7^(2 + n)*E^((8*a)/b
)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (3*
(a + b*ArcCosh[c*x]))/b] + 16*5^n*7^(1 + n)*E^((8*a)/b)*(-((a + b*ArcCosh[c
*x])/b))^(2*n)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (3*(a + b*Arc
Cosh[c*x]))/b] - 21^(1 + n)*E^((10*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*A
rcCosh[c*x])/b))^(3*n)*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b] + 3^(1 + n)
*5^n*E^((12*a)/b)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])/b))^(3*n)*
Gamma[1 + n, (7*(a + b*ArcCosh[c*x]))/b]))))/(128*5^n*c^2*E^((7*a)/b)*Sqrt[
d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^2/b^2))^(3*n))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^5-2\,c^2d^2x^3+d^2x\right)\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
```

```
[Out] integral((c^4*d^2*x^5 - 2*c^2*d^2*x^3 + d^2*x)*sqrt(-c^2*d*x^2 + d)*(b*arcc
osh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.28, size = 0, normalized size = 0.00

$$\int x\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{n}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(-c^{2} d x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n*x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(d-c^2\,d\,x^2\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.431 $\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$

Optimal. Leaf size=674

$$-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{16bc(n+1)\sqrt{cx-1}\sqrt{cx+1}}+\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(r\right.}{c\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -5/16*d^2*(a+b*arccosh(c*x))^(1+n)*(-c^2*d*x^2+d)^(1/2)/b/c/(1+n)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+2^(-7-n)*3^(-1-n)*d^2*(a+b*arccosh(c*x))^n*GAMMA(1+n,-6*
(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(6*a/b)/(((-a-b*arccosh(c*x
))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*2^(-7-2*n)*d^2*(a+b*arccosh(c*x))^n*
GAMMA(1+n,-4*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(4*a/b)/(((-a-
b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+15*2^(-7-n)*d^2*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/exp(2
*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-15*2^(-7-n)*d
^2*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(-c^2*
d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3*2
^(-7-2*n)*d^2*exp(4*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x)
)/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)-2^(-7-n)*3^(-1-n)*d^2*exp(6*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,6*(
a+b*arccosh(c*x))/b)*(-c^2*d*x^2+d)^(1/2)/c/(((a+b*arccosh(c*x))/b)^n)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 674, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {5713, 5701, 3312, 3307, 2181}

$$\frac{d^2 2^{-n-7}3^{-n-1}e^{-\frac{6a}{b}}\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{6\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c\sqrt{cx-1}\sqrt{cx+1}}-\frac{3d^2 2^{-2n-\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (-5*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(16*b*c*(1 + n)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]) + (2^(-7 - n)*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^
2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b])/(c*E^(
(6*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*2
^(-7 - 2*n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-4
*(a + b*ArcCosh[c*x]))/b])/(c*E^((4*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((
a + b*ArcCosh[c*x])/b))^n) + (15*2^(-7 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c*E^((2*a)/b)*S
qrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (15*2^(-7 - n)
*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2
*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[
c*x])/b)^n) + (3*2^(-7 - 2*n)*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (2^(-7 - n)*3^(-1 - n)*d^2*E^((6
*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (6*(a + b*Ar
cCosh[c*x]))/b])/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n
)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
```

```
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5701

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d1_) + (e1_.)*(x_))^(p_.)*(
(d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c, Subst[Int[(a
+ b*x)^n*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x]], x] /; FreeQ[{a, b, c, d1,
 e1, d2, e2, n}, x] && EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && IGtQ[p + 1/2, 0
] && (GtQ[d1, 0] && LtQ[d2, 0])
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\int \left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int(a+bx)^n\sinh^6(x)\,dx,x,\cosh^{-1}(cx)\right)}{c\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$=-\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\int\left(\frac{5}{16}(a+bx)^n-\frac{15}{32}(a+bx)^n\cosh(2x)+\right.\right.}{c\sqrt{-1+c}}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{32c}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\text{Subst}\left(\right.}{64c\sqrt{-}}$$

$$=-\frac{5d^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{16bc(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{2^{-7-n}3^{-1-n}d^2e^{-\frac{6a}{b}}\sqrt{d-c}}{}$$

Mathematica [A] time = 5.43, size = 538, normalized size = 0.80

$$d^3 2^{-2n-7} 3^{-n-1} e^{-\frac{6a}{b}} \sqrt{\frac{cx-1}{cx+1}} (cx+1) \left(a+b\cosh^{-1}(cx)\right)^n \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n} \left(-5b2^n 3^{n+2}(n+1)e^{\frac{4a}{b}} \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)}{b^2}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] (2^(-7 - 2*n)*3^(-1 - n)*d^3*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*Ar
cCosh[c*x])^n*(-(2^n*b*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh
[c*x])/b))^n*Gamma[1 + n, (-6*(a + b*ArcCosh[c*x]))/b]) + 3^(2 + n)*b*E^((2
*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])^(2*n)*(-((a + b*ArcCosh[c*x])/b))^n*Gam
ma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b] - 5*2^n*3^(2 + n)*b*E^((4*a)/b)*(1 +
 n)*(a/b + ArcCosh[c*x])^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (
-2*(a + b*ArcCosh[c*x]))/b] + 5*2^n*3^(2 + n)*b*E^((8*a)/b)*(1 + n)*(-((a +
 b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (2*(a
 + b*ArcCosh[c*x]))/b] - 3^(2 + n)*b*E^((10*a)/b)*(1 + n)*(a/b + ArcCosh[c*
x])^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]
))/b] + 2^n*E^((6*a)/b)*(5*2^(3 + n)*3^(1 + n)*(a + b*ArcCosh[c*x])*(-((a +
 b*ArcCosh[c*x])^2/b^2))^(2*n) + b*E^((6*a)/b)*(1 + n)*(a/b + ArcCosh[c*x])
^n*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (6*(a + b*ArcCosh[c*x]))/
b])))/(b*c*E^((6*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])^
2/b^2))^(2*n))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^4 - 2\,c^2d^2x^2 + d^2\right)\sqrt{-c^2dx^2 + d}\,(b\,\text{arcosh}\,(cx) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
```

```
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int \left(-c^2 d\,x^2 + d\right)^{\frac{5}{2}} (a + b\,\text{arccosh}\,(cx))^n \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^2dx^2+d\right)^{\frac{5}{2}}(b\,\mathrm{arcosh}\,(cx)+a)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}\,(c\,x))^n\,\left(d-c^2\,d\,x^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n,x)
[Out] Timed out
```

3.432 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$

Optimal. Leaf size=805

$$-\frac{5^{-n-1}d^3e^{-\frac{5a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}}{32\sqrt{d-c^2dx^2}}-\frac{5\,3^{-n-1}d^3e^{-\frac{3a}{b}}\sqrt{}}{}$$

```
[Out] -1/32*5^(-1-n)*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-5*(a+b*arccosh(c*x))/b)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(5*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*
x^2+d)^(1/2)-5/32*3^(-1-n)*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arcco
sh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)
^n)/(-c^2*d*x^2+d)^(1/2)+1/8*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arc
cosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(3^n)/exp(3*a/b)/(((-a-b*arccosh(
c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-11/16*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,(
-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(a/b)/(((-a-b*arccosh(
c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+11/16*d^3*exp(a/b)*(a+b*arccosh(c*x))^n*GA
MMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*arccosh(c*
x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+5/32*3^(-1-n)*d^3*exp(3*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(((a+b*
arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-1/8*d^3*exp(3*a/b)*(a+b*arccosh(c*
x))^n*GAMMA(1+n,3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(3^n)/(
((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+1/32*5^(-1-n)*d^3*exp(5*a/b)
*(a+b*arccosh(c*x))^n*GAMMA(1+n,5*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+
1)^(1/2)/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((
a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 2.55, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
```

```
[Out] (5^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-5
*(a + b*ArcCosh[c*x]))/b])/(32*E^((5*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-(
(a + b*ArcCosh[c*x])/b))^n) + (5*3^(-1 - n)*d^2*Sqrt[d - c^2*d*x^2]*(a + b*
ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b])/(32*E^((3*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (d^2*Sqrt[d -
 c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b
])/(8*3^n*E^((3*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/
b))^n) + (11*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -(
(a + b*ArcCosh[c*x])/b)])/(16*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a +
b*ArcCosh[c*x])/b))^n) - (11*d^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(16*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*((a + b*ArcCosh[c*x])/b)^n) - (5*3^(-1 - n)*d^2*E^((3*a)/b)*Sqrt[d - c
^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCosh[c*x]))/b])/
(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) + (d^2*E^((3*a
)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcC
osh[c*x]))/b])/(8*3^n*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)
^n) - (5^(-1 - n)*d^2*E^((5*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^
n*Gamma[1 + n, (5*(a + b*ArcCosh[c*x]))/b])/(32*Sqrt[-1 + c*x]*Sqrt[1 + c*x
]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*Sqrt[d - c^2*d*x^2]*Defer[Int][(a + b*
```

```
ArcCosh[c*x])^n/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{3c^2x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3c^4x^3\left(a+b\right.}{\sqrt{-1+}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2d^2\sqrt{d-c^2dx^2}\right)\int \frac{x\left(a+\right.}{\sqrt{-}}}{\sqrt{-1+cx}\,\sqrt{1+}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int\right.}{\sqrt{-}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\right.}{2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3d^2e^{-\frac{a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,\right.}{2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{5^{-1-n}d^2e^{-\frac{5a}{b}}\sqrt{d-c^2dx^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1\right.}{32\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x,x]
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^4d^2x^4 - 2\,c^2d^2x^2 + d^2\right)\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="fricas"
)
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x, x)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^{2} d x^{2}+d\right)^{\frac{5}{2}} \left(b\,\mathrm{arcosh}\left(c x\right)+a\right)^{n}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x,x, algorithm="maxima"
)
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(d-c^2\,d\,x^2\right)}^{5/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x,x)
```

```
[Out] Timed out
```

3.433 $\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$

Optimal. Leaf size=486

$$d^3\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)-\frac{15cd^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{8b(n+1)\sqrt{d-c^2dx^2}}-\frac{cd^3 2^{-2(n+3)}e^{-\frac{4a}{b}}\sqrt{cx-1}\sqrt{cx+1}}{}$$

```
[Out] -15/8*c*d^3*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/(1+n)/(-
c^2*d*x^2+d)^(1/2)-c*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+n,-4*(a+b*arccosh(c*x
))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(2^(6+2*n))/exp(4*a/b)/(((-a-b*arccosh(c*
x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+2^(-2-n)*c*d^3*(a+b*arccosh(c*x))^n*GAMMA(1+
n,-2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/exp(2*a/b)/(((-a-b*a
rccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-2^(-2-n)*c*d^3*exp(2*a/b)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(
((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+c*d^3*exp(4*a/b)*(a+b*arccos
h(c*x))^n*GAMMA(1+n,4*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(2^
(6+2*n))/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)+d^3*Unintegrable((
a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 2.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
```

```
[Out] (15*c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^(1 + n))/(8*b*(1 + n)*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) + (c*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x
])^n*Gamma[1 + n, (-4*(a + b*ArcCosh[c*x]))/b])/(2^(2*(3 + n))*E^((4*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-2 - n)*c
*d^2*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*Arc
Cosh[c*x]))/b])/(E^((2*a)/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(-((a + b*ArcCosh
[c*x])/b))^n) + (2^(-2 - n)*c*d^2*E^((2*a)/b)*Sqrt[d - c^2*d*x^2]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (2*(a + b*ArcCosh[c*x]))/b])/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (c*d^2*E^((4*a)/b)*Sqrt[d - c^2*d*
x^2]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (4*(a + b*ArcCosh[c*x]))/b])/(2^(2
*(3 + n))*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*((a + b*ArcCosh[c*x])/b)^n) - (d^2*S
qrt[d - c^2*d*x^2]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]), x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx = \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{(-1+cx)^{5/2}(1+cx)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \left(\frac{3c^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{3c^4x^2\left(a+b\cosh\right.}{\sqrt{-1+cx}\,\sqrt{1+}}\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{\left(3c^2d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cos\right.}{\sqrt{-1+c}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-}\right.}{x^2\sqrt{-1+cx}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{3cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh^{-}\right.}{x^2\sqrt{-1+cx}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh\right.}{x^2\sqrt{-1+c}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{\left(d^2\sqrt{d-c^2dx^2}\right)\int \frac{\left(a+b\cosh\right.}{x^2\sqrt{-1+c}}}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{15cd^2\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{8b(1+n)\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{4^{-3-n}cd^2e^{-\frac{4a}{b}}\sqrt{d-c^2dx^2}\,(}{}$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{\left(d-c^2dx^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^n}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2,x]
[Out] Integrate[((d - c^2*d*x^2)^(5/2)*(a + b*ArcCosh[c*x])^n)/x^2, x]
```

fricas [A] time = 0.62, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(c^4d^2x^4 - 2\,c^2d^2x^2 + d^2\right)\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^n}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="frica
s")
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*sqrt(-c^2*d*x^2 + d)*(b*arccos
h(c*x) + a)^n/x^2, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2 d\,x^2+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

```
[Out] int((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(-c^2dx^2+d\right)^{\frac{5}{2}}\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^n}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c^2*d*x^2+d)^(5/2)*(a+b*arccosh(c*x))^n/x^2,x, algorithm="maxim
a")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^n/x^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^n\,\left(d-c^2\,d\,x^2\right)^{5/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(5/2))/x^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c**2*d*x**2+d)**(5/2)*(a+b*acosh(c*x))**n/x**2,x)
```

```
[Out] Timed out
```

3.434 $\int \frac{x^3(a+b\cosh^{-1}(cx))^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=323

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-cx}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\,\left(a+b\cosh^{-1}(\right.}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c*x-1
)^(1/2)/c^4/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)+3/8*(a+b*
arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/exp(a/b)
/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-3/8*exp(a/b)*(a+b*arccosh(c*x))
^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b
)^n)/(-c*x+1)^(1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,
3*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c*x+
1)^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 375, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{1-c^2x^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[1 - c^2*x^2]*(-((a
+ b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c
*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[1 - c^2
*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c
*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4*Sqr
t[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])/(8*c^4*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)^n\cosh^3(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{3}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)^n\cosh(3x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{1-c^2x^2}}+\frac{\left(3\sqrt{-1\right.}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^4\sqrt{1-c^2x^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{\right.}$$
$$= \frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\right.}{8c^4\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.31, size = 292, normalized size = 0.90

$$3^{-n-1}e^{-\frac{3a}{b}}\sqrt{1-c^2x^2}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{n+2}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\Gamma\left(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[1 - c^2*x^2]*(a + b*ArcCosh[c*x])^n*(3^(2 + n)*E^((4*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 +
n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-((a + b*ArcCosh[c*x])^2/
b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] + 3^(2 + n)*E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)] -
```

```
E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma[1 + n, (3*(a + b*ArcCos
h[c*x]))/b])))/(8*c^4*E^((3*a)/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((
a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)^n x^3}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^3/(c^2*x^2 - 1), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a+b\,\text{arccosh}\,(cx))^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^3}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))^n}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.435 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=211

$$\frac{\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{2bc^3(n+1)\sqrt{1-cx}}+\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{1-cx}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c^3/(1+n)/(-c*x+1)^(1/2)+2^(-3
-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c
^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-2^(-3-n)*exp(2*a/b
)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^3/
(((a+b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.62, antiderivative size = 250, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{1-c^2x^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n
)*Sqrt[1 - c^2*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*Sqr
t[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)*Sqr
t[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b*Arc
Cosh[c*x]))/b])/(c^3*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
```

```
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^3\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx, x, \cosh$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+\right.}{2c^3\sqrt{1-}}$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-2x}\right.}{4c^3\sqrt{1-c}}$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{1-c^2x^2}} + \frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\right.}{}$$

Mathematica [A] time = 0.81, size = 212, normalized size = 1.00

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{n+2}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right.\right.}{bc^3(n+}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[1 - c^2*x^2]*(-((a
+ b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)^n x^2}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x^2/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2x^2 + 1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(c\,x))^n}{\sqrt{1 - c^2\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^n}{\sqrt{-(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.436 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=154

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-cx}}-\frac{e^{a/b}\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^n\left(\right.}{2c^2}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2
/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c*x+1)^(1/2)-1/2*exp(a/b)*(a+b*arcc
osh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)/c^2/(((a+b*arccos
h(c*x))/b)^n)/(-c*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 180, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {5798, 5781, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{1-c^2x^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[1 - c^2*x^2]*(-((a + b*ArcCosh[c*x])
/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamm
a[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[1 - c^2*x^2]*((a + b*ArcCosh[
c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh(x)\,dx, x, \cosh^{-1}(cx)\right)}{c^2\sqrt{1-c^2x^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx, x, \cosh^{-1}(cx)\right)}{2c^2\sqrt{1-c^2x^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1+c\right.}$$
$$= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}(c\right.}{b}}{2c^2\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.25, size = 154, normalized size = 1.00

$$-\frac{e^{-\frac{a}{b}}\sqrt{-((cx-1)(cx+1))}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-\right.}{2c^2\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] -1/2*(Sqrt[-((-1 + c*x)*(1 + c*x))]*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)*(
-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + Arc
Cosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(c^2*E^(a/b)*Sqrt[(-
1 + c*x)/(1 + c*x)]*(1 + c*x)*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)^n x}{c^2x^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n*x/(c^2*x^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

maple [F] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x\,(a + b\,\mathrm{arccosh}\,(cx))^{n}}{\sqrt{-c^{2}x^{2} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)^{n}x}{\sqrt{-c^{2}x^{2} + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(a + b\,\mathrm{acosh}\,(c\,x))^{n}}{\sqrt{1 - c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2),x)
[Out] int((x*(a + b*acosh(c*x))^n)/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\,(a + b\,\mathrm{acosh}\,(cx))^{n}}{\sqrt{-(cx - 1)\,(cx + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.437 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=43

$$\frac{\sqrt{cx-1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-cx}}$$

[Out] (a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)/b/c/(1+n)/(-c*x+1)^(1/2)

Rubi [A] time = 0.21, antiderivative size = 56, normalized size of antiderivative = 1.30, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]

[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sqrt[1 - c^2*x^2])

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}\\ &= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{1-c^2x^2}}\end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 1.30

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{1-c^2x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[1 - c^2*x^2])
```

fricas [B] time = 0.55, size = 213, normalized size = 4.95

$$\frac{\left(\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\sqrt{-c^2x^2+1}\,a\right)\cosh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right.\right.\right.}{bcn -}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + sqr
t(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^2 -
1)) + a)) + (sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*b*log(c*x + sqrt(c^2*x^2
- 1)) + sqrt(c^2*x^2 - 1)*sqrt(-c^2*x^2 + 1)*a)*sinh(n*log(b*log(c*x + sqrt
(c^2*x^2 - 1)) + a)))/(b*c*n - (b*c^3*n + b*c^3)*x^2 + b*c)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

maple [A] time = 0.06, size = 53, normalized size = 1.23

$$\frac{(a+b\,\mathrm{arccosh}\,(cx))^{1+n}\sqrt{cx-1}\,\sqrt{cx+1}}{b\,(1+n)\,c\sqrt{-(cx-1)\,(cx+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-(c*x-1)*(c*x+1))^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^n}{\sqrt{1-c^2\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(1 - c^2*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{\sqrt{-(cx-1)(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.438 $\int \frac{(a+b\cosh^{-1}(cx))^n}{x\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=31

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 2.66, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[1 - c^2*x^2]), x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2x^3-x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^3 - x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

maple [A] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}}{x\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{\sqrt{-c^{2}x^{2}+1}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}}{x\,\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^{n}}{x\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

3.439 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=31

$$\operatorname{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.45, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 1.40, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[1 - c^2*x^2]), x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,(b\operatorname{arcosh}(cx)+a)^n}{c^2x^4-x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(b*arccosh(c*x) + a)^n/(c^2*x^4 - x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2x^2+1}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}}{x^{2}\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{\sqrt{-c^{2}x^{2}+1}\,x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*x^2+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*x^2 + 1)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}}{x^{2}\,\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x^2*(1 - c^2*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^{n}}{x^{2}\sqrt{-(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-(c*x - 1)*(c*x + 1))), x)
```

3.440 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=379

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx-1}\,\sqrt{c}}{}$$

```
[Out] 1/8*3^(-1-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-3*(a+b*arccosh(c*x))/b)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^4/exp(3*a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^
2+d)^(1/2)+3/8*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^4/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+
d)^(1/2)-3/8*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(
1/2)-1/8*3^(-1-n)*exp(3*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,3*(a+b*arccosh(
c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^4/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d
*x^2+d)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{3e^{-\frac{a}{b}}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (3^(-1 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n
, (-3*(a + b*ArcCosh[c*x]))/b])/(8*c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])/b))^n) + (3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh
[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)])/(8*c^4*E^(a/b)*Sqrt[d - c
^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (3*E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (a + b*ArcCosh[c*x])/b])/(8*c^4
*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n) - (3^(-1 - n)*E^((3*a)/b)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (3*(a + b*
ArcCosh[c*x]))/b])/(8*c^4*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)^n\cosh^3(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^4\sqrt{d-c^2dx^2}}$$
$$=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int\left(\frac{3}{4}(a+bx)^n\cosh(x)+\frac{1}{4}(a+bx)^n\cosh(3x)\right)dx,x,\right.}{c^4\sqrt{d-c^2dx^2}}$$
$$=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int(a+bx)^n\cosh(3x)\,dx,x,\cosh^{-1}(cx)\right)}{4c^4\sqrt{d-c^2dx^2}}+\frac{\left(3\sqrt{-1+c\right.}{}$$
$$=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-3x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{8c^4\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+\right.}{}$$
$$=\frac{3^{-1-n}e^{-\frac{3a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{3(a+\right.}{}}{8c^4\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 1.10, size = 291, normalized size = 0.77

$$-\frac{3^{-n-1}e^{-\frac{3a}{b}}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-2n}\left(3^{n+2}e^{\frac{4a}{b}}\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] -1/8*(3^(-1 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^
n*(3^(2 + n)*E^((4*a)/b)*(-((a + b*ArcCosh[c*x])/b))^n*(-((a + b*ArcCosh[c*
x])^2/b^2))^n*Gamma[1 + n, a/b + ArcCosh[c*x]] - (a/b + ArcCosh[c*x])^n*((-
((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, (-3*(a + b*ArcCosh[c*x]))/b] +
 3^(2 + n)*E^((2*a)/b)*(-((a + b*ArcCosh[c*x])^2/b^2))^n*Gamma[1 + n, -((a
+ b*ArcCosh[c*x])/b)] - E^((6*a)/b)*(-((a + b*ArcCosh[c*x])/b))^(2*n)*Gamma
```

```
[1 + n, (3*(a + b*ArcCosh[c*x]))/b])))/(c^4*E^((3*a)/b)*Sqrt[d - c^2*d*x^2]
*(-((a + b*ArcCosh[c*x])^2/b^2))^(2*n))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x^3}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^3/(c^2*d*x^2 - d),
x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\,(a+b\,\text{arccosh}\,(cx))^n}{\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^3}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^3/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(a+b\,\text{acosh}\,(c\,x))^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^3*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \left(a + b \operatorname{acosh}\left(cx\right)\right)^n}{\sqrt{-d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**3*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.441 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=253

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{2bc^3(n+1)\sqrt{d-c^2dx^2}}+\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(\right.}{c^3\sqrt{d-c^2dx^2}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c^3/(1+n)/(-c^2*
d*x^2+d)^(1/2)+2^(-3-n)*(a+b*arccosh(c*x))^n*GAMMA(1+n,-2*(a+b*arccosh(c*x)
)/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/exp(2*a/b)/(((-a-b*arccosh(c*x))/b)^n)
/(-c^2*d*x^2+d)^(1/2)-2^(-3-n)*exp(2*a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,2*
(a+b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/(((a+b*arccosh(c*x))/
b)^n)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {5798, 5781, 3312, 3307, 2181}

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{2\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{c^3\sqrt{d-c^2dx^2}}-\frac{2^{-n-3}e^{\frac{2a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(2*b*c^3*(1 + n
)*Sqrt[d - c^2*d*x^2]) + (2^(-3 - n)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*Ar
cCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b])/(c^3*E^((2*a)/b)*S
qrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x])/b))^n) - (2^(-3 - n)*E^((2*a)/b)
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, (2*(a + b
*ArcCosh[c*x]))/b])/(c^3*Sqrt[d - c^2*d*x^2]*((a + b*ArcCosh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
```

```
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh^2(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int \left(\frac{1}{2}(a+bx)^n+\frac{1}{2}(a+bx)^n\cosh(2x)\right)dx,x,\cosh^{-1}(c\right.}{c^3\sqrt{d-c^2dx^2}}$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx\right.}{2c^3\sqrt{d-c^2}}$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-2x}(a\right.}{4c^3\sqrt{d-c^2dx}}$$
$$= \frac{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{2bc^3(1+n)\sqrt{d-c^2dx^2}}+\frac{2^{-3-n}e^{-\frac{2a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\,\text{co}\right.}{}$$

Mathematica [A] time = 0.77, size = 213, normalized size = 0.84

$$\frac{2^{-n-3}e^{-\frac{2a}{b}}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(2^{n+2}e^{\frac{2a}{b}}\left(a+b\cosh^{-1}(cx)\right)\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^n\right.}{bc^3(n+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (2^(-3 - n)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(2^
(2 + n)*E^((2*a)/b)*(a + b*ArcCosh[c*x])*(-((a + b*ArcCosh[c*x])^2/b^2))^n
+ b*(1 + n)*(a/b + ArcCosh[c*x])^n*Gamma[1 + n, (-2*(a + b*ArcCosh[c*x]))/b
] - b*E^((4*a)/b)*(1 + n)*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, (2*(a
+ b*ArcCosh[c*x]))/b]))/(b*c^3*E^((2*a)/b)*(1 + n)*Sqrt[d - c^2*d*x^2]*(-((
a + b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x^2}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^2*d*x^2 - d),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{arccosh}(cx))^n}{\sqrt{-c^2 d\, x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n x^2}{\sqrt{-c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(c\, x))^n}{\sqrt{d - c^2\, d\, x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (a + b \operatorname{acosh}(cx))^n}{\sqrt{-d (cx - 1) (cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.442 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=182

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\,c\right.}{}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^n*GAMMA(1+n,(-a-b*arccosh(c*x))/b)*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/c^2/exp(a/b)/(((-a-b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)-1/
2*exp(a/b)*(a+b*arccosh(c*x))^n*GAMMA(1+n,(a+b*arccosh(c*x))/b)*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/c^2/(((a+b*arccosh(c*x))/b)^n)/(-c^2*d*x^2+d)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {5798, 5781, 3307, 2181}

$$\frac{e^{-\frac{a}{b}}\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\text{Gamma}\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{2c^2\sqrt{d-c^2dx^2}}-\frac{e^{a/b}\sqrt{cx-1}\sqrt{cx+1}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Gamma[1 + n, -((a + b*
ArcCosh[c*x])/b)])/(2*c^2*E^(a/b)*Sqrt[d - c^2*d*x^2]*(-((a + b*ArcCosh[c*x
])/b))^n) - (E^(a/b)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^n*Ga
mma[1 + n, (a + b*ArcCosh[c*x])/b])/(2*c^2*Sqrt[d - c^2*d*x^2]*((a + b*ArcC
osh[c*x])/b)^n)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rule 5798

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_), x_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p
```

```
])/((1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(f*x)^m*(1 + c*x)^p*
(-1 + c*x)^p*(a + b*ArcCosh[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && EqQ[c^2*d + e, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int (a+bx)^n\cosh(x)\,dx,x,\cosh^{-1}(cx)\right)}{c^2\sqrt{d-c^2dx^2}}$$
$$= \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\text{Subst}\left(\int e^{-x}(a+bx)^n\,dx,x,\cosh^{-1}(cx)\right)}{2c^2\sqrt{d-c^2dx^2}} + \frac{\left(\sqrt{-1+cx}\,\sqrt{1\right.}{}$$
$$= \frac{e^{-\frac{a}{b}}\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{a+b\cosh^{-1}(cx)}{b}\right)^{-n}\Gamma\left(1+n,-\frac{a+b\cosh^{-1}}{b}\right.}{2c^2\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.23, size = 153, normalized size = 0.84

$$\frac{e^{-\frac{a}{b}}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\left(a+b\cosh^{-1}(cx)\right)^n\left(-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}\right)^{-n}\left(\left(\frac{a}{b}+\cosh^{-1}(cx)\right)^n\Gamma\left(n+1,-\frac{a+b\cosh^{-1}(cx)}{b}\right)-e^{\frac{2a}{b}}\left(\right.\right.}{2c^2\sqrt{-d(cx-1)(cx+1)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*(a + b*ArcCosh[c*x])^n*(-(E^((2*a)/b)
*(-((a + b*ArcCosh[c*x])/b))^n*Gamma[1 + n, a/b + ArcCosh[c*x]]) + (a/b + A
rcCosh[c*x])^n*Gamma[1 + n, -((a + b*ArcCosh[c*x])/b)]))/(2*c^2*E^(a/b)*Sqr
t[-(d*(-1 + c*x)*(1 + c*x))]*(-((a + b*ArcCosh[c*x])^2/b^2))^n)
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^2*d*x^2 - d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x \left(a+b \,\mathrm{arccosh}\left(c x \right)\right)^{n}}{\sqrt{-c^{2} d \,x^{2}+d}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x}{\sqrt{-c^{2} d x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}}{\sqrt{- d \left(c x - 1\right) \left(c x + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.443 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=57

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

```
[Out] (a+b*arccosh(c*x))^(1+n)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(1+n)/(-c^2*d*x^2+
d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {5713, 5676}

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

Rule 5676

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(Sqrt[(d1_) + (e1_.)*(x_)]*Sq
rt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(a + b*ArcCosh[c*x])^(n + 1)/(b
*c*Sqrt[-(d1*d2)]*(n + 1)), x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] &&
 EqQ[e1, c*d1] && EqQ[e2, -(c*d2)] && GtQ[d1, 0] && LtQ[d2, 0] && NeQ[n, -1
]
```

Rule 5713

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_), x
_Symbol] :> Dist[((-d)^IntPart[p]*(d + e*x^2)^FracPart[p])/((1 + c*x)^FracP
art[p]*(-1 + c*x)^FracPart[p]), Int[(1 + c*x)^p*(-1 + c*x)^p*(a + b*ArcCosh
[c*x])^n, x], x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[c^2*d + e, 0] &&
  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx &= \frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}\\ &= \frac{\sqrt{-1+cx}\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^{1+n}}{bc(1+n)\sqrt{d-c^2dx^2}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 1.00

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)^{n+1}}{bc(n+1)\sqrt{d-c^2dx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(1 + n))/(b*c*(1 + n)*Sq
rt[d - c^2*d*x^2])
```

fricas [B] time = 0.57, size = 221, normalized size = 3.88

$$\frac{\left(\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,b\log\left(cx+\sqrt{c^2x^2-1}\right)+\sqrt{-c^2dx^2+d}\,\sqrt{c^2x^2-1}\,a\right)\cosh\left(n\log\left(b\log\left(cx+\sqrt{c^2x^2-1}\right.\right.\right.}{bcdn+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] ((sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^2*x^2 - 1)) + s
qrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*cosh(n*log(b*log(c*x + sqrt(c^2*x^
2 - 1)) + a)) + (sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*b*log(c*x + sqrt(c^
2*x^2 - 1)) + sqrt(-c^2*d*x^2 + d)*sqrt(c^2*x^2 - 1)*a)*sinh(n*log(b*log(c*
x + sqrt(c^2*x^2 - 1)) + a)))/(b*c*d*n + b*c*d - (b*c^3*d*n + b*c^3*d)*x^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

maple [A] time = 0.06, size = 54, normalized size = 0.95

$$\frac{(a+b\operatorname{arccosh}(cx))^{1+n}\sqrt{cx-1}\,\sqrt{cx+1}}{b(1+n)c\sqrt{-(cx-1)(cx+1)d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] (a+b*arccosh(c*x))^(1+n)/b/(1+n)/c/(-(c*x-1)*(c*x+1)*d)^(1/2)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx)+a)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+b\operatorname{acosh}(c\,x))^n}{\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2),x)
[Out] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))^n}{\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
[Out] Integral((a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.444 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*Sqrt[d - c^2*d*x^2]), x]
```

fricas [A] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2dx^3-dx},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^3 - d*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^n}{x\sqrt{-c^2d\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^n}{x\,\sqrt{d-c^2\,d\,x^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^n}{x\sqrt{-d\,(cx-1)\,(cx+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.445 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.46, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*Sqrt[d - c^2*d*x^2]), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{c^2dx^4-dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="frica
s")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^2*d*x^4 - d*x^2),
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2dx^2+d}\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(cx))^n}{x^2 \sqrt{-c^2 d x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^n}{\sqrt{-c^2 d x^2 + d}\, x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/(sqrt(-c^2*d*x^2 + d)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a + b \operatorname{acosh}(c\,x))^n}{x^2\,\sqrt{d - c^2\,d\,x^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^n}{x^2 \sqrt{-d(cx - 1)(cx + 1)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x**2*sqrt(-d*(c*x - 1)*(c*x + 1))), x)
```

3.446 $\int \frac{x^2(a+b\cosh^{-1}(cx))^n}{(d-c^2dx^2)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(x^2*(a + b*ArcCosh[c*x])^n)/((-
1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x^2*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x^2}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x^2/(c^4*d^2*x^4 - 2*c
^2*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x^2}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a+b \,\mathrm{arccosh}\left(c x \right)\right)^{n}}{\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x^{2}}{{\left(-c^{2} d x^{2} + d\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n*x^2/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x^2\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int((x^2*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2} \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.447 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=30

$$\text{Int}\left(\frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(x*(a + b*ArcCosh[c*x])^n)/((-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx = -\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(x*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n x}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n*x/(c^4*d^2*x^4 - 2*c^2
*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n x}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x \left(a+b \,\mathrm{arccosh}\left(c x \right)\right)^{n}}{\left(-c^{2} d \,x^{2}+d \right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
[Out] int(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)}^{n} x}{{\left(-c^{2} d x^{2} + d\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
[Out] integrate((b*arccosh(c*x) + a)^n*x/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n}{{\left(d-c^2\,d\,x^2\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2),x)
[Out] int((x*(a + b*acosh(c*x))^n)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{n}}{\left(- d \left(c x - 1\right) \left(c x + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral(x*(a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.448 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/((-1 + c*
x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(d - c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d
^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}}{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
[Out] int((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}}{\left(d-c^{2}\,d\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2),x)
[Out] int((a + b*acosh(c*x))^n/(d - c^2*d*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^{n}}{(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
[Out] Integral((a + b*acosh(c*x))**n/(-d*(c*x - 1)*(c*x + 1))**(3/2), x)
```

3.449 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x*(d - c^2*d*x^2)^(3/2)), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\text{arcosh}\,(cx)+a)^n}{c^4d^2x^5-2\,c^2d^2x^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^5 - 2*c^2*d
^2*x^3 + d^2*x), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}}{x\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}}{x\left(d-c^{2}\,d\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^{n}}{x\,(-d\,(cx-1)\,(cx+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**n/(x*(-d*(c*x - 1)*(c*x + 1))**(3/2)), x)
```

3.450 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=32

$$\mathrm{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.52, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][(a + b*ArcCosh[c*x])^n/(x^2*(-1
+ c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^n}{x^2\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])^n/(x^2*(d - c^2*d*x^2)^(3/2)), x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,(b\,\mathrm{arcosh}\,(cx)+a)^n}{c^4d^2x^6-2\,c^2d^2x^4+d^2x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="frica
s")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^6 - 2*c^2*d
^2*x^4 + d^2*x^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^n}{\left(-c^2dx^2+d\right)^{\frac{3}{2}}x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="giac"
)
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{n}}{x^{2}\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^n/x^2/(-c^2*d*x^2+d)^(3/2),x, algorithm="maxim
a")
```

```
[Out] integrate((b*arccosh(c*x) + a)^n/((-c^2*d*x^2 + d)^(3/2)*x^2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{n}}{x^{2}\,\left(d-c^{2}\,d\,x^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)),x)
```

```
[Out] int((a + b*acosh(c*x))^n/(x^2*(d - c^2*d*x^2)^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**n/x**2/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.451 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$

Optimal. Leaf size=33

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

Rubi [A] time = 0.42, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[1 - c^2*x^2]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx=\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{1-c^2x^2}}$$

Mathematica [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{1-c^2x^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[1 - c^2*x^2], x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2x^2+1}\,\left(fx\right)^m(b\,\text{arcosh}\,(cx)+a)^n}{c^2x^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(-sqrt(-c^2*x^2 + 1)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*x^2 - 1),
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m(b\,\text{arcosh}\,(cx)+a)^n}{\sqrt{-c^2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{\sqrt{-c^{2}x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*x^2+1)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*x^2 + 1), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{n}\,\left(f\,x\right)^{m}}{\sqrt{1-c^{2}\,x^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(1 - c^2*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{n}}{\sqrt{-\left(cx-1\right)\left(cx+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*x**2+1)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-(c*x - 1)*(c*x + 1)), x)
```

3.452 $\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=32

$$\text{Int}\left(\left(d-c^2dx^2\right)^2 (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n ,x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Mathematica [A] time = 1.22, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^2*(a + b*ArcCosh[c*x])^n, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c^4d^2x^4-2\,c^2d^2x^2+d^2\right)\left(fx\right)^m \left(b\,\text{arcosh}\left(cx\right)+a\right)^n ,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="frica
s")
[Out] integral((c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)*(f*x)^m*(b*arccosh(c*x) + a)^n
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="giac"
)
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(-c^2 d x^2 + d\right)^2 (a + b \operatorname{arccosh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c^2 dx^2 - d\right)^2 (fx)^m (b \operatorname{arcosh}(cx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^2*(a+b*arccosh(c*x))^n,x, algorithm="maxim
a")
```

```
[Out] integrate((c^2*d*x^2 - d)^2*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int (a + b \operatorname{acosh}(c\, x))^n \left(d - c^2\, d\, x^2\right)^2 \left(f\, x\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^2*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**2*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.453 $\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=30

$$\text{Int}\left(\left(d-c^2dx^2\right)(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
[Out] Defer[Int][(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx = \int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Mathematica [A] time = 0.62, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d-c^2dx^2\right) \left(a+b\cosh^{-1}(cx)\right)^n \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)*(a + b*ArcCosh[c*x])^n, x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2dx^2-d\right)\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="fricas"
)
[Out] integral(-(c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2} d\,x^{2}+d\right) \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(c^{2}dx^{2}-d\right) \left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)*(a+b*arccosh(c*x))^n,x, algorithm="maxima"
)
```

```
[Out] -integrate((c^2*d*x^2 - d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\left(d-c^2\,d\,x^2\right)\,{\left(f\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.454 $\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n \,dx$

Optimal. Leaf size=19
$$\mathrm{Int}\left((fx)^m \left(a+b\cosh^{-1}(cx)\right)^n, x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}
$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n \,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
[Out] Defer[Int][(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

Rubi steps
$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n \,dx = \int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n \,dx$$

Mathematica [A] time = 0.02, size = 0, normalized size = 0.00
$$\int (fx)^m \left(a+b\cosh^{-1}(cx)\right)^n \,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*(a + b*ArcCosh[c*x])^n, x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00
$$\mathrm{integral}\left(\left(fx\right)^m (b\,\mathrm{arcosh}\,(cx)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="fricas")
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.13, size = 0, normalized size = 0.00
$$\int \left(fx\right)^m (a+b\,\mathrm{arccosh}\,(cx))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (b \operatorname{arcosh}(cx) + a)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n,x, algorithm="maxima")
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int (a + b \operatorname{acosh}(c\,x))^n \left(f\,x\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(f*x)^m,x)
[Out] int((a + b*acosh(c*x))^n*(f*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{acosh}(cx))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n,x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n, x)
```

3.455 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Mathematica [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{d-c^2dx^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="fricas"
)
[Out] integral(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^2dx^2-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(-(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}}{-c^{2} d\,x^{2}+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{c^{2} d x^{2}-d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d),x, algorithm="maxima"
)
```

```
[Out] -integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(f\,x\right)}^m}{d-c^2\,d\,x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{n}}{c^{2} x^{2}-1}\,dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d),x)
```

```
[Out] -Integral((f*x)**m*(a + b*acosh(c*x))**n/(c**2*x**2 - 1), x)/d
```

3.456 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$

Optimal. Leaf size=32

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

Rubi [A] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
```

```
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Mathematica [A] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2,x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^2, x]
```

fricas [A] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="frica
s")
```

```
[Out] integral((f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 - 2*c^2*d^2*x^2 + d^2)
, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{\left(c^2dx^2-d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="giac"
)
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

maple [A] time = 0.48, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}}{\left(-c^{2}d\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}}{\left(c^{2}dx^{2}-d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^2,x, algorithm="maxim
a")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 - d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)^{n}\,\left(f\,x\right)^{m}}{\left(d-c^{2}\,d\,x^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2,x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.457 $\int (fx)^m \left(d-c^2dx^2\right)^{3/2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=34

$$\text{Int}\left(\left(d-c^2dx^2\right)^{3/2}(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] -((d*Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*(-1 + c*x)^(3/2)*(1 + c*x)^(3/2
)*(a + b*ArcCosh[c*x])^n, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

Rubi steps

$$\int (fx)^m\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=-\frac{\left(d\sqrt{d-c^2dx^2}\right)\int (fx)^m(-1+cx)^{3/2}(1+cx)^{3/2}\left(a+b\cosh^{-1}(cx\right.}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.67, size = 0, normalized size = 0.00

$$\int (fx)^m\left(d-c^2dx^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n,x]
```

```
[Out] Integrate[(f*x)^m*(d - c^2*d*x^2)^(3/2)*(a + b*ArcCosh[c*x])^n, x]
```

fricas [A] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(c^2dx^2-d\right)\sqrt{-c^2dx^2+d}\,(fx)^m\,(b\,\text{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
```

```
[Out] integral(-(c^2*d*x^2 - d)*sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)
^n, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
```

```
[Out] Timed out
```

maple [A] time = 0.51, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}} \left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(3/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate((-c^2*d*x^2 + d)^(3/2)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(d-c^2\,d\,x^2\right)}^{3/2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(3/2)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(3/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.458 $\int (fx)^m \sqrt{d-c^2dx^2} \left(a+b\cosh^{-1}(cx)\right)^n \, dx$

Optimal. Leaf size=34

$$\text{Int}\left(\sqrt{d-c^2dx^2}\,(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n,x\right)$$

```
[Out] Unintegrable((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

Rubi [A] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
[Out] (Sqrt[d - c^2*d*x^2]*Defer[Int][(f*x)^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b
*ArcCosh[c*x])^n, x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rubi steps

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx=\frac{\sqrt{d-c^2dx^2}\int (fx)^m\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)^n\,dx}{\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.13, size = 0, normalized size = 0.00

$$\int (fx)^m\sqrt{d-c^2dx^2}\left(a+b\cosh^{-1}(cx)\right)^n\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n,x]
[Out] Integrate[(f*x)^m*Sqrt[d - c^2*d*x^2]*(a + b*ArcCosh[c*x])^n, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-c^2dx^2+d}\,\left(fx\right)^m(b\,\text{arcosh}\,(cx)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="f
ricas")
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="g
iac")
[Out] Timed out
```

maple [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \sqrt{-c^{2} d\,x^{2}+d}\, \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

```
[Out] int((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-c^{2}dx^{2}+d} \left(fx\right)^{m} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(-c^2*d*x^2+d)^(1/2)*(a+b*arccosh(c*x))^n,x, algorithm="m
axima")
```

```
[Out] integrate(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,\sqrt{d-c^2\,d\,x^2}\,{\left(f\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m,x)
```

```
[Out] int((a + b*acosh(c*x))^n*(d - c^2*d*x^2)^(1/2)*(f*x)^m, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(-c**2*d*x**2+d)**(1/2)*(a+b*acosh(c*x))**n,x)
```

```
[Out] Timed out
```

3.459 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] (Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x])/Sqrt[d - c^2*d*x^2]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx = \frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\sqrt{d-c^2dx^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2],x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/Sqrt[d - c^2*d*x^2], x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-c^2dx^2+d}\,\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^2dx^2-d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(-sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^2*d*x^2 -
d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{\sqrt{-c^2dx^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b \,\mathrm{arccosh}\left(cx\right)\right)^{n}}{\sqrt{-c^{2} d \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} {\left(b \operatorname{arcosh}\left(cx\right) + a\right)}^{n}}{\sqrt{-c^{2} d x^{2} + d}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/sqrt(-c^2*d*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(f\,x\right)}^m}{\sqrt{d-c^2\,d\,x^2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{n}}{\sqrt{- d \left(cx - 1\right) \left(cx + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))**n/sqrt(-d*(c*x - 1)*(c*x + 1)), x)
```

3.460 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x])^n)
/((-1 + c*x)^(3/2)*(1 + c*x)^(3/2)), x])/(d*Sqrt[d - c^2*d*x^2]))
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx=-\frac{\left(\sqrt{-1+cx}\,\sqrt{1+cx}\right)\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{(-1+cx)^{3/2}(1+cx)^{3/2}}\,dx}{d\sqrt{d-c^2dx^2}}$$

Mathematica [A] time = 0.72, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)^n}{\left(d-c^2dx^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2),x]
```

```
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x])^n)/(d - c^2*d*x^2)^(3/2), x]
```

fricas [A] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-c^2dx^2+d}\,\left(fx\right)^m\left(b\,\text{arcosh}\left(cx\right)+a\right)^n}{c^4d^2x^4-2\,c^2d^2x^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(-c^2*d*x^2 + d)*(f*x)^m*(b*arccosh(c*x) + a)^n/(c^4*d^2*x^4 -
 2*c^2*d^2*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="g
iac")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(a+b\operatorname{arccosh}\left(cx\right)\right)^{n}}{\left(-c^{2}d\,x^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m}\left(b\operatorname{arcosh}\left(cx\right)+a\right)^{n}}{\left(-c^{2}dx^{2}+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))^n/(-c^2*d*x^2+d)^(3/2),x, algorithm="m
axima")
```

```
[Out] integrate((f*x)^m*(b*arccosh(c*x) + a)^n/(-c^2*d*x^2 + d)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^n\,{\left(f\,x\right)}^m}{{\left(d-c^2\,d\,x^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2),x)
```

```
[Out] int(((a + b*acosh(c*x))^n*(f*x)^m)/(d - c^2*d*x^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))**n/(-c**2*d*x**2+d)**(3/2),x)
```

```
[Out] Timed out
```

3.461 $\int x^4 \left(d+ex^2\right) \left(a+b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=177

$$\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{8b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(49c^2d+30e\right)}{3675c^7}-\frac{4bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(49c\right.}{3675c^5}$$

```
[Out] 1/5*d*x^5*(a+b*arccosh(c*x))+1/7*e*x^7*(a+b*arccosh(c*x))-8/3675*b*(49*c^2*
d+30*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-4/3675*b*(49*c^2*d+30*e)*x^2*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c^5-1/1225*b*(49*c^2*d+30*e)*x^4*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c^3-1/49*b*e*x^6*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5786, 460, 100, 12, 74}

$$\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{bx^4\sqrt{cx-1}\,\sqrt{cx+1}\,\left(49c^2d+30e\right)}{1225c^3}-\frac{4bx^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(49\right.}{3675c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-8*b*(49*c^2*d + 30*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^7) - (4*b*(49
*c^2*d + 30*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(3675*c^5) - (b*(49*c^2*d
+ 30*e)*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(1225*c^3) - (b*e*x^6*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/(49*c) + (d*x^5*(a + b*ArcCosh[c*x]))/5 + (e*x^7*(a + b*
ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x^4\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{35}(bc)\int\frac{x^5(}{\sqrt{-1}}$$

$$= -\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+cx}}{49c}+\frac{1}{5}dx^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}ex^7\left(a+b\cos\right.$$

$$= -\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+cx}}{49c}+\frac{1}{5}$$

$$= -\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+cx}\,\sqrt{1+cx}}{1225c^3}-\frac{bex^6\sqrt{-1+cx}\,\sqrt{1+cx}}{49c}+\frac{1}{5}$$

$$= -\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+c}}{1225c^3}$$

$$= -\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^5}-\frac{b\left(49c^2d+30e\right)x^4\sqrt{-1+c}}{1225c^3}$$

$$= -\frac{8b\left(49c^2d+30e\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{3675c^7}-\frac{4b\left(49c^2d+30e\right)x^2\sqrt{-1+cx}}{3675c^5}$$

Mathematica [A] time = 0.11, size = 122, normalized size = 0.69

$$\frac{1}{35}ax^5\left(7d+5ex^2\right)-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\left(3c^6\left(49dx^4+25ex^6\right)+2c^4\left(98dx^2+45ex^4\right)+8c^2\left(49d+15ex^2\right)+240\right.}{3675c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (a*x^5*(7*d + 5*e*x^2))/35 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e + 8*c^2
*(49*d + 15*e*x^2) + 2*c^4*(98*d*x^2 + 45*e*x^4) + 3*c^6*(49*d*x^4 + 25*e*x
^6)))/(3675*c^7) + (b*x^5*(7*d + 5*e*x^2)*ArcCosh[c*x])/35
```

fricas [A] time = 0.69, size = 140, normalized size = 0.79

$$\frac{525\,ac^7ex^7+735\,ac^7dx^5+105\left(5\,bc^7ex^7+7\,bc^7dx^5\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(75\,bc^6ex^6+3\left(49\,bc^6d+30\,bc^4e\right.\right.}{3675\,c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e*x^7 + 735*a*c^7*d*x^5 + 105*(5*b*c^7*e*x^7 + 7*b*c^7*d*
x^5)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e*x^6 + 3*(49*b*c^6*d + 30*b*
c^4*e)*x^4 + 392*b*c^2*d + 4*(49*b*c^4*d + 30*b*c^2*e)*x^2 + 240*b*e)*sqrt(
c^2*x^2 - 1))/c^7
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.03, size = 133, normalized size = 0.75

$$\frac{\frac{a\left(\frac{1}{7}e\,c^{7}x^{7}+\frac{1}{5}c^{7}x^{5}d\right)}{c^{2}}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e\,c^{7}x^{7}}{7}+\frac{\operatorname{arccosh}(cx)c^{7}x^{5}d}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(75c^{6}e\,x^{6}+147c^{6}d\,x^{4}+90c^{4}e\,x^{4}+196c^{4}d\,x^{2}+120c^{2}x^{2}e+392c^{2}d+240e\right)}{3675}\right)}{c^{2}}}{c^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^2*(1/7*e*c^7*x^7+1/5*c^7*x^5*d)+b/c^2*(1/7*arccosh(c*x)*e*c^7*x^
7+1/5*arccosh(c*x)*c^7*x^5*d-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(75*c^6*e*x
^6+147*c^6*d*x^4+90*c^4*e*x^4+196*c^4*d*x^2+120*c^2*e*x^2+392*c^2*d+240*e))
)
```

maxima [A] time = 0.45, size = 178, normalized size = 1.01

$$\frac{1}{7}aex^{7}+\frac{1}{5}adx^{5}+\frac{1}{75}\left(15\,x^{5}\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^{2}x^{2}-1}\,x^{4}}{c^{2}}+\frac{4\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{4}}+\frac{8\sqrt{c^{2}x^{2}-1}}{c^{6}}\right)c\right)bd+\frac{1}{245}\left(35\,x^{7}\operatorname{arco}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e*x^7 + 1/5*a*d*x^5 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1
)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d +
 1/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2
 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*
b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{4}\,\left(a+b\operatorname{acosh}\left(c\,x\right)\right)\,\left(e\,x^{2}+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 5.64, size = 230, normalized size = 1.30

$$\begin{cases}\frac{adx^{5}}{5}+\frac{aex^{7}}{7}+\frac{bdx^{5}\operatorname{acosh}(cx)}{5}+\frac{bex^{7}\operatorname{acosh}(cx)}{7}-\frac{bdx^{4}\sqrt{c^{2}x^{2}-1}}{25c}-\frac{bex^{6}\sqrt{c^{2}x^{2}-1}}{49c}-\frac{4bdx^{2}\sqrt{c^{2}x^{2}-1}}{75c^{3}}-\frac{6bex^{4}\sqrt{c^{2}x^{2}-1}}{245c^{3}}-\frac{8bd\sqrt{c^{2}x^{2}-1}}{75c^{5}}- \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^{5}}{5}+\frac{ex^{7}}{7}\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**5/5 + a*e*x**7/7 + b*d*x**5*acosh(c*x)/5 + b*e*x**7*acosh
(c*x)/7 - b*d*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b*e*x**6*sqrt(c**2*x**2 - 1
)/(49*c) - 4*b*d*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e*x**4*sqrt(c**2*
x**2 - 1)/(245*c**3) - 8*b*d*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e*x**2*sqr
t(c**2*x**2 - 1)/(245*c**5) - 16*b*e*sqrt(c**2*x**2 - 1)/(245*c**7), Ne(c,
0)), ((a + I*pi*b/2)*(d*x**5/5 + e*x**7/7), True))
```

3.462 $\int x^3 \left(d + ex^2\right) \left(a + b\cosh^{-1}(cx)\right) \, dx$

Optimal. Leaf size=161

$$\frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{6}ex^6 \left(a + b\cosh^{-1}(cx)\right) - \frac{b\left(9c^2d + 5e\right)\cosh^{-1}(cx)}{96c^6} - \frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^2d + 5e\right)}{96c^5} - \frac{bx}{}$$

```
[Out] -1/96*b*(9*c^2*d+5*e)*arccosh(c*x)/c^6+1/4*d*x^4*(a+b*arccosh(c*x))+1/6*e*x
^6*(a+b*arccosh(c*x))-1/96*b*(9*c^2*d+5*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
5-1/144*b*(9*c^2*d+5*e)*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*b*e*x^5*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.14, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5786, 460, 100, 12, 90, 52}

$$\frac{1}{4}dx^4 \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{6}ex^6 \left(a + b\cosh^{-1}(cx)\right) - \frac{bx^3\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^2d + 5e\right)}{144c^3} - \frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^2d + \right.}{96c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*(9*c^2*d + 5*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96*c^5) - (b*(9*c^2*d
+ 5*e)*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e*x^5*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])/(36*c) - (b*(9*c^2*d + 5*e)*ArcCosh[c*x])/(96*c^6) + (d*x^
4*(a + b*ArcCosh[c*x]))/4 + (e*x^6*(a + b*ArcCosh[c*x]))/6
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{24}(bc)\int\frac{x^4\left(\right.}{\sqrt{-1}}$$
$$=-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}ex^6\left(a+b\cos\right.$$
$$=-\frac{b\left(9c^2d+5e\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4$$
$$=-\frac{b\left(9c^2d+5e\right)x^3\sqrt{-1+cx}\sqrt{1+cx}}{144c^3}-\frac{bex^5\sqrt{-1+cx}\sqrt{1+cx}}{36c}+\frac{1}{4}dx^4$$
$$=-\frac{b\left(9c^2d+5e\right)x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}-\frac{b\left(9c^2d+5e\right)x^3\sqrt{-1+cx}\sqrt{1+}}{144c^3}$$
$$=-\frac{b\left(9c^2d+5e\right)x\sqrt{-1+cx}\sqrt{1+cx}}{96c^5}-\frac{b\left(9c^2d+5e\right)x^3\sqrt{-1+cx}\sqrt{1+}}{144c^3}$$

Mathematica [A] time = 0.20, size = 140, normalized size = 0.87

$$\frac{24ac^6x^4\left(3d+2ex^2\right)+24bc^6x^4\cosh^{-1}(cx)\left(3d+2ex^2\right)-6b\left(9c^2d+5e\right)\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)-bcx\sqrt{cx-1}\sqrt{cx+}}{288c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (24*a*c^6*x^4*(3*d + 2*e*x^2) - b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15*e +
c^2*(27*d + 10*e*x^2) + 2*c^4*(9*d*x^2 + 4*e*x^4)) + 24*b*c^6*x^4*(3*d + 2*
e*x^2)*ArcCosh[c*x] - 6*b*(9*c^2*d + 5*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)
]])/(288*c^6)
```

fricas [A] time = 0.61, size = 136, normalized size = 0.84

$$\frac{48\,ac^6ex^6+72\,ac^6dx^4+3\left(16\,bc^6ex^6+24\,bc^6dx^4-9\,bc^2d-5\,be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(8\,bc^5ex^5+2\left(9\,bc^5d\right.\right.}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*e*x^6 + 72*a*c^6*d*x^4 + 3*(16*b*c^6*e*x^6 + 24*b*c^6*d*x^4
 - 9*b*c^2*d - 5*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e*x^5 + 2*(9*
b*c^5*d + 5*b*c^3*e)*x^3 + 3*(9*b*c^3*d + 5*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^
6
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 250, normalized size = 1.55

$$\frac{ae\,x^{6}}{6}+\frac{a\,x^{4}d}{4}+\frac{b\,\mathrm{arccosh}\left(cx\right)e\,x^{6}}{6}+\frac{b\,\mathrm{arccosh}\left(cx\right)x^{4}d}{4}-\frac{be\,x^{5}\sqrt{cx-1}\,\sqrt{cx+1}}{36c}-\frac{bd\,x^{3}\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{5b\sqrt{cx-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*e*x^6+1/4*a*x^4*d+1/6*b*arccosh(c*x)*e*x^6+1/4*b*arccosh(c*x)*x^4*d-1
/36*b*e*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/16*b*d*x^3*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/c-5/144/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x^3-3/32*b*d*x*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^3-3/32/c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^
(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d-5/96/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*
x-5/96/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^
2-1)^(1/2))
```

maxima [A] time = 0.36, size = 196, normalized size = 1.22

$$\frac{1}{6}\,aex^{6}+\frac{1}{4}\,adx^{4}+\frac{1}{32}\left(8\,x^{4}\,\mathrm{arcosh}\left(cx\right)-\left(\frac{2\,\sqrt{c^{2}x^{2}-1}\,x^{3}}{c^{2}}+\frac{3\,\sqrt{c^{2}x^{2}-1}\,x}{c^{4}}+\frac{3\,\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\,c\right)}{c^{5}}\right)c\right)bd+\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e*x^6 + 1/4*a*d*x^4 + 1/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)
*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*
c)/c^5)*c)*b*d + 1/288*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2
+ 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*
x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^{3}\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^{2}+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 3.83, size = 212, normalized size = 1.32

$$\begin{cases} \frac{adx^4}{4} + \frac{aex^6}{6} + \frac{bdx^4 \operatorname{acosh}(cx)}{4} + \frac{bex^6 \operatorname{acosh}(cx)}{6} - \frac{bdx^3\sqrt{c^2x^2-1}}{16c} - \frac{bex^5\sqrt{c^2x^2-1}}{36c} - \frac{3bdx\sqrt{c^2x^2-1}}{32c^3} - \frac{5bex^3\sqrt{c^2x^2-1}}{144c^3} - \frac{3bd \operatorname{acosh}(cx}{32c^4} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{dx^4}{4} + \frac{ex^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**4/4 + a*e*x**6/6 + b*d*x**4*acosh(c*x)/4 + b*e*x**6*acosh
(c*x)/6 - b*d*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*e*x**5*sqrt(c**2*x**2 - 1
)/(36*c) - 3*b*d*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*e*x**3*sqrt(c**2*x**
2 - 1)/(144*c**3) - 3*b*d*acosh(c*x)/(32*c**4) - 5*b*e*x*sqrt(c**2*x**2 - 1
)/(96*c**5) - 5*b*e*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**
4/4 + e*x**6/6), True))
```

3.463 $\int x^2 (d+ex^2) (a+b\cosh^{-1}(cx)) \, dx$

Optimal. Leaf size=138

$$\frac{1}{3}dx^3 \left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5 \left(a+b\cosh^{-1}(cx)\right)-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+12e\right)}{225c^5}-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(25c^2\right.}{225c^3}$$

```
[Out] 1/3*d*x^3*(a+b*arccosh(c*x))+1/5*e*x^5*(a+b*arccosh(c*x))-2/225*b*(25*c^2*d
+12*e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-1/225*b*(25*c^2*d+12*e)*x^2*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c^3-1/25*b*e*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.12, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5786, 460, 100, 12, 74}

$$\frac{1}{3}dx^3 \left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5 \left(a+b\cosh^{-1}(cx)\right)-\frac{bx^2\sqrt{cx-1}\sqrt{cx+1}\left(25c^2d+12e\right)}{225c^3}-\frac{2b\sqrt{cx-1}\sqrt{cx+1}\left(25c^2\right.}{225c^5}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*b*(25*c^2*d + 12*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^5) - (b*(25*c^
2*d + 12*e)*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(225*c^3) - (b*e*x^4*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(25*c) + (d*x^3*(a + b*ArcCosh[c*x]))/3 + (e*x^5*(a +
 b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x^2\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{15}(bc)\int\frac{x^3(\ldots}{\sqrt{-1\ldots}}$$

$$= -\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}dx^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}ex^5\left(a+b\cos\ldots\right.$$

$$= -\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}\ldots$$

$$= -\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\sqrt{1+cx}}{225c^3}-\frac{bex^4\sqrt{-1+cx}\sqrt{1+cx}}{25c}+\frac{1}{3}\ldots$$

$$= -\frac{2b\left(25c^2d+12e\right)\sqrt{-1+cx}\sqrt{1+cx}}{225c^5}-\frac{b\left(25c^2d+12e\right)x^2\sqrt{-1+cx}\ldots}{225c^3}$$

Mathematica [A] time = 0.10, size = 101, normalized size = 0.73

$$\frac{1}{225}\left(15ax^3\left(5d+3ex^2\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(25dx^2+9ex^4\right)+2c^2\left(25d+6ex^2\right)+24e\right)}{c^5}+15bx^3\cosh^{-1}(cx\ldots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x^3*(5*d + 3*e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e + 2*c^2*(
25*d + 6*e*x^2) + c^4*(25*d*x^2 + 9*e*x^4)))/c^5 + 15*b*x^3*(5*d + 3*e*x^2)
*ArcCosh[c*x])/225
```

fricas [A] time = 0.60, size = 119, normalized size = 0.86

$$\frac{45\,ac^5ex^5+75\,ac^5dx^3+15\left(3\,bc^5ex^5+5\,bc^5dx^3\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc^4ex^4+50\,bc^2d+\left(25\,bc^4d+12\,b\ldots\right.\right.}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e*x^5 + 75*a*c^5*d*x^3 + 15*(3*b*c^5*e*x^5 + 5*b*c^5*d*x^3)
*log(c*x + sqrt(c^2*x^2 - 1)) - (9*b*c^4*e*x^4 + 50*b*c^2*d + (25*b*c^4*d +
 12*b*c^2*e)*x^2 + 24*b*e)*sqrt(c^2*x^2 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 115, normalized size = 0.83

$$\frac{\frac{a\left(\frac{1}{5}c^5x^5e+\frac{1}{3}c^5x^3d\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^5x^5e}{5}+\frac{\operatorname{arccosh}(cx)c^5x^3d}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4ex^4+25c^4dx^2+12c^2x^2e+50c^2d+24e\right)}{225}\right)}{c^2}}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^2*(1/5*c^5*x^5*e+1/3*c^5*x^3*d)+b/c^2*(1/5*arccosh(c*x)*c^5*x^5*
e+1/3*arccosh(c*x)*c^5*x^3*d-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e*x^4
+25*c^4*d*x^2+12*c^2*e*x^2+50*c^2*d+24*e)))
```

maxima [A] time = 0.38, size = 139, normalized size = 1.01

$$\frac{1}{5}\,aex^5+\frac{1}{3}\,adx^3+\frac{1}{9}\left(3\,x^3\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-1}\,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e*x^5 + 1/3*a*d*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d + 1/75*(15*x^5*arccosh(c*x) - (3*sq
rt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)
/c^6)*c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 2.15, size = 178, normalized size = 1.29

$$\begin{cases} \frac{adx^3}{3}+\frac{aex^5}{5}+\frac{bdx^3\operatorname{acosh}\left(cx\right)}{3}+\frac{bex^5\operatorname{acosh}\left(cx\right)}{5}-\frac{bdx^2\sqrt{c^2x^2-1}}{9c}-\frac{bex^4\sqrt{c^2x^2-1}}{25c}-\frac{2bd\sqrt{c^2x^2-1}}{9c^3}-\frac{4bex^2\sqrt{c^2x^2-1}}{75c^3}-\frac{8be\sqrt{c^2x^2-1}}{75c^5} & \text{fo} \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^3}{3}+\frac{ex^5}{5}\right) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**3/3 + a*e*x**5/5 + b*d*x**3*acosh(c*x)/3 + b*e*x**5*acosh
(c*x)/5 - b*d*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e*x**4*sqrt(c**2*x**2 - 1)
/(25*c) - 2*b*d*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e*x**2*sqrt(c**2*x**2 -
1)/(75*c**3) - 8*b*e*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c, 0)), ((a + I*pi*b
/2)*(d*x**3/3 + e*x**5/5), True))
```

3.464 $\int x\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=122

$$\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{b\left(8c^2d+3e\right)\cosh^{-1}(cx)}{32c^4}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8c^2d+3e\right)}{32c^3}$$

```
[Out] -1/32*b*(8*c^2*d+3*e)*arccosh(c*x)/c^4+1/2*d*x^2*(a+b*arccosh(c*x))+1/4*e*x
^4*(a+b*arccosh(c*x))-1/32*b*(8*c^2*d+3*e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^
3-1/16*b*e*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.11, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {5786, 460, 90, 52}

$$\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,\left(8c^2d+3e\right)}{32c^3}-\frac{b\left(8c^2d+3e\right)\cosh^{-1}(cx)}{32c^4}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*(8*c^2*d + 3*e)*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(32*c^3) - (b*e*x^3*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (b*(8*c^2*d + 3*e)*ArcCosh[c*x])/(32*c^
4) + (d*x^2*(a + b*ArcCosh[c*x]))/2 + (e*x^4*(a + b*ArcCosh[c*x]))/4
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int x\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx=\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}(cx)\right)-\frac{1}{8}(bc)\int\frac{x^2\left(4d+\right.}{\sqrt{-1+cx}\,\sqrt{}}$$

$$=-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+\frac{1}{2}dx^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}ex^4\left(a+b\cosh^{-1}\right.$$

$$=-\frac{b\left(8c^2d+3e\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+\frac{1}{2}dx^2\left(a+\right.$$

$$=-\frac{b\left(8c^2d+3e\right)x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{bex^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{b\left(8c^2d+\right.}{}$$

Mathematica [A] time = 0.14, size = 120, normalized size = 0.98

$$\frac{cx\left(8ac^3x\left(2d+ex^2\right)-b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(2c^2\left(4d+ex^2\right)+3e\right)\right)+8bc^4x^2\cosh^{-1}(cx)\left(2d+ex^2\right)-2b\left(8c^2d+3e\right)}{32c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(8*a*c^3*x*(2*d + e*x^2) - b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3*e + 2*c^2
*(4*d + e*x^2))) + 8*b*c^4*x^2*(2*d + e*x^2)*ArcCosh[c*x] - 2*b*(8*c^2*d +
3*e)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(32*c^4)
```

fricas [A] time = 0.59, size = 114, normalized size = 0.93

$$\frac{8\,ac^4ex^4+16\,ac^4dx^2+\left(8\,bc^4ex^4+16\,bc^4dx^2-8\,bc^2d-3\,be\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(2\,bc^3ex^3+\left(8\,bc^3d+3\,bce\right)\right.}{32\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/32*(8*a*c^4*e*x^4 + 16*a*c^4*d*x^2 + (8*b*c^4*e*x^4 + 16*b*c^4*d*x^2 - 8*
b*c^2*d - 3*b*e)*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^3*e*x^3 + (8*b*c^3*d
 + 3*b*c*e)*x)*sqrt(c^2*x^2 - 1))/c^4
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 202, normalized size = 1.66

$$\frac{a\,x^4e}{4}+\frac{a\,x^2d}{2}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^4e}{4}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^2d}{2}-\frac{be\,x^3\sqrt{cx-1}\,\sqrt{cx+1}}{16c}-\frac{bdx\sqrt{cx-1}\,\sqrt{cx+1}}{4c}-\frac{b\sqrt{cx-1}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/4*a*x^4*e+1/2*a*x^2*d+1/4*b*arccosh(c*x)*x^4*e+1/2*b*arccosh(c*x)*x^2*d-1
/16*b*e*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4*b*d*x*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c-1/4/c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*
x^2-1)^(1/2))*d-3/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e*x-3/32/c^4*b*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.46, size = 156, normalized size = 1.28

$$\frac{1}{4}\,aex^4+\frac{1}{2}\,adx^2+\frac{1}{4}\left(2\,x^2\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd+\frac{1}{32}\left(8\,x^4\,\mathrm{arcosh}\,(cx)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*e*x^4 + 1/2*a*d*x^2 + 1/4*(2*x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*
x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d + 1/32*(8*x^4*arccos
h(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2
*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\left(a+b\,\mathrm{acosh}\,(c\,x)\right)\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 1.19, size = 160, normalized size = 1.31

$$\begin{cases} \frac{adx^2}{2}+\frac{aex^4}{4}+\frac{bdx^2\,\mathrm{acosh}\,(cx)}{2}+\frac{bex^4\,\mathrm{acosh}\,(cx)}{4}-\frac{bdx\sqrt{c^2x^2-1}}{4c}-\frac{bex^3\sqrt{c^2x^2-1}}{16c}-\frac{bd\,\mathrm{acosh}\,(cx)}{4c^2}-\frac{3bex\sqrt{c^2x^2-1}}{32c^3}-\frac{3be\,\mathrm{acosh}\,(cx)}{32c^4} \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{dx^2}{2}+\frac{ex^4}{4}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x**2/2 + a*e*x**4/4 + b*d*x**2*acosh(c*x)/2 + b*e*x**4*acosh
(c*x)/4 - b*d*x*sqrt(c**2*x**2 - 1)/(4*c) - b*e*x**3*sqrt(c**2*x**2 - 1)/(1
6*c) - b*d*acosh(c*x)/(4*c**2) - 3*b*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 3*
b*e*acosh(c*x)/(32*c**4), Ne(c, 0)), ((a + I*pi*b/2)*(d*x**2/2 + e*x**4/4),
 True))
```

3.465 $\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=94

$$dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+2e\right)}{9c^3}-\frac{bex^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

```
[Out] d*x*(a+b*arccosh(c*x))+1/3*e*x^3*(a+b*arccosh(c*x))-1/9*b*(9*c^2*d+2*e)*(c*
x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/9*b*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.08, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {5705, 460, 74}

$$dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(9c^2d+2e\right)}{9c^3}-\frac{bex^2\sqrt{cx-1}\sqrt{cx+1}}{9c}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*(9*c^2*d + 2*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c^3) - (b*e*x^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x])/(9*c) + d*x*(a + b*ArcCosh[c*x]) + (e*x^3*(a + b*Ar
cCosh[c*x]))/3
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx = dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)-(bc)\int \frac{x\left(d+\frac{ex^2}{3}\right)}{\sqrt{-1+cx}\,\sqrt{1-}$$

$$=-\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+dx\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(c\right.$$

$$=-\frac{b\left(9c^2d+2e\right)\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3}-\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9c}+dx\left(a+b\right.$$

Mathematica [A] time = 0.09, size = 76, normalized size = 0.81

$$\frac{1}{9}\left(3ax\left(3d+ex^2\right)-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2\left(9d+ex^2\right)+2e\right)}{c^3}+3bx\cosh^{-1}(cx)\left(3d+ex^2\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3*a*x*(3*d + e*x^2) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*
x^2)))/c^3 + 3*b*x*(3*d + e*x^2)*ArcCosh[c*x])/9
```

fricas [A] time = 0.47, size = 94, normalized size = 1.00

$$\frac{3\,ac^3ex^3+9\,ac^3dx+3\left(bc^3ex^3+3\,bc^3dx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(bc^2ex^2+9\,bc^2d+2\,be\right)\sqrt{c^2x^2-1}}{9\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e*x^3 + 9*a*c^3*d*x + 3*(b*c^3*e*x^3 + 3*b*c^3*d*x)*log(c*x +
sqrt(c^2*x^2 - 1)) - (b*c^2*e*x^2 + 9*b*c^2*d + 2*b*e)*sqrt(c^2*x^2 - 1))/c
^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 90, normalized size = 0.96

$$\frac{\frac{a\left(\frac{1}{3}c^3x^3e+c^3dx\right)}{c^2}+\frac{b\left(\frac{\operatorname{arccosh}(cx)c^3x^3e}{3}+\operatorname{arccosh}(cx)c^3dx-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2x^2e+9c^2d+2e\right)}{9}\right)}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b/c^2*(1/3*arccosh(c*x)*c^3*x^3*e+arccos
h(c*x)*c^3*d*x-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^2*d+2*e)))
```

maxima [A] time = 0.39, size = 91, normalized size = 0.97

$$\frac{1}{3}aex^3+\frac{1}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)be+adx+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)bd}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*e*x^3 + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sq
rt(c^2*x^2 - 1)/c^4))*b*e + a*d*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*
b*d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2), x)
```

sympy [A] time = 0.56, size = 116, normalized size = 1.23

$$\begin{cases} adx+\frac{aex^3}{3}+bdx\operatorname{acosh}(cx)+\frac{bex^3\operatorname{acosh}(cx)}{3}-\frac{bd\sqrt{c^2x^2-1}}{c}-\frac{bex^2\sqrt{c^2x^2-1}}{9c}-\frac{2be\sqrt{c^2x^2-1}}{9c^3} & \text{for } c\neq 0 \\ \left(a+\frac{i\pi b}{2}\right)\left(dx+\frac{ex^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d*x + a*e*x**3/3 + b*d*x*acosh(c*x) + b*e*x**3*acosh(c*x)/3 -
b*d*sqrt(c**2*x**2 - 1)/c - b*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*e*sqrt
(c**2*x**2 - 1)/(9*c**3), Ne(c, 0)), ((a + I*pi*b/2)*(d*x + e*x**3/3), True
))
```

3.466 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=264

$$d\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}\,\mathrm{Li}_2\left(e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{ibd\sqrt{1-c^2x^2}\,\sin^{-1}(cx)^2}{2\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] -1/4*b*e*arccosh(c*x)/c^2+1/2*e*x^2*(a+b*arccosh(c*x))+d*(a+b*arccosh(c*x))
*ln(x)-1/4*b*e*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*I*b*d*arcsin(c*x)^2*(-c^
2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d*arcsin(c*x)*ln(1-(I*c*x+(-c^
2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d*arcsi
n(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d*polyl
og(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)
^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.684, Rules used = {14, 5790, 12, 6742, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\,\sqrt{cx+1}}+d\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd\sqrt{1-c^2x^2}\,\mathrm{s}}{2\sqrt{cx-1}\,\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(b*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (b*e*ArcCosh[c*x])/(4*c^2) +
(e*x^2*(a + b*ArcCosh[c*x]))/2 - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2
)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1
 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d*(a + b*ArcCos
h[c*x])*Log[x] - (b*d*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]
*Sqrt[1 + c*x]) - ((I/2)*b*d*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c
*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int \frac{ex^2+}{2\sqrt{-1+}}$$

$$=\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int \frac{ex^2+}{\sqrt{-1+}}$$

$$=\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-\frac{1}{2}(bc)\int\left(\frac{}{\sqrt{-1}}\right.$$

$$=\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bcd)\int \frac{\mathrm{lo}}{\sqrt{-1+}}$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(a+b\cosh^{-1}(cx\right.$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(\right.$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)+d\left(\right.$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ib}{}$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ib}{}$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ib}{}$$

$$=-\frac{bex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{be\cosh^{-1}(cx)}{4c^2}+\frac{1}{2}ex^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ib}{}$$

Mathematica [A] time = 0.27, size = 119, normalized size = 0.45

$$\frac{1}{2}\left(2ad\log(x)+aex^2-\frac{be\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{2c^2}-bd\mathrm{Li}_2\left(-e^{-2\cosh^{-1}(cx)}\right)+bd\cosh^{-1}(cx)\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (a*e*x^2 + b*e*x^2*ArcCosh[c*x] - (b*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] +
2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(2*c^2) + b*d*ArcCosh[c*x]*(ArcCosh
[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*d*Log[x] - b*d*PolyLog[2, -E^
(-2*ArcCosh[c*x])])/2
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{aex^2+ad+\left(bex^2+bd\right)\mathrm{arcosh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(b \operatorname{arcosh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x, x)
```

maple [A] time = 0.32, size = 130, normalized size = 0.49

$$\frac{a\,x^2e}{2}+da\ln\left(cx\right)-\frac{b\mathrm{arccosh}\left(cx\right)^2 d}{2}-\frac{bex\sqrt{cx-1}\,\sqrt{cx+1}}{4c}+\frac{b\,\mathrm{arccosh}\left(cx\right)x^2e}{2}-\frac{be\,\mathrm{arccosh}\left(cx\right)}{4c^2}+bd\,\mathrm{arccosh}\left(cx\right)\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/2*a*x^2*e+d*a*ln(c*x)-1/2*b*arccosh(c*x)^2*d-1/4*b*e*x*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c+1/2*b*arccosh(c*x)*x^2*e-1/4*b*e*arccosh(c*x)/c^2+b*d*arccosh(c
*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*d*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,aex^2+ad\log(x)+\int bex\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\frac{bd\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/2*a*e*x^2 + a*d*log(x) + integrate(b*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + b*d*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(d+ex^2\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x, x)
```

3.467 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=75

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}$$

```
[Out] -d*(a+b*arccosh(c*x))/x+e*x*(a+b*arccosh(c*x))+b*c*d*arctan((c*x-1)^(1/2)*(
c*x+1)^(1/2))-b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5786, 460, 92, 205}

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{x}+ex\left(a+b\cosh^{-1}(cx)\right)+bcd\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c) - (d*(a + b*ArcCosh[c*x]))/x + e*x*
(a + b*ArcCosh[c*x]) + b*c*d*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^2}\,dx = -\frac{d(a+b\cosh^{-1}(cx))}{x} + ex(a+b\cosh^{-1}(cx)) + (bc)\int \frac{d-ex^2}{x\sqrt{-1+cx}\sqrt{1+cx}}$$
$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c} - \frac{d(a+b\cosh^{-1}(cx))}{x} + ex(a+b\cosh^{-1}(cx)) + (bc$$
$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c} - \frac{d(a+b\cosh^{-1}(cx))}{x} + ex(a+b\cosh^{-1}(cx)) + (bc$$
$$= -\frac{be\sqrt{-1+cx}\sqrt{1+cx}}{c} - \frac{d(a+b\cosh^{-1}(cx))}{x} + ex(a+b\cosh^{-1}(cx)) + bc$$

Mathematica [A] time = 0.13, size = 105, normalized size = 1.40

$$-\frac{ad}{x}+aex+\frac{bcd\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}}-\frac{bd\cosh^{-1}(cx)}{x}-\frac{be\sqrt{cx-1}\sqrt{cx+1}}{c}+bex\cosh^{-1}(cx)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d)/x) + a*e*x - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c - (b*d*ArcCosh[c*
x])/x + b*e*x*ArcCosh[c*x] + (b*c*d*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2
*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

fricas [A] time = 0.69, size = 132, normalized size = 1.76

$$\frac{2\,bc^2dx\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+acex^2-\sqrt{c^2x^2-1}\,bex-acd+(bcd-bce)x\log\left(-cx+\sqrt{c^2x^2-1}\right)+\left(bcex^2\right.}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] (2*b*c^2*d*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + a*c*e*x^2 - sqrt(c^2*x^2 -
1)*b*e*x - a*c*d + (b*c*d - b*c*e)*x*log(-c*x + sqrt(c^2*x^2 - 1)) + (b*c*e
*x^2 - b*c*d + (b*c*d - b*c*e)*x)*log(c*x + sqrt(c^2*x^2 - 1)))/(c*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(b\operatorname{arcosh}(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^2, x)
```

maple [A] time = 0.02, size = 95, normalized size = 1.27

$$axe-\frac{ad}{x}+b\operatorname{arccosh}(cx)\,xe-\frac{b\operatorname{arccosh}(cx)\,d}{x}-\frac{cb\sqrt{cx-1}\,\sqrt{cx+1}\,d\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{\sqrt{c^2x^2-1}}-\frac{be\sqrt{cx-1}\,\sqrt{cx+1}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] a*x*e-a*d/x+b*arccosh(c*x)*x*e-b*arccosh(c*x)*d/x-c*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*d*arctan(1/(c^2*x^2-1)^(1/2))-b*e*(c*x-1)^(1/2)*(c
*x+1)^(1/2)/c
```

maxima [A] time = 0.49, size = 63, normalized size = 0.84

$$-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}(cx)}{x}\right)bd+aex+\frac{\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)be}{c}-\frac{ad}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] -(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d + a*e*x + (c*x*arccosh(c*x)
- sqrt(c^2*x^2 - 1))*b*e/c - a*d/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(a+b\operatorname{acosh}(cx))\left(d+ex^2\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**2, x)
```

3.468 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=251

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{ibe\sqrt{1-c^2x^2}\operatorname{Li}_2\left(e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx)^2}{2\sqrt{cx-1}\sqrt{cx+1}}+\frac{be\sqrt{}}{}$$

```
[Out] -1/2*d*(a+b*arccosh(c*x))/x^2+e*(a+b*arccosh(c*x))*ln(x)+1/2*b*c*d*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/x-1/2*I*b*e*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1
/2)/(c*x+1)^(1/2)+b*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*e*arcsin(c*x)*ln(x)*(-c^2*x^2+1)
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*e*polylog(2,(I*c*x+(-c^2*x^2+1)^
(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.61, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.579, Rules used = {14, 5790, 6742, 95, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibe\sqrt{1-c^2x^2}\operatorname{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{ibe\sqrt{1-c^2x^2}\sin^{-1}(cx}{2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (d*(a + b*ArcCosh[c*x]))/(2*x^
2) - ((I/2)*b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c
*x]) + (b*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + e*(a + b*ArcCosh[c*x])*Log[x] - (b*e*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*e
*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int \frac{-\frac{d}{2x^2}+e\log}{\sqrt{-1+cx}\,\sqrt{1}}$$
$$=-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)-(bc)\int\left(-\frac{}{2x^2\sqrt{-1+}}\right.$$
$$=-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)+\frac{1}{2}(bcd)\int\frac{1}{x^2\sqrt{-1+c}}$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+e\left(a+b\cosh^{-1}(cx)\right)\log(x)$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$
$$=\frac{bcd\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2x^2}-\frac{ibe\sqrt{1-c^2x^2}\,\sin^{-1}(cx)^2}{2\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

Mathematica [A] time = 0.14, size = 101, normalized size = 0.40

$$\frac{-ad+2aex^2\log(x)-b\cosh^{-1}(cx)\left(d-2ex^2\log\left(e^{-2\cosh^{-1}(cx)}+1\right)\right)+bcdx\sqrt{cx-1}\,\sqrt{cx+1}-bex^2\mathrm{Li}_2\left(-e^{-2\cosh^{-}}\right.}{2x^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (-(a*d) + b*c*d*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + b*e*x^2*ArcCosh[c*x]^2 - b
*ArcCosh[c*x]*(d - 2*e*x^2*Log[1 + E^(-2*ArcCosh[c*x])]) + 2*a*e*x^2*Log[x]
 - b*e*x^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/(2*x^2)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{aex^2+ad+\left(bex^2+bd\right)\mathrm{arcosh}\,(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)\left(b\,\mathrm{arcosh}\,(cx)+a\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^3, x)
```

maple [A] time = 0.61, size = 126, normalized size = 0.50

$$ae\ln(cx) - \frac{ad}{2x^2} - \frac{b\,\mathrm{arccosh}(cx)^2 e}{2} + \frac{bcd\sqrt{cx-1}\,\sqrt{cx+1}}{2x} - \frac{c^2bd}{2} - \frac{b\,\mathrm{arccosh}(cx)\,d}{2x^2} + be\,\mathrm{arccosh}(cx)\ln\left(1 + \left(cx + \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] a*e*ln(c*x)-1/2*a*d/x^2-1/2*b*arccosh(c*x)^2*e+1/2*b*c*d*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/x-1/2*c^2*b*d-1/2*b*arccosh(c*x)*d/x^2+b*e*arccosh(c*x)*ln(1+(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)+1/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,bd\left(\frac{\sqrt{c^2x^2-1}\,c}{x} - \frac{\mathrm{arcosh}(cx)}{x^2}\right) + be\int \frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{x}\,dx + ae\log(x) - \frac{ad}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*d*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + b*e*integrate(log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/x, x) + a*e*log(x) - 1/2*a*d/x^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}(c\,x))\,\left(e\,x^2 + d\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}(cx))\left(d + ex^2\right)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**3, x)
```

3.469 $\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=94

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d+6e\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

```
[Out] -1/3*d*(a+b*arccosh(c*x))/x^3-e*(a+b*arccosh(c*x))/x+1/6*b*c*(c^2*d+6*e)*ar
ctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5786, 454, 92, 205}

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{x}+\frac{1}{6}bc\left(c^2d+6e\right)\tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*c*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*x^2) - (d*(a + b*ArcCosh[c*x]))/(3*
x^3) - (e*(a + b*ArcCosh[c*x]))/x + (b*c*(c^2*d + 6*e)*ArcTan[Sqrt[-1 + c*x
]*Sqrt[1 + c*x]])/6
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e*(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^n*(
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+b\cosh^{-1}(cx))}{x^4}\,dx = -\frac{d(a+b\cosh^{-1}(cx))}{3x^3} - \frac{e(a+b\cosh^{-1}(cx))}{x} - \frac{1}{3}(bc)\int \frac{-d-3ex^2}{x^3\sqrt{-1+cx}\sqrt{1}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d(a+b\cosh^{-1}(cx))}{3x^3} - \frac{e(a+b\cosh^{-1}(cx))}{x} + \frac{1}{6}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d(a+b\cosh^{-1}(cx))}{3x^3} - \frac{e(a+b\cosh^{-1}(cx))}{x} + \frac{1}{6}$$

$$= \frac{bcd\sqrt{-1+cx}\sqrt{1+cx}}{6x^2} - \frac{d(a+b\cosh^{-1}(cx))}{3x^3} - \frac{e(a+b\cosh^{-1}(cx))}{x} + \frac{1}{6}$$

Mathematica [A] time = 0.27, size = 128, normalized size = 1.36

$$\frac{\frac{-2a\sqrt{cx-1}\sqrt{cx+1}(d+3ex^2)+bcx^3\sqrt{c^2x^2-1}(c^2d+6e)\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+bcdx(c^2x^2-1)}{\sqrt{cx-1}\sqrt{cx+1}} - 2b\cosh^{-1}(cx)(d+3ex^2)}{6x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (-2*b*(d + 3*e*x^2)*ArcCosh[c*x] + (b*c*d*x*(-1 + c^2*x^2) - 2*a*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(d + 3*e*x^2) + b*c*(c^2*d + 6*e)*x^3*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(6*x^3)
```

fricas [A] time = 0.88, size = 139, normalized size = 1.48

$$\frac{2(bc^3d+6bce)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2(bd+3be)x^3\log\left(-cx+\sqrt{c^2x^2-1}\right)+\sqrt{c^2x^2-1}\,bcdx-6ae}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(2*(b*c^3*d + 6*b*c*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*d +
3*b*e)*x^3*log(-c*x + sqrt(c^2*x^2 - 1)) + sqrt(c^2*x^2 - 1)*b*c*d*x - 6*a*
e*x^2 - 2*a*d - 2*(3*b*e*x^2 - (b*d + 3*b*e)*x^3 + b*d)*log(c*x + sqrt(c^2*
x^2 - 1)))/x^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)(b\,\mathrm{arcosh}(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)/x^4, x)
```

maple [A] time = 0.02, size = 146, normalized size = 1.55

$$-\frac{ae}{x}-\frac{da}{3x^3}-\frac{b\,\mathrm{arccosh}(cx)\,e}{x}-\frac{db\,\mathrm{arccosh}(cx)}{3x^3}-\frac{c^3db\sqrt{cx-1}\sqrt{cx+1}\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)}{6\sqrt{c^2x^2-1}}+\frac{bcd\sqrt{cx-1}\sqrt{cx+1}}{6x^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] -a*e/x-1/3*d*a/x^3-b*arccosh(c*x)*e/x-1/3*d*b*arccosh(c*x)/x^3-1/6*c^3*d*b*
(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))+1
/6*b*c*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c
^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*e
```

maxima [A] time = 0.47, size = 85, normalized size = 0.90

$$-\frac{1}{6}\left(\left(c^{2}\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^{2}x^{2}-1}}{x^{2}}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^{3}}\right)bd-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)be-\frac{ae}{x}-\frac{ad}{3\,x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)
/x^3)*b*d - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*e - a*e/x - 1/3*a*d
/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2+d\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2))/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2))/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))\left(d+ex^{2}\right)}{x^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)/x**4, x)
```

3.470 $\int x^4\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=319

$$\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx)\right)-\frac{2be\left(1-c^2x^2\right)^4\left(9c^2d+14e\right)}{441c^9\sqrt{cx-1}\sqrt{cx+1}}+$$

```
[Out] 1/5*d^2*x^5*(a+b*arccosh(c*x))+2/7*d*e*x^7*(a+b*arccosh(c*x))+1/9*e^2*x^9*(
a+b*arccosh(c*x))+1/315*b*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)/c^9/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-2/945*b*(63*c^4*d^2+135*c^2*d*e+70*e^2)*(-c^2*x^
2+1)^2/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*(21*c^4*d^2+90*c^2*d*e+70*e^
2)*(-c^2*x^2+1)^3/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-2/441*b*e*(9*c^2*d+14*e)*
(-c^2*x^2+1)^4/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*e^2*(-c^2*x^2+1)^5/c^
9/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5790, 12, 520, 1251, 897, 1153}

$$\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)^3\left(21c^4d^2+90c^2d\right.}{525c^9\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (2*b*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^2
)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(21*c^4*d^2 + 90*c^2*d*e + 70
*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*e*(9*c
^2*d + 14*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^5*(a + b
*ArcCosh[c*x]))/5 + (2*d*e*x^7*(a + b*ArcCosh[c*x]))/7 + (e^2*x^9*(a + b*Ar
cCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^4\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{1}{5}d^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{7}dex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^2x^9\left(a+b\cos\right.$$

$$=\frac{b\left(63c^4d^2+90c^2de+35e^2\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}}-\frac{2b\left(63c^4d^2+135c^2de+70e^2\right)\left(\right.}{945c^9\sqrt{-1+cx}\sqrt{1+c}}$$

Mathematica [A] time = 0.28, size = 192, normalized size = 0.60

$$\frac{315ax^5\left(63d^2+90dex^2+35e^2x^4\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^8\left(3969d^2x^4+4050dex^6+1225e^2x^8\right)+4c^6\left(1323d^2x^2+1215dex^4+350e^2x^6\right)+24c^4\left(4\cdots\right.\right.}{c^9}}{99225}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^5*(63*d^2 + 90*d*e*x^2 + 35*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(4480*e^2 + 160*c^2*e*(81*d + 14*e*x^2) + 24*c^4*(441*d^2 + 270*d*e*x^
2 + 70*e^2*x^4) + 4*c^6*(1323*d^2*x^2 + 1215*d*e*x^4 + 350*e^2*x^6) + c^8*(
3969*d^2*x^4 + 4050*d*e*x^6 + 1225*e^2*x^8)))/c^9 + 315*b*x^5*(63*d^2 + 90*
d*e*x^2 + 35*e^2*x^4)*ArcCosh[c*x])/99225
```

fricas [A] time = 0.55, size = 231, normalized size = 0.72

$$\frac{11025\,ac^9e^2x^9+28350\,ac^9dex^7+19845\,ac^9d^2x^5+315\left(35\,bc^9e^2x^9+90\,bc^9dex^7+63\,bc^9d^2x^5\right)\log\left(cx+\sqrt{c^2x\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^2*x^9 + 28350*a*c^9*d*e*x^7 + 19845*a*c^9*d^2*x^5 +
315*(35*b*c^9*e^2*x^9 + 90*b*c^9*d*e*x^7 + 63*b*c^9*d^2*x^5)*log(c*x + sqrt
(c^2*x^2 - 1)) - (1225*b*c^8*e^2*x^8 + 10584*b*c^4*d^2 + 50*(81*b*c^8*d*e +
 28*b*c^6*e^2)*x^6 + 12960*b*c^2*d*e + 3*(1323*b*c^8*d^2 + 1620*b*c^6*d*e +
 560*b*c^4*e^2)*x^4 + 4480*b*e^2 + 4*(1323*b*c^6*d^2 + 1620*b*c^4*d*e + 560
*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 227, normalized size = 0.71

$$\frac{\frac{a\left(\frac{1}{9}e^2c^9x^9+\frac{2}{7}c^9de\,x^7+\frac{1}{5}c^9x^5d^2\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^2c^9x^9}{9}+\frac{2\operatorname{arccosh}(cx)c^9de\,x^7}{7}+\frac{\operatorname{arccosh}(cx)c^9x^5d^2}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1225c^8e^2x^8+4050c^8de\,x^6+3969c^8d^2x^4+1400\cdots\right.}{\cdots}\right.}{c^4}}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^4*(1/9*e^2*c^9*x^9+2/7*c^9*d*e*x^7+1/5*c^9*x^5*d^2)+b/c^4*(1/9*a
rccosh(c*x)*e^2*c^9*x^9+2/7*arccosh(c*x)*c^9*d*e*x^7+1/5*arccosh(c*x)*c^9*x
^5*d^2-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^2*x^8+4050*c^8*d*e*x
^6+3969*c^8*d^2*x^4+1400*c^6*e^2*x^6+4860*c^6*d*e*x^4+5292*c^6*d^2*x^2+1680
*c^4*e^2*x^4+6480*c^4*d*e*x^2+10584*c^4*d^2+2240*c^2*e^2*x^2+12960*c^2*d*e+
4480*e^2)))
```

maxima [A] time = 0.34, size = 305, normalized size = 0.96

$$\frac{1}{9}ae^2x^9+\frac{2}{7}adex^7+\frac{1}{5}ad^2x^5+\frac{1}{75}\left(15x^5\operatorname{arcosh}(cx)-\left(\frac{3\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\sqrt{c^2x^2-1}}{c^6}\right)c\right)bd^2+\frac{2}{245}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^2*x^9 + 2/7*a*d*e*x^7 + 1/5*a*d^2*x^5 + 1/75*(15*x^5*arccosh(c*x) -
 (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*d^2 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^
6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt
(c^2*x^2 - 1)/c^8)*c)*b*d*e + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x
^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c
^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 16.21, size = 422, normalized size = 1.32

$$\begin{cases} \frac{ad^2x^5}{5}+\frac{2adex^7}{7}+\frac{ae^2x^9}{9}+\frac{bd^2x^5\operatorname{acosh}(cx)}{5}+\frac{2bdex^7\operatorname{acosh}(cx)}{7}+\frac{be^2x^9\operatorname{acosh}(cx)}{9}-\frac{bd^2x^4\sqrt{c^2x^2-1}}{25c}-\frac{2bdex^6\sqrt{c^2x^2-1}}{49c}-\frac{be^2x^8\sqrt{c^2x^2-1}}{81c} \\ \left(a+\frac{i\pi b}{2}\right)\left(\frac{d^2x^5}{5}+\frac{2dex^7}{7}+\frac{e^2x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**5/5 + 2*a*d*e*x**7/7 + a*e**2*x**9/9 + b*d**2*x**5*aco
sh(c*x)/5 + 2*b*d*e*x**7*acosh(c*x)/7 + b*e**2*x**9*acosh(c*x)/9 - b*d**2*x
**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*b*d*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) -
b*e**2*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 4*b*d**2*x**2*sqrt(c**2*x**2 - 1)/
(75*c**3) - 12*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**2*x**6*sq
rt(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*sqrt(c**2*x**2 - 1)/(75*c**5) - 16*
b*d*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*x**4*sqrt(c**2*x**2 -
 1)/(945*c**5) - 32*b*d*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**2*x**2*s
qrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**2*sqrt(c**2*x**2 - 1)/(2835*c**9)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**5/5 + 2*d*e*x**7/7 + e**2*x**9/9), Tr
ue))
```

3.471 $\int x^3 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=341

$$\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(cx)\right)+\frac{be^2x^7\left(1-c^2x^2\right)}{64c\sqrt{cx-1}\sqrt{cx+1}}+\frac{bex^5\left(1\right.}{1152}$$

```
[Out] 1/4*d^2*x^4*(a+b*arccosh(c*x))+1/3*d*e*x^6*(a+b*arccosh(c*x))+1/8*e^2*x^8*(
a+b*arccosh(c*x))+1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)*x*(-c^2*x^2+1)
/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/4608*b*(288*c^4*d^2+320*c^2*d*e+105*e^2)
*x^3*(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1152*b*e*(64*c^2*d+21*e
)*x^5*(-c^2*x^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/64*b*e^2*x^7*(-c^2*x^2
+1)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/3072*b*(288*c^4*d^2+320*c^2*d*e+105*e^2
)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^8/(c*x-1)^(1/2)/(c*x+1
)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {266, 43, 5790, 12, 520, 1267, 459, 321, 217, 206}

$$\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\cosh^{-1}(cx)\right)+\frac{bx^3\left(1-c^2x^2\right)\left(288c^4d^2+320\right.}{4608c^5\sqrt{cx-1}\sqrt{cx}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x*(1 - c^2*x^2))/(3072*c^7*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*x^3*(1 -
c^2*x^2))/(4608*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(64*c^2*d + 21*e)*
x^5*(1 - c^2*x^2))/(1152*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*x^7*(1
- c^2*x^2))/(64*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^4*(a + b*ArcCosh[c
*x]))/4 + (d*e*x^6*(a + b*ArcCosh[c*x]))/3 + (e^2*x^8*(a + b*ArcCosh[c*x]))
/8 - (b*(288*c^4*d^2 + 320*c^2*d*e + 105*e^2)*Sqrt[-1 + c^2*x^2]*ArcTanh[(c
*x)/Sqrt[-1 + c^2*x^2]])/(3072*c^8*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$=\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{8}e^2x^8\left(a+b\right.$$

$$=\frac{be^2x^7\left(1-c^2x^2\right)}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}dex^6\left(a+b\,\mathrm{co}\right.$$

$$=\frac{be\left(64c^2d+21e\right)x^5\left(1-c^2x^2\right)}{1152c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^2x^7\left(1-c^2x^2\right)}{64c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{1}{4}d^2x^4\left(a+\right.$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x^3\left(1-c^2x^2\right)}{4608c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be\left(64c^2d+21e\right)x^5\left(1\right.}{1152c^3\sqrt{-1+cx}\sqrt{\ }}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2\right.\right.}{4608c^5\sqrt{-1+}}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2\right.\right.}{4608c^5\sqrt{-1+}}$$

$$=\frac{b\left(288c^4d^2+5e\left(64c^2d+21e\right)\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}}+\frac{b\left(288c^4d^2+5e\left(64c^2\right.\right.}{4608c^5\sqrt{-1+}}$$

Mathematica [A] time = 0.35, size = 214, normalized size = 0.63

$$\frac{384ac^8x^4\left(6d^2+8dex^2+3e^2x^4\right)+384bc^8x^4\cosh^{-1}(cx)\left(6d^2+8dex^2+3e^2x^4\right)-6b\left(288c^4d^2+320c^2de+105\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (384*a*c^8*x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4) - b*c*x*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]*(315*e^2 + 30*c^2*e*(32*d + 7*e*x^2) + 8*c^4*(108*d^2 + 80*d*e*x^2
 + 21*e^2*x^4) + 16*c^6*(36*d^2*x^2 + 32*d*e*x^4 + 9*e^2*x^6)) + 384*b*c^8*
x^4*(6*d^2 + 8*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x] - 6*b*(288*c^4*d^2 + 320*c
^2*d*e + 105*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(9216*c^8)
```

fricas [A] time = 0.70, size = 227, normalized size = 0.67

$$\frac{1152\,ac^8e^2x^8+3072\,ac^8dex^6+2304\,ac^8d^2x^4+3\left(384\,bc^8e^2x^8+1024\,bc^8dex^6+768\,bc^8d^2x^4-288\,bc^4d^2-32\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/9216*(1152*a*c^8*e^2*x^8 + 3072*a*c^8*d*e*x^6 + 2304*a*c^8*d^2*x^4 + 3*(3
84*b*c^8*e^2*x^8 + 1024*b*c^8*d*e*x^6 + 768*b*c^8*d^2*x^4 - 288*b*c^4*d^2 -
 320*b*c^2*d*e - 105*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (144*b*c^7*e^2*x
^7 + 8*(64*b*c^7*d*e + 21*b*c^5*e^2)*x^5 + 2*(288*b*c^7*d^2 + 320*b*c^5*d*e
```

```
 + 105*b*c^3*e^2)*x^3 + 3*(288*b*c^5*d^2 + 320*b*c^3*d*e + 105*b*c*e^2)*x)*
sqrt(c^2*x^2 - 1))/c^8
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 440, normalized size = 1.29

$$\frac{a\,e^{2}x^{8}}{8}+\frac{ade\,x^{6}}{3}+\frac{a\,x^{4}d^{2}}{4}+\frac{b\,\mathrm{arccosh}\left(cx\right)e^{2}x^{8}}{8}+\frac{b\,\mathrm{arccosh}\left(cx\right)de\,x^{6}}{3}+\frac{b\,\mathrm{arccosh}\left(cx\right)x^{4}d^{2}}{4}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,e^{2}x^{7}}{64c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*a*e^2*x^8+1/3*a*d*e*x^6+1/4*a*x^4*d^2+1/8*b*arccosh(c*x)*e^2*x^8+1/3*b*
arccosh(c*x)*d*e*x^6+1/4*b*arccosh(c*x)*x^4*d^2-1/64/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^2*x^7-1/18/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e-1/16*b*d^2*x
^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-7/384/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^
2*x^5-5/72/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x^3-3/32*b*d^2*x*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/c^3-3/32/c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(
1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-35/1536/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*e^2*x^3-5/48/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x-5/48/c^6*b*(c*x-1)^(
1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-35/1024/
c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x-35/1024/c^8*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.33, size = 332, normalized size = 0.97

$$\frac{1}{8}\,ae^{2}x^{8}+\frac{1}{3}\,adex^{6}+\frac{1}{4}\,ad^{2}x^{4}+\frac{1}{32}\left(8\,x^{4}\,\mathrm{arcosh}\left(cx\right)-\left(\frac{2\,\sqrt{c^{2}x^{2}-1}\,x^{3}}{c^{2}}+\frac{3\,\sqrt{c^{2}x^{2}-1}\,x}{c^{4}}+\frac{3\,\log\left(2\,c^{2}x+2\,\sqrt{c^{2}x^{2}-1}\right.}{c^{5}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^2*x^8 + 1/3*a*d*e*x^6 + 1/4*a*d^2*x^4 + 1/32*(8*x^4*arccosh(c*x) -
(2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x +
2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^2 + 1/144*(48*x^6*arccosh(c*x) - (8*sqrt
(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)
*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e + 1/3072*(38
4*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x
^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*l
og(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^{3}\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^{2}+d\right)}^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 11.23, size = 389, normalized size = 1.14

$$\begin{cases} \frac{ad^2x^4}{4} + \frac{adex^6}{3} + \frac{ae^2x^8}{8} + \frac{bd^2x^4\operatorname{acosh}(cx)}{4} + \frac{bdex^6\operatorname{acosh}(cx)}{3} + \frac{be^2x^8\operatorname{acosh}(cx)}{8} - \frac{bd^2x^3\sqrt{c^2x^2-1}}{16c} - \frac{bdex^5\sqrt{c^2x^2-1}}{18c} - \frac{be^2x^7\sqrt{c^2x^2-}}{64c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^4}{4} + \frac{dex^6}{3} + \frac{e^2x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**4/4 + a*d*e*x**6/3 + a*e**2*x**8/8 + b*d**2*x**4*acosh
(c*x)/4 + b*d*e*x**6*acosh(c*x)/3 + b*e**2*x**8*acosh(c*x)/8 - b*d**2*x**3*
sqrt(c**2*x**2 - 1)/(16*c) - b*d*e*x**5*sqrt(c**2*x**2 - 1)/(18*c) - b*e**2
*x**7*sqrt(c**2*x**2 - 1)/(64*c) - 3*b*d**2*x*sqrt(c**2*x**2 - 1)/(32*c**3)
 - 5*b*d*e*x**3*sqrt(c**2*x**2 - 1)/(72*c**3) - 7*b*e**2*x**5*sqrt(c**2*x**
2 - 1)/(384*c**3) - 3*b*d**2*acosh(c*x)/(32*c**4) - 5*b*d*e*x*sqrt(c**2*x**
2 - 1)/(48*c**5) - 35*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e
*acosh(c*x)/(48*c**6) - 35*b*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*
e**2*acosh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**4/4 + d*e*
x**6/3 + e**2*x**8/8), True))
```

3.472 $\int x^2 \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=260

$$\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x^2\right)^3\left(14c^2d+15e\right)}{175c^7\sqrt{cx-1}\sqrt{cx+1}}-\frac{}{49}$$

```
[Out] 1/3*d^2*x^3*(a+b*arccosh(c*x))+2/5*d*e*x^5*(a+b*arccosh(c*x))+1/7*e^2*x^7*(
a+b*arccosh(c*x))+1/105*b*(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)/c^7/(
c*x-1)^(1/2)/(c*x+1)^(1/2)-1/315*b*(35*c^4*d^2+84*c^2*d*e+45*e^2)*(-c^2*x^2
+1)^2/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/175*b*e*(14*c^2*d+15*e)*(-c^2*x^2+1
)^3/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/49*b*e^2*(-c^2*x^2+1)^4/c^7/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 520, 1251, 771}

$$\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{b\left(1-c^2x^2\right)^2\left(35c^4d^2+84c^2de+\right.}{315c^7\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(1 - c^2*x^2))/(105*c^7*Sqrt[-1 + c*x
]*Sqrt[1 + c*x]) - (b*(35*c^4*d^2 + 84*c^2*d*e + 45*e^2)*(1 - c^2*x^2)^2)/(
315*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e*(14*c^2*d + 15*e)*(1 - c^2*x^2
)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(1 - c^2*x^2)^4)/(49*c
^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2*x^3*(a + b*ArcCosh[c*x]))/3 + (2*d*
e*x^5*(a + b*ArcCosh[c*x]))/5 + (e^2*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 771

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^2\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$=\frac{1}{3}d^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{5}dex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^2x^7\left(a+b\right.$$

$$=\frac{b\left(35c^4d^2+42c^2de+15e^2\right)\left(1-c^2x^2\right)}{105c^7\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\left(35c^4d^2+84c^2de+45e^2\right)(}{315c^7\sqrt{-1+cx}\sqrt{1+}}$$

Mathematica [A] time = 0.26, size = 163, normalized size = 0.63

$$\frac{105ax^3\left(35d^2+42dex^2+15e^2x^4\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^6\left(1225d^2x^2+882dex^4+225e^2x^6\right)+2c^4\left(1225d^2+588dex^2+135e^2x^4\right)+24c^2e\left(98d+1\right.\right.}{c^7}}{11025}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (105*a*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(720*e^2 + 24*c^2*e*(98*d + 15*e*x^2) + 2*c^4*(1225*d^2 + 588*d*e*x^2
+ 135*e^2*x^4) + c^6*(1225*d^2*x^2 + 882*d*e*x^4 + 225*e^2*x^6)))/c^7 + 105
*b*x^3*(35*d^2 + 42*d*e*x^2 + 15*e^2*x^4)*ArcCosh[c*x])/11025
```

fricas [A] time = 0.56, size = 198, normalized size = 0.76

$$1575\,ac^7e^2x^7+4410\,ac^7dex^5+3675\,ac^7d^2x^3+105\left(15\,bc^7e^2x^7+42\,bc^7dex^5+35\,bc^7d^2x^3\right)\log\left(cx+\sqrt{c^2x^2-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/11025*(1575*a*c^7*e^2*x^7 + 4410*a*c^7*d*e*x^5 + 3675*a*c^7*d^2*x^3 + 105
*(15*b*c^7*e^2*x^7 + 42*b*c^7*d*e*x^5 + 35*b*c^7*d^2*x^3)*log(c*x + sqrt(c^
2*x^2 - 1)) - (225*b*c^6*e^2*x^6 + 2450*b*c^4*d^2 + 2352*b*c^2*d*e + 18*(49
*b*c^6*d*e + 15*b*c^4*e^2)*x^4 + 720*b*e^2 + (1225*b*c^6*d^2 + 1176*b*c^4*d
*e + 360*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 195, normalized size = 0.75

$$\frac{\frac{a\left(\frac{1}{7}e^{2}c^{7}x^{7}+\frac{2}{5}c^{7}de\,x^{5}+\frac{1}{3}x^{3}c^{7}d^{2}\right)}{c^{4}}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^{2}c^{7}x^{7}}{7}+\frac{2\operatorname{arccosh}(cx)c^{7}de\,x^{5}}{5}+\frac{\operatorname{arccosh}(cx)c^{7}x^{3}d^{2}}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(225c^{6}e^{2}x^{6}+882c^{6}de\,x^{4}+1225c^{6}d^{2}x^{2}+270c^{4}e^{2}x^{4}+\right.}{11025}}{c^{4}}}{c^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^4*(1/7*e^2*c^7*x^7+2/5*c^7*d*e*x^5+1/3*x^3*c^7*d^2)+b/c^4*(1/7*a
rccosh(c*x)*e^2*c^7*x^7+2/5*arccosh(c*x)*c^7*d*e*x^5+1/3*arccosh(c*x)*c^7*x
^3*d^2-1/11025*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(225*c^6*e^2*x^6+882*c^6*d*e*x^4
+1225*c^6*d^2*x^2+270*c^4*e^2*x^4+1176*c^4*d*e*x^2+2450*c^4*d^2+360*c^2*e^2
*x^2+2352*c^2*d*e+720*e^2)))
```

maxima [A] time = 0.36, size = 247, normalized size = 0.95

$$\frac{1}{7}ae^{2}x^{7}+\frac{2}{5}adex^{5}+\frac{1}{3}ad^{2}x^{3}+\frac{1}{9}\left(3x^{3}\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{2}}+\frac{2\sqrt{c^{2}x^{2}-1}}{c^{4}}\right)\right)bd^{2}+\frac{2}{75}\left(15x^{5}\operatorname{arcosh}(cx)-\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^2*x^7 + 2/5*a*d*e*x^5 + 1/3*a*d^2*x^3 + 1/9*(3*x^3*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2 + 2/75*(15*x^
5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4
 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e + 1/245*(35*x^7*arccosh(c*x) - (5*sqrt
(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x
^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 6.04, size = 340, normalized size = 1.31

$$\begin{cases} \frac{ad^2x^3}{3} + \frac{2adex^5}{5} + \frac{ae^2x^7}{7} + \frac{bd^2x^3\operatorname{acosh}(cx)}{3} + \frac{2bdex^5\operatorname{acosh}(cx)}{5} + \frac{be^2x^7\operatorname{acosh}(cx)}{7} - \frac{bd^2x^2\sqrt{c^2x^2-1}}{9c} - \frac{2bdex^4\sqrt{c^2x^2-1}}{25c} - \frac{be^2x^6\sqrt{c^2\cdots}}{49c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^3}{3} + \frac{2dex^5}{5} + \frac{e^2x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**3/3 + 2*a*d*e*x**5/5 + a*e**2*x**7/7 + b*d**2*x**3*aco
sh(c*x)/3 + 2*b*d*e*x**5*acosh(c*x)/5 + b*e**2*x**7*acosh(c*x)/7 - b*d**2*x
**2*sqrt(c**2*x**2 - 1)/(9*c) - 2*b*d*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**2*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*sqrt(c**2*x**2 - 1)/(9*c**
3) - 8*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 6*b*e**2*x**4*sqrt(c**2*x
**2 - 1)/(245*c**3) - 16*b*d*e*sqrt(c**2*x**2 - 1)/(75*c**5) - 8*b*e**2*x**
2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**2*sqrt(c**2*x**2 - 1)/(245*c**7)
, Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**3/3 + 2*d*e*x**5/5 + e**2*x**7/7), Tr
ue))
```

3.473 $\int x\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=269

$$\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{36c\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bx\left(1-c^2x^2\right)\left(2c^2d+e\right)\left(d+ex^2\right)}{144c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\left(2c^2d+\right.}{}$$

```
[Out] 1/6*(e*x^2+d)^3*(a+b*arccosh(c*x))/e+1/288*b*(44*c^4*d^2+44*c^2*d*e+15*e^2)
*x*(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+5/144*b*(2*c^2*d+e)*x*(-c^2
*x^2+1)*(e*x^2+d)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/36*b*x*(-c^2*x^2+1)*(e*
x^2+d)^2/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/96*b*(2*c^2*d+e)*(8*c^4*d^2+8*c^2*
d*e+5*e^2)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^6/e/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5788, 902, 416, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}+\frac{bx\left(1-c^2x^2\right)\left(44c^4d^2+44c^2de+15e^2\right)}{288c^5\sqrt{cx-1}\sqrt{cx+1}}-\frac{b\sqrt{c^2x^2-1}\left(2c^2d+e\right)\left(8c^4d^2+8c^2de+\right.}{96c^6e\sqrt{cx-1}\sqrt{cx+}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(44*c^4*d^2 + 44*c^2*d*e + 15*e^2)*x*(1 - c^2*x^2))/(288*c^5*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]) + (5*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2))/(144*c
^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(36*c*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + ((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/(6*e)
- (b*(2*c^2*d + e)*(8*c^4*d^2 + 8*c^2*d*e + 5*e^2)*Sqrt[-1 + c^2*x^2]*ArcTa
nh[(c*x)/Sqrt[-1 + c^2*x^2]])/(96*c^6*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 902

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^FracPart[m])/(d*
f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)dx=\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{(bc)\int\frac{\left(d+ex^2\right)^3}{\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{6e}$$

$$=\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{\left(d+ex^2\right)^3}{\sqrt{-1+c^2x^2}}\,dx}{6e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$=\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{6e}-\frac{\left(b\sqrt{-1+c\right.}}{}$$

$$=\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{144c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{36c\sqrt{-1+cx}\sqrt{1+cx}}+\frac{\left(d+\right.}{}$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(\right.}{144c^3\sqrt{-1+cx}\sqrt{1+\right.}$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(\right.}{144c^3\sqrt{-1+cx}\sqrt{1+\right.}$$

$$=\frac{b\left(44c^4d^2+44c^2de+15e^2\right)x\left(1-c^2x^2\right)}{288c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{5b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(\right.}{144c^3\sqrt{-1+cx}\sqrt{1+\right.}$$

Mathematica [A] time = 0.34, size = 183, normalized size = 0.68

$$\frac{cx\left(48ac^5x\left(3d^2+3dex^2+e^2x^4\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(4c^4\left(18d^2+9dex^2+2e^2x^4\right)+2c^2e\left(27d+5ex^2\right)+15e^2\right)\right.}{288c^6}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(48*a*c^5*x*(3*d^2 + 3*d*e*x^2 + e^2*x^4) - b*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(15*e^2 + 2*c^2*e*(27*d + 5*e*x^2) + 4*c^4*(18*d^2 + 9*d*e*x^2 + 2*e^2
*x^4))) + 48*b*c^6*x^2*(3*d^2 + 3*d*e*x^2 + e^2*x^4)*ArcCosh[c*x] - 6*b*(24
*c^4*d^2 + 18*c^2*d*e + 5*e^2)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(288*c^
6)
```

fricas [A] time = 0.67, size = 195, normalized size = 0.72

$$\frac{48\,ac^6e^2x^6 + 144\,ac^6dex^4 + 144\,ac^6d^2x^2 + 3\left(16\,bc^6e^2x^6 + 48\,bc^6dex^4 + 48\,bc^6d^2x^2 - 24\,bc^4d^2 - 18\,bc^2de - 5\,be^2\right)}{288\,c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/288*(48*a*c^6*e^2*x^6 + 144*a*c^6*d*e*x^4 + 144*a*c^6*d^2*x^2 + 3*(16*b*c
^6*e^2*x^6 + 48*b*c^6*d*e*x^4 + 48*b*c^6*d^2*x^2 - 24*b*c^4*d^2 - 18*b*c^2*
d*e - 5*b*e^2)*log(c*x + sqrt(c^2*x^2 - 1)) - (8*b*c^5*e^2*x^5 + 2*(18*b*c^
5*d*e + 5*b*c^3*e^2)*x^3 + 3*(24*b*c^5*d^2 + 18*b*c^3*d*e + 5*b*c*e^2)*x)*s
qrt(c^2*x^2 - 1))/c^6
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 363, normalized size = 1.35

$$\frac{a\,e^2x^6}{6}+\frac{ade\,x^4}{2}+\frac{a\,x^2d^2}{2}+\frac{b\operatorname{arccosh}(cx)\,e^2x^6}{6}+\frac{b\operatorname{arccosh}(cx)\,de\,x^4}{2}+\frac{b\operatorname{arccosh}(cx)\,x^2d^2}{2}-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,e^2x^5}{36c}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/6*a*e^2*x^6+1/2*a*d*e*x^4+1/2*a*x^2*d^2+1/6*b*arccosh(c*x)*e^2*x^6+1/2*b*
arccosh(c*x)*d*e*x^4+1/2*b*arccosh(c*x)*x^2*d^2-1/36/c*b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*e^2*x^5-1/8/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x^3-1/4*b*d^2*x*(
c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2
-1)^(1/2)*d^2*ln(c*x+(c^2*x^2-1)^(1/2))-5/144/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*e^2*x^3-3/16/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e*x-3/16/c^4*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d*e*ln(c*x+(c^2*x^2-1)^(1/2))-5/96/
c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2*x-5/96/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1
/2)/(c^2*x^2-1)^(1/2)*e^2*ln(c*x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.98, size = 273, normalized size = 1.01

$$\frac{1}{6}\,ae^2x^6+\frac{1}{2}\,adex^4+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(2\,x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2\,c^2x+2\,\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^2+\frac{1}{16}\left(8\,x^4\operatorname{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/6*a*e^2*x^6 + 1/2*a*d*e*x^4 + 1/2*a*d^2*x^2 + 1/4*(2*x^2*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^3))*b*d^
2 + 1/16*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^
2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d*e + 1/288
*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)
*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1
)*c)/c^7)*c)*b*e^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 4.20, size = 306, normalized size = 1.14

$$\begin{cases} \frac{ad^2x^2}{2} + \frac{adex^4}{2} + \frac{ae^2x^6}{6} + \frac{bd^2x^2\operatorname{acosh}(cx)}{2} + \frac{bdex^4\operatorname{acosh}(cx)}{2} + \frac{be^2x^6\operatorname{acosh}(cx)}{6} - \frac{bd^2x\sqrt{c^2x^2-1}}{4c} - \frac{bdex^3\sqrt{c^2x^2-1}}{8c} - \frac{be^2x^5\sqrt{c^2x^2-1}}{36c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^2x^2}{2} + \frac{dex^4}{2} + \frac{e^2x^6}{6}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x**2/2 + a*d*e*x**4/2 + a*e**2*x**6/6 + b*d**2*x**2*acosh
(c*x)/2 + b*d*e*x**4*acosh(c*x)/2 + b*e**2*x**6*acosh(c*x)/6 - b*d**2*x*sqr
t(c**2*x**2 - 1)/(4*c) - b*d*e*x**3*sqrt(c**2*x**2 - 1)/(8*c) - b*e**2*x**5
*sqrt(c**2*x**2 - 1)/(36*c) - b*d**2*acosh(c*x)/(4*c**2) - 3*b*d*e*x*sqrt(c
**2*x**2 - 1)/(16*c**3) - 5*b*e**2*x**3*sqrt(c**2*x**2 - 1)/(144*c**3) - 3*
b*d*e*acosh(c*x)/(16*c**4) - 5*b*e**2*x*sqrt(c**2*x**2 - 1)/(96*c**5) - 5*b
*e**2*acosh(c*x)/(96*c**6), Ne(c, 0)), ((a + I*pi*b/2)*(d**2*x**2/2 + d*e*x
**4/2 + e**2*x**6/6), True))
```

3.474 $\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=196

$$d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)-\frac{2be\left(1-c^2x^2\right)^2\left(5c^2d+3e\right)}{45c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{be^2}{25c^5\sqrt{}}$$

```
[Out] d^2*x*(a+b*arccosh(c*x))+2/3*d*e*x^3*(a+b*arccosh(c*x))+1/5*e^2*x^5*(a+b*ar
ccosh(c*x))+1/15*b*(15*c^4*d^2+10*c^2*d*e+3*e^2)*(-c^2*x^2+1)/c^5/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-2/45*b*e*(5*c^2*d+3*e)*(-c^2*x^2+1)^2/c^5/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+1/25*b*e^2*(-c^2*x^2+1)^3/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 520, 1247, 698}

$$d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{b\left(1-c^2x^2\right)\left(15c^4d^2+10c^2de+3e^2\right.}{15c^5\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(15*c^4*d^2 + 10*c^2*d*e + 3*e^2)*(1 - c^2*x^2))/(15*c^5*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (2*b*e*(5*c^2*d + 3*e)*(1 - c^2*x^2)^2)/(45*c^5*Sqrt[-1 +
c*x]*Sqrt[1 + c*x]) + (b*e^2*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]) + d^2*x*(a + b*ArcCosh[c*x]) + (2*d*e*x^3*(a + b*ArcCosh[c*x]))/3
+ (e^2*x^5*(a + b*ArcCosh[c*x]))/5
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 698

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)\,dx = d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh\right.$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^2x^5\left(a+b\cosh\right.$$

$$= \frac{b\left(15c^4d^2+10c^2de+3e^2\right)\left(1-c^2x^2\right)}{15c^5\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{2be\left(5c^2d+3e\right)\left(1-c^2x^2\right)^2}{45c^5\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{}{25}$$

Mathematica [A] time = 0.21, size = 130, normalized size = 0.66

$$\frac{1}{225}\left(15ax\left(15d^2+10dex^2+3e^2x^4\right)-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)+4c^2e\left(25d+3ex^2\right)+2\right.}{c^5}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (15*a*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4
)))/c^5 + 15*b*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x])/225
```

fricas [A] time = 0.45, size = 163, normalized size = 0.83

$$\frac{45\,ac^5e^2x^5+150\,ac^5dex^3+225\,ac^5d^2x+15\left(3\,bc^5e^2x^5+10\,bc^5dex^3+15\,bc^5d^2x\right)\log\left(cx+\sqrt{c^2x^2-1}\right)-\left(9\,bc\right.}{225\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/225*(45*a*c^5*e^2*x^5 + 150*a*c^5*d*e*x^3 + 225*a*c^5*d^2*x + 15*(3*b*c^5
*e^2*x^5 + 10*b*c^5*d*e*x^3 + 15*b*c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1))
```

```
- (9*b*c^4*e^2*x^4 + 225*b*c^4*d^2 + 100*b*c^2*d*e + 24*b*e^2 + 2*(25*b*c^4
*d*e + 6*b*c^2*e^2)*x^2)*sqrt(c^2*x^2 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 157, normalized size = 0.80

$$\frac{\frac{a\left(\frac{1}{5}e^2c^5x^5+\frac{2}{3}c^5de\,x^3+x\,c^5d^2\right)}{c^4}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^2c^5x^5}{5}+\frac{2\operatorname{arccosh}(cx)c^5de\,x^3}{3}+\operatorname{arccosh}(cx)c^5x\,d^2-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(9c^4e^2x^4+50c^4de\,x^2+225d^2c^4+12c^2e^2x^2+100c^2de+\right.}{225}\right.}{c^4}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b/c^4*(1/5*arccosh(c
*x)*e^2*c^5*x^5+2/3*arccosh(c*x)*c^5*d*e*x^3+arccosh(c*x)*c^5*x*d^2-1/225*(
c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+225*c^4*d^2+12*c^2
*e^2*x^2+100*c^2*d*e+24*e^2)))
```

maxima [A] time = 0.47, size = 180, normalized size = 0.92

$$\frac{1}{5}\,ae^2x^5+\frac{2}{3}\,adex^3+\frac{2}{9}\left(3\,x^3\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bde+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}\left(cx\right)-\left(\frac{3\,\sqrt{c^2x^2-}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/5*a*e^2*x^5 + 2/3*a*d*e*x^3 + 2/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 -
 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d*e + 1/75*(15*x^5*arccosh(c*x) -
 (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^
2 - 1)/c^6)*c)*b*e^2 + a*d^2*x + (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d
^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^2, x)
```

sympy [A] time = 2.22, size = 246, normalized size = 1.26

$$\begin{cases} ad^2x+\frac{2adex^3}{3}+\frac{ae^2x^5}{5}+bd^2x\operatorname{acosh}(cx)+\frac{2bdex^3\operatorname{acosh}(cx)}{3}+\frac{be^2x^5\operatorname{acosh}(cx)}{5}-\frac{bd^2\sqrt{c^2x^2-1}}{c}-\frac{2bdex^2\sqrt{c^2x^2-1}}{9c}-\frac{be^2x^4\sqrt{c^2x^2-}}{25c} \\ \left(a+\frac{i\pi b}{2}\right)\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**2*x + 2*a*d*e*x**3/3 + a*e**2*x**5/5 + b*d**2*x*acosh(c*x)
+ 2*b*d*e*x**3*acosh(c*x)/3 + b*e**2*x**5*acosh(c*x)/5 - b*d**2*sqrt(c**2*x
**2 - 1)/c - 2*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c) - b*e**2*x**4*sqrt(c**2
*x**2 - 1)/(25*c) - 4*b*d*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*e**2*x**2*sq
rt(c**2*x**2 - 1)/(75*c**3) - 8*b*e**2*sqrt(c**2*x**2 - 1)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

3.475 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x}\,dx$

Optimal. Leaf size=342

$$d^2\log(x)\left(a+b\cosh^{-1}(cx)\right)+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{ibd^2\sqrt{1-c^2x^2}\,\mathrm{Li}_2\left(e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}-$$

```
[Out] -1/32*b*e*(16*c^2*d+3*e)*arccosh(c*x)/c^4+d*e*x^2*(a+b*arccosh(c*x))+1/4*e^
2*x^4*(a+b*arccosh(c*x))+d^2*(a+b*arccosh(c*x))*ln(x)-1/32*b*e*(16*c^2*d+3*
e)*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/16*b*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)/c-1/2*I*b*d^2*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)+b*d^2*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*d^2*arcsin(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c
*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*I*b*d^2*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^
2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.79, antiderivative size = 369, normalized size of antiderivative = 1.08, number of steps used = 16, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 6742, 90, 52, 100, 12, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd^2\sqrt{1-c^2x^2}\,\mathrm{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+d^2\log(x)\left(a+b\cosh^{-1}(cx)\right)+dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cos\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] -(b*d*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*c) - (3*b*e^2*x*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/(32*c^3) - (b*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) -
(b*d*e*ArcCosh[c*x])/(2*c^2) - (3*b*e^2*ArcCosh[c*x])/(32*c^4) + d*e*x^2*(a
 + b*ArcCosh[c*x]) + (e^2*x^4*(a + b*ArcCosh[c*x]))/4 - ((I/2)*b*d^2*Sqrt[1
 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*d^2*Sqrt[1 -
 c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + d^2*(a + b*ArcCosh[c*x])*Log[x] - (b*d^2*Sqrt[1 - c^2*x^2]*ArcS
in[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((I/2)*b*d^2*Sqrt[1 - c^2*
x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= dex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^2x^4\left(a+b\cosh^{-1}(cx)\right)+d^2\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}+dex^2\left(a+b\cosh^{-1}(\right.$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1+cx}}{16c}-\frac{bde\cosh^{-1}(cx)}{2c^2}+d$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1}}{16c}$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1}}{16c}$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1}}{16c}$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1}}{16c}$$

$$= -\frac{bdex\sqrt{-1+cx}\,\sqrt{1+cx}}{2c}-\frac{3be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{32c^3}-\frac{be^2x^3\sqrt{-1+cx}\,\sqrt{1}}{16c}$$

Mathematica [A] time = 0.44, size = 217, normalized size = 0.63

$$ad^2\log(x)+adex^2+\frac{1}{4}ae^2x^4-\frac{bde\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{2c^2}-\frac{be^2\left(cx\sqrt{cx-1}\sqrt{cx+1}\left(2c^2x^2+\right.\right.}{32c^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] a*d*e*x^2 + (a*e^2*x^4)/4 + b*d*e*x^2*ArcCosh[c*x] + (b*e^2*x^4*ArcCosh[c*x
])/4 - (b*d*e*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sqrt[(-1 + c*x)
/(1 + c*x)]]))/(2*c^2) - (b*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3 + 2*c^
2*x^2) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(32*c^4) + (b*d^2*ArcCosh[
c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]))/2 + a*d^2*Log[x] - (b
*d^2*PolyLog[2, -E^(-2*ArcCosh[c*x])])/2
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arcosh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^2\left(b\,\text{arcosh}\left(cx\right)+a\right)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x, x)
```

maple [A] time = 0.34, size = 225, normalized size = 0.66

$$\frac{a\,e^2x^4}{4}+ade\,x^2+a\,d^2\ln\left(cx\right)+\frac{b\,\text{arccosh}\left(cx\right)e^2x^4}{4}-\frac{bdex\sqrt{cx-1}\,\sqrt{cx+1}}{2c}+b\,\text{arccosh}\left(cx\right)de\,x^2+\frac{b\,d^2\,\text{polylog}\left(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/4*a*e^2*x^4+a*d*e*x^2+a*d^2*ln(c*x)+1/4*b*arccosh(c*x)*e^2*x^4-1/2*b*d*e*
x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+b*arccosh(c*x)*d*e*x^2+1/2*b*d^2*polylog(2,
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/2*b*d*e*arccosh(c*x)/c^2-1/16*b*e^2
*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-3/32*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3-1/2*b*arccosh(c*x)^2*d^2+b*d^2*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2)-3/32*b*e^2*arccosh(c*x)/c^4
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}ae^2x^4+adex^2+ad^2\log(x)+\int be^2x^3\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+2\,bdex\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\frac{bd^2\log}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^2*x^4 + a*d*e*x^2 + a*d^2*log(x) + integrate(b*e^2*x^3*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)) + 2*b*d*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
)) + b*d^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2 + d\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^2}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x, x)
```

3.476 $\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx$

Optimal. Leaf size=160

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x^2\right)\left(6c^2d+e\right)}{3c^3\sqrt{cx-1}\sqrt{cx+1}}-\frac{be^2(}{9c^3\sqrt{cx}}$$

```
[Out] -d^2*(a+b*arccosh(c*x))/x+2*d*e*x*(a+b*arccosh(c*x))+1/3*e^2*x^3*(a+b*arcco
sh(c*x))+b*c*d^2*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))+1/3*b*e*(6*c^2*d+e)*(-
c^2*x^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/9*b*e^2*(-c^2*x^2+1)^2/c^3/(c*
x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 185, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {270, 5790, 520, 1251, 897, 1153, 205}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^2\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right.}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(6*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b
*e^2*(1 - c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*Ar
cCosh[c*x]))/x + 2*d*e*x*(a + b*ArcCosh[c*x]) + (e^2*x^3*(a + b*ArcCosh[c*x
]))/3 + (b*c*d^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 +
c*x]*Sqrt[1 + c*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
```

```
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}+2dex\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^2x^3\left(a+b\cosh^{-1}(c\right.$$

$$= \frac{be\left(6c^2d+e\right)\left(1-c^2x^2\right)}{3c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{be^2\left(1-c^2x^2\right)^2}{9c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-$$

$$= \frac{be\left(6c^2d+e\right)\left(1-c^2x^2\right)}{3c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{be^2\left(1-c^2x^2\right)^2}{9c^3\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{x}-$$

Mathematica [A] time = 0.26, size = 128, normalized size = 0.80

$$\frac{1}{3}\left(-\frac{3ad^2}{x}+6adex+ae^2x^3-\frac{be\sqrt{cx-1}\sqrt{cx+1}\left(c^2\left(18d+ex^2\right)+2e\right)}{3c^3}+\frac{b\cosh^{-1}(cx)\left(-3d^2+6dex^2+e^2x^4\right)}{x}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] ((-3*a*d^2)/x + 6*a*d*e*x + a*e^2*x^3 - (b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(
2*e + c^2*(18*d + e*x^2)))/(3*c^3) + (b*(-3*d^2 + 6*d*e*x^2 + e^2*x^4)*ArcC
osh[c*x])/x - 3*b*c*d^2*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])/3
```

fricas [A] time = 0.86, size = 236, normalized size = 1.48

$$\frac{3\,ac^3e^2x^4+18\,bc^4d^2x\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+18\,ac^3dex^2-9\,ac^3d^2+3\left(3\,bc^3d^2-6\,bc^3de-bc^3e^2\right)x\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/9*(3*a*c^3*e^2*x^4 + 18*b*c^4*d^2*x*arctan(-c*x + sqrt(c^2*x^2 - 1)) + 18
*a*c^3*d*e*x^2 - 9*a*c^3*d^2 + 3*(3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x*
log(-c*x + sqrt(c^2*x^2 - 1)) + 3*(b*c^3*e^2*x^4 + 6*b*c^3*d*e*x^2 - 3*b*c^
3*d^2 + (3*b*c^3*d^2 - 6*b*c^3*d*e - b*c^3*e^2)*x)*log(c*x + sqrt(c^2*x^2 -
 1)) - (b*c^2*e^2*x^3 + 2*(9*b*c^2*d*e + b*e^2)*x)*sqrt(c^2*x^2 - 1))/(c^3*
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^2(b\,\mathrm{arcosh}\,(cx)+a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^2, x)
```

maple [A] time = 0.02, size = 177, normalized size = 1.11

$$\frac{a\,x^3e^2}{3}+2adex-\frac{d^2a}{x}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^3e^2}{3}+2b\,\mathrm{arccosh}\,(cx)\,dex-\frac{d^2b\,\mathrm{arccosh}\,(cx)}{x}-\frac{c\,d^2b\sqrt{cx-1}\,\sqrt{cx+1}\,\ \mathrm{arcta}}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/3*a*x^3*e^2+2*a*d*e*x-d^2*a/x+1/3*b*arccosh(c*x)*x^3*e^2+2*b*arccosh(c*x)
*d*e*x-d^2*b*arccosh(c*x)/x-c*d^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)
^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))-1/9*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*
e^2-2*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e-2/9*b/c^3*(c*x-1)^(1/2)*(c*x+1)^(
1/2)*e^2
```

maxima [A] time = 0.77, size = 134, normalized size = 0.84

$$\frac{1}{3}\,ae^2x^3-\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^2+\frac{1}{9}\left(3\,x^3\,\mathrm{arcosh}\,(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)be^2+2\,ade$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^2*x^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d^2 + 1/9*(3*x^
3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b
*e^2 + 2*a*d*e*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d*e/c - a*d^2
/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b \operatorname{acosh}(c x)) \left(e x^2 + d\right)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx)) \left(d + ex^2\right)^2}{x^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**2, x)
```

3.477 $\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx$

Optimal. Leaf size=321

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+2de\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)-\frac{ibde\sqrt{1-c^2x^2}\,\text{Li}_2\left(e^{2i\sin^{-1}(c\right.}}{\sqrt{cx-1}\,\sqrt{cx+1}}$$

```
[Out] -1/4*b*e^2*arccosh(c*x)/c^2-1/2*d^2*(a+b*arccosh(c*x))/x^2+1/2*e^2*x^2*(a+b
*arccosh(c*x))+2*d*e*(a+b*arccosh(c*x))*ln(x)+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/x-1/4*b*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-I*b*d*e*arcsin(c*x)^
2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+2*b*d*e*arcsin(c*x)*ln(1-(
I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-2*b*d*e*arcsin(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-I
*b*d*e*polylog(2,(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.81, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 12, 6742, 95, 90, 52, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibde\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{\sqrt{cx-1}\,\sqrt{cx+1}}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+2de\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{2}e^2x^2\left(a+b\,\text{c}\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] (b*c*d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*x) - (b*e^2*x*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])/(4*c) - (b*e^2*ArcCosh[c*x])/(4*c^2) - (d^2*(a + b*ArcCosh[c*x])
)/(2*x^2) + (e^2*x^2*(a + b*ArcCosh[c*x]))/2 - (I*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (2*b*d*e*Sqrt[1 - c^2*x^2]*A
rcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) +
 2*d*e*(a + b*ArcCosh[c*x])*Log[x] - (2*b*d*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]
*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (I*b*d*e*Sqrt[1 - c^2*x^2]*PolyLo
g[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
```

```
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
```

```
x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)+2de\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)+2de\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)+2de\left(a+b\cosh^{-1}\right.$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{1}{2}e^2x^2\left(a+b\cosh^{-1}(cx)\right)+2de\left(a+b\cosh^{-1}\right.$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{d^2\left(a+b\cosh^{-1}(cx\right.}{2x^2}$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

$$= \frac{bcd^2\sqrt{-1+cx}\,\sqrt{1+cx}}{2x}-\frac{be^2x\sqrt{-1+cx}\,\sqrt{1+cx}}{4c}-\frac{be^2\cosh^{-1}(cx)}{4c^2}-$$

Mathematica [A] time = 0.45, size = 173, normalized size = 0.54

$$\frac{1}{4}\left(-\frac{2ad^2}{x^2}+8ade\log(x)+2ae^2x^2+\frac{be^2\left(2c^2x^2\cosh^{-1}(cx)-cx\sqrt{cx-1}\sqrt{cx+1}-2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{c^2}+\frac{2bd^2(cx\ldots}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^2)/x^2 + 2*a*e^2*x^2 + (2*b*d^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
- ArcCosh[c*x]))/x^2 + (b*e^2*(-(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + 2*c^2*
x^2*ArcCosh[c*x] - 2*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/c^2 + 8*a*d*e*Lo
g[x] + 4*b*d*e*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]
) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/4
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\mathrm{arcosh}\,(cx)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(ex^2+d\right)^2(b\,\mathrm{arcosh}\,(cx)+a)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^3, x)
```

maple [A] time = 0.54, size = 198, normalized size = 0.62

$$\frac{a\,x^2e^2}{2}+2ade\ln(cx)-\frac{a\,d^2}{2x^2}-bde\,\mathrm{arccosh}\,(cx)^2-\frac{b\,e^2x\sqrt{cx-1}\,\sqrt{cx+1}}{4c}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^2e^2}{2}-\frac{b\,e^2\mathrm{arccosh}\,(cx)}{4c^2}+\frac{bc\,d\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/2*a*x^2*e^2+2*a*d*e*ln(c*x)-1/2*a*d^2/x^2-b*d*e*arccosh(c*x)^2-1/4*b*e^2*
x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c+1/2*b*arccosh(c*x)*x^2*e^2-1/4*b*e^2*arccos
h(c*x)/c^2+1/2*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x-1/2*c^2*b*d^2-1/2*b*ar
ccosh(c*x)*d^2/x^2+2*b*d*e*arccosh(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2)+b*d*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,ae^2x^2+\frac{1}{2}\,bd^2\left(\frac{\sqrt{c^2x^2-1}\,c}{x}-\frac{\mathrm{arcosh}\,(cx)}{x^2}\right)+2\,ade\log(x)-\frac{ad^2}{2\,x^2}+\int be^2x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\frac{2\,bde\log(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*a*e^2*x^2 + 1/2*b*d^2*(sqrt(c^2*x^2 - 1)*c/x - arccosh(c*x)/x^2) + 2*a*
d*e*log(x) - 1/2*a*d^2/x^2 + integrate(b*e^2*x*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 2*b*d*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2 + d\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^2}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**3,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**3, x)
```

3.478 $\int \frac{(d+ex^2)^2(a+b\cosh^{-1}(cx))}{x^4}\,dx$

Optimal. Leaf size=184

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd\sqrt{c^2x^2-1}\,(c^2}{6\sqrt{c}}$$

```
[Out] -1/3*d^2*(a+b*arccosh(c*x))/x^3-2*d*e*(a+b*arccosh(c*x))/x+e^2*x*(a+b*arcco
sh(c*x))+b*e^2*(-c^2*x^2+1)/c/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c*d^2*(-c^2
*x^2+1)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/6*b*c*d*(c^2*d+12*e)*arctan((c^2*
x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {270, 5790, 520, 1251, 897, 1157, 388, 205}

$$-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}+e^2x\left(a+b\cosh^{-1}(cx)\right)-\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{cx-1}\sqrt{cx+1}}+\frac{bcd\sqrt{c^2x^2-1}\,(c^2}{6\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*e^2*(1 - c^2*x^2))/(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*d^2*(1 - c^2*
x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^2*(a + b*ArcCosh[c*x]))/(3*
x^3) - (2*d*e*(a + b*ArcCosh[c*x]))/x + e^2*x*(a + b*ArcCosh[c*x]) + (b*c*d
*(c^2*d + 12*e)*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 +
 c*x]*Sqrt[1 + c*x])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}{x^4}\,dx = -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x} + e^2x\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x} + e^2x\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x} + e^2x\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x} + e^2x\left(a+b\cosh^{-1}(cx)\right)$$

$$= -\frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - \frac{2de\left(a+b\cosh^{-1}(cx)\right)}{x}$$

$$= \frac{be^2\left(1-c^2x^2\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - 2a$$

$$= \frac{be^2\left(1-c^2x^2\right)}{c\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^2\left(1-c^2x^2\right)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{3x^3} - 2a$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.72

$$-\frac{ad^2}{3x^3}-\frac{2ade}{x}+ae^2x-\frac{1}{6}bcd\left(c^2d+12e\right)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right)+b\sqrt{cx-1}\sqrt{cx+1}\left(\frac{cd^2}{6x^2}-\frac{e^2}{c}\right)-\frac{b\cosh^{-1}(cx)\left(d^2\right.}{3x}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] -1/3*(a*d^2)/x^3 - (2*a*d*e)/x + a*e^2*x + b*(-(e^2/c) + (c*d^2)/(6*x^2))*S
qrt[-1 + c*x]*Sqrt[1 + c*x] - (b*(d^2 + 6*d*e*x^2 - 3*e^2*x^4)*ArcCosh[c*x]
)/(3*x^3) - (b*c*d*(c^2*d + 12*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])])
/6
```

fricas [A] time = 0.77, size = 216, normalized size = 1.17

$$\frac{6\,ace^2x^4-12\,acdex^2+2\left(bc^4d^2+12\,bc^2de\right)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right)+2\left(bcd^2+6\,bcde-3\,bce^2\right)x^3\log\left(-cx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*c*e^2*x^4 - 12*a*c*d*e*x^2 + 2*(b*c^4*d^2 + 12*b*c^2*d*e)*x^3*arct
an(-c*x + sqrt(c^2*x^2 - 1)) + 2*(b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3*log(
-c*x + sqrt(c^2*x^2 - 1)) - 2*a*c*d^2 + 2*(3*b*c*e^2*x^4 - 6*b*c*d*e*x^2 -
b*c*d^2 + (b*c*d^2 + 6*b*c*d*e - 3*b*c*e^2)*x^3)*log(c*x + sqrt(c^2*x^2 - 1
)) + (b*c^2*d^2*x - 6*b*e^2*x^3)*sqrt(c^2*x^2 - 1))/(c*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2\left(b\operatorname{arcosh}\left(cx\right)+a\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)/x^4, x)
```

maple [A] time = 0.02, size = 196, normalized size = 1.07

$$ax\,e^2-\frac{2ade}{x}-\frac{a\,d^2}{3x^3}+b\operatorname{arccosh}\left(cx\right)x\,e^2-\frac{2b\operatorname{arccosh}\left(cx\right)de}{x}-\frac{b\operatorname{arccosh}\left(cx\right)d^2}{3x^3}-\frac{c^3b\sqrt{cx-1}\,\sqrt{cx+1}\,d^2\arctan\left(\right.}{6\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] a*x*e^2-2*a*d*e/x-1/3*a*d^2/x^3+b*arccosh(c*x)*x*e^2-2*b*arccosh(c*x)*d*e/x
-1/3*b*arccosh(c*x)*d^2/x^3-1/6*c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-
1)^(1/2)*d^2*arctan(1/(c^2*x^2-1)^(1/2))+1/6*b*c*d^2*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/x^2-2*c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2
*x^2-1)^(1/2))*d*e-1/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2
```

maxima [A] time = 0.55, size = 126, normalized size = 0.68

$$-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\operatorname{arcosh}\left(cx\right)}{x^3}\right)bd^2-2\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\operatorname{arcosh}\left(cx\right)}{x}\right)bde+ae^2x+\frac{\left(cx\,\mathrm{ar}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] -1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c + 2*arccosh(c*x)
/x^3)*b*d^2 - 2*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d*e + a*e^2*x +
 (c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*e^2/c - 2*a*d*e/x - 1/3*a*d^2/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+b\operatorname{acosh}\left(c\,x\right)\right)\,\left(e\,x^2+d\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^2)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\operatorname{acosh}\left(cx\right)\right)\left(d+ex^2\right)^2}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**2/x**4, x)
```

3.479 $\int x^4 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=435

$$\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{11}e^3x^{11}\left(a+b\cosh^{-1}(cx)\right)+-$$

```
[Out] 1/5*d^3*x^5*(a+b*arccosh(c*x))+3/7*d^2*e*x^7*(a+b*arccosh(c*x))+1/3*d*e^2*x
^9*(a+b*arccosh(c*x))+1/11*e^3*x^11*(a+b*arccosh(c*x))+1/1155*b*(231*c^6*d^
3+495*c^4*d^2*e+385*c^2*d*e^2+105*e^3)*(-c^2*x^2+1)/c^11/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/3465*b*(462*c^6*d^3+1485*c^4*d^2*e+1540*c^2*d*e^2+525*e^3)*(-c^
2*x^2+1)^2/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1925*b*(77*c^6*d^3+495*c^4*d^
2*e+770*c^2*d*e^2+350*e^3)*(-c^2*x^2+1)^3/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/2)-
1/1617*b*e*(99*c^4*d^2+308*c^2*d*e+210*e^2)*(-c^2*x^2+1)^4/c^11/(c*x-1)^(1/
2)/(c*x+1)^(1/2)+1/297*b*e^2*(11*c^2*d+15*e)*(-c^2*x^2+1)^5/c^11/(c*x-1)^(1
/2)/(c*x+1)^(1/2)-1/121*b*e^3*(-c^2*x^2+1)^6/c^11/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A] time = 0.62, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 1610, 1799, 1620}

$$\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{11}e^3x^{11}\left(a+b\cosh^{-1}(cx)\right)--$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(231*c^6*d^3 + 495*c^4*d^2*e + 385*c^2*d*e^2 + 105*e^3)*(1 - c^2*x^2))/(
1155*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(462*c^6*d^3 + 1485*c^4*d^2*e
+ 1540*c^2*d*e^2 + 525*e^3)*(1 - c^2*x^2)^2)/(3465*c^11*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]) + (b*(77*c^6*d^3 + 495*c^4*d^2*e + 770*c^2*d*e^2 + 350*e^3)*(1 -
 c^2*x^2)^3)/(1925*c^11*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(99*c^4*d^2 +
308*c^2*d*e + 210*e^2)*(1 - c^2*x^2)^4)/(1617*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (b*e^2*(11*c^2*d + 15*e)*(1 - c^2*x^2)^5)/(297*c^11*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) - (b*e^3*(1 - c^2*x^2)^6)/(121*c^11*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]) + (d^3*x^5*(a + b*ArcCosh[c*x]))/5 + (3*d^2*e*x^7*(a + b*ArcCosh[c*x]
))/7 + (d*e^2*x^9*(a + b*ArcCosh[c*x]))/3 + (e^3*x^11*(a + b*ArcCosh[c*x]))
/11
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^4 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+\right.$$

$$=\frac{1}{5}d^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}d^2ex^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}de^2x^9\left(a+\right.$$

$$=\frac{b\left(231c^6d^3+495c^4d^2e+385c^2de^2+105e^3\right)\left(1-c^2x^2\right)}{1155c^{11}\sqrt{-1+cx}\sqrt{1+cx}}-\frac{b\left(462c^6d^3\right.}{}$$

Mathematica [A] time = 0.41, size = 276, normalized size = 0.63

$$\frac{3465ax^5\left(231d^3+495d^2ex^2+385de^2x^4+105e^3x^6\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^{10}x^4\left(160083d^3+245025d^2ex^2+148225de^2x^4+33075e^3x^6\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (3465*a*x^5*(231*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6) - (b*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*(134400*e^3 + 4480*c^2*e^2*(121*d + 15*e*x^2) +
```

```
80*c^4*e*(9801*d^2 + 3388*d*e*x^2 + 630*e^2*x^4) + 24*c^6*(17787*d^3 + 1633
5*d^2*e*x^2 + 8470*d*e^2*x^4 + 1750*e^3*x^6) + c^10*x^4*(160083*d^3 + 24502
5*d^2*e*x^2 + 148225*d*e^2*x^4 + 33075*e^3*x^6) + 2*c^8*(106722*d^3*x^2 + 1
47015*d^2*e*x^4 + 84700*d*e^2*x^6 + 18375*e^3*x^8)))/c^11 + 3465*b*x^5*(231
*d^3 + 495*d^2*e*x^2 + 385*d*e^2*x^4 + 105*e^3*x^6)*ArcCosh[c*x])/4002075
```

fricas [A] time = 0.80, size = 334, normalized size = 0.77

$$\frac{363825\, ac^{11}e^3x^{11} + 1334025\, ac^{11}de^2x^9 + 1715175\, ac^{11}d^2ex^7 + 800415\, ac^{11}d^3x^5 + 3465\left(105\, bc^{11}e^3x^{11} + 385\, bc^{11}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/4002075*(363825*a*c^11*e^3*x^11 + 1334025*a*c^11*d*e^2*x^9 + 1715175*a*c^
11*d^2*e*x^7 + 800415*a*c^11*d^3*x^5 + 3465*(105*b*c^11*e^3*x^11 + 385*b*c^
11*d*e^2*x^9 + 495*b*c^11*d^2*e*x^7 + 231*b*c^11*d^3*x^5)*log(c*x + sqrt(c^
2*x^2 - 1)) - (33075*b*c^10*e^3*x^10 + 426888*b*c^6*d^3 + 1225*(121*b*c^10*
d*e^2 + 30*b*c^8*e^3)*x^8 + 784080*b*c^4*d^2*e + 25*(9801*b*c^10*d^2*e + 67
76*b*c^8*d*e^2 + 1680*b*c^6*e^3)*x^6 + 542080*b*c^2*d*e^2 + 3*(53361*b*c^10
*d^3 + 98010*b*c^8*d^2*e + 67760*b*c^6*d*e^2 + 16800*b*c^4*e^3)*x^4 + 13440
0*b*e^3 + 4*(53361*b*c^8*d^3 + 98010*b*c^6*d^2*e + 67760*b*c^4*d*e^2 + 1680
0*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^11
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 335, normalized size = 0.77

$$\frac{\frac{a\left(\frac{1}{11}e^3c^{11}x^{11}+\frac{1}{3}d\,e^2c^{11}x^9+\frac{3}{7}c^{11}d^2e\,x^7+\frac{1}{5}c^{11}x^5d^3\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^3c^{11}x^{11}}{11}+\frac{\operatorname{arccosh}(cx)d\,e^2c^{11}x^9}{3}+\frac{3\operatorname{arccosh}(cx)c^{11}d^2e\,x^7}{7}+\frac{\operatorname{arccosh}(cx)c^{11}x^5d^3}{5}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^5*(a/c^6*(1/11*e^3*c^11*x^11+1/3*d*e^2*c^11*x^9+3/7*c^11*d^2*e*x^7+1/5*
c^11*x^5*d^3)+b/c^6*(1/11*arccosh(c*x)*e^3*c^11*x^11+1/3*arccosh(c*x)*d*e^2
*c^11*x^9+3/7*arccosh(c*x)*c^11*d^2*e*x^7+1/5*arccosh(c*x)*c^11*x^5*d^3-1/4
002075*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(33075*c^10*e^3*x^10+148225*c^10*d*e^2*x
^8+245025*c^10*d^2*e*x^6+36750*c^8*e^3*x^8+160083*c^10*d^3*x^4+169400*c^8*d
*e^2*x^6+294030*c^8*d^2*e*x^4+42000*c^6*e^3*x^6+213444*c^8*d^3*x^2+203280*c
^6*d*e^2*x^4+392040*c^6*d^2*e*x^2+50400*c^4*e^3*x^4+426888*c^6*d^3+271040*c
^4*d*e^2*x^2+784080*c^4*d^2*e+67200*c^2*e^3*x^2+542080*c^2*d*e^2+134400*e^3
)))
```

maxima [A] time = 0.53, size = 451, normalized size = 1.04

$$\frac{1}{11}\,ae^3x^{11}+\frac{1}{3}\,ade^2x^9+\frac{3}{7}\,ad^2ex^7+\frac{1}{5}\,ad^3x^5+\frac{1}{75}\left(15\,x^5\operatorname{arcosh}(cx)-\left(\frac{3\,\sqrt{c^2x^2-1}\,x^4}{c^2}+\frac{4\,\sqrt{c^2x^2-1}\,x^2}{c^4}+\frac{8\,\sqrt{c^2x^2-}}{c^6}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/11*a*e^3*x^11 + 1/3*a*d*e^2*x^9 + 3/7*a*d^2*e*x^7 + 1/5*a*d^3*x^5 + 1/75*
(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x
^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^3 + 3/245*(35*x^7*arccosh(c*x) - (
5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2
- 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d^2*e + 1/945*(315*x^9*arccos
h(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 - 1)*x^6/c^4 + 48*
sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 + 128*sqrt(c^2*x^2
 - 1)/c^10)*c)*b*d*e^2 + 1/7623*(693*x^11*arccosh(c*x) - (63*sqrt(c^2*x^2 -
 1)*x^10/c^2 + 70*sqrt(c^2*x^2 - 1)*x^8/c^4 + 80*sqrt(c^2*x^2 - 1)*x^6/c^6
+ 96*sqrt(c^2*x^2 - 1)*x^4/c^8 + 128*sqrt(c^2*x^2 - 1)*x^2/c^10 + 256*sqrt(
c^2*x^2 - 1)/c^12)*c)*b*e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^4*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 38.83, size = 638, normalized size = 1.47

$$\begin{cases} \frac{ad^3x^5}{5} + \frac{3ad^2ex^7}{7} + \frac{ade^2x^9}{3} + \frac{ae^3x^{11}}{11} + \frac{bd^3x^5\operatorname{acosh}(cx)}{5} + \frac{3bd^2ex^7\operatorname{acosh}(cx)}{7} + \frac{bde^2x^9\operatorname{acosh}(cx)}{3} + \frac{be^3x^{11}\operatorname{acosh}(cx)}{11} - \frac{bd^3x^4\sqrt{c^2x^2-}}{25c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^5}{5} + \frac{3d^2ex^7}{7} + \frac{de^2x^9}{3} + \frac{e^3x^{11}}{11}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**5/5 + 3*a*d**2*e*x**7/7 + a*d*e**2*x**9/3 + a*e**3*x**
11/11 + b*d**3*x**5*acosh(c*x)/5 + 3*b*d**2*e*x**7*acosh(c*x)/7 + b*d*e**2*
x**9*acosh(c*x)/3 + b*e**3*x**11*acosh(c*x)/11 - b*d**3*x**4*sqrt(c**2*x**2
 - 1)/(25*c) - 3*b*d**2*e*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*d*e**2*x**8*s
qrt(c**2*x**2 - 1)/(27*c) - b*e**3*x**10*sqrt(c**2*x**2 - 1)/(121*c) - 4*b*
d**3*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 18*b*d**2*e*x**4*sqrt(c**2*x**2 -
 1)/(245*c**3) - 8*b*d*e**2*x**6*sqrt(c**2*x**2 - 1)/(189*c**3) - 10*b*e**3
*x**8*sqrt(c**2*x**2 - 1)/(1089*c**3) - 8*b*d**3*sqrt(c**2*x**2 - 1)/(75*c*
*5) - 24*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*d*e**2*x**4*sq
rt(c**2*x**2 - 1)/(315*c**5) - 80*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(7623*c**
5) - 48*b*d**2*e*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(945*c**7) - 32*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(2541*c**7) - 1
28*b*d*e**2*sqrt(c**2*x**2 - 1)/(945*c**9) - 128*b*e**3*x**2*sqrt(c**2*x**2
 - 1)/(7623*c**9) - 256*b*e**3*sqrt(c**2*x**2 - 1)/(7623*c**11), Ne(c, 0)),
 ((a + I*pi*b/2)*(d**3*x**5/5 + 3*d**2*e*x**7/7 + d*e**2*x**9/3 + e**3*x**1
1/11), True))
```

3.480 $\int x^3 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=494

$$\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^4}{100ce\sqrt{cx-1}\sqrt{cx+1}}+\frac{bx\left(1-c^2x^2\right)\left(11c^2d+\cdots\right.}{1600c^3e\sqrt{cx-1}\cdots}$$

```
[Out] -1/8*d*(e*x^2+d)^4*(a+b*arccosh(c*x))/e^2+1/10*(e*x^2+d)^5*(a+b*arccosh(c*x
))/e^2-1/76800*b*(1232*c^8*d^4-2536*c^6*d^3*e-7758*c^4*d^2*e^2-6615*c^2*d*e
^3-1890*e^4)*x*(-c^2*x^2+1)/c^9/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/38400*b*(13
6*c^6*d^3-1096*c^4*d^2*e-1617*c^2*d*e^2-630*e^3)*x*(-c^2*x^2+1)*(e*x^2+d)/c
^7/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/9600*b*(26*c^4*d^2+201*c^2*d*e+126*e^2)*
x*(-c^2*x^2+1)*(e*x^2+d)^2/c^5/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1600*b*(11*c
^2*d+18*e)*x*(-c^2*x^2+1)*(e*x^2+d)^3/c^3/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1
00*b*x*(-c^2*x^2+1)*(e*x^2+d)^4/c/e/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/5120*b*(1
28*c^10*d^5-480*c^6*d^3*e^2-800*c^4*d^2*e^3-525*c^2*d*e^4-126*e^5)*arctanh(
c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^10/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A] time = 0.65, antiderivative size = 494, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {266, 43, 5790, 12, 566, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{bx\left(1-c^2x^2\right)\left(26c^4d^2+201c^2de+126e^2\right)\left(d+ex\cdots\right.}{9600c^5e\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(b*(1232*c^8*d^4 - 2536*c^6*d^3*e - 7758*c^4*d^2*e^2 - 6615*c^2*d*e^3 - 18
90*e^4)*x*(1 - c^2*x^2))/(76800*c^9*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(1
36*c^6*d^3 - 1096*c^4*d^2*e - 1617*c^2*d*e^2 - 630*e^3)*x*(1 - c^2*x^2)*(d
+ e*x^2))/(38400*c^7*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(26*c^4*d^2 + 201
*c^2*d*e + 126*e^2)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(9600*c^5*e*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*(11*c^2*d + 18*e)*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(16
00*c^3*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^4)/
(100*c*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d*(d + e*x^2)^4*(a + b*ArcCosh[c*
x]))/(8*e^2) + ((d + e*x^2)^5*(a + b*ArcCosh[c*x]))/(10*e^2) + (b*(128*c^10
*d^5 - 480*c^6*d^3*e^2 - 800*c^4*d^2*e^3 - 525*c^2*d*e^4 - 126*e^5)*Sqrt[-1
 + c^2*x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(5120*c^10*e^2*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 566

```
Int[((e1_) + (f1_.)*(x_)^(n2_.))^(r_.)*((e2_) + (f2_.)*(x_)^(n2_.))^(r_.)*(
(a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :>
Dist[((e1 + f1*x^(n/2))^FracPart[r]*(e2 + f2*x^(n/2))^FracPart[r])/(e1*e2 +
 f1*f2*x^n)^FracPart[r], Int[(a + b*x^n)^p*(c + d*x^n)^q*(e1*e2 + f1*f2*x^n
)^r, x], x] /; FreeQ[{a, b, c, d, e1, f1, e2, f2, n, p, q, r}, x] && EqQ[n2
, n/2] && EqQ[e2*f1 + e1*f2, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^3\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx = -\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-(bc)$$

$$= -\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{(bc)}{}$$

$$= -\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5\left(a+b\cosh^{-1}(cx)\right)}{10e^2}-\frac{(bc}{}$$

$$= \frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e^2}+\frac{\left(d+ex^2\right)^5}{}$$

$$= \frac{b\left(11c^2d+18e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{1600c^3e\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^4}{100ce\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{d(}{}$$

$$= \frac{b\left(26c^4d^2+201c^2de+126e^2\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{9600c^5e\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{b\left(11c^2d+18e\right)}{1600c^3e\sqrt{}}$$

$$= -\frac{b\left(136c^6d^3-1096c^4d^2e-1617c^2de^2-630e^3\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{38400c^7e\sqrt{-1+cx}\,\sqrt{1+cx}}+$$

$$= -\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2\right.}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2\right.}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{b\left(1232c^8d^4-2536c^6d^3e-7758c^4d^2e^2-6615c^2de^3-1890e^4\right)x\left(1-c^2\right.}{76800c^9e\sqrt{-1+cx}\,\sqrt{1+cx}}$$

Mathematica [A] time = 0.55, size = 294, normalized size = 0.60

$$\frac{1920ax^4\left(10d^3+20d^2ex^2+15de^2x^4+4e^3x^6\right)-\frac{30b\left(480c^6d^3+800c^4d^2e+525c^2de^2+126e^3\right)\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)}{c^{10}}-\frac{bx\sqrt{cx-1}\,\sqrt{cx+1}\,(16c^8}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (1920*a*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*x^4 + 4*e^3*x^6) - (b*x*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]*(1890*e^3 + 315*c^2*e^2*(25*d + 4*e*x^2) + 6*c^4*e*
(2000*d^2 + 875*d*e*x^2 + 168*e^2*x^4) + 8*c^6*(900*d^3 + 1000*d^2*e*x^2 +
525*d*e^2*x^4 + 108*e^3*x^6) + 16*c^8*(300*d^3*x^2 + 400*d^2*e*x^4 + 225*d*
e^2*x^6 + 48*e^3*x^8)))/c^9 + 1920*b*x^4*(10*d^3 + 20*d^2*e*x^2 + 15*d*e^2*
x^4 + 4*e^3*x^6)*ArcCosh[c*x] - (30*b*(480*c^6*d^3 + 800*c^4*d^2*e + 525*c^
2*d*e^2 + 126*e^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/c^10)/76800
```

fricas [A] time = 0.81, size = 330, normalized size = 0.67

$$\frac{7680\,ac^{10}e^3x^{10}+28800\,ac^{10}de^2x^8+38400\,ac^{10}d^2ex^6+19200\,ac^{10}d^3x^4+15\left(512\,bc^{10}e^3x^{10}+1920\,bc^{10}de^2x^8+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/76800*(7680*a*c^10*e^3*x^10 + 28800*a*c^10*d*e^2*x^8 + 38400*a*c^10*d^2*e
*x^6 + 19200*a*c^10*d^3*x^4 + 15*(512*b*c^10*e^3*x^10 + 1920*b*c^10*d*e^2*x
^8 + 2560*b*c^10*d^2*e*x^6 + 1280*b*c^10*d^3*x^4 - 480*b*c^6*d^3 - 800*b*c^
4*d^2*e - 525*b*c^2*d*e^2 - 126*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (768*
b*c^9*e^3*x^9 + 144*(25*b*c^9*d*e^2 + 6*b*c^7*e^3)*x^7 + 8*(800*b*c^9*d^2*e
 + 525*b*c^7*d*e^2 + 126*b*c^5*e^3)*x^5 + 10*(480*b*c^9*d^3 + 800*b*c^7*d^2
*e + 525*b*c^5*d*e^2 + 126*b*c^3*e^3)*x^3 + 15*(480*b*c^7*d^3 + 800*b*c^5*d
^2*e + 525*b*c^3*d*e^2 + 126*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^10
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 659, normalized size = 1.33

$$-\frac{b\sqrt{cx-1}\,\sqrt{cx+1}\,x^3d^3}{16c}-\frac{9b\sqrt{cx-1}\,\sqrt{cx+1}\,e^3x^7}{800c^3}-\frac{21b\sqrt{cx-1}\,\sqrt{cx+1}\,e^3x^5}{1600c^5}-\frac{21b\sqrt{cx-1}\,\sqrt{cx+1}\,e^3x^3}{1280c^7}-\frac{63}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] -1/16/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*d^3-9/800/c^3*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*e^3*x^7-21/1600/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^5-21/128
0/c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^3-63/2560/c^9*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*e^3*x-1/100/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^9-3/64/c*b*(c*
x-1)^(1/2)*(c*x+1)^(1/2)*x^7*d*e^2-35/512/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*d*e^2-63/2560/c^10*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e^3
*ln(c*x+(c^2*x^2-1)^(1/2))-3/32/c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-
1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^3-1/12/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^5*d^2*e-7/128/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e^2-105/1024/c^7*b
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e^2-5/32/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x*d^2*e-5/48/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*d^2*e+1/10*b*arccosh(c*
x)*e^3*x^10+1/4*b*arccosh(c*x)*x^4*d^3+3/8*a*d*e^2*x^8+1/2*a*d^2*e*x^6-3/32
*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3+1/2*b*arccosh(c*x)*d^2*e*x^6-5/32/
c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2
))*d^2*e+1/10*a*e^3*x^10+1/4*a*x^4*d^3-105/1024/c^8*b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d*e^2+3/8*b*arccosh(c*x)
*d*e^2*x^8
```

maxima [A] time = 0.40, size = 487, normalized size = 0.99

$$\frac{1}{10}\,ae^3x^{10}+\frac{3}{8}\,ade^2x^8+\frac{1}{2}\,ad^2ex^6+\frac{1}{4}\,ad^3x^4+\frac{1}{32}\left(8\,x^4\,\text{arcosh}\,(cx)-\left(\frac{2\,\sqrt{c^2x^2-1}\,x^3}{c^2}+\frac{3\,\sqrt{c^2x^2-1}\,x}{c^4}+\frac{3\,\log\left(2\,c\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/10*a*e^3*x^10 + 3/8*a*d*e^2*x^8 + 1/2*a*d^2*e*x^6 + 1/4*a*d^3*x^4 + 1/32*
(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3/c^2 + 3*sqrt(c^2*x^2 - 1)*x/
c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^5)*c)*b*d^3 + 1/96*(48*x^6*a
rccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 + 10*sqrt(c^2*x^2 - 1)*x^3/c^4 +
 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*
c)*b*d^2*e + 1/1024*(384*x^8*arccosh(c*x) - (48*sqrt(c^2*x^2 - 1)*x^7/c^2 +
 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*x^2 - 1)*x^3/c^6 + 105*sqrt(c^2
*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c^9)*c)*b*d*e^2
+ 1/12800*(1280*x^10*arccosh(c*x) - (128*sqrt(c^2*x^2 - 1)*x^9/c^2 + 144*sq
rt(c^2*x^2 - 1)*x^7/c^4 + 168*sqrt(c^2*x^2 - 1)*x^5/c^6 + 210*sqrt(c^2*x^2
- 1)*x^3/c^8 + 315*sqrt(c^2*x^2 - 1)*x/c^10 + 315*log(2*c^2*x + 2*sqrt(c^2*
x^2 - 1)*c)/c^11)*c)*b*e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^3*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 28.22, size = 604, normalized size = 1.22

$$\begin{cases} \frac{ad^3x^4}{4} + \frac{ad^2ex^6}{2} + \frac{3ade^2x^8}{8} + \frac{ae^3x^{10}}{10} + \frac{bd^3x^4\,\mathrm{acosh}\,(cx)}{4} + \frac{bd^2ex^6\,\mathrm{acosh}\,(cx)}{2} + \frac{3bde^2x^8\,\mathrm{acosh}\,(cx)}{8} + \frac{be^3x^{10}\,\mathrm{acosh}\,(cx)}{10} - \frac{bd^3x^3\sqrt{c^2x^2-1}}{16c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^4}{4} + \frac{d^2ex^6}{2} + \frac{3de^2x^8}{8} + \frac{e^3x^{10}}{10}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**4/4 + a*d**2*e*x**6/2 + 3*a*d*e**2*x**8/8 + a*e**3*x**
10/10 + b*d**3*x**4*acosh(c*x)/4 + b*d**2*e*x**6*acosh(c*x)/2 + 3*b*d*e**2*
x**8*acosh(c*x)/8 + b*e**3*x**10*acosh(c*x)/10 - b*d**3*x**3*sqrt(c**2*x**2
 - 1)/(16*c) - b*d**2*e*x**5*sqrt(c**2*x**2 - 1)/(12*c) - 3*b*d*e**2*x**7*s
qrt(c**2*x**2 - 1)/(64*c) - b*e**3*x**9*sqrt(c**2*x**2 - 1)/(100*c) - 3*b*d
**3*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(
48*c**3) - 7*b*d*e**2*x**5*sqrt(c**2*x**2 - 1)/(128*c**3) - 9*b*e**3*x**7*s
qrt(c**2*x**2 - 1)/(800*c**3) - 3*b*d**3*acosh(c*x)/(32*c**4) - 5*b*d**2*e*
x*sqrt(c**2*x**2 - 1)/(32*c**5) - 35*b*d*e**2*x**3*sqrt(c**2*x**2 - 1)/(512
*c**5) - 21*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(1600*c**5) - 5*b*d**2*e*acosh(
c*x)/(32*c**6) - 105*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 21*b*e**3
*x**3*sqrt(c**2*x**2 - 1)/(1280*c**7) - 105*b*d*e**2*acosh(c*x)/(1024*c**8)
 - 63*b*e**3*x*sqrt(c**2*x**2 - 1)/(2560*c**9) - 63*b*e**3*acosh(c*x)/(2560
*c**10), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**4/4 + d**2*e*x**6/2 + 3*d*e**2
*x**8/8 + e**3*x**10/10), True))
```

3.481 $\int x^2 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=365

$$\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\cosh^{-1}(cx)\right)-$$

```
[Out] 1/3*d^3*x^3*(a+b*arccosh(c*x))+3/5*d^2*e*x^5*(a+b*arccosh(c*x))+3/7*d*e^2*x
^7*(a+b*arccosh(c*x))+1/9*e^3*x^9*(a+b*arccosh(c*x))+1/315*b*(105*c^6*d^3+1
89*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)/c^9/(c*x-1)^(1/2)/(c*x+1)^(
1/2)-1/945*b*(105*c^6*d^3+378*c^4*d^2*e+405*c^2*d*e^2+140*e^3)*(-c^2*x^2+1)
^2/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/525*b*e*(63*c^4*d^2+135*c^2*d*e+70*e^2
)*(-c^2*x^2+1)^3/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/441*b*e^2*(27*c^2*d+28*e
)*(-c^2*x^2+1)^4/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/81*b*e^3*(-c^2*x^2+1)^5/
c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.54, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {270, 5790, 12, 1610, 1799, 1620}

$$\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}e^3x^9\left(a+b\cosh^{-1}(cx)\right)+$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(105*c^6*d^3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2))/(3
15*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*(105*c^6*d^3 + 378*c^4*d^2*e + 40
5*c^2*d*e^2 + 140*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*e*(63*c^4*d^2 + 135*c^2*d*e + 70*e^2)*(1 - c^2*x^2)^3)/(525*c^9*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^2*(27*c^2*d + 28*e)*(1 - c^2*x^2)^4)/(44
1*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^5)/(81*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (d^3*x^3*(a + b*ArcCosh[c*x]))/3 + (3*d^2*e*x^5*
(a + b*ArcCosh[c*x]))/5 + (3*d*e^2*x^7*(a + b*ArcCosh[c*x]))/7 + (e^3*x^9*(
a + b*ArcCosh[c*x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] && !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int x^2 \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\,c\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\,c\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\,c\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\,c\right.$$

$$= \frac{1}{3}d^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}d^2ex^5\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{7}de^2x^7\left(a+b\,c\right.$$

$$= \frac{b\left(105c^6d^3+189c^4d^2e+135c^2de^2+35e^3\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\,\sqrt{1+cx}} - \frac{b\left(105c^6d^3+37\right.}{9}$$

Mathematica [A] time = 0.33, size = 236, normalized size = 0.65

$$\frac{315ax^3\left(105d^3+189d^2ex^2+135de^2x^4+35e^3x^6\right) - \frac{b\sqrt{cx-1}\,\sqrt{cx+1}\left(c^8\left(11025d^3x^2+11907d^2ex^4+6075de^2x^6+1225e^3x^8\right)+2c^6\left(11025d^3\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x^3*(105*d^3 + 189*d^2*e*x^2 + 135*d*e^2*x^4 + 35*e^3*x^6) - (b*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^3 + 80*c^2*e^2*(243*d + 28*e*x^2) + 24*c^4
*e*(1323*d^2 + 405*d*e*x^2 + 70*e^2*x^4) + 2*c^6*(11025*d^3 + 7938*d^2*e*x^
2 + 3645*d*e^2*x^4 + 700*e^3*x^6) + c^8*(11025*d^3*x^2 + 11907*d^2*e*x^4 +
6075*d*e^2*x^6 + 1225*e^3*x^8)))/c^9 + 315*b*x^3*(105*d^3 + 189*d^2*e*x^2 +
 135*d*e^2*x^4 + 35*e^3*x^6)*ArcCosh[c*x])/99225
```

fricas [A] time = 0.64, size = 289, normalized size = 0.79

$$\frac{11025\,ac^9e^3x^9 + 42525\,ac^9de^2x^7 + 59535\,ac^9d^2ex^5 + 33075\,ac^9d^3x^3 + 315\left(35\,bc^9e^3x^9 + 135\,bc^9de^2x^7 + 189\,bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^3*x^9 + 42525*a*c^9*d*e^2*x^7 + 59535*a*c^9*d^2*e*x^
5 + 33075*a*c^9*d^3*x^3 + 315*(35*b*c^9*e^3*x^9 + 135*b*c^9*d*e^2*x^7 + 189
*b*c^9*d^2*e*x^5 + 105*b*c^9*d^3*x^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*
b*c^8*e^3*x^8 + 22050*b*c^6*d^3 + 31752*b*c^4*d^2*e + 25*(243*b*c^8*d*e^2 +
 56*b*c^6*e^3)*x^6 + 19440*b*c^2*d*e^2 + 3*(3969*b*c^8*d^2*e + 2430*b*c^6*d
*e^2 + 560*b*c^4*e^3)*x^4 + 4480*b*e^3 + (11025*b*c^8*d^3 + 15876*b*c^6*d^2
*e + 9720*b*c^4*d*e^2 + 2240*b*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 289, normalized size = 0.79

$$\frac{\frac{a\left(\frac{1}{9}e^3c^9x^9+\frac{3}{7}d\,e^2c^9x^7+\frac{3}{5}c^9d^2e\,x^5+\frac{1}{3}x^3c^9d^3\right)}{c^6}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^3c^9x^9}{9}+\frac{3\operatorname{arccosh}(cx)d\,e^2c^9x^7}{7}+\frac{3\operatorname{arccosh}(cx)c^9d^2e\,x^5}{5}+\frac{\operatorname{arccosh}(cx)c^9x^3d^3}{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,\left(1225\right.}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c^3*(a/c^6*(1/9*e^3*c^9*x^9+3/7*d*e^2*c^9*x^7+3/5*c^9*d^2*e*x^5+1/3*x^3*c
^9*d^3)+b/c^6*(1/9*arccosh(c*x)*e^3*c^9*x^9+3/7*arccosh(c*x)*d*e^2*c^9*x^7+
3/5*arccosh(c*x)*c^9*d^2*e*x^5+1/3*arccosh(c*x)*c^9*x^3*d^3-1/99225*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^3*x^8+6075*c^8*d*e^2*x^6+11907*c^8*d^2*e*x
^4+1400*c^6*e^3*x^6+11025*c^8*d^3*x^2+7290*c^6*d*e^2*x^4+15876*c^6*d^2*e*x^
2+1680*c^4*e^3*x^4+22050*c^6*d^3+9720*c^4*d*e^2*x^2+31752*c^4*d^2*e+2240*c^
2*e^3*x^2+19440*c^2*d*e^2+4480*e^3)))
```

maxima [A] time = 0.37, size = 374, normalized size = 1.02

$$\frac{1}{9}\,ae^3x^9+\frac{3}{7}\,ade^2x^7+\frac{3}{5}\,ad^2ex^5+\frac{1}{3}\,ad^3x^3+\frac{1}{9}\left(3\,x^3\,\operatorname{arcosh}\left(cx\right)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3+\frac{1}{25}\left(15\,x^5\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^3*x^9 + 3/7*a*d*e^2*x^7 + 3/5*a*d^2*e*x^5 + 1/3*a*d^3*x^3 + 1/9*(3*
x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))
*b*d^3 + 1/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(
c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e + 3/245*(35*x^7*
arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 +
```

```
8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^2 + 1/283
5*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c^2*x^2 -
 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x^2/c^8 +
 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x^2*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 16.71, size = 532, normalized size = 1.46

$$\begin{cases} \frac{ad^3x^3}{3} + \frac{3ad^2ex^5}{5} + \frac{3ade^2x^7}{7} + \frac{ae^3x^9}{9} + \frac{bd^3x^3\operatorname{acosh}(cx)}{3} + \frac{3bd^2ex^5\operatorname{acosh}(cx)}{5} + \frac{3bde^2x^7\operatorname{acosh}(cx)}{7} + \frac{be^3x^9\operatorname{acosh}(cx)}{9} - \frac{bd^3x^2\sqrt{c^2x^2-1}}{9c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^3}{3} + \frac{3d^2ex^5}{5} + \frac{3de^2x^7}{7} + \frac{e^3x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**3/3 + 3*a*d**2*e*x**5/5 + 3*a*d*e**2*x**7/7 + a*e**3*x
**9/9 + b*d**3*x**3*acosh(c*x)/3 + 3*b*d**2*e*x**5*acosh(c*x)/5 + 3*b*d*e**
2*x**7*acosh(c*x)/7 + b*e**3*x**9*acosh(c*x)/9 - b*d**3*x**2*sqrt(c**2*x**2
 - 1)/(9*c) - 3*b*d**2*e*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 3*b*d*e**2*x**6*
sqrt(c**2*x**2 - 1)/(49*c) - b*e**3*x**8*sqrt(c**2*x**2 - 1)/(81*c) - 2*b*d
**3*sqrt(c**2*x**2 - 1)/(9*c**3) - 4*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(25*
c**3) - 18*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*b*e**3*x**6*sqr
t(c**2*x**2 - 1)/(567*c**3) - 8*b*d**2*e*sqrt(c**2*x**2 - 1)/(25*c**5) - 24
*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*x**4*sqrt(c**2*x*
*2 - 1)/(945*c**5) - 48*b*d*e**2*sqrt(c**2*x**2 - 1)/(245*c**7) - 64*b*e**3
*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**3*sqrt(c**2*x**2 - 1)/(283
5*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**3/3 + 3*d**2*e*x**5/5 + 3*d*e*
*2*x**7/7 + e**3*x**9/9), True))
```

3.482 $\int x\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=358

$$\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}+\frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{cx-1}\sqrt{cx+1}}+\frac{7bx\left(1-c^2x^2\right)\left(2c^2d+e\right)\left(d+ex^2\right)^2}{384c^3\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bx\left(1-c^2x^2\right)\left(\right.}{3\ldots}$$

```
[Out] 1/8*(e*x^2+d)^4*(a+b*arccosh(c*x))/e+5/3072*b*(2*c^2*d+e)*(40*c^4*d^2+40*c^
2*d*e+21*e^2)*x*(-c^2*x^2+1)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/1536*b*(104*
c^4*d^2+104*c^2*d*e+35*e^2)*x*(-c^2*x^2+1)*(e*x^2+d)/c^5/(c*x-1)^(1/2)/(c*x
+1)^(1/2)+7/384*b*(2*c^2*d+e)*x*(-c^2*x^2+1)*(e*x^2+d)^2/c^3/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+1/64*b*x*(-c^2*x^2+1)*(e*x^2+d)^3/c/(c*x-1)^(1/2)/(c*x+1)^(1/
2)-1/1024*b*(128*c^8*d^4+256*c^6*d^3*e+288*c^4*d^2*e^2+160*c^2*d*e^3+35*e^4
)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^8/e/(c*x-1)^(1/2)/(c*x
+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {5788, 902, 416, 528, 388, 217, 206}

$$\frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e}+\frac{bx\left(1-c^2x^2\right)\left(104c^4d^2+104c^2de+35e^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{cx-1}\sqrt{cx+1}}+\frac{5bx\left(1-c^2x^2\right)\left(2c^2d+e\right)\ldots}{3072c^7\sqrt{cx\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (5*b*(2*c^2*d + e)*(40*c^4*d^2 + 40*c^2*d*e + 21*e^2)*x*(1 - c^2*x^2))/(307
2*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*(104*c^4*d^2 + 104*c^2*d*e + 35*e^
2)*x*(1 - c^2*x^2)*(d + e*x^2))/(1536*c^5*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
7*b*(2*c^2*d + e)*x*(1 - c^2*x^2)*(d + e*x^2)^2)/(384*c^3*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) + (b*x*(1 - c^2*x^2)*(d + e*x^2)^3)/(64*c*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]) + ((d + e*x^2)^4*(a + b*ArcCosh[c*x]))/(8*e) - (b*(128*c^8*d^4 +
256*c^6*d^3*e + 288*c^4*d^2*e^2 + 160*c^2*d*e^3 + 35*e^4)*Sqrt[-1 + c^2*x^2
]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(1024*c^8*e*Sqrt[-1 + c*x]*Sqrt[1 + c*
x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 902

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (c_.)*(x_)^2
)^(p_), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^FracPart[m])/(d*
f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + c*x^2)^p, x], x] /; Fr
eeQ[{a, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e*f + d*g, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int x\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)dx = \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{(bc)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{8e}$$

$$= \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{\left(d+ex^2\right)^4}{\sqrt{-1+c^2x^2}}dx}{8e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{\left(d+ex^2\right)^4\left(a+b\cosh^{-1}(cx)\right)}{8e} - \frac{\left(b\sqrt{-1+c\right.}{}$$

$$= \frac{7b\left(2c^2d+e\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)^2}{384c^3\sqrt{-1+cx}\sqrt{1+cx}} + \frac{bx\left(1-c^2x^2\right)\left(d+ex^2\right)^3}{64c\sqrt{-1+cx}\sqrt{1+cx}} + \frac{(d}{}$$

$$= \frac{b\left(104c^4d^2+104c^2de+35e^2\right)x\left(1-c^2x^2\right)\left(d+ex^2\right)}{1536c^5\sqrt{-1+cx}\sqrt{1+cx}} + \frac{7b\left(2c^2d+e\right)x}{384c^3\sqrt{-}}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{15}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{15}$$

$$= \frac{5b\left(2c^2d+e\right)\left(40c^4d^2+40c^2de+21e^2\right)x\left(1-c^2x^2\right)}{3072c^7\sqrt{-1+cx}\sqrt{1+cx}} + \frac{b\left(104c^4d^2+10\right.}{15}$$

Mathematica [A] time = 0.41, size = 256, normalized size = 0.72

$$\frac{cx\left(384ac^7x\left(4d^3+6d^2ex^2+4de^2x^4+e^3x^6\right)-b\sqrt{cx-1}\sqrt{cx+1}\left(16c^6\left(48d^3+36d^2ex^2+16de^2x^4+3e^3x^6\right)+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (c*x*(384*a*c^7*x*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*x^6) - b*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]*(105*e^3 + 10*c^2*e^2*(48*d + 7*e*x^2) + 8*c^4*e*(108
*d^2 + 40*d*e*x^2 + 7*e^2*x^4) + 16*c^6*(48*d^3 + 36*d^2*e*x^2 + 16*d*e^2*x
^4 + 3*e^3*x^6))) + 384*b*c^8*x^2*(4*d^3 + 6*d^2*e*x^2 + 4*d*e^2*x^4 + e^3*
x^6)*ArcCosh[c*x] - 6*b*(256*c^6*d^3 + 288*c^4*d^2*e + 160*c^2*d*e^2 + 35*e
^3)*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]])/(3072*c^8)
```

fricas [A] time = 0.54, size = 286, normalized size = 0.80

$$\frac{384\,ac^8e^3x^8+1536\,ac^8de^2x^6+2304\,ac^8d^2ex^4+1536\,ac^8d^3x^2+3\left(128\,bc^8e^3x^8+512\,bc^8de^2x^6+768\,bc^8d^2ex^4\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3072*(384*a*c^8*e^3*x^8 + 1536*a*c^8*d*e^2*x^6 + 2304*a*c^8*d^2*e*x^4 + 1
536*a*c^8*d^3*x^2 + 3*(128*b*c^8*e^3*x^8 + 512*b*c^8*d*e^2*x^6 + 768*b*c^8*
d^2*e*x^4 + 512*b*c^8*d^3*x^2 - 256*b*c^6*d^3 - 288*b*c^4*d^2*e - 160*b*c^2
*d*e^2 - 35*b*e^3)*log(c*x + sqrt(c^2*x^2 - 1)) - (48*b*c^7*e^3*x^7 + 8*(32
```

```
*b*c^7*d*e^2 + 7*b*c^5*e^3)*x^5 + 2*(288*b*c^7*d^2*e + 160*b*c^5*d*e^2 + 35
*b*c^3*e^3)*x^3 + 3*(256*b*c^7*d^3 + 288*b*c^5*d^2*e + 160*b*c^3*d*e^2 + 35
*b*c*e^3)*x)*sqrt(c^2*x^2 - 1))/c^8
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.02, size = 553, normalized size = 1.54

$$\frac{a e^3 x^8}{8}+\frac{a d e^2 x^6}{2}+\frac{3a d^2 e x^4}{4}+\frac{d^3 a x^2}{2}+\frac{b \operatorname{arccosh}(cx) e^3 x^8}{8}+\frac{b \operatorname{arccosh}(cx) d e^2 x^6}{2}+\frac{3b \operatorname{arccosh}(cx) d^2 e x^4}{4}+\frac{d^3 b \operatorname{arcc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/8*a*e^3*x^8+1/2*a*d*e^2*x^6+3/4*a*d^2*e*x^4+1/2*d^3*a*x^2+1/8*b*arccosh(c
*x)*e^3*x^8+1/2*b*arccosh(c*x)*d*e^2*x^6+3/4*b*arccosh(c*x)*d^2*e*x^4+1/2*d
^3*b*arccosh(c*x)*x^2-1/64/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^7-1/12/c*b
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^5*d*e^2-3/16/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*d^2*e-1/4*b*d^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/4/c^2*d^3*b*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))-7/384/c^3*b
*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^5-5/48/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)
*x^3*d*e^2-9/32/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d^2*e-9/32/c^4*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*ln(c*x+(c^2*x^2-1)^(1/2))*d^2*e-35/
1536/c^5*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^3-5/32/c^5*b*(c*x-1)^(1/2)*(c*
x+1)^(1/2)*x*d*e^2-5/32/c^6*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)
*ln(c*x+(c^2*x^2-1)^(1/2))*d*e^2-35/1024/c^7*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*
e^3*x-35/1024/c^8*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*e^3*ln(c*
x+(c^2*x^2-1)^(1/2))
```

maxima [A] time = 0.46, size = 409, normalized size = 1.14

$$\frac{1}{8}ae^3x^8+\frac{1}{2}ade^2x^6+\frac{3}{4}ad^2ex^4+\frac{1}{2}ad^3x^2+\frac{1}{4}\left(2x^2\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x}{c^2}+\frac{\log\left(2c^2x+2\sqrt{c^2x^2-1}\,c\right)}{c^3}\right)\right)bd^3+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/8*a*e^3*x^8 + 1/2*a*d*e^2*x^6 + 3/4*a*d^2*e*x^4 + 1/2*a*d^3*x^2 + 1/4*(2*
x^2*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x/c^2 + log(2*c^2*x + 2*sqrt(c^2*x^
2 - 1)*c)/c^3))*b*d^3 + 3/32*(8*x^4*arccosh(c*x) - (2*sqrt(c^2*x^2 - 1)*x^3
/c^2 + 3*sqrt(c^2*x^2 - 1)*x/c^4 + 3*log(2*c^2*x + 2*sqrt(c^2*x^2 - 1)*c)/c
^5)*c)*b*d^2*e + 1/96*(48*x^6*arccosh(c*x) - (8*sqrt(c^2*x^2 - 1)*x^5/c^2 +
 10*sqrt(c^2*x^2 - 1)*x^3/c^4 + 15*sqrt(c^2*x^2 - 1)*x/c^6 + 15*log(2*c^2*x
 + 2*sqrt(c^2*x^2 - 1)*c)/c^7)*c)*b*d*e^2 + 1/3072*(384*x^8*arccosh(c*x) -
(48*sqrt(c^2*x^2 - 1)*x^7/c^2 + 56*sqrt(c^2*x^2 - 1)*x^5/c^4 + 70*sqrt(c^2*
x^2 - 1)*x^3/c^6 + 105*sqrt(c^2*x^2 - 1)*x/c^8 + 105*log(2*c^2*x + 2*sqrt(c
^2*x^2 - 1)*c)/c^9)*c)*b*e^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int(x*(a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 11.72, size = 490, normalized size = 1.37

$$\begin{cases} \frac{ad^3x^2}{2} + \frac{3ad^2ex^4}{4} + \frac{ade^2x^6}{2} + \frac{ae^3x^8}{8} + \frac{bd^3x^2\,\mathrm{acosh}\,(cx)}{2} + \frac{3bd^2ex^4\,\mathrm{acosh}\,(cx)}{4} + \frac{bde^2x^6\,\mathrm{acosh}\,(cx)}{2} + \frac{be^3x^8\,\mathrm{acosh}\,(cx)}{8} - \frac{bd^3x\sqrt{c^2x^2-1}}{4c} \\ \left(a + \frac{i\pi b}{2}\right)\left(\frac{d^3x^2}{2} + \frac{3d^2ex^4}{4} + \frac{de^2x^6}{2} + \frac{e^3x^8}{8}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x**2/2 + 3*a*d**2*e*x**4/4 + a*d*e**2*x**6/2 + a*e**3*x**
8/8 + b*d**3*x**2*acosh(c*x)/2 + 3*b*d**2*e*x**4*acosh(c*x)/4 + b*d*e**2*x**
*6*acosh(c*x)/2 + b*e**3*x**8*acosh(c*x)/8 - b*d**3*x*sqrt(c**2*x**2 - 1)/(
4*c) - 3*b*d**2*e*x**3*sqrt(c**2*x**2 - 1)/(16*c) - b*d*e**2*x**5*sqrt(c**2
*x**2 - 1)/(12*c) - b*e**3*x**7*sqrt(c**2*x**2 - 1)/(64*c) - b*d**3*acosh(c
*x)/(4*c**2) - 9*b*d**2*e*x*sqrt(c**2*x**2 - 1)/(32*c**3) - 5*b*d*e**2*x**3
*sqrt(c**2*x**2 - 1)/(48*c**3) - 7*b*e**3*x**5*sqrt(c**2*x**2 - 1)/(384*c**
3) - 9*b*d**2*e*acosh(c*x)/(32*c**4) - 5*b*d*e**2*x*sqrt(c**2*x**2 - 1)/(32
*c**5) - 35*b*e**3*x**3*sqrt(c**2*x**2 - 1)/(1536*c**5) - 5*b*d*e**2*acosh(
c*x)/(32*c**6) - 35*b*e**3*x*sqrt(c**2*x**2 - 1)/(1024*c**7) - 35*b*e**3*ac
osh(c*x)/(1024*c**8), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x**2/2 + 3*d**2*e*x*
*4/4 + d*e**2*x**6/2 + e**3*x**8/8), True))
```

3.483 $\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=287

$$d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{3be^2\left(1\right.}{175c^7}$$

```
[Out] d^3*x*(a+b*arccosh(c*x))+d^2*e*x^3*(a+b*arccosh(c*x))+3/5*d*e^2*x^5*(a+b*ar
ccosh(c*x))+1/7*e^3*x^7*(a+b*arccosh(c*x))+1/35*b*(35*c^6*d^3+35*c^4*d^2*e+
21*c^2*d*e^2+5*e^3)*(-c^2*x^2+1)/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/105*b*e*
(35*c^4*d^2+42*c^2*d*e+15*e^2)*(-c^2*x^2+1)^2/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/
2)+3/175*b*e^2*(7*c^2*d+5*e)*(-c^2*x^2+1)^3/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)
-1/49*b*e^3*(-c^2*x^2+1)^4/c^7/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+d^3x\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{7}e^3x^7\left(a+b\cosh^{-1}(cx)\right)-\frac{be\left(1-c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(35*c^6*d^3 + 35*c^4*d^2*e + 21*c^2*d*e^2 + 5*e^3)*(1 - c^2*x^2))/(35*c^
7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e*(35*c^4*d^2 + 42*c^2*d*e + 15*e^2)*(
1 - c^2*x^2)^2)/(105*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(7*c^2*d
+ 5*e)*(1 - c^2*x^2)^3)/(175*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1
- c^2*x^2)^4)/(49*c^7*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*x*(a + b*ArcCosh[
c*x]) + d^2*e*x^3*(a + b*ArcCosh[c*x]) + (3*d*e^2*x^5*(a + b*ArcCosh[c*x]))
/5 + (e^3*x^7*(a + b*ArcCosh[c*x]))/7
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx = d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh\right.$$

$$= d^3x\left(a+b\cosh^{-1}(cx)\right)+d^2ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{5}de^2x^5\left(a+b\cosh\right.$$

$$= \frac{b\left(35c^6d^3+35c^4d^2e+21c^2de^2+5e^3\right)\left(1-c^2x^2\right)}{35c^7\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be\left(35c^4d^2+42c^2de\right.}{105c^7\sqrt{-1+}}$$

Mathematica [A] time = 0.29, size = 193, normalized size = 0.67

$$a\left(d^3x+d^2ex^3+\frac{3}{5}de^2x^5+\frac{e^3x^7}{7}\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^6\left(3675d^3+1225d^2ex^2+441de^2x^4+75e^3x^6\right)+2c^4e\left(1\right.\right.}{3675c^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] a*(d^3*x + d^2*e*x^3 + (3*d*e^2*x^5)/5 + (e^3*x^7)/7) - (b*Sqrt[-1 + c*x]*S
qrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 2
94*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
 75*e^3*x^6)))/(3675*c^7) + (b*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*
e^3*x^6)*ArcCosh[c*x])/35
```

fricas [A] time = 0.62, size = 241, normalized size = 0.84

$$\frac{525\,ac^7e^3x^7+2205\,ac^7de^2x^5+3675\,ac^7d^2ex^3+3675\,ac^7d^3x+105\left(5\,bc^7e^3x^7+21\,bc^7de^2x^5+35\,bc^7d^2ex^3+3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/3675*(525*a*c^7*e^3*x^7 + 2205*a*c^7*d*e^2*x^5 + 3675*a*c^7*d^2*e*x^3 + 3
675*a*c^7*d^3*x + 105*(5*b*c^7*e^3*x^7 + 21*b*c^7*d*e^2*x^5 + 35*b*c^7*d^2*
e*x^3 + 35*b*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (75*b*c^6*e^3*x^6 +
3675*b*c^6*d^3 + 2450*b*c^4*d^2*e + 1176*b*c^2*d*e^2 + 9*(49*b*c^6*d*e^2 +
10*b*c^4*e^3)*x^4 + 240*b*e^3 + (1225*b*c^6*d^2*e + 588*b*c^4*d*e^2 + 120*b
*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))/c^7
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 235, normalized size = 0.82

$$\frac{\frac{a\left(\frac{1}{7}e^{3}c^{7}x^{7}+\frac{3}{5}c^{7}d\,e^{2}x^{5}+c^{7}d^{2}e\,x^{3}+x\,c^{7}d^{3}\right)}{c^{6}}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^{3}c^{7}x^{7}}{7}+\frac{3\operatorname{arccosh}(cx)d\,e^{2}c^{7}x^{5}}{5}+\operatorname{arccosh}(cx)c^{7}d^{2}e\,x^{3}+\operatorname{arccosh}(cx)c^{7}x\,d^{3}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}\,(75c^{6}e^{3}x^{6}+\cdots}{\cdots}\right)}{c^{6}}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b/c^
6*(1/7*arccosh(c*x)*e^3*c^7*x^7+3/5*arccosh(c*x)*d*e^2*c^7*x^5+arccosh(c*x)
*c^7*d^2*e*x^3+arccosh(c*x)*c^7*x*d^3-1/3675*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(7
5*c^6*e^3*x^6+441*c^6*d*e^2*x^4+1225*c^6*d^2*e*x^2+90*c^4*e^3*x^4+3675*c^6*
d^3+588*c^4*d*e^2*x^2+2450*c^4*d^2*e+120*c^2*e^3*x^2+1176*c^2*d*e^2+240*e^3
)))
```

maxima [A] time = 0.43, size = 287, normalized size = 1.00

$$\frac{1}{7}ae^{3}x^{7}+\frac{3}{5}ade^{2}x^{5}+ad^{2}ex^{3}+\frac{1}{3}\left(3x^{3}\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^{2}x^{2}-1}\,x^{2}}{c^{2}}+\frac{2\sqrt{c^{2}x^{2}-1}}{c^{4}}\right)\right)bd^{2}e+\frac{1}{25}\left(15x^{5}\operatorname{arcosh}(cx)-\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/7*a*e^3*x^7 + 3/5*a*d*e^2*x^5 + a*d^2*e*x^3 + 1/3*(3*x^3*arccosh(c*x) - c
*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*b*d^2*e + 1/25*(15*
x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c
^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d*e^2 + 1/245*(35*x^7*arccosh(c*x) - (5*
sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 -
1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*e^3 + a*d^3*x + (c*x*arccosh(c*
x) - sqrt(c^2*x^2 - 1))*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^{2}+d\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^3, x)
```

sympy [A] time = 6.38, size = 396, normalized size = 1.38

$$\begin{cases} ad^3x + ad^2ex^3 + \frac{3ade^2x^5}{5} + \frac{ae^3x^7}{7} + bd^3x\operatorname{acosh}(cx) + bd^2ex^3\operatorname{acosh}(cx) + \frac{3bde^2x^5\operatorname{acosh}(cx)}{5} + \frac{be^3x^7\operatorname{acosh}(cx)}{7} - \frac{bd^3 \cdots}{} \\ \left(a + \frac{i\pi b}{2}\right)\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**3*x + a*d**2*e*x**3 + 3*a*d*e**2*x**5/5 + a*e**3*x**7/7 + b
*d**3*x*acosh(c*x) + b*d**2*e*x**3*acosh(c*x) + 3*b*d*e**2*x**5*acosh(c*x)/
5 + b*e**3*x**7*acosh(c*x)/7 - b*d**3*sqrt(c**2*x**2 - 1)/c - b*d**2*e*x**2
*sqrt(c**2*x**2 - 1)/(3*c) - 3*b*d*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - b
*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - 2*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c
**3) - 4*b*d*e**2*x**2*sqrt(c**2*x**2 - 1)/(25*c**3) - 6*b*e**3*x**4*sqrt(c
**2*x**2 - 1)/(245*c**3) - 8*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 8*b*e
**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7), Ne(c, 0)), ((a + I*pi*b/2)*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5
 + e**3*x**7/7), True))
```

3.484 $\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x} \, dx$

Optimal. Leaf size=509

$$d^3 \log(x) \left(a + b\cosh^{-1}(cx)\right) + \frac{3}{2} d^2 e x^2 \left(a + b\cosh^{-1}(cx)\right) + \frac{3}{4} d e^2 x^4 \left(a + b\cosh^{-1}(cx)\right) + \frac{1}{6} e^3 x^6 \left(a + b\cosh^{-1}(cx)\right) - \frac{5}{...}$$

```
[Out] -3/4*b*d^2*e*arccosh(c*x)/c^2-9/32*b*d*e^2*arccosh(c*x)/c^4-5/96*b*e^3*arcc
osh(c*x)/c^6+3/2*d^2*e*x^2*(a+b*arccosh(c*x))+3/4*d*e^2*x^4*(a+b*arccosh(c*
x))+1/6*e^3*x^6*(a+b*arccosh(c*x))+d^3*(a+b*arccosh(c*x))*ln(x)-3/4*b*d^2*e
*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-9/32*b*d*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)
/c^3-5/96*b*e^3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-3/16*b*d*e^2*x^3*(c*x-1)^
(1/2)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-1/36*
b*e^3*x^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-1/2*I*b*d^3*polylog(2,(I*c*x+(-c^2*
x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*d^3*arcsi
n(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)-b*d^3*arcsin(c*x)*ln(x)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x
+1)^(1/2)-1/2*I*b*d^3*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1
)^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {266, 43, 5790, 12, 6742, 90, 52, 100, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{ibd^3\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}} + \frac{3}{2}d^2 e x^2 \left(a+b\cosh^{-1}(cx)\right) + d^3\log(x)\left(a+b\cosh^{-1}(cx)\right) + \frac{3}{4} d e^2 x^4 \left(a+b ...\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (-3*b*d^2*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c) - (9*b*d*e^2*x*Sqrt[-1 +
c*x]*Sqrt[1 + c*x])/(32*c^3) - (5*b*e^3*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(96
*c^5) - (3*b*d*e^2*x^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*c) - (5*b*e^3*x^3*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(144*c^3) - (b*e^3*x^5*Sqrt[-1 + c*x]*Sqrt[1
+ c*x])/(36*c) - (3*b*d^2*e*ArcCosh[c*x])/(4*c^2) - (9*b*d*e^2*ArcCosh[c*x]
)/(32*c^4) - (5*b*e^3*ArcCosh[c*x])/(96*c^6) + (3*d^2*e*x^2*(a + b*ArcCosh[
c*x]))/2 + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/4 + (e^3*x^6*(a + b*ArcCosh[c
*x]))/6 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[-1 + c*x]*Sqr
t[1 + c*x]) + (b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[
c*x])])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^3*(a + b*ArcCosh[c*x])*Log[x] -
(b*d^3*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 - ((I/2)*b*d^3*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{(d+ex^2)^3 \left(a+b\cosh^{-1}(cx)\right)}{x}\,dx = \frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$=\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$=\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$=\frac{3}{2}d^2ex^2\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{4}de^2x^4\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{6}e^3x^6\left(a+b\right.$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{be^3x^5\sqrt{-1+}}{3}$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{3bde^2x^3\sqrt{-1+cx}\sqrt{1+cx}}{16c}-\frac{be^3x^5\sqrt{-1+}}{3}$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{3bde^2x^3\sqrt{-1}}{1}$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{3bde^2x^3\sqrt{-1}}{1}$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+}}{96}$$

$$=-\frac{3bd^2ex\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+}}{96}$$

$$=-\frac{3bd^2e\sqrt{-1+cx}\sqrt{1+cx}}{4c}-\frac{9bde^2x\sqrt{-1+cx}\sqrt{1+cx}}{32c^3}-\frac{5be^3x\sqrt{-1+}}{96}$$

Mathematica [A] time = 0.76, size = 314, normalized size = 0.62

$$ad^3\log(x)+\frac{3}{2}ad^2ex^2+\frac{3}{4}ade^2x^4+\frac{1}{6}ae^3x^6-\frac{3bd^2e\left(-2c^2x^2\cosh^{-1}(cx)+cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right.}{4c^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x,x]
```

```
[Out] (3*a*d^2*e*x^2)/2 + (3*a*d*e^2*x^4)/4 + (a*e^3*x^6)/6 - (3*b*d^2*e*(c*x*Sqr
t[-1 + c*x]*Sqrt[1 + c*x] - 2*c^2*x^2*ArcCosh[c*x] + 2*ArcTanh[Sqrt[(-1 + c
*x)/(1 + c*x)]]))/(4*c^2) - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(3
 + 2*c^2*x^2) - 8*c^4*x^4*ArcCosh[c*x] + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x
)]]))/(32*c^4) - (b*e^3*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(15 + 10*c^2*x^2
+ 8*c^4*x^4) - 48*c^6*x^6*ArcCosh[c*x] + 30*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*
x)]]))/(288*c^6) + a*d^3*Log[x] + (b*d^3*(ArcCosh[c*x]*(ArcCosh[c*x] + 2*Lo
g[1 + E^(-2*ArcCosh[c*x])]) - PolyLog[2, -E^(-2*ArcCosh[c*x])]))/2
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\text{arcosh}\left(cx\right)}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^3 (b \operatorname{arcosh}(cx) + a)}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x, x)
```

maple [A] time = 0.30, size = 351, normalized size = 0.69

$$\frac{a e^3 x^6}{6} + \frac{3ad\, e^2 x^4}{4} + \frac{3a\, d^2 e\, x^2}{2} + d^3 a \ln(cx) + \frac{3b \operatorname{arccosh}(cx)\, d\, e^2 x^4}{4} + \frac{3b \operatorname{arccosh}(cx)\, d^2 e\, x^2}{2} + \frac{b \operatorname{arccosh}(cx)\, e^3 x^6}{6} + \frac{d^3}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x)
```

```
[Out] 1/6*a*e^3*x^6+3/4*a*d*e^2*x^4+3/2*a*d^2*e*x^2+d^3*a*ln(c*x)+3/4*b*arccosh(c
*x)*d*e^2*x^4+3/2*b*arccosh(c*x)*d^2*e*x^2+1/6*b*arccosh(c*x)*e^3*x^6+1/2*d
^3*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/4*b*d^2*e*arccosh(c*
x)/c^2-9/32*b*d*e^2*arccosh(c*x)/c^4-1/2*d^3*b*arccosh(c*x)^2+d^3*b*arccosh
(c*x)*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-1/36*b*e^3*x^5*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c-5/144*b*e^3*x^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-5/96*b*e^
3*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-3/16*b*d*e^2*x^3*(c*x-1)^(1/2)*(c*x+1)^
(1/2)/c-9/32*b*d*e^2*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-3/4*b*d^2*e*x*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/c-5/96*b*e^3*arccosh(c*x)/c^6
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{6} ae^3 x^6 + \frac{3}{4} ade^2 x^4 + \frac{3}{2} ad^2 ex^2 + ad^3 \log(x) + \int be^3 x^5 \log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right) + 3\, bde^2 x^3 \log\left(cx + \sqrt{cx+1}\sqrt{cx-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x,x, algorithm="maxima")
```

```
[Out] 1/6*a*e^3*x^6 + 3/4*a*d*e^2*x^4 + 3/2*a*d^2*e*x^2 + a*d^3*log(x) + integrat
e(b*e^3*x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x^3*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d^2*e*x*log(c*x + sqrt(c*x + 1)*sqrt(c
*x - 1)) + b*d^3*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\, x)) \left(e\, x^2 + d\right)^3}{x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^3}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x, x)
```

3.485 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x^2}\,dx$

Optimal. Leaf size=265

$$-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\cosh^{-1}(cx)\right)+de^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^3\sqrt{c^2}}{}$$

```
[Out] -d^3*(a+b*arccosh(c*x))/x+3*d^2*e*x*(a+b*arccosh(c*x))+d*e^2*x^3*(a+b*arcco
sh(c*x))+1/5*e^3*x^5*(a+b*arccosh(c*x))+1/5*b*e*(15*c^4*d^2+5*c^2*d*e+e^2)*
(-c^2*x^2+1)/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/15*b*e^2*(5*c^2*d+2*e)*(-c^2
*x^2+1)^2/c^5/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/25*b*e^3*(-c^2*x^2+1)^3/c^5/(c*
x-1)^(1/2)/(c*x+1)^(1/2)+b*c*d^3*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2
)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {270, 5790, 1610, 1799, 1620, 63, 205}

$$3d^2ex\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+de^2x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{5}e^3x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{be\left(1-c^2x\right.}{5c^5}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] (b*e*(15*c^4*d^2 + 5*c^2*d*e + e^2)*(1 - c^2*x^2))/(5*c^5*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]) - (b*e^2*(5*c^2*d + 2*e)*(1 - c^2*x^2)^2)/(15*c^5*Sqrt[-1 + c*
x]*Sqrt[1 + c*x]) + (b*e^3*(1 - c^2*x^2)^3)/(25*c^5*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]) - (d^3*(a + b*ArcCosh[c*x]))/x + 3*d^2*e*x*(a + b*ArcCosh[c*x]) + d*
e^2*x^3*(a + b*ArcCosh[c*x]) + (e^3*x^5*(a + b*ArcCosh[c*x]))/5 + (b*c*d^3*
Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x
])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
```

```
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3\left(a+b\cosh^{-1}(cx)\right)}{x^2}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\cosh^{-1}(cx)\right)+de^2x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\cosh^{-1}(cx)\right)+de^2x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\cosh^{-1}(cx)\right)+de^2x^3\left(a+b\cosh\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{x}+3d^2ex\left(a+b\cosh^{-1}(cx)\right)+de^2x^3\left(a+b\cosh\right.$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{}{25c}$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{}{25c}$$

$$= \frac{be\left(15c^4d^2+5c^2de+e^2\right)\left(1-c^2x^2\right)}{5c^5\sqrt{-1+cx}\sqrt{1+cx}}-\frac{be^2\left(5c^2d+2e\right)\left(1-c^2x^2\right)^2}{15c^5\sqrt{-1+cx}\sqrt{1+cx}}+\frac{}{25c}$$

Mathematica [A] time = 0.37, size = 182, normalized size = 0.69

$$-\frac{ad^3}{x}+3ad^2ex+ade^2x^3+\frac{1}{5}ae^3x^5-\frac{be\sqrt{cx-1}\sqrt{cx+1}\left(c^4\left(225d^2+25dex^2+3e^2x^4\right)+2c^2e\left(25d+2ex^2\right)+8e^2\right)}{75c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^2,x]
```

```
[Out] -((a*d^3)/x) + 3*a*d^2*e*x + a*d*e^2*x^3 + (a*e^3*x^5)/5 - (b*e*Sqrt[-1 + c
*x]*Sqrt[1 + c*x]*(8*e^2 + 2*c^2*e*(25*d + 2*e*x^2) + c^4*(225*d^2 + 25*d*e
*x^2 + 3*e^2*x^4)))/(75*c^5) + (b*(-5*d^3 + 15*d^2*e*x^2 + 5*d*e^2*x^4 + e^
3*x^6)*ArcCosh[c*x])/(5*x) - b*c*d^3*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]
)]
```

fricas [A] time = 0.75, size = 327, normalized size = 1.23

$$15\,ac^5e^3x^6 + 75\,ac^5de^2x^4 + 150\,bc^6d^3x\arctan\left(-cx + \sqrt{c^2x^2-1}\right) + 225\,ac^5d^2ex^2 - 75\,ac^5d^3 + 15\left(5\,bc^5d^3 - 15\,bc\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="fricas")
```

```
[Out] 1/75*(15*a*c^5*e^3*x^6 + 75*a*c^5*d*e^2*x^4 + 150*b*c^6*d^3*x*arctan(-c*x +
 sqrt(c^2*x^2 - 1)) + 225*a*c^5*d^2*e*x^2 - 75*a*c^5*d^3 + 15*(5*b*c^5*d^3
- 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x*log(-c*x + sqrt(c^2*x^2 - 1
)) + 15*(b*c^5*e^3*x^6 + 5*b*c^5*d*e^2*x^4 + 15*b*c^5*d^2*e*x^2 - 5*b*c^5*d
^3 + (5*b*c^5*d^3 - 15*b*c^5*d^2*e - 5*b*c^5*d*e^2 - b*c^5*e^3)*x)*log(c*x
+ sqrt(c^2*x^2 - 1)) - (3*b*c^4*e^3*x^5 + (25*b*c^4*d*e^2 + 4*b*c^2*e^3)*x^
3 + (225*b*c^4*d^2*e + 50*b*c^2*d*e^2 + 8*b*e^3)*x)*sqrt(c^2*x^2 - 1))/(c^5
*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^3 (b\operatorname{arcosh}(cx) + a)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^2, x)
```

maple [A] time = 0.02, size = 282, normalized size = 1.06

$$\frac{a\,e^3x^5}{5} + ad\,e^2x^3 + 3a\,d^2ex - \frac{d^3a}{x} + \frac{b\operatorname{arccosh}(cx)\,e^3x^5}{5} + b\operatorname{arccosh}(cx)\,d\,e^2x^3 + 3b\operatorname{arccosh}(cx)\,d^2ex - \frac{d^3b\operatorname{arccosh}(cx)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x)
```

```
[Out] 1/5*a*e^3*x^5+a*d*e^2*x^3+3*a*d^2*e*x-d^3*a/x+1/5*b*arccosh(c*x)*e^3*x^5+b*
arccosh(c*x)*d*e^2*x^3+3*b*arccosh(c*x)*d^2*e*x-d^3*b*arccosh(c*x)/x-c*d^3*
b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))
-1/25*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^4*e^3-1/3*b/c*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*x^2*d*e^2-3*b/c*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d^2*e-4/75*b/c^3*(c*x-1)
^(1/2)*(c*x+1)^(1/2)*x^2*e^3-2/3*b/c^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)*d*e^2-8/
75*b/c^5*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3
```

maxima [A] time = 0.41, size = 222, normalized size = 0.84

$$\frac{1}{5}\,ae^3x^5 + ade^2x^3 - \left(c\arcsin\left(\frac{1}{c|x|}\right) + \frac{\operatorname{arcosh}(cx)}{x}\right)bd^3 + \frac{1}{3}\left(3\,x^3\operatorname{arcosh}(cx) - c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2} + \frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bde^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^2,x, algorithm="maxima")
```

```
[Out] 1/5*a*e^3*x^5 + a*d*e^2*x^3 - (c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*d
^3 + 1/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^
2 - 1)/c^4))*b*d*e^2 + 1/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4
/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*e^3 + 3*
a*d^2*e*x + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*b*d^2*e/c - a*d^3/x
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2 + d\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^3}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**2, x)
```

3.486 $\int \frac{(d+ex^2)^3(a+b\cosh^{-1}(cx))}{x^3}\,dx$

Optimal. Leaf size=476

$$-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+3d^2e\log(x)\left(a+b\cosh^{-1}(cx)\right)+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cosh^{-1}(cx)\right)-\frac{}{2}$$

```
[Out] -1/2*d^3*(a+b*arccosh(c*x))/x^2+3/2*d*e^2*x^2*(a+b*arccosh(c*x))+1/4*e^3*x^
4*(a+b*arccosh(c*x))+3*d^2*e*(a+b*arccosh(c*x))*ln(x)-1/2*b*c*d^3*(-c^2*x^2
+1)/x/(c*x-1)^(1/2)/(c*x+1)^(1/2)+3/32*b*e^2*(8*c^2*d+e)*x*(-c^2*x^2+1)/c^3
/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/16*b*e^3*x^3*(-c^2*x^2+1)/c/(c*x-1)^(1/2)/(c
*x+1)^(1/2)-3/2*I*b*d^2*e*arcsin(c*x)^2*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c
*x+1)^(1/2)+3*b*d^2*e*arcsin(c*x)*ln(1-(I*c*x+(-c^2*x^2+1)^(1/2))^2)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3*b*d^2*e*arcsin(c*x)*ln(x)*(-c^2*
x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/2*I*b*d^2*e*polylog(2,(I*c*x+(-c
^2*x^2+1)^(1/2))^2)*(-c^2*x^2+1)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-3/32*b*e
^2*(8*c^2*d+e)*arctanh(c*x/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/c^4/(c*x-1)
^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.76, antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 19, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.905, Rules used = {266, 43, 5790, 12, 6742, 1610, 1807, 1584, 459, 321, 217, 206, 2328, 2326, 4625, 3717, 2190, 2279, 2391}

$$-\frac{3ibd^2e\sqrt{1-c^2x^2}\,\text{PolyLog}\left(2,e^{2i\sin^{-1}(cx)}\right)}{2\sqrt{cx-1}\sqrt{cx+1}}+3d^2e\log(x)\left(a+b\cosh^{-1}(cx)\right)-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\right.$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] -(b*c*d^3*(1 - c^2*x^2))/(2*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*e^2*(8*c
^2*d + e)*x*(1 - c^2*x^2))/(32*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*x
^3*(1 - c^2*x^2))/(16*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh
[c*x]))/(2*x^2) + (3*d*e^2*x^2*(a + b*ArcCosh[c*x]))/2 + (e^3*x^4*(a + b*Ar
cCosh[c*x]))/4 - (((3*I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]^2)/(Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) - (3*b*e^2*(8*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTan
h[(c*x)/Sqrt[-1 + c^2*x^2]])/(32*c^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*b*d
^2*e*Sqrt[1 - c^2*x^2]*ArcSin[c*x]*Log[1 - E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x]) + 3*d^2*e*(a + b*ArcCosh[c*x])*Log[x] - (3*b*d^2*e*S
qrt[1 - c^2*x^2]*ArcSin[c*x]*Log[x])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (((3*
I)/2)*b*d^2*e*Sqrt[1 - c^2*x^2]*PolyLog[2, E^((2*I)*ArcSin[c*x])])/(Sqrt[-1
 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
 :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2326

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/Sqrt[(d_) + (e_.)*(x_)^2], x_Symb
ol] :> Simp[(ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]*(a + b*Log[c*x^n]))/Rt[-e, 2], x
] - Dist[(b*n)/Rt[-e, 2], Int[ArcSin[(Rt[-e, 2]*x)/Sqrt[d]]/x, x], x] /; Fr
eeQ[{a, b, c, d, e, n}, x] && GtQ[d, 0] && NegQ[e]
```

Rule 2328

```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(Sqrt[(d1_) + (e1_.)*(x_)]*Sqrt[(
d2_) + (e2_.)*(x_)]), x_Symbol] :> Dist[Sqrt[1 + (e1*e2*x^2)/(d1*d2)]/(Sqrt
[d1 + e1*x]*Sqrt[d2 + e2*x]), Int[(a + b*Log[c*x^n])/Sqrt[1 + (e1*e2*x^2)/(
d1*d2)], x], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[d2*e1 + d1
*e2, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4625

```
Int[((a_.) + ArcSin[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[(
a + b*x)^n/Tan[x], x], x, ArcSin[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)}{x^3}\,dx = -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cos\right.$$

$$= -\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{4}e^3x^4\left(a+b\cos\right.$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}\right.$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{2x^2}+\frac{3}{2}de^2x^2\left(a+b\cosh^{-1}\right.$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c^2x^2\right)}{16c\sqrt{-1+cx}\sqrt{1+cx}}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right.}{2x^2}$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c\right.}{16c\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c\right.}{16c\sqrt{-1+cx}\,\sqrt{}}$$

$$= -\frac{bcd^3\left(1-c^2x^2\right)}{2x\sqrt{-1+cx}\sqrt{1+cx}}+\frac{3be^2\left(8c^2d+e\right)x\left(1-c^2x^2\right)}{32c^3\sqrt{-1+cx}\sqrt{1+cx}}+\frac{be^3x^3\left(1-c\right.}{16c\sqrt{-1+cx}\,\sqrt{}}$$

Mathematica [A] time = 0.71, size = 267, normalized size = 0.56

$$\frac{1}{4}\left(-\frac{2ad^3}{x^2}+12ad^2e\log(x)+6ade^2x^2+ae^3x^4-\frac{3bde^2\left(cx\sqrt{cx-1}\sqrt{cx+1}+2\tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right)\right)}{c^2}-\frac{be^3\left(cx\sqrt{cx}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^3,x]
```

```
[Out] ((-2*a*d^3)/x^2 + 6*a*d*e^2*x^2 + a*e^3*x^4 + (2*b*d^3*(c*x*Sqrt[-1 + c*x]*
Sqrt[1 + c*x] - ArcCosh[c*x]))/x^2 + 6*b*d*e^2*x^2*ArcCosh[c*x] + b*e^3*x^4
*ArcCosh[c*x] - (3*b*d*e^2*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] + 2*ArcTanh[Sq
rt[(-1 + c*x)/(1 + c*x)]]))/c^2 - (b*e^3*(c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*
(3 + 2*c^2*x^2) + 6*ArcTanh[Sqrt[(-1 + c*x)/(1 + c*x)]]))/(8*c^4) + 6*b*d^2
*e*ArcCosh[c*x]*(ArcCosh[c*x] + 2*Log[1 + E^(-2*ArcCosh[c*x])]) + 12*a*d^2*
e*Log[x] - 6*b*d^2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])])/4
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\mathrm{arcosh}\left(cx\right)}{x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))/x^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ex^2+d)^3 (b \operatorname{arcosh}(cx) + a)}{x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^3, x)
```

maple [A] time = 0.43, size = 296, normalized size = 0.62

$$\frac{a\,x^4e^3}{4}+\frac{3a\,x^2d\,e^2}{2}+3a\,d^2e\ln(cx)-\frac{d^3a}{2x^2}-\frac{d^3b\operatorname{arccosh}(cx)}{2x^2}-\frac{3b\,d^2e\operatorname{arccosh}(cx)^2}{2}+3b\,d^2e\operatorname{arccosh}(cx)\ln\Bigg(1+\Big(cx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x)
```

```
[Out] 1/4*a*x^4*e^3+3/2*a*x^2*d*e^2+3*a*d^2*e*ln(c*x)-1/2*d^3*a/x^2-1/2*d^3*b*arc
cosh(c*x)/x^2-3/2*b*d^2*e*arccosh(c*x)^2+3*b*d^2*e*arccosh(c*x)*ln(1+(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))^2)-3/32/c^4*b*arccosh(c*x)*e^3+1/2*c*d^3*b/x*(c
*x+1)^(1/2)*(c*x-1)^(1/2)-1/16/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x^3-3/32
/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3*x+3/2*b*d^2*e*polylog(2,-(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2)+1/4*b*arccosh(c*x)*x^4*e^3-1/2*d^3*b*c^2+3/2*b*ar
ccosh(c*x)*x^2*d*e^2-3/4/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*d*e^2-3/4/c^2*b*
arccosh(c*x)*d*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,ae^3x^4+\frac{3}{2}\,ade^2x^2+\frac{1}{2}\,bd^3\left(\frac{\sqrt{c^2x^2-1}\,c}{x}-\frac{\operatorname{arcosh}(cx)}{x^2}\right)+3\,ad^2e\log(x)-\frac{ad^3}{2\,x^2}+\int be^3x^3\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*e^3*x^4 + 3/2*a*d*e^2*x^2 + 1/2*b*d^3*(sqrt(c^2*x^2 - 1)*c/x - arccos
h(c*x)/x^2) + 3*a*d^2*e*log(x) - 1/2*a*d^3/x^2 + integrate(b*e^3*x^3*log(c*
x + sqrt(c*x + 1)*sqrt(c*x - 1)) + 3*b*d*e^2*x*log(c*x + sqrt(c*x + 1)*sqrt
(c*x - 1)) + 3*b*d^2*e*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/x, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}(c\,x))\,(e\,x^2+d)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\,\mathrm{acosh}\,(cx))\left(d + ex^2\right)^3}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**3,x)
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**3, x)
```

3.487 $\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^4} dx$

Optimal. Leaf size=260

$$-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x}+3de^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\cosh^{-1}(cx)\right)-\frac{bcd^3(1}{6x^2\sqrt{cx-}}$$

```
[Out] -1/3*d^3*(a+b*arccosh(c*x))/x^3-3*d^2*e*(a+b*arccosh(c*x))/x+3*d*e^2*x*(a+b
*arccosh(c*x))+1/3*e^3*x^3*(a+b*arccosh(c*x))+1/3*b*e^2*(9*c^2*d+e)*(-c^2*x
^2+1)/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/6*b*c*d^3*(-c^2*x^2+1)/x^2/(c*x-1)^
(1/2)/(c*x+1)^(1/2)-1/9*b*e^3*(-c^2*x^2+1)^2/c^3/(c*x-1)^(1/2)/(c*x+1)^(1/2
)+1/6*b*c*d^2*(c^2*d+18*e)*arctan((c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/(c*x
-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {270, 5790, 12, 1610, 1799, 1621, 897, 1153, 205}

$$-\frac{3d^2e\left(a+b\cosh^{-1}(cx)\right)}{x}-\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{3x^3}+3de^2x\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{3}e^3x^3\left(a+b\cosh^{-1}(cx)\right)+\frac{bcd^2\sqrt{c^2x}}{}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] (b*e^2*(9*c^2*d + e)*(1 - c^2*x^2))/(3*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) -
(b*c*d^3*(1 - c^2*x^2))/(6*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (b*e^3*(1 -
c^2*x^2)^2)/(9*c^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (d^3*(a + b*ArcCosh[c*x]
))/(3*x^3) - (3*d^2*e*(a + b*ArcCosh[c*x]))/x + 3*d*e^2*x*(a + b*ArcCosh[c*
x]) + (e^3*x^3*(a + b*ArcCosh[c*x]))/3 + (b*c*d^2*(c^2*d + 18*e)*Sqrt[-1 +
c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(6*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 897

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2)^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1621

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :
> With[{Qx = PolynomialQuotient[Px, a + b*x, x], R = PolynomialRemainder[Px
, a + b*x, x]}, Simp[(R*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((m + 1)*(b*c
- a*d)), x] + Dist[1/((m + 1)*(b*c - a*d)), Int[(a + b*x)^(m + 1)*(c + d*x)
^n*ExpandToSum[(m + 1)*(b*c - a*d)*Qx - d*R*(m + n + 2), x], x], x]] /; Fre
eQ[{a, b, c, d, n}, x] && PolyQ[Px, x] && ILtQ[m, -1] && GtQ[Expon[Px, x],
2]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{(d+ex^2)^3 (a+b\cosh^{-1}(cx))}{x^4}\,dx = -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(c$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(c$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(c$$

$$= -\frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x} + 3de^2x(a+b\cosh^{-1}(c$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x}$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x}$$

$$= -\frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{d^3(a+b\cosh^{-1}(cx))}{3x^3} - \frac{3d^2e(a+b\cosh^{-1}(cx))}{x}$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{be^2(9c^2d+e)(1-c^2x^2)}{3c^3\sqrt{-1+cx}\sqrt{1+cx}} - \frac{bcd^3(1-c^2x^2)}{6x^2\sqrt{-1+cx}\sqrt{1+cx}} - \frac{be^3(1-c^2x^2)^2}{9c^3\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.39, size = 184, normalized size = 0.71

$$\frac{1}{6}\left(-\frac{2ad^3}{x^3} - \frac{18ad^2e}{x} + 18ade^2x + 2ae^3x^3 - bcd^2(c^2d+18e)\tan^{-1}\left(\frac{1}{\sqrt{cx-1}\sqrt{cx+1}}\right) - \frac{b\sqrt{cx-1}\sqrt{cx+1}\,(-3c^4d$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^3*(a + b*ArcCosh[c*x]))/x^4,x]
```

```
[Out] ((-2*a*d^3)/x^3 - (18*a*d^2*e)/x + 18*a*d*e^2*x + 2*a*e^3*x^3 - (b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]*(-3*c^4*d^3 + 4*e^3*x^2 + 2*c^2*e^2*x^2*(27*d + e*x^2)
))/(3*c^3*x^2) + (2*b*(-d^3 - 9*d^2*e*x^2 + 9*d*e^2*x^4 + e^3*x^6)*ArcCosh[
c*x])/x^3 - b*c*d^2*(c^2*d + 18*e)*ArcTan[1/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])]
)/6
```

fricas [A] time = 0.81, size = 322, normalized size = 1.24

$$\frac{6\,ac^3e^3x^6 + 54\,ac^3de^2x^4 - 54\,ac^3d^2ex^2 - 6\,ac^3d^3 + 6\left(bc^6d^3 + 18\,bc^4d^2e\right)x^3\arctan\left(-cx+\sqrt{c^2x^2-1}\right) + 6\left(bc^3d^3 +\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/18*(6*a*c^3*e^3*x^6 + 54*a*c^3*d*e^2*x^4 - 54*a*c^3*d^2*e*x^2 - 6*a*c^3*d
^3 + 6*(b*c^6*d^3 + 18*b*c^4*d^2*e)*x^3*arctan(-c*x + sqrt(c^2*x^2 - 1)) +
6*(b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3*log(-c*x + sq
rt(c^2*x^2 - 1)) + 6*(b*c^3*e^3*x^6 + 9*b*c^3*d*e^2*x^4 - 9*b*c^3*d^2*e*x^2
 - b*c^3*d^3 + (b*c^3*d^3 + 9*b*c^3*d^2*e - 9*b*c^3*d*e^2 - b*c^3*e^3)*x^3)
*log(c*x + sqrt(c^2*x^2 - 1)) - (2*b*c^2*e^3*x^5 - 3*b*c^4*d^3*x + 2*(27*b*
c^2*d*e^2 + 2*b*e^3)*x^3)*sqrt(c^2*x^2 - 1))/(c^3*x^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^3 (b\,\mathrm{arcosh}\,(cx)+a)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)/x^4, x)
```

maple [A] time = 0.02, size = 278, normalized size = 1.07

$$\frac{a\,x^3e^3}{3}+3axd\,e^2-\frac{3a\,d^2e}{x}-\frac{a\,d^3}{3x^3}+\frac{b\,\mathrm{arccosh}\,(cx)\,x^3e^3}{3}+3b\,\mathrm{arccosh}\,(cx)\,xd\,e^2-\frac{3b\,\mathrm{arccosh}\,(cx)\,d^2e}{x}-\frac{b\,\mathrm{arccosh}\,(cx)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x)
```

```
[Out] 1/3*a*x^3*e^3+3*a*x*d*e^2-3*a*d^2*e/x-1/3*a*d^3/x^3+1/3*b*arccosh(c*x)*x^3*
e^3+3*b*arccosh(c*x)*x*d*e^2-3*b*arccosh(c*x)*d^2*e/x-1/3*b*arccosh(c*x)*d^
3/x^3-1/6*c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*d^3*arctan(1/
(c^2*x^2-1)^(1/2))+1/6*b*c*d^3*(c*x-1)^(1/2)*(c*x+1)^(1/2)/x^2-3*c*b*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)*arctan(1/(c^2*x^2-1)^(1/2))*d^2*e-1
/9/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^2*e^3-3/c*b*(c*x-1)^(1/2)*(c*x+1)^(1/2
)*d*e^2-2/9/c^3*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3
```

maxima [A] time = 0.57, size = 197, normalized size = 0.76

$$\frac{1}{3}\,ae^3x^3-\frac{1}{6}\left(\left(c^2\arcsin\left(\frac{1}{c|x|}\right)-\frac{\sqrt{c^2x^2-1}}{x^2}\right)c+\frac{2\,\mathrm{arcosh}\,(cx)}{x^3}\right)bd^3-3\left(c\arcsin\left(\frac{1}{c|x|}\right)+\frac{\mathrm{arcosh}\,(cx)}{x}\right)bd^2e+\frac{1}{9}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*a*e^3*x^3 - 1/6*((c^2*arcsin(1/(c*abs(x))) - sqrt(c^2*x^2 - 1)/x^2)*c +
 2*arccosh(c*x)/x^3)*b*d^3 - 3*(c*arcsin(1/(c*abs(x))) + arccosh(c*x)/x)*b*
d^2*e + 1/9*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2
*x^2 - 1)/c^4))*b*e^3 + 3*a*d*e^2*x + 3*(c*x*arccosh(c*x) - sqrt(c^2*x^2 -
1))*b*d*e^2/c - 3*a*d^2*e/x - 1/3*a*d^3/x^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))\,\left(e\,x^2+d\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4,x)
```

```
[Out] int(((a + b*acosh(c*x))*(d + e*x^2)^3)/x^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\operatorname{acosh}(cx))\left(d + ex^2\right)^3}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))/x**4,x)
```

```
[Out] Integral((a + b*acosh(c*x))*(d + e*x**2)**3/x**4, x)
```

3.488 $\int (d+ex^2)^4 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=395

$$d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}$$

```
[Out] d^4*x*(a+b*arccosh(c*x))+4/3*d^3*e*x^3*(a+b*arccosh(c*x))+6/5*d^2*e^2*x^5*(
a+b*arccosh(c*x))+4/7*d*e^3*x^7*(a+b*arccosh(c*x))+1/9*e^4*x^9*(a+b*arccosh
(c*x))+1/315*b*(315*c^8*d^4+420*c^6*d^3*e+378*c^4*d^2*e^2+180*c^2*d*e^3+35*
e^4)*(-c^2*x^2+1)/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2)-4/945*b*e*(105*c^6*d^3+18
9*c^4*d^2*e+135*c^2*d*e^2+35*e^3)*(-c^2*x^2+1)^2/c^9/(c*x-1)^(1/2)/(c*x+1)^
(1/2)+2/525*b*e^2*(63*c^4*d^2+90*c^2*d*e+35*e^2)*(-c^2*x^2+1)^3/c^9/(c*x-1)
^(1/2)/(c*x+1)^(1/2)-4/441*b*e^3*(9*c^2*d+7*e)*(-c^2*x^2+1)^4/c^9/(c*x-1)^(
1/2)/(c*x+1)^(1/2)+1/81*b*e^4*(-c^2*x^2+1)^5/c^9/(c*x-1)^(1/2)/(c*x+1)^(1/2
)
```

Rubi [A] time = 0.47, antiderivative size = 395, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {194, 5705, 12, 1610, 1799, 1850}

$$\frac{6}{5}d^2e^2x^5\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{7}de^3x^7\left(a+b\cosh^{-1}(cx)\right)+\frac{1}{9}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*(315*c^8*d^4 + 420*c^6*d^3*e + 378*c^4*d^2*e^2 + 180*c^2*d*e^3 + 35*e^4)
*(1 - c^2*x^2))/(315*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e*(105*c^6*d^
3 + 189*c^4*d^2*e + 135*c^2*d*e^2 + 35*e^3)*(1 - c^2*x^2)^2)/(945*c^9*Sqrt[
-1 + c*x]*Sqrt[1 + c*x]) + (2*b*e^2*(63*c^4*d^2 + 90*c^2*d*e + 35*e^2)*(1 -
 c^2*x^2)^3)/(525*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (4*b*e^3*(9*c^2*d + 7
*e)*(1 - c^2*x^2)^4)/(441*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^4*(1 - c
^2*x^2)^5)/(81*c^9*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + d^4*x*(a + b*ArcCosh[c*x
]) + (4*d^3*e*x^3*(a + b*ArcCosh[c*x]))/3 + (6*d^2*e^2*x^5*(a + b*ArcCosh[c
*x]))/5 + (4*d*e^3*x^7*(a + b*ArcCosh[c*x]))/7 + (e^4*x^9*(a + b*ArcCosh[c*
x]))/9
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1799

```
Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[1/2, Su
bst[Int[x^((m - 1)/2)*SubstFor[x^2, Pq, x]*(a + b*x)^p, x], x, x^2], x] /;
FreeQ[{a, b, p}, x] && PolyQ[Pq, x^2] && IntegerQ[(m - 1)/2]
```

Rule 1850

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand
[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p
, 0] || EqQ[n, 1])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^4 \left(a+b\cosh^{-1}(cx)\right) dx = d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh\right.$$

$$= d^4x\left(a+b\cosh^{-1}(cx)\right)+\frac{4}{3}d^3ex^3\left(a+b\cosh^{-1}(cx)\right)+\frac{6}{5}d^2e^2x^5\left(a+b\cosh\right.$$

$$= \frac{b\left(315c^8d^4+420c^6d^3e+378c^4d^2e^2+180c^2de^3+35e^4\right)\left(1-c^2x^2\right)}{315c^9\sqrt{-1+cx}\sqrt{1+cx}} - \frac{4be\left(1\right.}{}$$

Mathematica [A] time = 0.40, size = 265, normalized size = 0.67

$$\frac{315ax\left(315d^4+420d^3ex^2+378d^2e^2x^4+180de^3x^6+35e^4x^8\right)-\frac{b\sqrt{cx-1}\sqrt{cx+1}\left(c^8\left(99225d^4+44100d^3ex^2+23814d^2e^2x^4+8100de^3\right.\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^4*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (315*a*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^
4*x^8) - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(4480*e^4 + 320*c^2*e^3*(81*d + 7*
e*x^2) + 48*c^4*e^2*(1323*d^2 + 270*d*e*x^2 + 35*e^2*x^4) + 8*c^6*e*(11025*
d^3 + 3969*d^2*e*x^2 + 1215*d*e^2*x^4 + 175*e^3*x^6) + c^8*(99225*d^4 + 441
00*d^3*e*x^2 + 23814*d^2*e^2*x^4 + 8100*d*e^3*x^6 + 1225*e^4*x^8)))/c^9 + 3
15*b*x*(315*d^4 + 420*d^3*e*x^2 + 378*d^2*e^2*x^4 + 180*d*e^3*x^6 + 35*e^4*
x^8)*ArcCosh[c*x])/99225
```

fricas [A] time = 0.64, size = 333, normalized size = 0.84

$$11025\,ac^9e^4x^9 + 56700\,ac^9de^3x^7 + 119070\,ac^9d^2e^2x^5 + 132300\,ac^9d^3ex^3 + 99225\,ac^9d^4x + 315\,\Big(35\,bc^9e^4x^9 + 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] 1/99225*(11025*a*c^9*e^4*x^9 + 56700*a*c^9*d*e^3*x^7 + 119070*a*c^9*d^2*e^2
*x^5 + 132300*a*c^9*d^3*e*x^3 + 99225*a*c^9*d^4*x + 315*(35*b*c^9*e^4*x^9 +
 180*b*c^9*d*e^3*x^7 + 378*b*c^9*d^2*e^2*x^5 + 420*b*c^9*d^3*e*x^3 + 315*b*
c^9*d^4*x)*log(c*x + sqrt(c^2*x^2 - 1)) - (1225*b*c^8*e^4*x^8 + 99225*b*c^8
*d^4 + 88200*b*c^6*d^3*e + 63504*b*c^4*d^2*e^2 + 25920*b*c^2*d*e^3 + 100*(8
1*b*c^8*d*e^3 + 14*b*c^6*e^4)*x^6 + 4480*b*e^4 + 6*(3969*b*c^8*d^2*e^2 + 16
20*b*c^6*d*e^3 + 280*b*c^4*e^4)*x^4 + 4*(11025*b*c^8*d^3*e + 7938*b*c^6*d^2
*e^2 + 3240*b*c^4*d*e^3 + 560*b*c^2*e^4)*x^2)*sqrt(c^2*x^2 - 1))/c^9
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.01, size = 331, normalized size = 0.84

$$\frac{a\left(\frac{1}{9}e^4c^9x^9+\frac{4}{7}c^9d\,e^3x^7+\frac{6}{5}c^9d^2e^2x^5+\frac{4}{3}c^9d^3e\,x^3+c^9d^4x\right)}{c^8}+\frac{b\left(\frac{\operatorname{arccosh}(cx)e^4c^9x^9}{9}+\frac{4\operatorname{arccosh}(cx)c^9d\,e^3x^7}{7}+\frac{6\operatorname{arccosh}(cx)c^9d^2e^2x^5}{5}+\frac{4\operatorname{arccosh}(cx)c^9d^3e\,x^3}{3}+\text{arcco}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^4*(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(a/c^8*(1/9*e^4*c^9*x^9+4/7*c^9*d*e^3*x^7+6/5*c^9*d^2*e^2*x^5+4/3*c^9*d
^3*e*x^3+c^9*d^4*x)+b/c^8*(1/9*arccosh(c*x)*e^4*c^9*x^9+4/7*arccosh(c*x)*c^
9*d*e^3*x^7+6/5*arccosh(c*x)*c^9*d^2*e^2*x^5+4/3*arccosh(c*x)*c^9*d^3*e*x^3
+arccosh(c*x)*c^9*d^4*x-1/99225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(1225*c^8*e^4*x
^8+8100*c^8*d*e^3*x^6+23814*c^8*d^2*e^2*x^4+1400*c^6*e^4*x^6+44100*c^8*d^3*
e*x^2+9720*c^6*d*e^3*x^4+99225*c^8*d^4+31752*c^6*d^2*e^2*x^2+1680*c^4*e^4*x
^4+88200*c^6*d^3*e+12960*c^4*d*e^3*x^2+63504*c^4*d^2*e^2+2240*c^2*e^4*x^2+2
5920*c^2*d*e^3+4480*e^4)))
```

maxima [A] time = 0.36, size = 415, normalized size = 1.05

$$\frac{1}{9}\,ae^4x^9+\frac{4}{7}\,ade^3x^7+\frac{6}{5}\,ad^2e^2x^5+\frac{4}{3}\,ad^3ex^3+\frac{4}{9}\left(3\,x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\,\sqrt{c^2x^2-1}}{c^4}\right)\right)bd^3e+\frac{2}{25}\Big(15\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^4*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] 1/9*a*e^4*x^9 + 4/7*a*d*e^3*x^7 + 6/5*a*d^2*e^2*x^5 + 4/3*a*d^3*e*x^3 + 4/9
*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c
^4))*b*d^3*e + 2/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4
```

```
*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*b*d^2*e^2 + 4/245*
(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)*x
^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c)*b*d*e^3
 + 1/2835*(315*x^9*arccosh(c*x) - (35*sqrt(c^2*x^2 - 1)*x^8/c^2 + 40*sqrt(c
^2*x^2 - 1)*x^6/c^4 + 48*sqrt(c^2*x^2 - 1)*x^4/c^6 + 64*sqrt(c^2*x^2 - 1)*x
^2/c^8 + 128*sqrt(c^2*x^2 - 1)/c^10)*c)*b*e^4 + a*d^4*x + (c*x*arccosh(c*x)
 - sqrt(c^2*x^2 - 1))*b*d^4/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{acosh}(c\,x))\,\left(e\,x^2 + d\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^4,x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^4, x)
```

sympy [A] time = 16.75, size = 600, normalized size = 1.52

$$\begin{cases} ad^4x + \frac{4ad^3ex^3}{3} + \frac{6ad^2e^2x^5}{5} + \frac{4ade^3x^7}{7} + \frac{ae^4x^9}{9} + bd^4x\operatorname{acosh}(cx) + \frac{4bd^3ex^3\operatorname{acosh}(cx)}{3} + \frac{6bd^2e^2x^5\operatorname{acosh}(cx)}{5} + \frac{4bde^3x^7\operatorname{acosh}(cx)}{7} \\ \left(a + \frac{i\pi b}{2}\right)\left(d^4x + \frac{4d^3ex^3}{3} + \frac{6d^2e^2x^5}{5} + \frac{4de^3x^7}{7} + \frac{e^4x^9}{9}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**4*(a+b*acosh(c*x)),x)
```

```
[Out] Piecewise((a*d**4*x + 4*a*d**3*e*x**3/3 + 6*a*d**2*e**2*x**5/5 + 4*a*d*e**3
*x**7/7 + a*e**4*x**9/9 + b*d**4*x*acosh(c*x) + 4*b*d**3*e*x**3*acosh(c*x)/
3 + 6*b*d**2*e**2*x**5*acosh(c*x)/5 + 4*b*d*e**3*x**7*acosh(c*x)/7 + b*e**4
*x**9*acosh(c*x)/9 - b*d**4*sqrt(c**2*x**2 - 1)/c - 4*b*d**3*e*x**2*sqrt(c*
*2*x**2 - 1)/(9*c) - 6*b*d**2*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 4*b*d*
e**3*x**6*sqrt(c**2*x**2 - 1)/(49*c) - b*e**4*x**8*sqrt(c**2*x**2 - 1)/(81*
c) - 8*b*d**3*e*sqrt(c**2*x**2 - 1)/(9*c**3) - 8*b*d**2*e**2*x**2*sqrt(c**2
*x**2 - 1)/(25*c**3) - 24*b*d*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 8*
b*e**4*x**6*sqrt(c**2*x**2 - 1)/(567*c**3) - 16*b*d**2*e**2*sqrt(c**2*x**2
- 1)/(25*c**5) - 32*b*d*e**3*x**2*sqrt(c**2*x**2 - 1)/(245*c**5) - 16*b*e**
4*x**4*sqrt(c**2*x**2 - 1)/(945*c**5) - 64*b*d*e**3*sqrt(c**2*x**2 - 1)/(24
5*c**7) - 64*b*e**4*x**2*sqrt(c**2*x**2 - 1)/(2835*c**7) - 128*b*e**4*sqrt(
c**2*x**2 - 1)/(2835*c**9), Ne(c, 0)), ((a + I*pi*b/2)*(d**4*x + 4*d**3*e*x
**3/3 + 6*d**2*e**2*x**5/5 + 4*d*e**3*x**7/7 + e**4*x**9/9), True))
```

3.489 $\int \frac{x^4(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=627

$$\frac{(-d)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{(-d)^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{5/2}}+\frac{(-d)}{}$$

```
[Out] -a*d*x/e^2-b*d*x*arccosh(c*x)/e^2+1/3*x^3*(a+b*arccosh(c*x))/e+1/2*(-d)^(3/
2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d
)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1+(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e
^(5/2)+1/2*(-d)^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)-1/2*(-d)^(3/2)*(a+b*a
rccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(
-c^2*d-e)^(1/2)))/e^(5/2)-1/2*b*(-d)^(3/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3
/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2
*d-e)^(1/2)))/e^(5/2)-1/2*b*(-d)^(3/2)*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)+1/2*b*(-d)^(3/2)*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e
)^(1/2)))/e^(5/2)+b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2-2/9*b*(c*x-1)^(1/2)
*(c*x+1)^(1/2)/c^3/e-1/9*b*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e
```

Rubi [A] time = 1.05, antiderivative size = 627, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5654, 74, 5662, 100, 12, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}+\frac{b(-d)^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{5/2}}-\frac{b(-d)^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cos}}{\sqrt{c^2(-d)}}\right)}{2e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) - (2*b*Sqrt[-1
+ c*x]*Sqrt[1 + c*x])/(9*c^3*e) - (b*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(9*c
*e) - (b*d*x*ArcCosh[c*x])/e^2 + (x^3*(a + b*ArcCosh[c*x]))/(3*e) + ((-d)^(
3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) +
((-d)^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - ((-d)^(3/2)*(a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(5
/2)) - (b*(-d)^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e]))])/(2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(5/2)) - (b*(-d)^(3/2)*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e^(5/2)) + (b*(-d)^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
+ Sqrt[-(c^2*d) - e])])/(2*e^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
```

```
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^4 \left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^2}+\frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{e}+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$= -\frac{d\int \left(a+b\cosh^{-1}(cx)\right)dx}{e^2}+\frac{d^2\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2}+\frac{\int x^2\left(a+b\cosh^{-1}(cx)\right)d}{e}$$

$$= -\frac{adx}{e^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{(bd)\int \cosh^{-1}(cx)\,dx}{e^2}+\frac{d^2\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}\right.}{}$$

$$= -\frac{adx}{e^2}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{-1}(cx)}{e^2}+\frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{3e}-\frac{(-}{}$$

$$= -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}-\frac{bdx\cosh^{-1}(cx)}{e^2}+\frac{x^3(}{}$$

$$= -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}$$

$$= -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}$$

$$= -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}$$

$$= -\frac{adx}{e^2}+\frac{bd\sqrt{-1+cx}\,\sqrt{1+cx}}{ce^2}-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}}{9c^3e}-\frac{bx^2\sqrt{-1+cx}\,\sqrt{1+cx}}{9ce}$$

Mathematica [C] time = 1.51, size = 524, normalized size = 0.84

$$\frac{ad^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}}-\frac{adx}{e^2}+\frac{ax^3}{3e}+\frac{b\left(-id^{3/2}\left(2\text{Li}_2\left(\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{dc^2+e}-c\sqrt{d}}\right)+2\text{Li}_2\left(-\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{d}\,c+\sqrt{dc^2+e}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -((a*d*x)/e^2) + (a*x^3)/(3*e) + (a*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^
(5/2) + (b*((4*d*Sqrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCos
h[c*x]))/c - (4*e^(3/2)*(Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2 + c^2*x^2) - 3*c^3
*x^3*ArcCosh[c*x]))/(9*c^3) - I*d^(3/2)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(L
og[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] + Log[1 +
(I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) + 2*PolyLog[2,
(I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + 2*PolyLog[2,
 ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])]) + I*d^(3/2)*
(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*S
qrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 + Sqrt[c^2*d + e])])) + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d]
 - Sqrt[c^2*d + e])] + 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
 Sqrt[c^2*d + e])])))/(4*e^(5/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4 \operatorname{arcosh}(cx) + ax^4}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^4}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d), x)
```

maple [C] time = 9.45, size = 364, normalized size = 0.58

$$\frac{a\,x^3}{3e} - \frac{adx}{e^2} + \frac{a\,d^2 \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e^2\sqrt{de}} - \frac{b\,x^2\sqrt{cx-1}\,\sqrt{cx+1}}{9ce} + \frac{cb\,d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)} \frac{\_R1\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R1-cx-}{}\right.\right.}{}\right.}{2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] 1/3*a*x^3/e-a*d*x/e^2+a*d^2/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-1/9*b*x
^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e+1/2*c*b*d^2/e^2*sum(_R1/(_R1^2*e+2*c^2*d
+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-
c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2
+e))-1/2*c*b*d^2/e^2*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/3*b/e*arccosh(c*x)*x^
3-2/9*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3/e+b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
/e^2-b*d*x*arccosh(c*x)/e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}a\left(\frac{3\,d^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2} + \frac{ex^3 - 3\,dx}{e^2}\right) + b\int \frac{x^4 \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*d^2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) + (e*x^3 - 3*d*x)/e^2) +
 b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,(a+b\,\mathrm{acosh}\,(c\,x))}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\,(a+b\,\mathrm{acosh}\,(cx))}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.490 $\int \frac{x^3(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=521

$$-\frac{d\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}-\frac{d\left(a+b\cosh^{-1}(c\right.}{}$$

```
[Out] -1/4*b*arccosh(c*x)/c^2/e+1/2*x^2*(a+b*arccosh(c*x))/e+1/2*d*(a+b*arccosh(c
*x))^2/b/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(
1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/
2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*d*(a+b*arccosh(c*x))*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1
/2*b*d*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*polylog(2,-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*d*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e
)^(1/2)))/e^2-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e
```

Rubi [A] time = 0.91, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5792, 5662, 90, 52, 5800, 5562, 2190, 2279, 2391}

$$-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}-\frac{bd\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}-\frac{bd\text{PolyL}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -(b*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(4*c*e) - (b*ArcCosh[c*x])/(4*c^2*e) +
(x^2*(a + b*ArcCosh[c*x]))/(2*e) + (d*(a + b*ArcCosh[c*x])^2)/(2*b*e^2) - (
d*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (d*(a + b*ArcCosh[c*
x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2
*e^2) - (d*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) - (b*d*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) - (b*d*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2)
 - (b*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*e^2)
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/
(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)
^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b
```

```
*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e} - \frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right) dx$$

$$= \frac{\int x\left(a+b\cosh^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}dx}{e}$$

$$= \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} - \frac{(bc)\int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{2e} - \frac{d\int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\cosh^{-1}}{2\sqrt{e}\left(\sqrt{-d}+\right.}\right.}{e}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}dx}{2e^{3/2}} - \frac{d\int \frac{a+b\cos}{\sqrt{-d}+}}{2e^3}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\,\mathrm{Subst}\left(\int \frac{(a+}{c\sqrt{-d}}\right.}{}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(\right.}{2be^2}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(\right.}{2be^2}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(\right.}{2be^2}$$

$$= -\frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{4ce} - \frac{b\cosh^{-1}(cx)}{4c^2e} + \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{2e} + \frac{d\left(a+b\cosh^{-1}(\right.}{2be^2}$$

Mathematica [A] time = 0.56, size = 512, normalized size = 0.98

$$-\frac{2ac^2d\log\left(d+ex^2\right) - 2ac^2ex^2 + 2bc^2d\,\mathrm{Li}_2\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right) + 2bc^2d\,\mathrm{Li}_2\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-c\sqrt{-d}}\right) + 2bc^2d\,\mathrm{Li}_2\left(-\frac{\sqrt{e}e^{\cosh^{-1}(cx}}{\sqrt{-d}c+\sqrt{-dc^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/4*(-2*a*c^2*e*x^2 + b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x] - 2*b*c^2*e*x^2
*ArcCosh[c*x] - 2*b*c^2*d*ArcCosh[c*x]^2 + 2*b*e*ArcTanh[Sqrt[(-1 + c*x)/(1
 + c*x)]] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt
[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^Arc
Cosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*ArcCosh[c*x]*L
og[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + 2*b*c^
2*d*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*
d) - e])] + 2*a*c^2*d*Log[d + e*x^2] + 2*b*c^2*d*PolyLog[2, (Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b*c^2*d*PolyLog
[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] + 2*b*c^
2*d*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(c^2*e^2)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^3\operatorname{arcosh}(cx) + ax^3}{ex^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(e*x^2 + d), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.68, size = 2912, normalized size = 5.59

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] -1/4*b*arccosh(c*x)/c^2/e+1/2*a*x^2/e-3*c^2*b*d^2/e^3/(c^2*d+e)*ln(1-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arc
cosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-2*c^4*b*d^3/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh
(c*x)*(c^2*d*(c^2*d+e))^(1/2)+1/2*b*arccosh(c*x)*x^2/e+b*arccosh(c*x)^2*d/e
^2-1/4*b/e^2*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*
(c^2*d*(c^2*d+e))^(1/2)-e))-1/2*a*d/e^2*ln(c^2*e*x^2+c^2*d)-1/2*b*d/e^2*sum
((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1
)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/8/c^2*b/e/(c^2*d+e)*polylog(2
,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-
e))*(c^2*d*(c^2*d+e))^(1/2)+5/4*c^2*b/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d^2-2*c^2*
b/e^4*d^2*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+c^6*b*d^4/e^4/(c^2*d+e)*po
lylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)-e))+2*c^4*b*d^3/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-5/2*c^2*b/e^2/(c^2*d+e)*a
rccosh(c*x)^2*d^2+1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,e
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)
)-2*c^6*b*d^4/e^4/(c^2*d+e)*arccosh(c*x)^2-4*c^4*b*d^3/e^3/(c^2*d+e)*arccos
h(c*x)^2+c^2*b/e^4*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-2*c^2*b/e^3*ln(
1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
-e))*arccosh(c*x)*d^2-2*c^4*b/e^4*d^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-3/4*b/e^2/(c^2*d
+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)*d+1/4*b*(c^2*d*(c^2*d+e))^(1/2)*d/
e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)-e))+b*(c^2*d*(c^2*d+e))^(1/2)*d/e^2/(c^2*d+e)*arccos
h(c*x)^2+b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*
d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d*(c^2*d+e))^(1/2)+1/2*b/e/(c^2*
d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)-e))*arccosh(c*x)*d+1/4*b/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d-1/2*b/e/(c^2*d
+e)*arccosh(c*x)^2*d-b/e^3*d*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)+1/2*b/e
^3*d*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^
```

```
2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-1/2*b/e^2*ln(1-e*(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d-
c^2*b/e^3*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(
c^2*d*(c^2*d+e))^(1/2)-e))-c^4*b/e^4*d^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+2*c^4*b/e^4*d^3*arcco
sh(c*x)^2+2*c^2*b/e^3*d^2*arccosh(c*x)^2-1/4*b*x*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c/e-3/2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)*d+1/
2*b*(c^2*d*(c^2*d+e))^(1/2)*d/e^2/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+3*c^2*b*d^
2/e^3/(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-c^4*b*d^3/e^4/(c^2*d
+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-3/2*c^2*b*d^2/e^3/(c^2*d+e)*polylo
g(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/
2)-e))*(c^2*d*(c^2*d+e))^(1/2)+1/4/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e
)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)-e))+2*c^4*b*d^3/e^4/(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2
*d+e))^(1/2)-1/4/c^2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(
1/2)+2*c^2*b/e^4*d^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2
*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+2*c^6*b*d
^4/e^4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^
2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)+4*c^4*b*d^3/e^3/(c^2*d+e)*ln(1-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arc
cosh(c*x)+5/2*c^2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^
2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{x^2}{e}-\frac{d\log\left(ex^2+d\right)}{e^2}\right)+b\int\frac{x^3\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] 1/2*a*(x^2/e - d*log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x + sqrt(c*x +
 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2),x)
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d),x)
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.491 $\int \frac{x^2(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx$

Optimal. Leaf size=544

$$\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^{3/2}}+\frac{\sqrt{-d}\left(a\right.}{}$$

```
[Out] a*x/e+b*x*arccosh(c*x)/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2
*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^
(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d
)^(1/2)/e^(3/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylo
g(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/
2)))*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-1/2*b*polylog(2,
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
*(-d)^(1/2)/e^(3/2)+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/
2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(3/2)-b*(c*x-1)^(1/2)*(c*x
+1)^(1/2)/c/e
```

Rubi [A] time = 0.90, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {5792, 5654, 74, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}+\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^{3/2}}-\frac{b\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{}}\right.}{2e^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] (a*x)/e - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e) + (b*x*ArcCosh[c*x])/e + (
Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
- Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2))
+ (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-
d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (Sqrt[-d]*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^(3/2
)) - (b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e]))])/(2*e^(3/2)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^(3/2)) - (b*Sqrt[-d]*PolyLog[2
, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^(3/2
)) + (b*Sqrt[-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*e^(3/2))
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{e} - \frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \left(a+b\cosh^{-1}(cx)\right)dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e}$$

$$= \frac{ax}{e} + \frac{b\int \cosh^{-1}(cx)\,dx}{e} - \frac{d\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{e}$$

$$= \frac{ax}{e} + \frac{bx\cosh^{-1}(cx)}{e} - \frac{(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx}{e} - \frac{\sqrt{-d}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e} - \frac{\sqrt{-d}}{}$$

$$= \frac{ax}{e} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce} + \frac{bx\cosh^{-1}(cx)}{e} - \frac{\sqrt{-d}\ \mathrm{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\,\cosh(x)}\,dx,\right.}{2e}$$

$$= \frac{ax}{e} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce} + \frac{bx\cosh^{-1}(cx)}{e} - \frac{\sqrt{-d}\ \mathrm{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\right.}{2e}$$

$$= \frac{ax}{e} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce} + \frac{bx\cosh^{-1}(cx)}{e} + \frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\right.}{2e^{3/2}}$$

$$= \frac{ax}{e} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce} + \frac{bx\cosh^{-1}(cx)}{e} + \frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\right.}{2e^{3/2}}$$

$$= \frac{ax}{e} - \frac{b\sqrt{-1+cx}\,\sqrt{1+cx}}{ce} + \frac{bx\cosh^{-1}(cx)}{e} + \frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\right.}{2e^{3/2}}$$

Mathematica [C] time = 0.75, size = 457, normalized size = 0.84

$$\frac{-4ac\sqrt{d}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) + 4ac\sqrt{e}\,x + ib\left(-c\sqrt{d}\left(-2\mathrm{Li}_2\left(\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{dc^2+e}-c\sqrt{d}}\right) - 2\mathrm{Li}_2\left(-\frac{i\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{d}\,c+\sqrt{dc^2+e}}\right) + \cosh^{-1}(cx)\left(\cosh^{-}\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] (4*a*c*Sqrt[e]*x - 4*a*c*Sqrt[d]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + I*b*((4*I)*S
qrt[e]*(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - c*x*ArcCosh[c*x]) - c*Sqrt[d
]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[d] - Sqrt[c^2*d + e])] + Log[1 + (I*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] +
 Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]
) + Sqrt[c^2*d + e])] - 2*PolyLog[2, ((-I)*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
d] + Sqrt[c^2*d + e])]) + c*Sqrt[d]*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1
+ (I*Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[d]) + Sqrt[c^2*d + e])] + Log[1 - (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])) - 2*PolyLog[2, (I
*Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] - Sqrt[c^2*d + e])] - 2*PolyLog[2, (I*S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[d] + Sqrt[c^2*d + e])])))/(4*c*e^(3/2))
```

fricas [F] time = 0.75, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^2\,\mathrm{arcosh}\,(cx) + ax^2}{ex^2+d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d), x)
```

maple [C] time = 1.36, size = 284, normalized size = 0.52

$$\frac{ax}{e} - \frac{ad \arctan\left(\frac{xe}{\sqrt{de}}\right)}{e\sqrt{de}} - \frac{b\sqrt{cx-1}\,\sqrt{cx+1}}{ce} + \frac{bx \operatorname{arccosh}(cx)}{e} - \frac{cbd\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)} \frac{\_R1\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R}{}\right.\right.}{}\right.}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x)
[Out] a*x/e-a*d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-b*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/c/e+b*x*arccosh(c*x)/e-1/2*c*b*d/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(
c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(
1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/2*c*b*
d/e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Ro
otOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{d \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e} - \frac{x}{e}\right) + b\int \frac{x^2 \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] -a*(d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e) - x/e) + b*integrate(x^2*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(a + b \operatorname{acosh}(c\,x)\right)}{e\,x^2 + d} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2),x)
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \left(a + b \operatorname{acosh}(cx)\right)}{d + ex^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.492 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=449

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{2}$$

[Out] -1/2*(a+b*arccosh(c*x))^2/b/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e

Rubi [A] time = 0.74, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {5792, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e}+b\text{PolyLog}\left(2,\right.$$

Antiderivative was successfully verified.

[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]

[Out] -(a + b*ArcCosh[c*x])^2/(2*b*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e)

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{e}}+\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{e}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{e}}+\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx,x,\text{co}\right.}{2\sqrt{e}}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx,x,\cosh^{-1}(cx)\right)}{2\sqrt{e}}-\frac{\text{Subst}}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cos\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cos\right.}{}$$

$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2e}+\frac{\left(a+b\cos\right.}{}$$

Mathematica [A] time = 0.14, size = 447, normalized size = 1.00

$$\frac{a\log\left(d+ex^2\right)}{2e}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2e}+\frac{b\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-d}\,c+\sqrt{-dc^2-e}}\right)}{2e}+$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
```

```
[Out] -1/2*(b*ArcCosh[c*x]^2)/e + (b*ArcCosh[c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*ArcCosh[
c*x]*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(
2*e) + (b*ArcCosh[c*x]*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*e) + (a*Log[d + e*x^2])/(2*e) + (b*PolyLog[2, -((Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e) + (b*PolyLog[2
, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e) + (b*P
olyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(
2*e) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
 e])])/(2*e)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx\,\text{arcosh}\,(cx)+ax}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*x*arccosh(c*x) + a*x)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\,\text{arcosh}\,(cx)+a)x}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d), x)
```

maple [C] time = 0.37, size = 2805, normalized size = 6.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] 3/2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c
^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-1/2*b*(c^2*d
*(c^2*d+e))^(1/2)/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+3/4*b/e/(c^2*d+e)*polylo
g(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/
2)-e))*(c^2*d*(c^2*d+e))^(1/2)-1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/(c^2*d+e
)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e))+1/8/c^2*b/d/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-2
*c^4*b/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d^2-5/4*c^2*b/e/(c^2*d+e)*polylog(2,e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*d-
c^6*b*d^3/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+4*c^4*b/e^2/(c^2*d+e)*arccosh(c*x)^2*d
^2+5/2*c^2*b/e/(c^2*d+e)*arccosh(c*x)^2*d+1/2*a/e*ln(c^2*e*x^2+c^2*d)+1/2*b
/(c^2*d+e)*arccosh(c*x)^2-1/4*b/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-b*arccosh(c*x)^2/e+2
```

```
*c^4*b*d^2/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2
*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+3*c^2
*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^
2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)*d-1/2*b/(c^2*
d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)-e))*arccosh(c*x)+b/e^2*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-1/2*
b/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+1/2*b/e*ln(1-e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-1/4
*b*(c^2*d*(c^2*d+e))^(1/2)/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))-b*(c^2*d*(c^2*d+e))^(1/
2)/e/(c^2*d+e)*arccosh(c*x)^2-b/e^2*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^
(1/2)-2*c^2*b*arccosh(c*x)^2*d/e^2+c^4*b/e^3*d^2*polylog(2,e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+c^2*b/e^2*d*p
olylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e)
)^(1/2)-e))-2*c^4*b/e^3*d^2*arccosh(c*x)^2+1/4*b/e*polylog(2,e*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+1/2*b*sum((
_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^
(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1))
,_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/e+2*c^6*b*d^3/e^3/(c^2*d+e)*arcco
sh(c*x)^2+2*c^4*b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-
2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2+2*c^2*b/e^2*ln(1-e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(
c*x)*d+2*c^2*b/e^3*d*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)-c^2*b/e^3*d*pol
ylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^
(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-4*c^4*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x
)*d^2-5/2*c^2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d+1/4/c^2*b/d/(c^2*d+e)*ln
(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2
)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-1/4/c^2*b*(c^2*d*(c^2*d+e))^(1/2
)/d/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))-2*c^4*b*d^2/e^3/(c^2*d+e)*arccosh(c*x)^2
*(c^2*d*(c^2*d+e))^(1/2)+3/2*c^2*b/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d
+e))^(1/2)*d+c^4*b*d^2/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-3
*c^2*b*(c^2*d*(c^2*d+e))^(1/2)*d/e^2/(c^2*d+e)*arccosh(c*x)^2-2*c^6*b*d^3/e
^3/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*
(c^2*d+e))^(1/2)-e))*arccosh(c*x)-2*c^2*b/e^3*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d*(c^2*d
*(c^2*d+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{x\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2+d}\,dx+\frac{a\log\left(ex^2+d\right)}{2\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(x*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + 1/2*
a*log(e*x^2 + d)/e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{e\,x^2+d} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}{d+ex^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.493 $\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx$

Optimal. Leaf size=501

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{\left(a+b\cosh^{-1}(cx)\right)}{2}$$

```
[Out] 1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/
(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*
arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1
/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+1/2*b
*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-
e)^(1/2)))/(-d)^(1/2)/e^(1/2)
```

Rubi [A] time = 0.73, antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2\sqrt{-d}\,\sqrt{e}}+b\text{PolyLo}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-
(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqr
t[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[
e]) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
- e]))])/(2*Sqrt[-d]*Sqrt[e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*Sqrt[-d]*Sqrt[
e]) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(2*Sqrt[-d]*Sqrt[e])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(c\right.}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}} - \frac{\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}+\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, c\right.}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right.}{2\sqrt{-d}\sqrt{e}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\sqrt{e}} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right.}{2\sqrt{-d}\sqrt{e}}$$

Mathematica [A] time = 0.35, size = 397, normalized size = 0.79

$$-\left(\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)\right)+\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}+1\right)+\left(a+b\cosh^{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(
-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh[c*x])
*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] + b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])] - b*Po
lyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - b
*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))]
+ b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*Sqrt[-d]*Sqrt[e])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

maple [C] time = 0.13, size = 232, normalized size = 0.46

$$\frac{a\arctan\left(\frac{xe}{\sqrt{de}}\right)}{\sqrt{de}}+\frac{cb\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\_R1\left(\text{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\,\sqrt{cx+1}}{\_R1}\right)+\text{dilog}\left(\frac{\_R1-cx-\sqrt{cx-1}\,\sqrt{cx+1}}{\_R1}\right)\right)}{\_R1^2e+2c^2d+e}\right)}{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d),x)
```

```
[Out] a/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/2*c*b*sum(_R1/(_R1^2*e+2*c^2*d+e)*(
arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(
c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-
1/2*c*b*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/
2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R
1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$b\int \frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2+d}\,dx + \frac{a\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d), x) + a*arct
an(e*x/sqrt(d*e))/sqrt(d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(c\,x)}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2), x)
```

3.494 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)}\,dx$

Optimal. Leaf size=489

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d}-\frac{\left(a+b\cosh^{-1}(cx\right.}{}$$

```
[Out] (a+b*arccosh(c*x))^2/b/d+(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/d-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-
1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(2
,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d-1/2*b*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d-1/2*b*polylog
(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/d-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e
^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d
```

Rubi **[A]** time = 0.92, antiderivative size = 472, normalized size of antiderivative = 0.97, number of steps used = 25, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d}-\frac{b\text{PolyLog}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d) - ((a + b*ArcCosh[c*x])*Log[
1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d) - ((
a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c
^2*d) - e])])/(2*d) + ((a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d
- (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]
))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^
2*d) - e])])/(2*d) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
 Sqrt[-(c^2*d) - e]))])/(2*d) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])
/(2*d)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)}\,dx = \int\left(\frac{a+b\cosh^{-1}(cx)}{dx} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d(d+ex^2)}\right)dx$$
$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d} - \frac{e\int \frac{x(a+b\cosh^{-1}(cx))}{d+ex^2}\,dx}{d}$$
$$= \frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d} - \frac{e\int\left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)} + \frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}\right.}{d}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx, x, \cosh^{-1}(cx)\right)}{d} + \frac{\sqrt{e}\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}}{2d}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd} + \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \log\left(1\right.\right.}{}$$
$$= \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2\cosh^{-1}(cx)}\right)}{2d} + $$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}}}{c\sqrt{-d}-\sqrt{-c}}\right.}{2d}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}}}{c\sqrt{-d}-\sqrt{-c}}\right.}{2d}$$
$$= -\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2d} - \frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+\frac{\sqrt{e}e^{\cosh^{-1}}}{c\sqrt{-d}-\sqrt{-c}}\right.}{2d}$$

Mathematica [C] time = 0.82, size = 418, normalized size = 0.85

$$\frac{-2a\log\left(d+ex^2\right)+4a\log(x)+b\left(\text{Li}_2\left(-\frac{\left(2dc^2+e-2\sqrt{c^2d\left(dc^2+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right)+\text{Li}_2\left(-\frac{\left(2dc^2+e+2\sqrt{c^2d\left(dc^2+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)),x]
```

```
[Out] (4*a*Log[x] - 2*a*Log[d + e*x^2] + b*((-4*I)*ArcSin[Sqrt[1 + (c^2*d)/e]]*Ar
cTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c^2*d*(c^2*d + e)]*x
)] + 4*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] - 2*ArcCosh[c*x]*Log[1 + (
2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)*Ar
cSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])
/(e*E^(2*ArcCosh[c*x]))] - 2*ArcCosh[c*x]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2
*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] - (2*I)*ArcSin[Sqrt[1 + (c^2*d)/e]
]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))]
 - 2*PolyLog[2, -E^(-2*ArcCosh[c*x])] + PolyLog[2, -((2*c^2*d + e - 2*Sqrt[
c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))] + PolyLog[2, -((2*c^2*d + e +
2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x])))]))/(4*d)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^3+dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^3 + d*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)x} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x), x)
```

maple [C] time = 0.36, size = 393, normalized size = 0.80

$$\frac{a \ln(cx)}{d} - \frac{a \ln\left(c^2x^2e + c^2d\right)}{2d} - \frac{b\left(\sum_{\_R1=\mathrm{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)} \frac{\left(\_R1^2+1\right)\left(\operatorname{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{cx-1}\,\sqrt{cx+1}}{\_R1}\right)+\operatorname{dilog}\left(\frac{\_R1-cx-\sqrt{c}}{\_R}\right.\right.}{\_R1^2e+2c^2d+e}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d),x)
```

```
[Out] a/d*ln(c*x)-1/2*a/d*ln(c^2*e*x^2+c^2*d)-1/4*b*sum((_R1^2+1)/(_R1^2*e+2*c^2*
d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^
2+e))*e/d+b/d*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*ar
ccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1+I*(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2)))+b/d*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))
-1/4*b*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{\log\left(ex^2+d\right)}{d} - \frac{2\log(x)}{d}\right) + b\int \frac{\log\left(cx + \sqrt{cx+1}\sqrt{cx-1}\right)}{ex^3+dx} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] -1/2*a*(log(e*x^2 + d)/d - 2*log(x)/d) + b*integrate(log(c*x + sqrt(c*x + 1
)*sqrt(c*x - 1))/(e*x^3 + d*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{x\,\left(e\,x^2 + d\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\operatorname{acosh}(cx)}{x\left(d + ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)), x)
```

3.495 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)}\,dx$

Optimal. Leaf size=543

$$\frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{3/2}}+\frac{\sqrt{e}\left(a+b\cos\right.}{}$$

```
[Out] (-a-b*arccosh(c*x))/d/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d+1/2*(a+b*
arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-
(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(
-d)^(3/2)+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(
1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*(a+b*arccosh(c
*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e
)^(1/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*poly
log(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1
/2)))*e^(1/2)/(-d)^(3/2)-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(3/2)+1/2*b*polylog(
2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2))
)*e^(1/2)/(-d)^(3/2)
```

Rubi [A] time = 0.91, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5662, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{e}\,\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{3/2}}-\frac{b\sqrt{e}\,\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{3/2}}+\frac{b\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/
d + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh[c*x])*
Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d
)^(3/2)) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (Sqrt[e]*(a + b*ArcCosh
[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])
/(2*(-d)^(3/2)) - (b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2)) - (b*S
qrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e]))])/(2*(-d)^(3/2)) + (b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*S
qrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(3/2))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d} - \frac{e\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(c\right.}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{\left(bc^2\right)\text{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}}{2(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\,\text{Subst}\left(\int \frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, c\right.}{2(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} - \frac{e\,\text{Subst}\left(\int \frac{e^x(a+bx)}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx,\right.}{2(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{}{c}\right.}{2(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{}{c}\right.}{2(-d)^{3/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{dx} + \frac{bc\tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right)}{d} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{}{c}\right.}{2(-d)^{3/2}}$$

Mathematica [A] time = 1.69, size = 549, normalized size = 1.01

$$\frac{1}{2}\left(\frac{d\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{(-d)^{5/2}} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}+1\right)}{(-d)^{3/2}} + \frac{\sqrt{e}\left(a\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x) + (2*b*c*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1
 + c^2*x^2]])/(d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d*Sqrt[e]*(a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(-d)^(5/2) + (Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])
/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(3/2) + (Sqrt[e]*(a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(3/2) + (d*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2,
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(-d)^(3/2) +
(b*d*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^
2*d) - e])])/(-d)^(5/2) + (b*d*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (b*Sqrt[e]*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(3/2))/2
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{ex^4+dx^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(e*x^4 + d*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{(ex^2+d)x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^2), x)
```

maple [C] time = 1.58, size = 329, normalized size = 0.61

$$-\frac{a}{dx}-\frac{ae\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d\sqrt{de}}-\frac{b\,\text{arccosh}\,(cx)}{dx}-\frac{be\left(\sum_{\_R1=\text{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_Z^2+e\right)}\frac{\left(4\_R1^2c^2d+\_R1^2e+e\right)\left(\text{arccosh}(cx)\ln\left(\frac{\_R1-cx-\sqrt{}}{}\right.\right.}{\_R1\left(\_R1^2e+\right.}\right.}{8c\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d),x)
[Out] -a/d/x-a*e/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-b*arccosh(c*x)/d/x-1/8*b/c
/d^2*e*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/8*b/c/d^2*e
*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-
(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*c*b/d*arctan(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-a\left(\frac{e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}+\frac{1}{dx}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^4+dx^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d),x, algorithm="maxima")
[Out] -a*(e*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d) + 1/(d*x)) + b*integrate(log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^4 + d*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\text{acosh}\,(c\,x)}{x^2\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)),x)
[Out] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^2 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d),x)
[Out] Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)), x)
```

3.496 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=550

$$\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}+\frac{e\left(a+b\cosh^{-1}(\right.}{}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d/x^2-e*(a+b*arccosh(c*x))^2/b/d^2-e*(a+b*arccosh(c
*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2+1/2*e*(a+b*arccosh(c*x
))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^
(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1
-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))
/d^2+1/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,-1/(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2)/d^2+1/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2+1/2*
b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^
2*d-e)^(1/2)))/d^2+1/2*b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1
/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2+1/2*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/d/x
```

Rubi [A] time = 0.95, antiderivative size = 531, normalized size of antiderivative = 0.97, number of steps used = 27, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5800, 5562}

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}+\frac{be\text{PolyL}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d*x) - (a + b*ArcCosh[c*x])/(2*d*x^2)
 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (e*(a + b*ArcCos
h[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^2) + (e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) - (e*(a + b*ArcCosh[c*x])*Log[1 + E^(
2*ArcCosh[c*x])])/d^2 + (b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) + (b*e*PolyLog[2, -((Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^2) + (b*e*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2)
 - (b*e*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
```

```
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^3} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2} + \frac{e^2\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d} - \frac{e\,\text{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \text{co}\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} - \frac{(2e)\,\text{Subst}\left(\int \frac{e^{2}}{}\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2} - \frac{e\left(a+b\cosh^{-1}(\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d}}\right.}{2d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d}}\right.}{2d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2dx} - \frac{a+b\cosh^{-1}(cx)}{2dx^2} + \frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d}}\right.}{2d^2}$$

Mathematica [C] time = 1.44, size = 479, normalized size = 0.87

$$2ae\log\left(d+ex^2\right) - \frac{2ad}{x^2} - 4ae\log(x) + b\left(-e\text{Li}_2\left(-\frac{\left(2dc^2+e-2\sqrt{c^2d\left(dc^2+e\right)}\right)e^{-2\cosh^{-1}(cx)}}{e}\right) - e\text{Li}_2\left(-\frac{\left(2dc^2+e+2\sqrt{c^2d\left(dc^2+e\right)}\right)}{e}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-2*a*d)/x^2 - 4*a*e*Log[x] + 2*a*e*Log[d + e*x^2] + b*((2*c*d*Sqrt[(-1 +
c*x)/(1 + c*x)]*(1 + c*x))/x - (2*d*ArcCosh[c*x])/x^2 + (4*I)*e*ArcSin[Sqrt
[1 + (c^2*d)/e]]*ArcTanh[(c*d*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(Sqrt[c
^2*d*(c^2*d + e)]*x)] - 4*e*ArcCosh[c*x]*Log[1 + E^(-2*ArcCosh[c*x])] + 2*e
*ArcCosh[c*x]*Log[1 + (2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*Arc
Cosh[c*x]))] - (2*I)*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e - 2
*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*ArcCosh[c*x]*Log[1
+ (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))] + (2*I)
*e*ArcSin[Sqrt[1 + (c^2*d)/e]]*Log[1 + (2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d +
 e)])/(e*E^(2*ArcCosh[c*x]))] + 2*e*PolyLog[2, -E^(-2*ArcCosh[c*x])] - e*Po
lyLog[2, -((2*c^2*d + e - 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCosh[c*x]))
```

```
)] - e*PolyLog[2, -((2*c^2*d + e + 2*Sqrt[c^2*d*(c^2*d + e)])/(e*E^(2*ArcCo
sh[c*x]))))]))/(4*d^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcosh}(cx) + a}{ex^5 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^5 + d*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)x^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^3), x)
```

maple [C] time = 0.42, size = 462, normalized size = 0.84

$$-\frac{a}{2d\,x^2} - \frac{ae\ln(cx)}{d^2} + \frac{ae\ln\left(c^2x^2e + c^2d\right)}{2d^2} + \frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{2dx} - \frac{c^2b}{2d} - \frac{b\operatorname{arccosh}(cx)}{2d\,x^2} + \frac{b\,e^2\left(\sum_{\_R1=\operatorname{RootOf}\left(e\_Z^4+\left(4c^2d+2e\right)\_\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d),x)
```

```
[Out] -1/2*a/d/x^2-a/d^2*e*ln(c*x)+1/2*a*e/d^2*ln(c^2*e*x^2+c^2*d)+1/2*b*c*(c*x-1
)^(1/2)*(c*x+1)^(1/2)/d/x-1/2*c^2*b/d-1/2*b*arccosh(c*x)/d/x^2+1/4*b/d^2*e^
2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=
RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b/d^2*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2)))-b/d^2*e*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2)))-b/d^2*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-b/d^2*e*d
ilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/4*b/d^2*e*sum((_R1^2*e+4*c^2*
d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z
^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{e\log\left(ex^2 + d\right)}{d^2} - \frac{2e\log(x)}{d^2} - \frac{1}{dx^2}\right) + b\int \frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^5 + dx^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/2*a*(e*log(e*x^2 + d)/d^2 - 2*e*log(x)/d^2 - 1/(d*x^2)) + b*integrate(log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^5 + d*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,\left(e\,x^2 + d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{x^3 \left(d + ex^2\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**3*(d + e*x**2)), x)
```

3.497 $\int \frac{a+b\cosh^{-1}(cx)}{x^4(d+ex^2)}\,dx$

Optimal. Leaf size=624

$$\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2(-d)^{5/2}}+\frac{e^{3/2}\left(a+b\cos\right.}{}$$

```
[Out] 1/3*(-a-b*arccosh(c*x))/d/x^3+e*(a+b*arccosh(c*x))/d^2/x+1/6*b*c^3*arctan((
c*x-1)^(1/2)*(c*x+1)^(1/2))/d-b*c*e*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2
+1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1
/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*e^(3/2)*(a+b*arccosh(c*
x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)
^(1/2)))/(-d)^(5/2)+1/2*e^(3/2)*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*e^(3
/2)*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)+
1/2*b*e^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(
1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)-1/2*b*e^(3/2)*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/2*
b*e^(3/2)*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(5/2)+1/6*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2
```

Rubi [A] time = 0.98, antiderivative size = 624, normalized size of antiderivative = 1.00, number of steps used = 28, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5662, 103, 12, 92, 205, 5707, 5800, 5562, 2190, 2279, 2391}

$$-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}+\frac{be^{3/2}\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2(-d)^{5/2}}-\frac{be^{3/2}\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2(-d)^{5/2}}+\frac{be}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(6*d*x^2) - (a + b*ArcCosh[c*x])/(3*d*x^
3) + (e*(a + b*ArcCosh[c*x]))/(d^2*x) + (b*c^3*ArcTan[Sqrt[-1 + c*x]*Sqrt[1
 + c*x]])/(6*d) - (b*c*e*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 + (e^(3/
2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) +
 (e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (e^(3/2)*(a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*(-d)^(
5/2)) - (b*e^(3/2)*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e]))])/(2*(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2)) - (b*e^(3/2)*Pol
yLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*
(-d)^(5/2)) + (b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*(-d)^(5/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 103

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[
m] && (IntegerQ[n] || IntegersQ[2*n, 2*p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
```

```
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^4\left(d+ex^2\right)}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{dx^4} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x^2} + \frac{e^2\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^4}\,dx}{d} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2} + \frac{e^2\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{(bc)\int \frac{1}{x^3\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{3d} - \frac{(bce)\int \frac{1}{x\sqrt{-1+cx}}}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{(bc)\int \frac{c^2}{x\sqrt{-1+cx}\sqrt{1+cx}}}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} - \frac{bce\tan^{-1}\left(\sqrt{-1+cx}\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} - \frac{bce\tan^{-1}\left(\sqrt{-1+cx}\right.}{d^2}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\right.}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\right.}{6d}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{6dx^2} - \frac{a+b\cosh^{-1}(cx)}{3dx^3} + \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2x} + \frac{bc^3\tan^{-1}\left(\sqrt{-1+cx}\right.}{6d}$$

Mathematica [A] time = 1.61, size = 641, normalized size = 1.03

$$\frac{1}{6}\left(-\frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{(-d)^{5/2}} + \frac{3e^{3/2}\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}-c\sqrt{-d}}+1\right)}{(-d)^{5/2}} + \frac{3e^{3/2}}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^4*(d + e*x^2)),x]
```

```
[Out] ((-2*(a + b*ArcCosh[c*x]))/(d*x^3) + (6*e*(a + b*ArcCosh[c*x]))/(d^2*x) - (
6*b*c*e*Sqrt[-1 + c^2*x^2]*ArcTan[Sqrt[-1 + c^2*x^2]])/(d^2*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]) + (b*c*(-1 + c^2*x^2 + c^2*x^2*Sqrt[-1 + c^2*x^2]*ArcTan[Sqr
t[-1 + c^2*x^2]]))/(d*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*e^(3/2)*(a + b
*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d)
 - e])])/(-d)^(5/2) + (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^Ar
cCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*e^(3/2)*
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(
c^2*d) - e])])/(-d)^(5/2) - (3*e^(3/2)*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(-(c*Sqrt[-
d]) + Sqrt[-(c^2*d) - e])])/(-d)^(5/2) - (3*b*e^(3/2)*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(-d)^(5/2) + (3*b*e^(
3/2)*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(-d)^(5/2))/6
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcosh}\left(cx\right) + a}{ex^{6} + dx^{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e*x^6 + d*x^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}\left(cx\right) + a}{\left(ex^{2} + d\right)x^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)*x^4), x)
```

maple [C] time = 1.79, size = 410, normalized size = 0.66

$$-\frac{a}{3d\,x^{3}}+\frac{ae}{d^{2}x}+\frac{a\,e^{2}\arctan\left(\frac{xe}{\sqrt{de}}\right)}{d^{2}\sqrt{de}}+\frac{bc\sqrt{cx-1}\,\sqrt{cx+1}}{6d\,x^{2}}+\frac{b\operatorname{arccosh}\left(cx\right)e}{d^{2}x}-\frac{b\operatorname{arccosh}\left(cx\right)}{3d\,x^{3}}-\frac{2cbe\arctan\left(cx+\sqrt{\ldots}\right.}{d^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^4/(e*x^2+d),x)
```

```
[Out] -1/3*a/d/x^3+a/d^2*e/x+a*e^2/d^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/6*b*
c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/d/x^2+b*arccosh(c*x)/d^2*e/x-1/3*b/d*arccosh(
c*x)/x^3-2*c*b/d^2*e*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-1/8/c*b/d^3*e^
2*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/3*c^3*b/d*arctan(c*x+(
c*x-1)^(1/2)*(c*x+1)^(1/2))+1/8/c*b/d^3*e^2*sum((4*_R1^2*c^2*d+_R1^2*e+e)/_
R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2
))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4
+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{3\,e^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}+\frac{3\,ex^2-d}{d^2x^3}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^6+dx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^4/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] 1/3*a*(3*e^2*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d^2) + (3*e*x^2 - d)/(d^2*x^3
)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^6 + d*x^4), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^4\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^4*(d + e*x^2)),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^4*(d + e*x^2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(cx\right)}{x^4\left(d+ex^2\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**4/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**4*(d + e*x**2)), x)
```

3.498 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=562

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^2}+\frac{\left(a+b\cosh^{-1}(cx)\right)}{}$$

```
[Out] 1/2*d*(a+b*arccosh(c*x))/e^2/(e*x^2+d)-1/2*(a+b*arccosh(c*x))^2/b/e^2+1/2*(
a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*(a+b*arccosh
(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d
-e)^(1/2)))/e^2+1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,-(c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^2+1/2*b*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))/e^2+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c
*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^2-1/2*b*c*arctanh(x*(c
^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*d^(1/2)*(c^2*x^2-1)^(1/2)/e^2/(c^2
*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.99, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5788, 519, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^2}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^2}+\frac{b\text{PolyLog}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (d*(a + b*ArcCosh[c*x]))/(2*e^2*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*
e^2) - (b*c*Sqrt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]
*Sqrt[-1 + c^2*x^2])])/(2*e^2*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
 + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + ((a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e
^2) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +
Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c
*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*PolyLog[2, (Sqrt[e]*E^ArcCo
sh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^2) + (b*PolyLog[2, -((Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^2) + (b*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int\left(-\frac{dx\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e}-\frac{d\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{(bcd)\int\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e^2}+\frac{\int\left(-\frac{a+b\cosh^{-1}(cx)}{2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\ldots\right.}{e}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{3/2}}+\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2e^{3/2}}-\frac{\left(bcd\sqrt{-1+c^2x^2}\ldots\right.}{2e^2\sqrt{-\ldots}}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\text{Subst}\left(\int\frac{(a+bx)\sinh(x)}{c\sqrt{-d}-\sqrt{e}\,\cosh(x)}\,dx,x,\cosh^{-1}(cx)\right)}{2e^{3/2}}+\frac{\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2}-\frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d\ldots}}{\sqrt{d}\,\sqrt{-1\ldots}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+\ldots}}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2}-\frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d\ldots}}{\sqrt{d}\,\sqrt{-1\ldots}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+\ldots}}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2}-\frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d\ldots}}{\sqrt{d}\,\sqrt{-1\ldots}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+\ldots}}$$

$$=\frac{d\left(a+b\cosh^{-1}(cx)\right)}{2e^2\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2be^2}-\frac{bc\sqrt{d}\,\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d\ldots}}{\sqrt{d}\,\sqrt{-1\ldots}}\right.}{2e^2\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+\ldots}}$$

Mathematica [C] time = 2.15, size = 693, normalized size = 1.23

$$\frac{\frac{2ad}{d+ex^2}+2a\log\left(d+ex^2\right)+b\left(2\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-ic\sqrt{d}}\right)+2\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-ic\sqrt{d}}\right)+2\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{i\sqrt{d}\,c+\sqrt{-dc^2-e}}\right)+2\text{Li}_2\left(\frac{\sqrt{e}\ldots}{i\sqrt{d}\ldots}\right.\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((2*a*d)/(d + e*x^2) + 2*a*Log[d + e*x^2] + b*(-2*ArcCosh[c*x]^2 + 2*ArcCos
h[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
) + 2*ArcCosh[c*x]*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]) - I*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2
*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1
```

```
+ c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e
]) - I*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqr
t[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/
(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Po
lyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]
 + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])]))/(4*e^2)
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^3 \operatorname{arcosh}(cx) + ax^3}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^3*arccosh(c*x) + a*x^3)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 0.93, size = 2964, normalized size = 5.27

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*a/e^2*ln(c^2*e*x^2+c^2*d)+1/4*b/e^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-b*arccosh(c*x)^2/e^2+1
/2*b/e^2*sum((_R1^2*e+4*c^2*d+2*e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1
)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/4/c^2*b/d/e/(c^2*
d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+2*c^4*b*d^2/e^4/(c^2*d+e)
*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(
1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)-1/4/c^2*b*(c^2*d*(c^2*d+e))^(
1/2)/d/e/(c^2*d+e)*arccosh(c*x)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/
(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))+3*c^2*b*d/e^3/(c^2*d+e)*ln(1-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arc
cosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+1/2*c^2*a/e^2*d/(c^2*e*x^2+c^2*d)+c^2*b/e
^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*
d+e))^(1/2)-e))*d-2*c^2*b/e^3*arccosh(c*x)^2*d-2*c^4*b/e^4*d^2*arccosh(c*x)
^2+c^4*b/e^4*d^2*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-
2*(c^2*d*(c^2*d+e))^(1/2)-e))-1/2*b/e/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-b/e^3*
ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1
/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+e))^(1/2)+3/4*b/e^2/(c^2*d+e)*polylog(2,
e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e
```

```
))*(c^2*d*(c^2*d+e))^(1/2)+1/2*b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arct
anh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*
d*e)^(1/2))-1/4*b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))-b*(c^2
*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arccosh(c*x)^2+1/2*b/e^2*ln(1-e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(
c*x)-1/2*b/e^3*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*
(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)+b/e^3*arccosh(c*x)^2*(c
^2*d*(c^2*d+e))^(1/2)+1/2*b/e/(c^2*d+e)*arccosh(c*x)^2-1/4*b/e/(c^2*d+e)*po
lylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)-e))+1/2*c^2*b*arccosh(c*x)/e^2*d/(c^2*e*x^2+c^2*d)+2*c^2*b/e^3*ln(1-
e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e
))*arccosh(c*x)*d+2*c^4*b/e^4*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-
2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*d^2-c^2*b/e^4*d*polylog(
2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
-e))*(c^2*d*(c^2*d+e))^(1/2)+4*c^4*b/e^3/(c^2*d+e)*arccosh(c*x)^2*d^2-c^6*b
*d^3/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)-e))+2*c^2*b/e^4*d*arccosh(c*x)^2*(c^2*d*(c^2*d+
e))^(1/2)+2*c^6*b*d^3/e^4/(c^2*d+e)*arccosh(c*x)^2+5/2*c^2*b*d/e^2/(c^2*d+e
)*arccosh(c*x)^2-5/4*c^2*b*d/e^2/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))-2*c^4*b/e^3/(c^2*d+
e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*
d+e))^(1/2)-e))*d^2+3/2*b/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)*(c^2*d*(c^2*d+
e))^(1/2)-1/2*b*(c^2*d*(c^2*d+e))^(1/2)/e^2/(c^2*d+e)*arccosh(c*x)*ln(1-e*(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))-
1/8/c^2*b*(c^2*d*(c^2*d+e))^(1/2)/d/e/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))^2/(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e))-2*c^6*b*d^3/e^
4/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(
c^2*d+e))^(1/2)-e))*arccosh(c*x)-4*c^4*b*d^2/e^3/(c^2*d+e)*ln(1-e*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(
c*x)-5/2*c^2*b*d/e^2/(c^2*d+e)*ln(1-e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(
-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arccosh(c*x)-2*c^2*b/e^4*ln(1-e*(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*arc
cosh(c*x)*d*(c^2*d*(c^2*d+e))^(1/2)+1/8/c^2*b/d/e/(c^2*d+e)*polylog(2,e*(c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c
^2*d*(c^2*d+e))^(1/2)+3/2*c^2*b/e^3/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))^2/(-2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e)
)^(1/2)*d-3*c^2*b/e^3/(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)*d+c^
4*b*d^2/e^4/(c^2*d+e)*polylog(2,e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2/(-2*c
^2*d-2*(c^2*d*(c^2*d+e))^(1/2)-e))*(c^2*d*(c^2*d+e))^(1/2)-2*c^4*b*d^2/e^4/
(c^2*d+e)*arccosh(c*x)^2*(c^2*d*(c^2*d+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{d}{e^3x^2+de^2}+\frac{\log\left(ex^2+d\right)}{e^2}\right)+b\int\frac{x^3\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d/(e^3*x^2 + d*e^2) + log(e*x^2 + d)/e^2) + b*integrate(x^3*log(c*x
+ sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^3\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 (a + b \operatorname{acosh}(cx))}{(d + ex^2)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
[Out] Integral(x**3*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.499 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=113

$$\frac{bc\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}\,e\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d+e}}-\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/e/(e*x^2+d)+1/2*b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/
2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/e/d^(1/2)/(c^2*d+e)^(1/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {5788, 519, 377, 208}

$$\frac{bc\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{2\sqrt{d}\,e\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{c^2d+e}}-\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCosh[c*x])/(2*e*(d + e*x^2)) + (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(
Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*Sqrt[d]*e*Sqrt[c^2*d +
 e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{(bc)\int\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{2e}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int\frac{1}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)}\,dx}{2e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\text{Subst}\left(\int\frac{1}{d-\left(c^2d+e\right)x^2}\,dx,x,\frac{x}{\sqrt{-1+c^2x^2}}\right)}{2e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2e\left(d+ex^2\right)}+\frac{bc\sqrt{-1+c^2x^2}\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\sqrt{-1+c^2x^2}}\right)}{2\sqrt{d}\,e\sqrt{c^2d+e}\sqrt{-1+cx}\sqrt{1+cx}}$$

Mathematica [A] time = 0.33, size = 123, normalized size = 1.09

$$-\frac{\frac{a}{d+ex^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\sqrt{c^2x^2-1}\sqrt{c^2(-d)-e}}+\frac{b\cosh^{-1}(cx)}{d+ex^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] -1/2*(a/(d + e*x^2) + (b*ArcCosh[c*x])/(d + e*x^2) - (b*c*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(S
qrt[d]*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c^2*x^2]))/e
```

fricas [B] time = 0.54, size = 537, normalized size = 4.75

$$\left[-\frac{2\,ac^2d^2-2\left(bc^2de+be^2\right)x^2\log\left(cx+\sqrt{c^2x^2-1}\right)+2\,ade-\left(bcex^2+bcd\right)\sqrt{c^2d^2+de}\,\log\left(-\frac{2\,c^2d^2-\left(4\,c^4d^2+4\,c^2de+e\right.}{}\right.}{4\left(c^2d^3e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*c^2*d^2 - 2*(b*c^2*d*e + b*e^2)*x^2*log(c*x + sqrt(c^2*x^2 - 1))
 + 2*a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4*c
^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d + c*e
)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x + 2
*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 2*(b*c^2*d^2 + b*d*e + (b*c^2*d*e + b
*e^2)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e
^2 + d*e^3)*x^2), -1/2*(a*c^2*d^2 - (b*c^2*d*e + b*e^2)*x^2*log(c*x + sqrt(
c^2*x^2 - 1)) + a*d*e - (b*c*e*x^2 + b*c*d)*sqrt(-c^2*d^2 - d*e)*arctan((sq
rt(-c^2*d^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 +
c*d))/(c^2*d^2 + d*e)) - (b*c^2*d^2 + b*d*e + (b*c^2*d*e + b*e^2)*x^2)*log(
-c*x + sqrt(c^2*x^2 - 1)))/(c^2*d^3*e + d^2*e^2 + (c^2*d^2*e^2 + d*e^3)*x^2
)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)x}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^2, x)
```

maple [B] time = 0.04, size = 638, normalized size = 5.65

$$-\frac{c^2 a}{2e\left(c^2x^2e+c^2d\right)}-\frac{c^2 b \,\mathrm{arccosh}\,(cx)}{2e\left(c^2x^2e+c^2d\right)}-\frac{c^4 b\sqrt{cx-1}\,\sqrt{cx+1}\,\ln\left(\frac{2\sqrt{c^2x^2-1}\,\sqrt{-\frac{c^2d+e}{e}}\,e+2\sqrt{-c^2de}\,cx-2e}{cxe-\sqrt{-c^2de}}\right)d}{4\sqrt{c^2x^2-1}\,\left(\sqrt{-c^2de}+e\right)\left(e-\sqrt{-c^2de}\right)\sqrt{-c^2de}\,\sqrt{-\frac{c^2d+e}{e}}}+\frac{c^4 b\sqrt{cx-1}}{4\sqrt{c^2x^2-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e/(c^2*e*x^2+c^2*d)-1/2*c^2*b/e/(c^2*e*x^2+c^2*d)*arccosh(c*x)-1
/4*c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)
/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)*ln(2*((c^2*x^2-
1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(
1/2)))*d+1/4*c^4*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2)/((-c^2*d*e
)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)*ln(-2
*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+
(-c^2*d*e)^(1/2)))*d-1/4*c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x^2-1)^(1/2
)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)
^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-
e)/(c*x*e-(-c^2*d*e)^(1/2)))*e+1/4*c^2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(c^2*x
^2-1)^(1/2)/((-c^2*d*e)^(1/2)+e)/(e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c
^2*d+e)/e)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e
)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}\left(4\,c\int\frac{1}{2\left(c^3e^2x^5-cdex+\left(c^3de-ce^2\right)x^3+\left(c^2e^2x^4+\left(c^2de-e^2\right)x^2-de\right)e^{\left(\frac{1}{2}\log(cx+1)+\frac{1}{2}\log(cx-1)\right)}\right)}\,dx+\frac{c^2\log}{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(4*c*integrate(1/2/(c^3*e^2*x^5 - c*d*e*x + (c^3*d*e - c*e^2)*x^3 + (c
^2*e^2*x^4 + (c^2*d*e - e^2)*x^2 - d*e)*e^(1/2*log(c*x + 1) + 1/2*log(c*x -
 1))), x) + c^2*log(e*x^2 + d)/(c^2*d*e + e^2) + (2*(c^2*d + e)*log(c*x + s
qrt(c*x + 1)*sqrt(c*x - 1)) - (c^2*e*x^2 + c^2*d)*log(c*x + 1) - (c^2*e*x^2
 + c^2*d)*log(c*x - 1))/(c^2*d^2*e + d*e^2 + (c^2*d*e^2 + e^3)*x^2))*b - 1/
2*a/(e^2*x^2 + d*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(a+b\operatorname{acosh}\left(cx\right)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.500 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)^2}\,dx$

Optimal. Leaf size=598

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^2}-\frac{\left(a+b\cosh^{-1}(cx\right.}{}$$

```
[Out] 1/2*(a+b*arccosh(c*x))/d/(e*x^2+d)+(a+b*arccosh(c*x))^2/b/d^2+(a+b*arccosh(
c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^2-1/2*(a+b*arccosh(c*x)
)*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(
1/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^
(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*(a+b*arccosh(c*x))*ln(1-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-
1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))^2)/d^2-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^2-1/2*b*polylog(2,-
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/
d^2-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/d^2-1/2*b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-
1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(3/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(
1/2)
```

Rubi [A] time = 1.06, antiderivative size = 581, normalized size of antiderivative = 0.97, number of steps used = 29, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^2}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^2}-\frac{b\text{PolyLog}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] (a + b*ArcCosh[c*x])/(2*d*(d + e*x^2)) - (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(S
qrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(3/2)*Sqrt[c^2*d + e]
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[
c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/
(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])])/(2*d^2) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^2) + ((a + b*ArcCo
sh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^2 - (b*PolyLog[2, -((Sqrt[e]*E^ArcC
osh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^2) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^2) - (b*PolyL
og[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^
2) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e
])])/(2*d^2) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^2)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
```

```
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx=\int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x}-\frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2}-\frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^2}-\frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^2}-\frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}+\frac{\text{Subst}\left(\int (a+bx)\tanh(x)\,dx,x,\cosh^{-1}(cx)\right)}{d^2}-\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\,\sqrt{1+\cdots}}}{2d}$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2}+\frac{2\,\text{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,dx,x,\cosh^{-1}(cx)\right)}{d^2}+\cdots$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^2}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1+e^{2\cosh^{-1}(cx)}\right)}{d^2}-\cdots$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1\cdots\right.}{d^2}$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1\cdots\right.}{2d^2}$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1\cdots\right.}{2d^2}$$

$$=\frac{a+b\cosh^{-1}(cx)}{2d\left(d+ex^2\right)}-\frac{bc\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{c^2d+e}\,x}{\sqrt{d}\,\sqrt{-1+c^2x^2}}\right)}{2d^{3/2}\sqrt{c^2d+e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1\cdots\right.}{2d^2}$$

Mathematica [F] time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2),x]
```

```
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^2), x]
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^2x^5+2\,dex^3+d^2x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x), x)
```

maple [C] time = 0.44, size = 529, normalized size = 0.88

$$\frac{a\ln(cx)}{d^2}+\frac{a\,c^2}{2d\left(c^2x^2e+c^2d\right)}-\frac{a\ln\left(c^2x^2e+c^2d\right)}{2d^2}+\frac{b\,c^2\text{arccosh}\,(cx)}{2d\left(c^2x^2e+c^2d\right)}+\frac{b\sqrt{c^2d\left(c^2d+e\right)}\ \text{arctanh}\left(\frac{2\left(cx+\sqrt{cx-1}\ \sqrt{cx+1}\right)^2e-}{4\sqrt{d^2c^4+c^2de}}\right.}{2d^2\left(c^2d+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x)
```

```
[Out] a/d^2*ln(c*x)+1/2*a*c^2/d/(c^2*e*x^2+c^2*d)-1/2*a/d^2*ln(c^2*e*x^2+c^2*d)+1
/2*b*c^2*arccosh(c*x)/d/(c^2*e*x^2+c^2*d)+1/2*b*(c^2*d*(c^2*d+e))^(1/2)/d^2
/(c^2*d+e)*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e
)/(c^4*d^2+c^2*d*e)^(1/2))-1/4*b/d^2*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2
*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R
1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z
^2+e))+b/d^2*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1+I*(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^2*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-1/4*b/d^2*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{1}{dex^2+d^2}-\frac{\log\left(ex^2+d\right)}{d^2}+\frac{2\,\log(x)}{d^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^5+2\,dex^3+d^2x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(1/(d*e*x^2 + d^2) - log(e*x^2 + d)/d^2 + 2*log(x)/d^2) + b*integrate
(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^5 + 2*d*e*x^3 + d^2*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{x\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{x \left(d + ex^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x*(d + e*x**2)**2), x)
```

3.501 $\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=634

$$\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{e\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{d^3}+\frac{e\left(a+b\cosh^{-1}(cx\right.}{}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d^2/x^2-1/2*e*(a+b*arccosh(c*x))/d^2/(e*x^2+d)-2*e*
(a+b*arccosh(c*x))^2/b/d^3-2*e*(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)
*(c*x+1)^(1/2))^2)/d^3+e*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2
)))/d^3+e*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)
/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+b*e*pol
ylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3+b*e*polylog(2,-(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3+b*e*p
olylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)
^(1/2)))/d^3+b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3+1/2*b*c*(c*x-1)^(1/2)*(c*
x+1)^(1/2)/d^2/x+1/2*b*c*e*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1
/2))*(c^2*x^2-1)^(1/2)/d^(5/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.09, antiderivative size = 616, normalized size of antiderivative = 0.97, number of steps used = 31, number of rules used = 14, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 377, 208, 5800, 5562}

$$\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{d^3}+\frac{be\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{d^3}+\frac{be\text{PolyLog}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^2*x) - (a + b*ArcCosh[c*x])/(2*d^2*
x^2) - (e*(a + b*ArcCosh[c*x]))/(2*d^2*(d + e*x^2)) + (b*c*e*Sqrt[-1 + c^2*
x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*
Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (e*(a + b*ArcCosh[c*x])*Log
[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (e*
(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 + (e*(a + b*ArcCosh[c*x])*Log[1
+ (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/d^3 - (2*e*(
a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^3 + (b*e*PolyLog[2, -((S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/d^3 + (b*e*Poly
Log[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/d^3 + (
b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
)])/d^3 + (b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/d^3 - (b*e*PolyLog[2, -E^(2*ArcCosh[c*x])])/d^3
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
```

```
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^3} - \frac{2e\left(a+b\cosh^{-1}(cx)\right)}{d^3x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} + \frac{2e^2x\left(a+\right.}{d^3\left(\right.}$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^2} - \frac{(2e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} + \frac{\left(2e^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} + \frac{e^2\int \frac{x(a+bc}{(d+}}{a}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{2d^2} - \frac{(2e)\,\mathrm{Subst}\left(\right.}{}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1}(c\right.}{bd^3}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{e\left(a+b\cosh^{-1}(c\right.}{bd^3}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\ \mathrm{ta}}{2d^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\ \mathrm{ta}}{2d^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\ \mathrm{ta}}{2d^{5/2}\sqrt{c^2d+e}}$$

$$= \frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{2d^2x} - \frac{a+b\cosh^{-1}(cx)}{2d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d^2\left(d+ex^2\right)} + \frac{bce\sqrt{-1+c^2x^2}\ \mathrm{ta}}{2d^{5/2}\sqrt{c^2d+e}}$$

Mathematica [F] time = 6.03, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^2), x]
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcosh}\,(cx)+a}{e^2x^7+2\,dex^5+d^2x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^7 + 2*d*e*x^5 + d^2*x^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{(ex^2 + d)^2 x^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^3), x)
```

maple [C] time = 0.64, size = 723, normalized size = 1.14

$$-\frac{a}{2d^2x^2} - \frac{2ae\ln(cx)}{d^3} - \frac{c^2ae}{2d^2(c^2x^2e + c^2d)} + \frac{ae\ln(c^2x^2e + c^2d)}{d^3} + \frac{c^3bx\sqrt{cx-1}\,\sqrt{cx+1}\,e}{2d^2(c^2x^2e + c^2d)} + \frac{c^3b\sqrt{cx-1}\,\sqrt{cx+1}}{2xd(c^2x^2e + c^2d)} - \frac{}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x)
```

```
[Out] -1/2*a/d^2/x^2-2*a/d^3*e*ln(c*x)-1/2*c^2*a*e/d^2/(c^2*e*x^2+c^2*d)+a*e/d^3*
ln(c^2*e*x^2+c^2*d)+1/2*c^3*b*x/d^2/(c^2*e*x^2+c^2*d)*(c*x-1)^(1/2)*(c*x+1)
^(1/2)*e+1/2*c^3*b/x/d/(c^2*e*x^2+c^2*d)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-1/2*c^
4*b*x^2/d^2/(c^2*e*x^2+c^2*d)*e-1/2*c^4*b/d/(c^2*e*x^2+c^2*d)-c^2*b*arccosh
(c*x)*e/d^2/(c^2*e*x^2+c^2*d)-1/2*c^2*b/x^2/d/(c^2*e*x^2+c^2*d)*arccosh(c*x
)-1/2*b*(c^2*d*(c^2*d+e))^(1/2)/d^3/(c^2*d+e)*e*arctanh(1/4*(2*(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))+1/2*b/d^3*e
*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_
R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-2*b/d^3*e*arccosh(c*x)*ln(1+I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-2*b/d^3*e*arccosh(c*x)*ln(1-I*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2)))-2*b/d^3*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2)))-2*b/d^3*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/2*b/d^3*e^2*
sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(
c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Ro
otOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}a\left(\frac{2\,ex^2 + d}{d^2ex^4 + d^3x^2} - \frac{2\,e\log(ex^2 + d)}{d^3} + \frac{4\,e\log(x)}{d^3}\right) + b\int \frac{\log(cx + \sqrt{cx+1}\,\sqrt{cx-1})}{e^2x^7 + 2\,dex^5 + d^2x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((2*e*x^2 + d)/(d^2*e*x^4 + d^3*x^2) - 2*e*log(e*x^2 + d)/d^3 + 4*e*
log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^7 +
 2*d*e*x^5 + d^2*x^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\operatorname{acosh}(c\,x)}{x^3\,(e\,x^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.502 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=839

$$\frac{x\cosh^{-1}(cx)b}{e^2}+\frac{cd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\,\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}c+\sqrt{e}}\,e^{5/2}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}c+\sqrt{e}}\,e^{5/2}}-\frac{3\sqrt{-d}\,\mathrm{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-}}\right.}{4e^{5/2}}$$

[Out] a*x/e^2+b*x*arccosh(c*x)/e^2+3/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)+3/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*(-d)^(1/2)/e^(5/2)-1/4*d*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)-x*e^(1/2))+1/4*d*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)+x*e^(1/2))-b*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/e^2+1/2*b*c*d*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)-1/2*b*c*d*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)

Rubi [A] time = 2.19, antiderivative size = 839, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 12, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {5792, 5654, 74, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{x\cosh^{-1}(cx)b}{e^2}+\frac{cd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}c+\sqrt{e}}\,\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}c+\sqrt{e}}\,e^{5/2}}-\frac{cd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)b}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}c+\sqrt{e}}\,e^{5/2}}-\frac{3\sqrt{-d}\,\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}}{c\sqrt{-}}\right.}{4e^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]

[Out] (a*x)/e^2 - (b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*e^2) + (b*x*ArcCosh[c*x])/e^2 - (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] - Sqrt[e]*x)) + (d*(a + b*ArcCosh[c*x]))/(4*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) - (b*c*d*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) + (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*Sqrt[-d]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] +

```
 Sqrt[-(c^2*d) - e])])/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[
-d]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])
/(4*e^(5/2)) - (3*b*Sqrt[-d]*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))])/(4*e^(5/2)) + (3*b*Sqrt[-d]*PolyLog[2, (Sqrt[e
]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*e^(5/2))
```

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ
[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{e^2}+\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}-\frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int\left(a+b\cosh^{-1}(cx)\right)dx}{e^2}-\frac{(2d)\int\frac{a+b\cosh^{-1}(cx)}{d+ex^2}dx}{e^2}+\frac{d^2\int\frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}dx}{e^2}$$

$$= \frac{ax}{e^2}+\frac{b\int\cosh^{-1}(cx)\,dx}{e^2}-\frac{(2d)\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{e^2}+$$

$$= \frac{ax}{e^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{(bc)\int\frac{x}{\sqrt{-1+cx}\sqrt{1+cx}}dx}{e^2}-\frac{\sqrt{-d}\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}dx}{e^2}-\frac{\sqrt{-d}}{}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

$$= \frac{ax}{e^2}-\frac{b\sqrt{-1+cx}\sqrt{1+cx}}{ce^2}+\frac{bx\cosh^{-1}(cx)}{e^2}-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{4e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{d\left(a+b\,\mathrm{co}\right.}{4e^{5/2}\left(\sqrt{-\right.}}$$

Mathematica [C] time = 2.41, size = 777, normalized size = 0.93

$$\frac{4ad\sqrt{e}\,x}{d+ex^2}-12a\sqrt{d}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+8a\sqrt{e}\,x+b\left(-3i\sqrt{d}\left(2\mathrm{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-ic\sqrt{d}}\right)+2\mathrm{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{i\sqrt{d}\,c+\sqrt{-dc^2-e}}\right)+\cosh^{-1}(cx\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (8*a*Sqrt[e]*x + (4*a*d*Sqrt[e]*x)/(d + e*x^2) - 12*a*Sqrt[d]*ArcTan[(Sqrt[
e]*x)/Sqrt[d]] + b*((8*Sqrt[e]*(-(Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)) + c
*x*ArcCosh[c*x]))/c + 2*d*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log
[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d)
- e]) + 2*d*(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(-Sqrt[e] -
 I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqr
t[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt
[d]*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c
*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqr
t[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)
*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[d]*(ArcCosh[c*x]*(-ArcC
osh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*
d) - e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])])))/(8*e^(5/2))
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{bx^4\,\mathrm{arcosh}\,(cx) + ax^4}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx) + a)x^4}{\left(ex^2 + d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^2, x)
```

maple [C] time = 7.08, size = 1749, normalized size = 2.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] a*x/e^2+1/2*c^2*a/e^2*d*x/(c^2*e*x^2+c^2*d)-3/2*a/e^2*d/(d*e)^(1/2)*arctan(
x*e/(d*e)^(1/2))-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arc
tanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)-e)*e)^(1/2))*d^2/e^5-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*
d+e))^(1/2)-e)*e)^(1/2))*d/e^4-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2))*d^2/e^5-1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1
/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^4+3/4*c*b/e^2*d*sum(1/_R1/(_R1^2*e+2*c
^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((
```

```
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*
_Z^2+e))-3/4*c*b/e^2*d*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*
x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1
/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-b*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c/e^2+b*x*arccosh(c*x)/e^2+1/2*c^2*b*arccosh(c*x)*d*x/e^2/(c^2*e*x^2
+c^2*d)+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*arctanh(
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)
*e)^(1/2))/e^5/(c^2*d+e)+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*d/e^5*(c^2*d*(c^2*d+e))^(1/2)+c^5*b*
((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*arctan((c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c
^2*d+e)+c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctan((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)
^(1/2))/e^4/(c^2*d+e)+c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))*d/e^5*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/(
(-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)*(c^2*d*(c^2*
d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)-e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)*(c^2*d*(
c^2*d+e))^(1/2)-1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}a\left(\frac{dx}{e^3x^2+de^2}-\frac{3\,d\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}+\frac{2\,x}{e^2}\right)+b\int\frac{x^4\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(d*x/(e^3*x^2 + d*e^2) - 3*d*arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e^2) +
2*x/e^2) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4
+ 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\left(a+b\,\mathrm{acosh}\left(cx\right)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**2,x)

[Out] Integral(x**4*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.503 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2} dx$

Optimal. Leaf size=792

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{4\sqrt{-d}e^{3/2}}+\frac{\left(a+b\cosh^{-1}(cx)\right)}{4}$$

```
[Out] 1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-
d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1+
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/
e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*(a+b*
arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1
/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)-1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*b
*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-
e)^(1/2)))/e^(3/2)/(-d)^(1/2)+1/4*(a+b*arccosh(c*x))/e^(3/2)/((-d)^(1/2)-x*
e^(1/2))+1/4*(-a-b*arccosh(c*x))/e^(3/2)/((-d)^(1/2)+x*e^(1/2))-1/2*b*c*arc
tanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)
+e^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2)
)^(1/2)+1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^
(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c
*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A] time = 1.97, antiderivative size = 792, normalized size of antiderivative = 1.00, number of steps used = 46, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {5792, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{4\sqrt{-d}e^{3/2}}-\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{4\sqrt{-d}e^{3/2}}+b\text{PolyLog}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] (a + b*ArcCosh[c*x])/(4*e^(3/2)*(Sqrt[-d] - Sqrt[e]*x)) - (a + b*ArcCosh[c*
x])/(4*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) - (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sq
rt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(2*Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) + (b*c*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])])/(2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/
2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 +
(Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^
(3/2)) + ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]
*e^(3/2)) - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c
^2*d) - e]))])/(4*Sqrt[-d]*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x]
```

```
)/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*Sqrt[-d]*e^(3/2)) - (b*PolyLog[2,
-((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*Sqrt[-d]
*e^(3/2)) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2
*d) - e])])/(4*Sqrt[-d]*e^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^2}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(\right.}{4d\left(\sqrt{-d}\,\sqrt{e}+\right.}\right.}{e}$$

$$= \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}dx + \frac{1}{4}\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}dx + \frac{1}{2}\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}dx -$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{1}{2}\int \left(-\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2de\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \sqrt{\right.$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{(bc)\,\text{Subst}\left(\int \frac{1}{c\sqrt{-d}\,\sqrt{e}+e-\left(c\sqrt{-d}\,\sqrt{e}-e\right)x^2}\right.}{2e}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}} +$$

$$= \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{a+b\cosh^{-1}(cx)}{4e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\,\sqrt{-1+cx}}\right)}{2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\sqrt{-d}+\sqrt{e}}\,e^{3/2}} +$$

Mathematica [C] time = 1.92, size = 720, normalized size = 0.91

$$-\frac{4a\sqrt{e}\,x}{d+ex^2}+\frac{4a\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}}+b\left(\frac{i\left(2\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-ic\sqrt{d}}\right)+2\text{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{i\sqrt{d}\,c+\sqrt{-dc^2-e}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+2\left(\log\left(1+\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{-\sqrt{c^2(-d)-e}+ic\sqrt{d}}\right)+\log\left(1+\right.\right.\right.\right.}{\sqrt{d}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
```

```
[Out] ((-4*a*Sqrt[e]*x)/(d + e*x^2) + (4*a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] +
 b*((-2*ArcCosh[c*x])/(I*Sqrt[d] + Sqrt[e]*x) - 2*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]) - (2*c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x +
 Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I
*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e] + (I*(ArcCosh[c*x]*(-ArcCosh[c*
x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])
] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))
+ 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
- e]))]))/Sqrt[d] + (I*(ArcCosh[c*x]*(ArcCosh[c*x] - 2*(Log[1 + (Sqrt[e]*E^
ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^A
rcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) - 2*PolyLog[2, (Sqrt[e]*
E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] - 2*PolyLog[2, (Sqrt[e]
*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/Sqrt[d]))/(8*e^(3/2)
)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2 \operatorname{arcosh}(cx) + ax^2}{e^2x^4 + 2\,dex^2 + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)x^2}{(ex^2 + d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^2, x)
```

maple [C] time = 2.92, size = 1689, normalized size = 2.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] -1/2*c^2*a/e*x/(c^2*e*x^2+c^2*d)+1/2*a/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2)
)-1/2*c^2*b*arccosh(c*x)*x/e/(c^2*e*x^2+c^2*d)-c^5*b*(-(2*c^2*d-2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/
((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)-c^3*b*(-(2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arctanh((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d
+e)*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e
)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*d-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))
^(1/2)+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+
```

```
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(
1/2))*d/e^4+c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/e^4*(c^2*d*(c^2*d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3-c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)+c^3*b*((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*
(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arct
an((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+
e)*e)^(1/2))/e^3/(c^2*d+e)*d+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^4-c*b*((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4*(c^2*d*(c^
2*d+e))^(1/2)+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arcta
n((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e
)*e)^(1/2))/e^3+1/4*c*b/e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/4*c*b/e*sum(1/_R1/
(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/
_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4
*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{x}{e^2x^2+de}-\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e}\right)+b\int\frac{x^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(x/(e^2*x^2 + d*e) - arctan(e*x/sqrt(d*e))/(sqrt(d*e)*e)) + b*integr
ate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2),
 x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2,x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2\left(a+b\,\mathrm{acosh}\,(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral(x**2*(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.504 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=804

$$-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(c}}{\sqrt{-d}\,c+\sqrt{-dc^2}}\right.}{4(-}$$

```
[Out] -1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b*arccosh(c*x))*ln(1
+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))
/(-d)^(3/2)/e^(1/2)-1/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(a+b
*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)-
1/4*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c
^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(1/2)-1/4*
b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d
-e)^(1/2)))/(-d)^(3/2)/e^(1/2)+1/4*(-a-b*arccosh(c*x))/d/e^(1/2)/((-d)^(1/2
)-x*e^(1/2))+1/4*(a+b*arccosh(c*x))/d/e^(1/2)/((-d)^(1/2)+x*e^(1/2))+1/2*b*
c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^
(1/2)+e^(1/2))^(1/2))/d/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+
e^(1/2))^(1/2)-1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(
c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))
^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 804, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}+\frac{\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{3/2}\sqrt{e}}-\frac{\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(c}}{\sqrt{-d}\,c+\sqrt{-dc^2}}\right.}{4(-}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] -(a + b*ArcCosh[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCosh
[c*x])/(4*d*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d]
 - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(2
*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) - (b*c*Ar
cTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]
]*Sqrt[-1 + c*x])]])/(2*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[
e]]*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcCosh[c*x
])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(4*
(-d)^(3/2)*Sqrt[e]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt[e]) + ((a + b*ArcC
osh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]
)]])/(4*(-d)^(3/2)*Sqrt[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sq
rt[-d] - Sqrt[-(c^2*d) - e]))]])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(4*(-d)^(3/2)*Sqrt
```

```
[e]) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d)
 - e]))])/(4*(-d)^(3/2)*Sqrt[e]) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(3/2)*Sqrt[e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx=\int\left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx$$

$$=-\frac{e\int\frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{4d}-\frac{e\int\frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}\,dx}{4d}-\frac{e\int\frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}\,dx}{2d}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{(bc)\int\frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(\sqrt{-d}\sqrt{e}-ex\right)}\,dx}{4d}-$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}x}\,dx}{4(-d)^{3/2}}+\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}x}}{4(-d)^{3/2}}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{4d\sqrt{e}\left(\sqrt{-d}+\sqrt{e}x\right)}+\frac{bc\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{1+cx}}{\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{-1+cx}}\right)}{2d\sqrt{c\sqrt{-d}-\sqrt{e}}\sqrt{c\sqrt{-d}+\sqrt{e}}\sqrt{e}}$$

Mathematica [C] time = 2.09, size = 734, normalized size = 0.91

$$\frac{1}{2}\left(\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{d^{3/2}\sqrt{e}}+\frac{ax}{d^2+dex^2}+\frac{b\left(i\left(2\text{Li}_2\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-ic\sqrt{d}}\right)+2\text{Li}_2\left(-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{i\sqrt{d}c+\sqrt{-dc^2-e}}\right)+\cosh^{-1}(cx)\left(-\cosh^{-1}(cx)+\cdots\right.\right.\right.}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^2,x]
```

```
[Out] ((a*x)/(d^2 + d*e*x^2) + (a*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*Sqrt[e])
+ (b*(2*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*S
qrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
)/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - 2*
Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] -
I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt
[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + I*(ArcCosh[
c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sq
rt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-
(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] +
Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d]
 + Sqrt[-(c^2*d) - e]))]) - I*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sq
rt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqr
t[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (
Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + 2*PolyLog[2,
(Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])))/(4*d^(3/2)*
Sqrt[e]))/2
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\,\text{arcosh}\,(cx)+a}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{b\,\text{arcosh}\,(cx)+a}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

maple [C] time = 1.93, size = 1695, normalized size = 2.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] 1/2*c^2*a*x/d/(c^2*e*x^2+c^2*d)+1/2*a/d/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))
+1/2*c^2*b*arccosh(c*x)*x/d/(c^2*e*x^2+c^2*d)+1/4*c*b/d*sum(_R1/(_R1^2*e+2*
c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog(
(_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)
*_Z^2+e))+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/e^3/(c^2*d+e)*d+c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*(-(2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^2+
1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))
/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3-c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e^3*(c^2*d*(c^2*d+e))^(1/2)-1/2*
c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e
^2+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e
^3/(c^2*d+e)*d-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan
((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)
*e)^(1/2))/e^3/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2-1/2*c*b*((2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c
^2*d*(c^2*d+e))^(1/2)-c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
1/2)+e)*e)^(1/2))/e^3+c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*a
rctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/
2)+e)*e)^(1/2))/d/e^3*(c^2*d*(c^2*d+e))^(1/2)-1/2*c*b*((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/e^2-1/4*c*b/d*sum(1/_R1/(_R1^
2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+
dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*
d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\,a\left(\frac{x}{dex^2+d^2}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^4+2\,dex^2+d^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*a*(x/(d*e*x^2 + d^2) + arctan(e*x/sqrt(d*e))/(sqrt(d*e)*d)) + b*integra
te(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{\left(d + ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.505 $\int \frac{a+b\cosh^{-1}(cx)}{x^2(d+ex^2)^2}\,dx$

Optimal. Leaf size=846

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(\right.}{}$$

```
[Out] (-a-b*arccosh(c*x))/d^2/x+b*c*arctan((c*x-1)^(1/2)*(c*x+1)^(1/2))/d^2-3/4*(
a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arccosh(c*x))*ln(1+(c*x+
(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/
2)/(-d)^(5/2)-3/4*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*(a+b*arcco
sh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2
*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*
polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e
)^(1/2)))*e^(1/2)/(-d)^(5/2)+3/4*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))*e^(1/2)/(-d)^(5/2)-3/4*b*poly
log(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1
/2)))*e^(1/2)/(-d)^(5/2)+1/4*(a+b*arccosh(c*x))*e^(1/2)/d^2/((-d)^(1/2)-x*e
^(1/2))-1/4*(a+b*arccosh(c*x))*e^(1/2)/d^2/((-d)^(1/2)+x*e^(1/2))-1/2*b*c*a
rctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/
2)+e^(1/2))^(1/2))*e^(1/2)/d^2/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e
^(1/2))^(1/2)+1/2*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c
*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))*e^(1/2)/d^2/(c*(-d)^(1/2)-e^(1/2)
)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
```

Rubi [A] time = 2.02, antiderivative size = 846, normalized size of antiderivative = 1.00, number of steps used = 49, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.619, Rules used = {5792, 5662, 92, 205, 5707, 5802, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{3\sqrt{e}\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}+\frac{3\sqrt{e}\log\left(\frac{e^{\cosh^{-1}(cx)}\sqrt{e}}{c\sqrt{-d}-\sqrt{-dc^2-e}}+1\right)\left(a+b\cosh^{-1}(cx)\right)}{4(-d)^{5/2}}-\frac{3\sqrt{e}\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -((a + b*ArcCosh[c*x])/(d^2*x)) + (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sq
rt[-d] - Sqrt[e]*x)) - (Sqrt[e]*(a + b*ArcCosh[c*x]))/(4*d^2*(Sqrt[-d] + Sq
rt[e]*x)) + (b*c*ArcTan[Sqrt[-1 + c*x]*Sqrt[1 + c*x]])/d^2 - (b*c*Sqrt[e]*A
rcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e
]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sq
rt[e]]) + (b*c*Sqrt[e]*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(
Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(2*d^2*Sqrt[c*Sqrt[-d] - Sqrt[
e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]) - (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2))
+ (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] - Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) - (3*Sqrt[e]*(a + b*ArcCosh[c*x]
)*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(
-d)^(5/2)) + (3*Sqrt[e]*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
```

```
])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(4*(-d
)^(5/2)) - (3*b*Sqrt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - S
qrt[-(c^2*d) - e])])/(4*(-d)^(5/2)) + (3*b*Sqrt[e]*PolyLog[2, -((Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(4*(-d)^(5/2)) - (3*b*Sq
rt[e]*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
])/(4*(-d)^(5/2))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^2\left(d+ex^2\right)^2}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^2x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{d^2} - \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{(bc)\int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}}\,dx}{d^2} - \frac{e\int\left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(c\right.}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right.}{d^2}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\left(bc^2\right)\mathrm{Subst}\left(\int \frac{1}{c+cx^2}\,dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right)}{d^2} + \frac{e\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}}{2(-d)^{5/2}}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{d^2x} + \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{\sqrt{e}\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(\sqrt{-d}+\sqrt{e}\,x\right)} + \frac{bc\tan^{-1}\left(\sqrt{-1}\right.}{d}$$

Mathematica [C] time = 2.85, size = 821, normalized size = 0.97

$$-12\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)a - \frac{8\sqrt{d}\,a}{x} - \frac{4\sqrt{d}\,exa}{ex^2+d} + b\left(8\sqrt{d}\left(\frac{c\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)}{\sqrt{cx-1}\sqrt{cx+1}} - \frac{\cosh^{-1}(cx)}{x}\right) - 2\sqrt{d}\sqrt{e}\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}\,x-i\sqrt{d}} + \frac{c\log\left(\right.}{}\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^2*(d + e*x^2)^2),x]
```

```
[Out] ((-8*a*Sqrt[d])/x - (4*a*Sqrt[d]*e*x)/(d + e*x^2) - 12*a*Sqrt[e]*ArcTan[(Sq
rt[e]*x)/Sqrt[d]] + b*(8*Sqrt[d]*(-(ArcCosh[c*x]/x) + (c*Sqrt[-1 + c^2*x^2]
*ArcTan[Sqrt[-1 + c^2*x^2]])/(Sqrt[-1 + c*x]*Sqrt[1 + c*x])) - 2*Sqrt[d]*Sq
rt[e]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c
^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[
-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) + 2*Sqrt[d]*Sq
rt[e]*(-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*
c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-
(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqrt[-(c^2*d) - e]) - (3*I)*Sqrt[e]
*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sq
rt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d
] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*
Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I
*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))]) + (3*I)*Sqrt[e]*(ArcCosh[c*x]*(-ArcCosh
[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e
])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d)
- e])] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d)
 - e])])))/(8*d^(5/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcosh}\left(cx\right) + a}{e^{2} x^{6} + 2\, d e x^{4} + d^{2} x^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^2*x^6 + 2*d*e*x^4 + d^2*x^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}\left(cx\right) + a}{{\left(e x^{2} + d\right)}^{2} x^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^2*x^2), x)
```

maple [C] time = 6.42, size = 1821, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x)
```

```
[Out] -a/d^2/x-1/2*a/d^2*e*c^2*x/(c^2*e*x^2+c^2*d)-3/2*a/d^2*e/(d*e)^(1/2)*arctan
(x*e/(d*e)^(1/2))-3/2*b*c^2*x*arccosh(c*x)/d^2/(c^2*e*x^2+c^2*d)*e-b*c^2/x*
arccosh(c*x)/d/(c^2*e*x^2+c^2*d)-b*c^5*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)/e^2-b*c^3*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+
e))^(1/2)-b*c^3*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e
)^(1/2))/d/(c^2*d+e)/e-1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
```

```
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/d^2/(c^2*d+e)/e*(c^2*d*(c^2*d+e))^(1/2)+b*c^3*(-(2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d/e^2+c*b*(-
(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(
c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d^2/e^2*(
c^2*d*(c^2*d+e))^(1/2)+1/2*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/d^2/e-b*c^5*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)/e^2+b*c^3*((2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)/e^2*(c^2*d*(c^2*d+e))^(1/2)
-b*c^3*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(
c^2*d+e)/e+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e
)^(1/2))/d^2/(c^2*d+e)/e*(c^2*d*(c^2*d+e))^(1/2)+b*c^3*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/e^2-c*b*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/e^2*(c^2*d*(c^2*d+e))^(1/2
)+1/2*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/
d^2/e+3/16*b/c/d^3*e*sum((_R1^2*e+4*c^2*d+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arcco
sh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1
)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+2*c*b
/d^2*arctan(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))-3/16*b/c/d^3*e*sum((4*_R1^2*c^
2*d+_R1^2*e+e)/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_
R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}\,a\left(\frac{3\,ex^2+2\,d}{d^2ex^3+d^3x}+\frac{3\,e\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^2x^6+2\,dex^4+d^2x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*((3*e*x^2 + 2*d)/(d^2*e*x^3 + d^3*x) + 3*e*arctan(e*x/sqrt(d*e))/(sq
rt(d*e)*d^2)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^2*x^6
 + 2*d*e*x^4 + d^2*x^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^2\,{\left(e\,x^2+d\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^2*(d + e*x^2)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\,\mathrm{acosh}\,(cx)}{x^2\left(d+ex^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**2/(e*x**2+d)**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))/(x**2*(d + e*x**2)**2), x)
```

3.506 $\int \frac{x^5(a+b\cosh^{-1}(cx))}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=737

$$\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2e^3}+\frac{\left(a+b\cosh^{-1}(cx)\right)\log}{2}$$

```
[Out] -1/4*d^2*(a+b*arccosh(c*x))/e^3/(e*x^2+d)^2+d*(a+b*arccosh(c*x))/e^3/(e*x^2
+d)-1/2*(a+b*arccosh(c*x))^2/b/e^3+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b
*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
-(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+1/2*(a+b*arccosh(c*
x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)
^(1/2)))/e^3+1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^3+1/2*b*polylog(2,-(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^3+
1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c
^2*d-e)^(1/2)))/e^3+1/8*b*c*d*x*(-c^2*x^2+1)/e^2/(c^2*d+e)/(e*x^2+d)/(c*x-1
)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)
/(c^2*x^2-1)^(1/2))*d^(1/2)*(c^2*x^2-1)^(1/2)/e^3/(c^2*d+e)^(3/2)/(c*x-1)^(
1/2)/(c*x+1)^(1/2)-b*c*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))
*d^(1/2)*(c^2*x^2-1)^(1/2)/e^3/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.18, antiderivative size = 737, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5792, 5788, 519, 382, 377, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2e^3}+\frac{b\text{PolyLog}\left(2,\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*d*x*(1 - c^2*x^2))/(8*e^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
 + e*x^2)) - (d^2*(a + b*ArcCosh[c*x]))/(4*e^3*(d + e*x^2)^2) + (d*(a + b*Ar
cCosh[c*x]))/(e^3*(d + e*x^2)) - (a + b*ArcCosh[c*x])^2/(2*b*e^3) - (b*c*Sq
rt[d]*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2
*x^2])])/(e^3*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*Sqrt[d]*
(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[
-1 + c^2*x^2])])/(8*e^3*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (
(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(
c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + ((a + b*ArcCosh[c*x])*Lo
g[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
+ ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt
[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c
*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*e^3) + (b*PolyLog[2, -((Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*e^3) + (b*PolyLog[
2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*e^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
```

```
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{x^5\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx=\int\left(\frac{d^2x\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3}-\frac{2dx\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)}\right)dx$$

$$=\frac{\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{e^2}-\frac{(2d)\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{e^2}+\frac{d^2\int\frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$=-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}-\frac{(bcd)\int\frac{1}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)}\,dx}{e^3}+$$

$$=-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e^3\left(d+ex^2\right)}-\frac{\int\frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2e^{5/2}}+\frac{\int\frac{a+b\cosh^{-1}(c}{\sqrt{-d}+\sqrt{e}\,x}}{2e^{5/2}}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh}{e^3\left(d+e\right.}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh}{e^3\left(d+e\right.}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh}{e^3\left(d+e\right.}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh}{e^3\left(d+e\right.}$$

$$=\frac{bcdx\left(1-c^2x^2\right)}{8e^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)}-\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{4e^3\left(d+ex^2\right)^2}+\frac{d\left(a+b\cosh}{e^3\left(d+e\right.}$$

Mathematica [C] time = 7.26, size = 1155, normalized size = 1.57

$$-\frac{ad^2}{4e^3\left(ex^2+d\right)^2}+\frac{ad}{e^3\left(ex^2+d\right)}+\frac{a\log\left(ex^2+d\right)}{2e^3}+b\left(-\frac{7i\sqrt{d}\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16e^3}-\cdots\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^5*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*d^2)/(e^3*(d + e*x^2)^2) + (a*d)/(e^3*(d + e*x^2)) + (a*Log[d + e*x
^2])/(2*e^3) + b*((((-7*I)/16)*Sqrt[d]*(ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt[e
]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])/S
qrt[-(c^2*d) - e]))/e^3 - (((7*I)/16)*Sqrt[d]*(-(ArcCosh[c*x]/(I*Sqrt[d] +
Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x
))])/Sqrt[-(c^2*d) - e]))/e^3 - (d*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*
d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqrt[d] +
Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e]
 - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d +
 I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(16*e^(5/2)) - (d*(
(c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[e]*x)) - Ar
cCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(Log[4] + Log
[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1
 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d
+ e)^(3/2))))/(16*e^(5/2)) + (ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqr
t[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])] + Log[1 + (Sqrt[e]
*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, (Sqrt
[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + 2*PolyLog[2, -
((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))])/(4*e^3) + (
ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*S
qrt[d] + Sqrt[-(c^2*d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[
d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)
*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/
(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])/(4*e^3))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^5\operatorname{arcosh}(cx)+ax^5}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^5*arccosh(c*x) + a*x^5)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [C] time = 1.56, size = 5196, normalized size = 7.05

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a\left(\frac{4\,dex^2+3\,d^2}{e^5x^4+2\,de^4x^2+d^2e^3}+\frac{2\,\log\left(ex^2+d\right)}{e^3}\right)+b\int\frac{x^5\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((4*d*e*x^2 + 3*d^2)/(e^5*x^4 + 2*d*e^4*x^2 + d^2*e^3) + 2*log(e*x^2
+ d)/e^3) + b*integrate(x^5*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e^3*x^6
 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^5\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^5*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.507 $\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=231

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{1-c^2x^2}\,\sin^{-1}(cx)}{4de^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc\sqrt{1-c^2x^2}\,\left(2c^2d+3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{1-c^2x^2}}\right)}{8\sqrt{d}\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)^{3/2}}-\frac{bcx\,(1}{8e\sqrt{cx-1}\,\sqrt{cx+}}$$

```
[Out] 1/4*x^4*(a+b*arccosh(c*x))/d/(e*x^2+d)^2-1/8*b*c*x*(-c^2*x^2+1)/e/(c^2*d+e)
/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/4*b*arcsin(c*x)*(-c^2*x^2+1)^(1/2)
/d/e^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+3*e)*arctan(x*(c^2*d+e)
^(1/2)/d^(1/2)/(-c^2*x^2+1)^(1/2))*(-c^2*x^2+1)^(1/2)/e^2/(c^2*d+e)^(3/2)/d
^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 9, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {264, 5790, 12, 519, 470, 523, 217, 206, 377, 208}

$$\frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2}-\frac{b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{4de^2\sqrt{cx-1}\,\sqrt{cx+1}}+\frac{bc\sqrt{c^2x^2-1}\,\left(2c^2d+3e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{8\sqrt{d}\,e^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2d+e\right)^{3/2}}-\frac{}{8e\sqrt{cx-}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*x*(1 - c^2*x^2))/(8*e*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e
*x^2)) + (x^4*(a + b*ArcCosh[c*x]))/(4*d*(d + e*x^2)^2) - (b*Sqrt[-1 + c^2*
x^2]*ArcTanh[(c*x)/Sqrt[-1 + c^2*x^2]])/(4*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*
x]) + (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(
Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*Sqrt[d]*e^2*(c^2*d + e)^(3/2)*Sqrt[-1 + c*
x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 264

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int \frac{x^3\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - (bc)\int \frac{x^4}{4d\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^2}\,dx$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{(bc)\int \frac{x^4}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x^4}{\sqrt{-1+c^2x^2}\,\left(d+ex^2\right)^2}\,dx}{4d\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+\right.}{8de\left(c^2\right.}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+\right.}{4de^2\sqrt{}}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{\left(bc\sqrt{-1+\right.}{}$$

$$= -\frac{bcx\left(1-c^2x^2\right)}{8e\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{4d\left(d+ex^2\right)^2} - \frac{b\sqrt{-1+c}}{4de^2}$$

Mathematica [A] time = 0.84, size = 192, normalized size = 0.83

$$\frac{\frac{\frac{bcex\sqrt{cx-1}\,\sqrt{cx+1}\left(d+ex^2\right)}{c^2d+e}-2a\left(d+2ex^2\right)}{\left(d+ex^2\right)^2} - \frac{bc\sqrt{cx-1}\,\sqrt{cx+1}\left(2c^2d+3e\right)\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\,\sqrt{c^2x^2-1}}\right)}{\sqrt{d}\,\sqrt{c^2x^2-1}\left(c^2(-d)-e\right)^{3/2}} - \frac{2b\cosh^{-1}(cx)\left(d+2ex^2\right)}{\left(d+ex^2\right)^2}}{8e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (((b*c*e*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(c^2*d + e) - 2*a*(d +
 2*e*x^2))/(d + e*x^2)^2 - (2*b*(d + 2*e*x^2)*ArcCosh[c*x])/(d + e*x^2)^2 -
 (b*c*(2*c^2*d + 3*e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) -
e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(Sqrt[d]*(-(c^2*d) - e)^(3/2)*Sqrt[-1
+ c^2*x^2]))/(8*e^2)
```

fricas [B] time = 0.84, size = 1217, normalized size = 5.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(2*a - b)*c^4*d^4 + 2*(4*a - b)*c^2*d^3*e - 4*(b*c^4*d^2*e^2 + 2*
b*c^2*d*e^3 + b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 4*a*d^2*e^2 - 2*(b*
c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*d^2
*e^2 + 2*a*d*e^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c
*e^3)*x^4 + 2*(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*log(-(
```

```
2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d*e + e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)
*((2*c^3*d + c*e)*x^2 - c*d) - 2*sqrt(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*
c^2*d + e)*x + 2*(c^3*d^2 + c*d*e)*x))/(e*x^2 + d)) - 4*(b*c^4*d^4 + 2*b*c^
2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^
4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - 2
*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^
2*e^2)*x))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^
2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^2), -1/8*(
(2*a - b)*c^4*d^4 + (4*a - b)*c^2*d^3*e - 2*(b*c^4*d^2*e^2 + 2*b*c^2*d*e^3
+ b*e^4)*x^4*log(c*x + sqrt(c^2*x^2 - 1)) + 2*a*d^2*e^2 - (b*c^4*d^2*e^2 +
b*c^2*d*e^3)*x^4 + 2*((2*a - b)*c^4*d^3*e + (4*a - b)*c^2*d^2*e^2 + 2*a*d*e
^3)*x^2 - (2*b*c^3*d^3 + 3*b*c*d^2*e + (2*b*c^3*d*e^2 + 3*b*c*e^3)*x^4 + 2*
(2*b*c^3*d^2*e + 3*b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*arctan((sqrt(-c^2*d
^2 - d*e)*sqrt(c^2*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 + c*d))/(c^
2*d^2 + d*e)) - 2*(b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 +
 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x
^2)*log(-c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c
*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^5*e^2 + 2*c^2*d^4*e^3
+ d^3*e^4 + (c^4*d^3*e^4 + 2*c^2*d^2*e^5 + d*e^6)*x^4 + 2*(c^4*d^4*e^3 + 2*
c^2*d^3*e^4 + d^2*e^5)*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [B] time = 0.04, size = 2499, normalized size = 10.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/2*c^2*a/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*a/e^2*d/(c^2*e*x^2+c^2*d)^2-1/2*c^
2*b*arccosh(c*x)/e^2/(c^2*e*x^2+c^2*d)+1/4*c^4*b*arccosh(c*x)/e^2*d/(c^2*e*
x^2+c^2*d)^2-1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1
/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^
2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^
(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2
)))*x^2*d^2-1/8*c^8*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2)
)/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d
*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/
2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))
*d^3+1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c
^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(
1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*
(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^
2*d^2+1/8*c^8*b*e*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^
2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1
/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(
-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d^3
+1/8*c^5*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(c*x*e+
(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x*d-5/16*c^
```

```
6*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/
2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-
c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e
)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2*d-5/16*c^6*
b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)
/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^
2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^
(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d^2+5/16*c^6*b*e^
3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(
c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*
e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1
/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2*d+5/16*c^6*b*e^
2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(
c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*
e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1
/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d^2+1/8*c^3*b*e^3*(
c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)^(1/2)
)/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x-3/16*c^4*b*e^4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^
(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^
2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2-3/16*c^4*b*e^3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2
)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c
^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(
1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d+3/16*c^4*b*e^4*(c*x+1)^(1/2)*(c*x-1
)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x
*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2
-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)
*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2+3/16*c^4*b*e^3*(c*x+1)^(1/2)*(c*x-1)^
(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e
+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1
)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c
*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}b\left(\frac{\left(c^4d+2c^2e\right)\log\left(ex^2+d\right)}{c^4d^2e^2+2c^2de^3+e^4}+\frac{c^4d^3+c^2d^2e+\left(c^4d^2e+c^2de^2\right)x^2+2\left(c^4d^3+2c^2d^2e+de^2+2\left(c^4d^2e+2c\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b*((c^4*d + 2*c^2*e)*log(e*x^2 + d)/(c^4*d^2*e^2 + 2*c^2*d*e^3 + e^4)
+ (c^4*d^3 + c^2*d^2*e + (c^4*d^2*e + c^2*d*e^2)*x^2 + 2*(c^4*d^3 + 2*c^2*d
^2*e + d*e^2 + 2*(c^4*d^2*e + 2*c^2*d*e^2 + e^3)*x^2)*log(c*x + sqrt(c*x +
1)*sqrt(c*x - 1)) - (c^4*d^3 + 2*c^2*d^2*e + (c^4*d*e^2 + 2*c^2*e^3)*x^4 +
2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x + 1) - (c^4*d^3 + 2*c^2*d^2*e + (c
^4*d*e^2 + 2*c^2*e^3)*x^4 + 2*(c^4*d^2*e + 2*c^2*d*e^2)*x^2)*log(c*x - 1))/
(c^4*d^4*e^2 + 2*c^2*d^3*e^3 + d^2*e^4 + (c^4*d^2*e^4 + 2*c^2*d*e^5 + e^6)*
x^4 + 2*(c^4*d^3*e^3 + 2*c^2*d^2*e^4 + d*e^5)*x^2) + 8*integrate(1/4*(2*c*e
*x^2 + c*d)/(c^3*e^4*x^7 - c*d^2*e^2*x + (2*c^3*d*e^3 - c*e^4)*x^5 + (c^3*d
^2*e^2 - 2*c*d*e^3)*x^3 + (c^2*e^4*x^6 + (2*c^2*d*e^3 - e^4)*x^4 - d^2*e^2
+ (c^2*d^2*e^2 - 2*d*e^3)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x)
) - 1/4*(2*e*x^2 + d)*a/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(a+b\,\mathrm{acosh}\,(c\,x))}{\left(e\,x^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^3*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.508 $\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=177

$$-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{3/2}e\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}+\frac{bcx\left(1-c^2x^2\right)}{8d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\left(d+ex^2\right)}$$

```
[Out] 1/4*(-a-b*arccosh(c*x))/e/(e*x^2+d)^2+1/8*b*c*x*(-c^2*x^2+1)/d/(c^2*d+e)/(e
*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/8*b*c*(2*c^2*d+e)*arctanh(x*(c^2*d+e)
^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(3/2)/e/(c^2*d+e)^(3/
2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {5788, 519, 382, 377, 208}

$$-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\sqrt{c^2x^2-1}\left(2c^2d+e\right)\tanh^{-1}\left(\frac{x\sqrt{c^2d+e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{8d^{3/2}e\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^{3/2}}+\frac{bcx\left(1-c^2x^2\right)}{8d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)\left(d+ex^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (b*c*x*(1 - c^2*x^2))/(8*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*
x^2)) - (a + b*ArcCosh[c*x])/(4*e*(d + e*x^2)^2) + (b*c*(2*c^2*d + e)*Sqrt[
-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8
*d^(3/2)*e*(c^2*d + e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
```

```
[n, 2] && IGtQ[q, 0])
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{(bc)\int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)^2}\,dx}{4e}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{1}{\sqrt{-1+c^2x^2}\left(d+ex^2\right)^2}\,dx}{4e\sqrt{-1+cx}\sqrt{1+cx}}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc\left(2c^2d+e\right)\sqrt{-1\ldots}\right.}{8de\left(c^2d+e\ldots\right.}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{\left(bc\left(2c^2d+e\right)\sqrt{-1\ldots}\right.}{8de\ldots}$$

$$= \frac{bcx\left(1-c^2x^2\right)}{8d\left(c^2d+e\right)\sqrt{-1+cx}\sqrt{1+cx}\left(d+ex^2\right)}-\frac{a+b\cosh^{-1}(cx)}{4e\left(d+ex^2\right)^2}+\frac{bc\left(2c^2d+e\right)\sqrt{-1+\ldots}}{8d^{3/2}e\left(c^2d+e\ldots\right.}$$

Mathematica [A] time = 0.94, size = 183, normalized size = 1.03

$$\frac{1}{8}\left(-\frac{\frac{2a}{e}+\frac{bcx\sqrt{cx-1}\sqrt{cx+1}\left(d+ex^2\right)}{d\left(c^2d+e\right)}}{\left(d+ex^2\right)^2}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}\left(2c^2d+e\right)\tan^{-1}\left(\frac{x\sqrt{c^2(-d)-e}}{\sqrt{d}\sqrt{c^2x^2-1}}\right)}{d^{3/2}e\sqrt{c^2x^2-1}\left(c^2(-d)-e\right)^{3/2}}-\frac{2b\cosh^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (-(((2*a)/e + (b*c*x*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d +
e)))/(d + e*x^2)^2) - (2*b*ArcCosh[c*x])/(e*(d + e*x^2)^2) - (b*c*(2*c^2*d
+ e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*ArcTan[(Sqrt[-(c^2*d) - e]*x)/(Sqrt[d]*Sq
rt[-1 + c^2*x^2])])/(d^(3/2)*(-(c^2*d) - e)^(3/2)*e*Sqrt[-1 + c^2*x^2]))/8
```

fricas [B] time = 1.24, size = 1233, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(2*a + b)*c^4*d^4 + 2*(4*a + b)*c^2*d^3*e + 4*a*d^2*e^2 + 2*(b*c^
4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 4*(b*c^4*d^3*e + b*c^2*d^2*e^2)*x^2 - (2*b*c
```

```
^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e + b*c
*d*e^2)*x^2)*sqrt(c^2*d^2 + d*e)*log(-(2*c^2*d^2 - (4*c^4*d^2 + 4*c^2*d*e +
 e^2)*x^2 + d*e - 2*sqrt(c^2*d^2 + d*e)*((2*c^3*d + c*e)*x^2 - c*d) - 2*sqr
t(c^2*x^2 - 1)*(sqrt(c^2*d^2 + d*e)*(2*c^2*d + e)*x + 2*(c^3*d^2 + c*d*e)*x
))/(e*x^2 + d)) - 4*((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4
*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1)) - 4*(
b*c^4*d^4 + 2*b*c^2*d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*
e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt
(c^2*x^2 - 1)) + 2*sqrt(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*
c^3*d^3*e + b*c*d^2*e^2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^
4*e^3 + 2*c^2*d^3*e^4 + d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3
*e^4)*x^2), -1/8*((2*a + b)*c^4*d^4 + (4*a + b)*c^2*d^3*e + 2*a*d^2*e^2 + (
b*c^4*d^2*e^2 + b*c^2*d*e^3)*x^4 + 2*(b*c^4*d^3*e + b*c^2*d^2*e^2)*x^2 - (2
*b*c^3*d^3 + b*c*d^2*e + (2*b*c^3*d*e^2 + b*c*e^3)*x^4 + 2*(2*b*c^3*d^2*e +
 b*c*d*e^2)*x^2)*sqrt(-c^2*d^2 - d*e)*arctan((sqrt(-c^2*d^2 - d*e)*sqrt(c^2
*x^2 - 1)*e*x - sqrt(-c^2*d^2 - d*e)*(c*e*x^2 + c*d))/(c^2*d^2 + d*e)) - 2*
((b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*d^3*e + 2*b*c^2*d^2
*e^2 + b*d*e^3)*x^2)*log(c*x + sqrt(c^2*x^2 - 1)) - 2*(b*c^4*d^4 + 2*b*c^2*
d^3*e + b*d^2*e^2 + (b*c^4*d^2*e^2 + 2*b*c^2*d*e^3 + b*e^4)*x^4 + 2*(b*c^4*
d^3*e + 2*b*c^2*d^2*e^2 + b*d*e^3)*x^2)*log(-c*x + sqrt(c^2*x^2 - 1)) + sqr
t(c^2*x^2 - 1)*((b*c^3*d^2*e^2 + b*c*d*e^3)*x^3 + (b*c^3*d^3*e + b*c*d^2*e^
2)*x))/(c^4*d^6*e + 2*c^2*d^5*e^2 + d^4*e^3 + (c^4*d^4*e^3 + 2*c^2*d^3*e^4
+ d^2*e^5)*x^4 + 2*(c^4*d^5*e^2 + 2*c^2*d^4*e^3 + d^3*e^4)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\operatorname{arcosh}(cx) + a)x}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x/(e*x^2 + d)^3, x)
```

maple [B] time = 0.03, size = 2443, normalized size = 13.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -1/4*c^4*a/e/(c^2*e*x^2+c^2*d)^2-1/4*c^4*b/e/(c^2*e*x^2+c^2*d)^2*arccosh(c*
x)+1/8*c^8*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2
*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/
2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-
(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2*
d+1/8*c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*
d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2
))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(
c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d^2-1
/8*c^8*b*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e
)^(1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^
2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d
+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2*d-1/8*
c^8*b*e^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(
1/2)/(-(c^2*d+e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/(
(-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)
/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d^2-1/8*c^5*b
*e^3*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(c*x*e+(-c^2*d*e)
```

```
^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2*x+3/16*c^6*b*e^4*(c*x
+1)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+
e)/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/
2)+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+
(-c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2+3/16*c^6*b*e^3*(c*x+1
)^(1/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)
/e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)
+e)^2/(c^2*x^2-1)^(1/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-
c^2*d*e)^(1/2)*c*x+e)/(c*x*e+(-c^2*d*e)^(1/2)))*d-3/16*c^6*b*e^4*(c*x+1)^(1
/2)*(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^
(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^
2/(c^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*
e)^(1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*x^2-3/16*c^6*b*e^3*(c*x+1)^(1/2)*
(c*x-1)^(1/2)/(c*x*e-(-c^2*d*e)^(1/2))/(-c^2*d*e)^(1/2)/(-(c^2*d+e)/e)^(1/2
)/(c*x*e+(-c^2*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c
^2*x^2-1)^(1/2)*ln(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(
1/2)*c*x-e)/(c*x*e-(-c^2*d*e)^(1/2)))*d-1/8*c^3*b*e^4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(c*x*e-(-c^2*d*e)^(1/2))/d/(e-(-c^2*d*e)^(1
/2))^2/((-c^2*d*e)^(1/2)+e)^2*x+1/16*c^4*b*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)/
(-c^2*d*e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2
*d*e)^(1/2))/d/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1
/2)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e
)/(c*x*e+(-c^2*d*e)^(1/2)))*x^2+1/16*c^4*b*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/
(-c^2*d*e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2
*d*e)^(1/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2
)*ln(-2*(-(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x+e)/
(c*x*e+(-c^2*d*e)^(1/2)))-1/16*c^4*b*e^5*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(-c^2*
d*e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^
(1/2))/d/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln
(2*((c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e
-(-c^2*d*e)^(1/2)))*x^2-1/16*c^4*b*e^4*(c*x+1)^(1/2)*(c*x-1)^(1/2)/(-c^2*d*
e)^(1/2)/(c*x*e+(-c^2*d*e)^(1/2))/(-(c^2*d+e)/e)^(1/2)/(c*x*e-(-c^2*d*e)^(1
/2))/(e-(-c^2*d*e)^(1/2))^2/((-c^2*d*e)^(1/2)+e)^2/(c^2*x^2-1)^(1/2)*ln(2*(
(c^2*x^2-1)^(1/2)*(-(c^2*d+e)/e)^(1/2)*e+(-c^2*d*e)^(1/2)*c*x-e)/(c*x*e-(-c
^2*d*e)^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\left(\frac{c^4\log\left(ex^2+d\right)}{c^4d^2e+2\,c^2de^2+e^3}+8\,c\int\frac{1}{4\left(c^3e^3x^7+\left(2\,c^3de^2-ce^3\right)x^5-cd^2ex+\left(c^3d^2e-2\,cde^2\right)x^3+\left(c^2e^3x^6+\left(2\,c^2de^2\right.\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(c^4*log(e*x^2 + d)/(c^4*d^2*e + 2*c^2*d*e^2 + e^3) + 8*c*integrate(1/
4/(c^3*e^3*x^7 + (2*c^3*d*e^2 - c*e^3)*x^5 - c*d^2*e*x + (c^3*d^2*e - 2*c*d
*e^2)*x^3 + (c^2*e^3*x^6 + (2*c^2*d*e^2 - e^3)*x^4 - d^2*e + (c^2*d^2*e - 2
*d*e^2)*x^2)*e^(1/2*log(c*x + 1) + 1/2*log(c*x - 1))), x) - (c^4*d^2 + c^2*
d*e + (c^4*d*e + c^2*e^2)*x^2 - 2*(c^4*d^2 + 2*c^2*d*e + e^2)*log(c*x + sqr
t(c*x + 1)*sqrt(c*x - 1)) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*log(c*x
 + 1) + (c^4*e^2*x^4 + 2*c^4*d*e*x^2 + c^4*d^2)*log(c*x - 1))/(c^4*d^4*e +
2*c^2*d^3*e^2 + d^2*e^3 + (c^4*d^2*e^3 + 2*c^2*d*e^4 + e^5)*x^4 + 2*(c^4*d^
3*e^2 + 2*c^2*d^2*e^3 + d*e^4)*x^2))*b - 1/4*a/(e^3*x^4 + 2*d*e^2*x^2 + d^2
*e)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.509 $\int \frac{a+b\cosh^{-1}(cx)}{x(d+ex^2)^3}\,dx$

Optimal. Leaf size=772

$$-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(1-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{\left(a+b\cosh^{-1}(cx)\right)\log\left(\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}+1\right)}{2d^3}-\frac{\left(a+b\cosh^{-1}(cx)\right)\mathrm{lo}}{}$$

```
[Out] 1/4*(a+b*arccosh(c*x))/d/(e*x^2+d)^2+1/2*(a+b*arccosh(c*x))/d^2/(e*x^2+d)+(
a+b*arccosh(c*x))^2/b/d^3+(a+b*arccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))^2)/d^3-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^3-1/2*(a+b*arccosh(c*x))*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/d^3-1/2*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2
)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*(a+b*arccosh(c*x))*ln(1+(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*
b*polylog(2,-1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^3-1/2*b*polylog(2,-(c
*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^
3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(
-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^
(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/2*b*polylog(2,(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^3-1/8*b*c*e*x*
(-c^2*x^2+1)/d^2/(c^2*d+e)/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/8*b*c*(2
*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^
(1/2)/d^(5/2)/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)-1/2*b*c*arctanh(x
*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(5/2)/(c^2*
d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.25, antiderivative size = 755, normalized size of antiderivative = 0.98, number of steps used = 34, number of rules used = 13, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.619$, Rules used = {5792, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$-\frac{b\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\mathrm{PolyLog}\left(2,\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{2d^3}-\frac{b\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}e^{\cosh^{-1}(cx)}}{\sqrt{c^2(-d)-e}+c\sqrt{-d}}\right)}{2d^3}-\frac{b\mathrm{PolyLog}\left(2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
```

```
[Out] -(b*c*e*x*(1 - c^2*x^2))/(8*d^2*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d
 + e*x^2)) + (a + b*ArcCosh[c*x])/(4*d*(d + e*x^2)^2) + (a + b*ArcCosh[c*x]
)/(2*d^2*(d + e*x^2)) - (b*c*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d + e]*x)
/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(2*d^(5/2)*Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*S
qrt[1 + c*x]) - (b*c*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqrt[c^2*d +
 e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*d^(5/2)*(c^2*d + e)^(3/2)*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) -
 ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(2*d^3) - ((a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^3) + ((a + b*ArcCosh[c*x])*
Log[1 + E^(2*ArcCosh[c*x])])/d^3 - (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*PolyLog[2, (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^3) - (b*PolyLog[2, -((
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^3) - (b*P
```

```
olyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d
^3) + (b*PolyLog[2, -E^(2*ArcCosh[c*x])])/(2*d^3)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d\left(d+ex^2\right)^3} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^2} - \frac{ex\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+\right.}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^3} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx}{d^2} - \frac{e\int \frac{x\left(a+b\cosh^{-1}(c\right.}{\left(d+ex^2\right)^3}}{d}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} + \frac{\mathrm{Subst}\left(\int (a+bx)\tanh(x)\,dx, x, \cosh^{-1}(cx)\right)}{d^3}$$

$$= \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)} - \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{2bd^3} + \frac{2\,\mathrm{Subst}\left(\int \frac{e^{2x}(a+bx)}{1+e^{2x}}\,d\right.}{d^3}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

$$= -\frac{bcex\left(1-c^2x^2\right)}{8d^2\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)} + \frac{a+b\cosh^{-1}(cx)}{4d\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{2d^2\left(d+ex^2\right)}$$

Mathematica [F] time = 8.34, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3),x]
[Out] Integrate[(a + b*ArcCosh[c*x])/(x*(d + e*x^2)^3), x]
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\,\mathrm{arcosh}\,(cx)+a}{e^3x^7+3\,de^2x^5+3\,d^2ex^3+d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\,(cx) + a}{\left(ex^2 + d\right)^3 x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x), x)
```

maple [C] time = 0.50, size = 1478, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x)
```

```
[Out] 1/2*a*c^2/d^2/(c^2*e*x^2+c^2*d)+1/4*a*c^4/d/(c^2*e*x^2+c^2*d)^2-1/8*b*c^6/(
c^2*d+e)/(c^2*e*x^2+c^2*d)^2+a/d^3*ln(c*x)-1/2*a/d^3*ln(c^2*e*x^2+c^2*d)+1/
8*b*c^5/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e+3/4
*b*c^6/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)+1/2*b*c^6/d/(c^2*d+e)/(c^
2*e*x^2+c^2*d)^2*arccosh(c*x)*x^2*e+1/2*b*c^4/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*
d)^2*arccosh(c*x)*x^2*e^2+1/8*b*c^5/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-
1)^(1/2)*(c*x+1)^(1/2)*x^3*e^2+5/8*b*(c^2*d*(c^2*d+e))^(1/2)/d^3/(c^2*d+e)^
2*e*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*
d^2+c^2*d*e)^(1/2))+b/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2)))+b/d^3/(c^2*d+e)*e*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))+3/4*b*c^2*(c^2*d*(c^2*d+e))^(1/2)/d^2/(c^2*d+e)^2*arctanh(1
/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^
(1/2))+b*c^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))+b*c^2/d^2/(c^2*d+e)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(
1/2)))-1/4*b*c^2/d^2/(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c
*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1
/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e-1/4*b/d
^3/(c^2*d+e)*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln
((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c
*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+b/d^3/(c^2*d+e)
*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+b/d^3/(c^2*d+e)*e*dilog(1-I
*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/4*b/d^3/(c^2*d+e)*sum((_R1^2+1)/(_R1^
2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+
dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*
d+2*e)*_Z^2+e))*e^2-1/4*b*c^2/d^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*
e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+di
log((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+
2*e)*_Z^2+e))+b*c^2/d^2/(c^2*d+e)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
)))+b*c^2/d^2/(c^2*d+e)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-1/8*b*
c^6/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e^2*x^4-1/4*b*c^6/d/(c^2*d+e)/(c^2*e*
x^2+c^2*d)^2*e*x^2+3/4*b*c^4/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*e
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}\,a\left(\frac{2\,ex^2 + 3\,d}{d^2e^2x^4 + 2\,d^3ex^2 + d^4} - \frac{2\,\log\left(ex^2 + d\right)}{d^3} + \frac{4\,\log(x)}{d^3}\right) + b\int \frac{\log\left(cx + \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{e^3x^7 + 3\,de^2x^5 + 3\,d^2ex^3 + d^3x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/4*a*((2*e*x^2 + 3*d)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) - 2*log(e*x^2 + d)
/d^3 + 4*log(x)/d^3) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(
e^3*x^7 + 3*d*e^2*x^5 + 3*d^2*e*x^3 + d^3*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{x\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x*(d + e*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.510 $\int \frac{a+b\cosh^{-1}(cx)}{x^3(d+ex^2)^3}\,dx$

Optimal. Leaf size=834

$$\frac{bcx(1-c^2x^2)e^2}{8d^3(dc^2+e)\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)}-\frac{3(a+b\cosh^{-1}(cx))^2e}{bd^4}-\frac{(a+b\cosh^{-1}(cx))e}{d^3(ex^2+d)}-\frac{(a+b\cosh^{-1}(cx))e}{4d^2(ex^2+d)^2}+\frac{bc}{\cdots}$$

```
[Out] 1/2*(-a-b*arccosh(c*x))/d^3/x^2-1/4*e*(a+b*arccosh(c*x))/d^2/(e*x^2+d)^2-e*
(a+b*arccosh(c*x))/d^3/(e*x^2+d)-3*e*(a+b*arccosh(c*x))^2/b/d^4-3*e*(a+b*ar
ccosh(c*x))*ln(1+1/(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4+3/2*e*(a+b*arcc
osh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^
2*d-e)^(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x
))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^
(1/2)))/d^4+3/2*e*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,-1/(c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))^2)/d^4+3/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,(
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/d
^4+3/2*b*e*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)+(-c^2*d-e)^(1/2)))/d^4+3/2*b*e*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2
))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/d^4+1/8*b*c*e^2*x*(-c^2*x^2+1)/
d^3/(c^2*d+e)/(e*x^2+d)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+1/2*b*c*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/d^3/x+1/8*b*c*e*(2*c^2*d+e)*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/
(c^2*x^2-1)^(1/2))*(c^2*x^2-1)^(1/2)/d^(7/2)/(c^2*d+e)^(3/2)/(c*x-1)^(1/2)/
(c*x+1)^(1/2)+b*c*e*arctanh(x*(c^2*d+e)^(1/2)/d^(1/2)/(c^2*x^2-1)^(1/2))*(c
^2*x^2-1)^(1/2)/d^(7/2)/(c^2*d+e)^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 1.30, antiderivative size = 815, normalized size of antiderivative = 0.98, number of steps used = 36, number of rules used = 15, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {5792, 5662, 95, 5660, 3718, 2190, 2279, 2391, 5788, 519, 382, 377, 208, 5800, 5562}

$$\frac{bcx(1-c^2x^2)e^2}{8d^3(dc^2+e)\sqrt{cx-1}\sqrt{cx+1}(ex^2+d)}-\frac{(a+b\cosh^{-1}(cx))e}{d^3(ex^2+d)}-\frac{(a+b\cosh^{-1}(cx))e}{4d^2(ex^2+d)^2}+\frac{bc(2dc^2+e)\sqrt{c^2x^2-1}\tan\cdots}{8d^{7/2}(dc^2+e)^{3/2}\sqrt{cx\cdots}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
```

```
[Out] (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(2*d^3*x) + (b*c*e^2*x*(1 - c^2*x^2))/(8
*d^3*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2)) - (a + b*ArcCosh
[c*x])/(2*d^3*x^2) - (e*(a + b*ArcCosh[c*x]))/(4*d^2*(d + e*x^2)^2) - (e*(a
 + b*ArcCosh[c*x]))/(d^3*(d + e*x^2)) + (b*c*e*Sqrt[-1 + c^2*x^2]*ArcTanh[(
Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(d^(7/2)*Sqrt[c^2*d + e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*c*e*(2*c^2*d + e)*Sqrt[-1 + c^2*x^2]*Arc
Tanh[(Sqrt[c^2*d + e]*x)/(Sqrt[d]*Sqrt[-1 + c^2*x^2])])/(8*d^(7/2)*(c^2*d +
 e)^(3/2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (3*e*(a + b*ArcCosh[c*x])*Log[1 -
 (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*
e*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[
-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^Arc
Cosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) + (3*e*(a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(2*d^4) - (3*e*(a + b*ArcCosh[c*x])*Log[1 + E^(2*ArcCosh[c*x])])/d^4 + (3
*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]
```

```
))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqr
t[-(c^2*d) - e])])/(2*d^4) + (3*b*e*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(2*d^4) + (3*b*e*PolyLog[2, (Sqrt[e]*E^
ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*d^4) - (3*b*e*PolyLog[
2, -E^(2*ArcCosh[c*x])])/(2*d^4)
```

Rule 95

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] /; FreeQ[{a, b, c, d, e, f
, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1), 0] && NeQ[m, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3718

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> -Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] + Dist[2*I, Int[((c
 + d*x)^m*E^(2*(-(I*e) + f*fz*x)))/(1 + E^(2*(-(I*e) + f*fz*x))), x], x] /;
 FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5660

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> Subst[Int[
(a + b*x)^n/Coth[x], x], x, ArcCosh[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n,
 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5788

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*(x_)*((d_) + (e_.)*(x_)^2)^(p_.), x
_Symbol] :> Simp[((d + e*x^2)^(p + 1)*(a + b*ArcCosh[c*x]))/(2*e*(p + 1)),
x] - Dist[(b*c)/(2*e*(p + 1)), Int[(d + e*x^2)^(p + 1)/(Sqrt[1 + c*x]*Sqrt[
-1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[c^2*d + e, 0] &&
NeQ[p, -1]
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx = \int \left(\frac{a+b\cosh^{-1}(cx)}{d^3x^3} - \frac{3e\left(a+b\cosh^{-1}(cx)\right)}{d^4x} + \frac{e^2x\left(a+b\cosh^{-1}(cx)\right)}{d^2\left(d+ex^2\right)^3} + \frac{2e^2x\left(a+\right.}{d^3\left(d\right.}\right.$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{x^3}\,dx}{d^3} - \frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{x}\,dx}{d^4} + \frac{\left(3e^2\right)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx}{d^4} + \frac{\left(2e^2\right)\int \frac{x(a}{}}{}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{d^3\left(d+ex^2\right)} + \frac{(bc)\int \frac{1}{x^2\sqrt{-1+cx}\,\sqrt{}}}{2d^3}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} - \frac{a+b\cosh^{-1}(cx)}{2d^3x^2} - \frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d^2\left(d+ex^2\right)^2} - \frac{e\left(a+b\cosh^{-1}(c\right.}{d^3\left(d+ex^2\right)}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(c}{2d^3x^2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(c}{2d^3x^2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(c}{2d^3x^2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(c}{2d^3x^2}$$

$$= \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{2d^3x} + \frac{bce^2x\left(1-c^2x^2\right)}{8d^3\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\left(d+ex^2\right)} - \frac{a+b\cosh^{-1}(c}{2d^3x^2}$$

Mathematica [F] time = 12.21, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{x^3\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3),x]
[Out] Integrate[(a + b*ArcCosh[c*x])/(x^3*(d + e*x^2)^3), x]
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b\operatorname{arcosh}(cx)+a}{e^3x^9+3\,de^2x^7+3\,d^2ex^5+d^3x^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^
3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx) + a}{(ex^2 + d)^3 x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/((e*x^2 + d)^3*x^3), x)
```

maple [C] time = 0.77, size = 1928, normalized size = 2.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x)
```

```
[Out] -1/2*a/d^3/x^2-1/2*c^4*b/x^2/(c^2*e*x^2+c^2*d)^2/d/(c^2*d+e)*arccosh(c*x)*e
-3/2*c^4*b*x^2/(c^2*e*x^2+c^2*d)^2/d^3/(c^2*d+e)*arccosh(c*x)*e^3-3/2*c^6*b
/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^2*e^2+7/8*c^5*b*x/(c^2*e*
x^2+c^2*d)^2/d^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^2+1/2*c^5*b/x/(c^2
*e*x^2+c^2*d)^2/d/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e+3/8*c^5*b*x^3/(c^
2*e*x^2+c^2*d)^2/d^3/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*e^3+1/2*c^7*b/d^
2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x^3*e^2+c^7*b/d
/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*x*e-1/2*c^8*b/(c
^2*d+e)/(c^2*e*x^2+c^2*d)^2-3*a/d^4*e*ln(c*x)+3/2*a*e/d^4*ln(c^2*e*x^2+c^2*
d)-3/4*c^6*b*x^2/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*e^2-1/2*c^8*b/d^2/(c^2*d
+e)/(c^2*e*x^2+c^2*d)^2*e^2*x^4-c^8*b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*e*x^2
-3/8*c^6*b*x^4/(c^2*e*x^2+c^2*d)^2/d^3/(c^2*d+e)*e^3-5/4*c^2*b*(c^2*d*(c^2*
d+e))^(1/2)/d^3/(c^2*d+e)^2*e*arctanh(1/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^(1/2))-3*c^2*b/d^3/(c^2*d+e)*e*arcco
sh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3*c^2*b/d^3/(c^2*d+e)*e*a
rccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+1/2*c^7*b/x/(c^2*e*x^
2+c^2*d)^2/(c^2*d+e)*(c*x-1)^(1/2)*(c*x+1)^(1/2)-9/4*c^6*b/d/(c^2*d+e)/(c^2
*e*x^2+c^2*d)^2*arccosh(c*x)*e-9/4*c^4*b/(c^2*e*x^2+c^2*d)^2/d^2/(c^2*d+e)*
arccosh(c*x)*e^2-c^2*a*e/d^3/(c^2*e*x^2+c^2*d)-1/4*c^4*a*e/d^2/(c^2*e*x^2+c
^2*d)^2-3*b/d^4*e^2/(c^2*d+e)*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+
3/4*b/d^4*e^2/(c^2*d+e)*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccos
h(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)
^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3*b/d^
4*e^2/(c^2*d+e)*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+3/4*b/d^4*e^3/
(c^2*d+e)*sum((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-
1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R
1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/8*c^6*b/(c^2*e*x^2+c^2*d)^2/
d/(c^2*d+e)*e-3*b/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1+I*(c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2)))-9/8*b*(c^2*d*(c^2*d+e))^(1/2)/d^4/(c^2*d+e)^2*e^2*arctanh(1
/4*(2*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))^2*e+4*c^2*d+2*e)/(c^4*d^2+c^2*d*e)^
(1/2))-3*b/d^4*e^2/(c^2*d+e)*arccosh(c*x)*ln(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2)))-1/2*c^6*b/x^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)+3/4*c^2*b
/d^3/(c^2*d+e)*e*sum((_R1^2*e+4*c^2*d+e)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3*c^2*b/d^3/(
c^2*d+e)*e*dilog(1+I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))-3*c^2*b/d^3/(c^2*d+
e)*e*dilog(1-I*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)))+3/4*c^2*b/d^3/(c^2*d+e)*s
um((_R1^2+1)/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c
```

```
*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=Roo
tOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))*e^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{4}a\left(\frac{6e^2x^4+9dex^2+2d^2}{d^3e^2x^6+2d^4ex^4+d^5x^2}-\frac{6e\log\left(ex^2+d\right)}{d^4}+\frac{12e\log(x)}{d^4}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^9+3de^2x^7+3d^2ex^5+d^3x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/x^3/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a*((6*e^2*x^4 + 9*d*e*x^2 + 2*d^2)/(d^3*e^2*x^6 + 2*d^4*e*x^4 + d^5*x^
2) - 6*e*log(e*x^2 + d)/d^4 + 12*e*log(x)/d^4) + b*integrate(log(c*x + sqrt
(c*x + 1)*sqrt(c*x - 1))/(e^3*x^9 + 3*d*e^2*x^7 + 3*d^2*e*x^5 + d^3*x^3), x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{x^3\,{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3),x)
```

```
[Out] int((a + b*acosh(c*x))/(x^3*(d + e*x^2)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/x**3/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.511 $\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1224

$$-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{5/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{5/2}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right.}{8\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+\sqrt{e}\cdots}}$$

```
[Out] 3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(a+b*arccosh(c*x))*ln(
1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2))
)/e^(5/2)/(-d)^(1/2)+3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*(
a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1
/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1
/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2
)-(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2
)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2
)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+
(-c^2*d-e)^(1/2)))/e^(5/2)/(-d)^(1/2)-1/8*b*c^3*d*arctanh((c*x+1)^(1/2)*(c*
(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/e^(5/
2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/8*b*c^3*d*ar
ctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2
)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2
))^(3/2)-1/16*(a+b*arccosh(c*x))*(-d)^(1/2)/e^(5/2)/((-d)^(1/2)-x*e^(1/2))^
2+5/16*(a+b*arccosh(c*x))/e^(5/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh(
c*x))*(-d)^(1/2)/e^(5/2)/((-d)^(1/2)+x*e^(1/2))^2-5/16*(a+b*arccosh(c*x))/e
^(5/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/e^2/(c^2*d+e)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(-d)^(1/2)*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/e^2/(c^2*d+e)/((-d)^(1/2)+x*e^(1/2))-5/8*b*c*arctanh((c*x+1)^(
1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2
))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)+5/8*b*
c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^
(1/2)-e^(1/2))^(1/2))/e^(5/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^
(1/2))^(1/2)
```

Rubi [A] time = 3.96, antiderivative size = 1224, normalized size of antiderivative = 1.00, number of steps used = 80, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5792, 5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{bd\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{5/2}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{5/2}}-\frac{5b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right.}{8\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+\sqrt{e}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d + e)*(Sqrt[-d]
- Sqrt[e]*x)) - (b*c*Sqrt[-d]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*e^2*(c^2*d
+ e)*(Sqrt[-d] + Sqrt[e]*x)) - (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*
(Sqrt[-d] - Sqrt[e]*x)^2) + (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d]
- Sqrt[e]*x)) + (Sqrt[-d]*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqr
```

```
t[e]*x)^2) - (5*(a + b*ArcCosh[c*x]))/(16*e^(5/2)*(Sqrt[-d] + Sqrt[e]*x)) -
 (b*c^3*d*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-
d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d]
 + Sqrt[e])^(3/2)*e^(5/2)) - (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d
] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e^(5/2)) + (b*c^3*d*ArcTanh[(Sqrt[c
*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c
*x])]])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(5/2)
) + (5*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[
-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[
-d] + Sqrt[e]]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCos
h[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*(a +
 b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*
d) - e])]])/(16*Sqrt[-d]*e^(5/2)) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]
*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2))
- (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sq
rt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) + (3
*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])]])/
(16*Sqrt[-d]*e^(5/2)) - (3*b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[
-d] + Sqrt[-(c^2*d) - e]))]])/(16*Sqrt[-d]*e^(5/2)) + (3*b*PolyLog[2, (Sqrt[
e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]])/(16*Sqrt[-d]*e^(5/2)
)
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^4\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^3} - \frac{2d\left(a+b\cosh^{-1}(cx)\right)}{e^2\left(d+ex^2\right)^2} + \frac{a+b\cosh^{-1}(cx)}{e^2\left(d+ex^2\right)}\right) dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{d+ex^2}\,dx}{e^2} - \frac{(2d)\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e^2} + \frac{d^2\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e^2}$$

$$= \frac{\int \left(\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right) dx}{e^2} - \frac{(2d)\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\sqrt{e}-ex\right)^2} - \frac{e\left(a+b\right.}{4d\left(\sqrt{-}\right.}\right.}{e^2}$$

$$= -\frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}\,e^2} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}\,e^2} - \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}\,dx}{16e} - \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}}{16e}$$

$$= -\frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2} + \frac{5\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{\sqrt{-d}\left(a+b\cosh^{-1}(cx)\right)}{16e^{5/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)^2} -$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

$$= -\frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-d}\sqrt{-1+cx}\sqrt{1+cx}}{16e^2\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{\sqrt{-d}\left(a+b\,\mathrm{co}\right.}{16e^{5/2}\left(\sqrt{-d}\right.}$$

Mathematica [C] time = 6.90, size = 1185, normalized size = 0.97

$$-\frac{5ax}{8e^2\left(ex^2+d\right)}+\frac{adx}{4e^2\left(ex^2+d\right)^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8\sqrt{d}e^{5/2}}+b\left(-\frac{5\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16e^{5/2}}+\frac{5\left(-\frac{\cos\ldots}{\sqrt{\ldots}}\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] (a*d*x)/(4*e^2*(d + e*x^2)^2) - (5*a*x)/(8*e^2*(d + e*x^2)) + (3*a*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[d]*e^(5/2)) + b*((-5*(ArcCosh[c*x]/((-I)*Sqrt[
d] + Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d)
- e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt
[e]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + (5*(-(ArcCosh[c*x]/(I*Sqrt[d]
 + Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) -
e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e
]*x))])/Sqrt[-(c^2*d) - e]))/(16*e^(5/2)) + ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/
(Sqrt[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt
[c^2*d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/
2))))/e^2 - ((I/16)*Sqrt[d]*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*
(I*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2)
 - (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d
]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sq
rt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/e^2 + (((3*I)/32)*(ArcCosh[c*x]*
(-ArcCosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(
c^2*d) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sq
rt[-(c^2*d) - e]))]))/(Sqrt[d]*e^(5/2)) - (((3*I)/32)*(ArcCosh[c*x]*(-ArcCo
sh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*
d) - e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
 e])])) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[
-(c^2*d) - e])]))/(Sqrt[d]*e^(5/2)))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^4\operatorname{arcosh}(cx)+ax^4}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^4*arccosh(c*x) + a*x^4)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)x^4}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^4/(e*x^2 + d)^3, x)
```

maple [C] time = 4.24, size = 3125, normalized size = 2.55

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 3/8*a/e^2/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))-5/4*c*b*((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))
^(1/2)-9/4*c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((
c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*
e)^(1/2))/e^4/(c^2*d+e)^2*d^2-5/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2
)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+7/4*c^3*b*(-(2*c^2*d-2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)*d-9/4*c
^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(
c^2*d+e)^2*d^2-5/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*ar
ctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+7/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1
/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2
*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)*d-c^7*b*(-(2*c^2*d-2*(c^2*d*
(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*
e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d+e)^2+c^5*b*(
-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*d^2/e^5/
(c^2*d+e)-c^7*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^3*arctan(
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2))/e^5/(c^2*d+e)^2+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2))*d^2/e^5/(c^2*d+e)-5/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+
e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*
d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^
(1/2)+5/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^
(1/2))/e^4/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+5/8*c*b*((2*c^2*d+2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^
2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e)
)^(1/2)-1/8*c^5*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(c*x+1)^(1/2)*(c*x-1)^(1/
2)*x^2*d-3/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e^2*d*x+5/8*c*b*((2*c^2*d+2*(c^2*d*(
c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)+5/8*c*b*(-(2*c^2
*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)
^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)+3
/16*c^3*b/e^2/(c^2*d+e)*d*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1
-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)
^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/16*c^3*b/e^2/(c^2*
d+e)*d*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2
)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1
=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-5/8*c^4*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+
e)*arccosh(c*x)*x^3+7/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/
2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))
^(1/2)+e)*e)^(1/2))/e^4/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)*d-c^5*b*(-(2*c^
2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*d^2*arctanh((c*x+(c*x-1)^(1/2)*(c
```

```
*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^5/(c^2*d
+e)^2*(c^2*d*(c^2*d+e))^(1/2)-1/8*c^5*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*d^2-5/8*c^6*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*ar
ccosh(c*x)*x^3*d-3/8*c^6*b/e^2/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x
*d^2-3/8*c^4*b/e/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x*d+c^3*b*(-(2*
c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x
+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))*d/e^5/(c^2*d
+e)*(c^2*d*(c^2*d+e))^(1/2)+c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)
^(1/2)*d^2*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c
^2*d+e))^(1/2)+e)*e)^(1/2))/e^5/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(
(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c
*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))*d/e^5/(c^2*
d+e)*(c^2*d*(c^2*d+e))^(1/2)-7/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^4/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)*d-5
/8*c^4*a/(c^2*e*x^2+c^2*d)^2*x^3/e-3/16*c*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*e+
2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilo
g((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*
e)*_Z^2+e))+3/16*c*b/e/(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*
ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*
(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{8}\,a\left(\frac{5\,ex^3+3\,dx}{e^4x^4+2\,de^3x^2+d^2e^2}-\frac{3\,\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,e^2}\right)+b\int\frac{x^4\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a*((5*e*x^3 + 3*d*x)/(e^4*x^4 + 2*d*e^3*x^2 + d^2*e^2) - 3*arctan(e*x/
sqrt(d*e))/(sqrt(d*e)*e^2)) + b*integrate(x^4*log(c*x + sqrt(c*x + 1)*sqrt(
c*x - 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^4\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^4*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.512 $\int \frac{x^2\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1234

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{3/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-}}\right)}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+}}$$

```
[Out] -1/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*(a+b*arccosh(c*x))*ln
(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/(-d)^(3/2)/e^(3/2)-1/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+
1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*
(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(
1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-(c*x+(c*x-1)^(
1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(
3/2)-1/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/
2)-(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/16*b*polylog(2,-(c*x+(c*x-1)^(1/
2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/
2)-1/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)
+(-c^2*d-e)^(1/2)))/(-d)^(3/2)/e^(3/2)+1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(
-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/e^(3/2
)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)-1/8*b*c^3*arcta
nh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e
^(1/2))^(1/2))/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(3/2)/(c*(-d)^(1/2)+e^(1/2))^
(3/2)+1/16*(-a-b*arccosh(c*x))/e^(3/2)/(-d)^(1/2)/((-d)^(1/2)-x*e^(1/2))^2+
1/16*(-a-b*arccosh(c*x))/d/e^(3/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh
(c*x))/e^(3/2)/(-d)^(1/2)/((-d)^(1/2)+x*e^(1/2))^2+1/16*(a+b*arccosh(c*x))/
d/e^(3/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/e/(c^
2*d+e)/(-d)^(1/2)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/
2)/e/(c^2*d+e)/(-d)^(1/2)/((-d)^(1/2)+x*e^(1/2))+1/8*b*c*arctanh((c*x+1)^(1
/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)
)/d/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)-1/8*b
*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)
^(1/2)-e^(1/2))^(1/2))/d/e^(3/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)
+e^(1/2))^(1/2)
```

Rubi [A] time = 2.92, antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 62, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {5792, 5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{3/2}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}e^{3/2}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-}}\right)}{8d\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)*(Sqrt[-d] -
Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*Sqrt[-d]*e*(c^2*d + e)
*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[
-d] - Sqrt[e]*x)^2) - (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] - Sqrt[e
]*x)) + (a + b*ArcCosh[c*x])/(16*Sqrt[-d]*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)^2)
```

```
 + (a + b*ArcCosh[c*x])/(16*d*e^(3/2)*(Sqrt[-d] + Sqrt[e]*x)) + (b*c^3*ArcT
anh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*
Sqrt[-1 + c*x])])/(8*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3
/2)*e^(3/2)) + (b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqr
t[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*S
qrt[c*Sqrt[-d] + Sqrt[e]]*e^(3/2)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt
[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])])/(8*(c*Sqr
t[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)*e^(3/2)) - (b*c*ArcTanh
[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqr
t[-1 + c*x])])/(8*d*Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*e
^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCosh[c*x])*L
og[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d
)^(3/2)*e^(3/2)) - ((a + b*ArcCosh[c*x])*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(
c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2)) + ((a + b*ArcCos
h[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])]
)/(16*(-d)^(3/2)*e^(3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqr
t[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(
3/2)) + (b*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d
) - e]))])/(16*(-d)^(3/2)*e^(3/2)) - (b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(3/2)*e^(3/2))
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5792

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_.)*((d_) + (e
_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n,
 (f*x)^m*(d + e*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[c^2*d
 + e, 0] && IGtQ[n, 0] && IntegerQ[p] && IntegerQ[m]
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{x^2 \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \left(-\frac{d\left(a+b\cosh^{-1}(cx)\right)}{e\left(d+ex^2\right)^3}+\frac{a+b\cosh^{-1}(cx)}{e\left(d+ex^2\right)^2}\right)dx$$

$$= \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^2}\,dx}{e} - \frac{d\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx}{e}$$

$$= \frac{\int \left(-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{4d\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}-\frac{e\left(a+b\cosh^{-1}(cx)\right)}{2d\left(-de-e^2x^2\right)}\right)dx}{e} - \frac{d\int \left(-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx\right.}{8(-d)^{3/2}\left(\sqrt{-d}\,\sqrt{e}-e\right.}\right.}{}$$

$$= \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx}{16d} + \frac{3\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx}{16d} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}-ex\right)^2}\,dx}{4d} - \frac{\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\,\sqrt{e}+ex\right)^2}\,dx}{4d}$$

$$= -\frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}\,e^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)^2} - \frac{a+b\cosh^{-1}(cx)}{16de^{3/2}\left(\sqrt{-d}-\sqrt{e}\,x\right)} + \frac{a+b\cosh^{-1}(cx)}{16\sqrt{-d}\,e^{3/2}\left(\sqrt{-d}+\sqrt{e}\,x\right)^2}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

$$= -\frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}\,x\right)} - \frac{bc\sqrt{-1+cx}\,\sqrt{1+cx}}{16\sqrt{-d}\,e\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}\,x\right)} - \frac{a+}{16\sqrt{-d}\,e^3}$$

Mathematica [C] time = 6.72, size = 1193, normalized size = 0.97

$$\frac{ax}{8de\left(ex^2+d\right)}-\frac{ax}{4e\left(ex^2+d\right)^2}+\frac{a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{3/2}e^{3/2}}+b\left(\frac{\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}}{16de^{3/2}}-\frac{\frac{\cosh^{-1}(cx)}{\sqrt{e}x+i\sqrt{d}}-\cdots}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
```

```
[Out] -1/4*(a*x)/(e*(d + e*x^2)^2) + (a*x)/(8*d*e*(d + e*x^2)) + (a*ArcTan[(Sqrt[
e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(3/2)) + b*((ArcCosh[c*x]/((-I)*Sqrt[d] + Sqrt
[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x))])
/Sqrt[-(c^2*d) - e])/(16*d*e^(3/2)) - (-(ArcCosh[c*x]/(I*Sqrt[d] + Sqrt[e]*
x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e]*Sqrt[-1
+ c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]*x))])/Sqr
t[-(c^2*d) - e])/(16*d*e^(3/2)) - ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*((-I)*Sqr
t[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*
Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c
^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e)
+ ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d] + Sqrt[
e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*Sqrt[d]*(L
og[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqrt[c^2*d +
 e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt
[e]*(c^2*d + e)^(3/2))))/(Sqrt[d]*e) + ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x]
 + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d) - e])]
+ Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) +
2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])
] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
e]))]))/(d^(3/2)*e^(3/2)) - ((I/32)*(ArcCosh[c*x]*(-ArcCosh[c*x] + 2*(Log[1
 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e])] + Log[1
- (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])])) + 2*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) - e]))] + 2
*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]))/
(d^(3/2)*e^(3/2)))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^2\operatorname{arcosh}(cx)+ax^2}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*x^2*arccosh(c*x) + a*x^2)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(b\operatorname{arcosh}(cx)+a)x^2}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*x^2/(e*x^2 + d)^3, x)
```

maple **[C]** time = 3.27, size = 2269, normalized size = 1.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] 1/4*c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*
x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2
))/e^3/(c^2*d+e)^2*d+1/4*c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1
/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*d+1/16*c^3*b/e/(c^2*d+e)*sum(_R1/(_R1^
2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+
dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*
d+2*e)*_Z^2+e))+1/8*a/d/e/(d*e)^(1/2)*arctan(x*e/(d*e)^(1/2))+1/8*c^4*a/(c^
2*e*x^2+c^2*d)^2/d*x^3-1/8*c^4*a/(c^2*e*x^2+c^2*d)^2/e*x+1/16*c*b/d/(c^2*d+
e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x
+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootO
f(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-1/16*c*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c
^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((
_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*
_Z^2+e))-1/16*c^3*b/e/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)
*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)
*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+1/8*c*b*(-(2
*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2
/d/e^2*(c^2*d*(c^2*d+e))^(1/2)-1/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)
+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)-1/8
*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*
d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)+1/4*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^
(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c
^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+
1/8*c^4*b*e/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x^3+1/8*c^5*b/e*d/
(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)-1/8*c^6*b/e/(c^2*
e*x^2+c^2*d)^2/(c^2*d+e)*arccosh(c*x)*x*d-1/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)-1/4*c^3*b*(-(2*c^2*
d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^
(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)+1/
4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-
1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))
/(c^2*d+e)^2/e^2+1/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*
arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1
/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2+1/8*c^6*b/(c^2*e*x^2+c^2*d)^2/(c^2*d+e)*ar
ccosh(c*x)*x^3-1/8*c^4*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x+1/4*c
^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^
(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^
3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-1/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e
))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2
*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/
2)-1/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/
2))/e^2/d/(c^2*d+e)-1/8*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)
*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(
```

```
1/2)+e)*e)^(1/2))/e^2/d/(c^2*d+e)+1/8*c^5*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(
c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}a\left(\frac{ex^3-dx}{de^3x^4+2d^2e^2x^2+d^3e}+\frac{\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,de}\right)+b\int\frac{x^2\log\left(cx+\sqrt{cx+1}\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((e*x^3 - d*x)/(d*e^3*x^4 + 2*d^2*e^2*x^2 + d^3*e) + arctan(e*x/sqrt(
d*e))/(sqrt(d*e)*d*e)) + b*integrate(x^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{x^2\,\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3,x)
```

```
[Out] int((x^2*(a + b*acosh(c*x)))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.513 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=1234

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\cdots}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{c\cdots}}\right)\cdots}{8d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c\cdots}}$$

[Out] 3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*(a+b*arccosh(c*x))*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*(a+b*arccosh(c*x))*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-3/16*b*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)+3/16*b*polylog(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(5/2)/e^(1/2)-1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(3/2)/e^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/8*b*c^3*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d/(c*(-d)^(1/2)-e^(1/2))^(3/2)/e^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(3/2)+1/16*(-a-b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)-x*e^(1/2))^2-3/16*(a+b*arccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)-x*e^(1/2))+1/16*(a+b*arccosh(c*x))/(-d)^(3/2)/e^(1/2)/((-d)^(1/2)+x*e^(1/2))^2+3/16*(a+b*arccosh(c*x))/d^2/e^(1/2)/((-d)^(1/2)+x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)-x*e^(1/2))-1/16*b*c*(c*x-1)^(1/2)*(c*x+1)^(1/2)/(-d)^(3/2)/(c^2*d+e)/((-d)^(1/2)+x*e^(1/2))+3/8*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2))/d^2/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)-3/8*b*c*arctanh((c*x+1)^(1/2)*(c*(-d)^(1/2)+e^(1/2))^(1/2)/(c*x-1)^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2))/d^2/e^(1/2)/(c*(-d)^(1/2)-e^(1/2))^(1/2)/(c*(-d)^(1/2)+e^(1/2))^(1/2)

Rubi [A] time = 1.46, antiderivative size = 1234, normalized size of antiderivative = 1.00, number of steps used = 34, number of rules used = 10, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {5707, 5802, 96, 93, 208, 5800, 5562, 2190, 2279, 2391}

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{cx+1}}{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{cx-1}}\right)c^3}{8d\left(c\sqrt{-d}-\sqrt{e}\right)^{3/2}\left(\sqrt{-d}\,c+\sqrt{e}\right)^{3/2}\sqrt{e}}+\frac{3b\tanh^{-1}\left(\frac{\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{c\cdots}}{\sqrt{\sqrt{-d}\,c+\sqrt{e}}\,\sqrt{c\cdots}}\right)\cdots}{8d^2\sqrt{c\sqrt{-d}-\sqrt{e}}\,\sqrt{\sqrt{-d}\,c\cdots}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]

[Out] -(b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] - Sqrt[e]*x)) - (b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(16*(-d)^(3/2)*(c^2*d + e)*(Sqrt[-d] + Sqrt[e]*x)) - (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)^2) - (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] - Sqrt[e]*x)) + (a + b*ArcCosh[c*x])/(16*(-d)^(3/2)*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x)^2) + (3*(a + b*ArcCosh[c*x]))/(16*d^2*Sqrt[e]*(Sqrt[-d] + Sqrt[e]*x

```
)) - (b*c^3*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt
[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt
[-d] + Sqrt[e])^(3/2)*Sqrt[e]) + (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] - Sqrt[e]]
*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt[c
*Sqrt[-d] - Sqrt[e]]*Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (b*c^3*ArcTanh[(
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt[c*Sqrt[-d] - Sqrt[e]]*Sqrt[
-1 + c*x])]])/(8*d*(c*Sqrt[-d] - Sqrt[e])^(3/2)*(c*Sqrt[-d] + Sqrt[e])^(3/2)
*Sqrt[e]) - (3*b*c*ArcTanh[(Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[1 + c*x])/(Sqrt
[c*Sqrt[-d] - Sqrt[e]]*Sqrt[-1 + c*x])]])/(8*d^2*Sqrt[c*Sqrt[-d] - Sqrt[e]]*
Sqrt[c*Sqrt[-d] + Sqrt[e]]*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*Log[1 - (Sqrt
[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt
[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 - Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) + (3*(a + b*ArcCosh[c*x])*
Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-
d)^(5/2)*Sqrt[e]) - (3*(a + b*ArcCosh[c*x])*Log[1 + (Sqrt[e]*E^ArcCosh[c*x]
)/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLo
g[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(16*(-
d)^(5/2)*Sqrt[e]) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] -
Sqrt[-(c^2*d) - e])])/(16*(-d)^(5/2)*Sqrt[e]) - (3*b*PolyLog[2, -((Sqrt[e]*
E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(16*(-d)^(5/2)*Sqrt[e]
) + (3*b*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) -
e])])/(16*(-d)^(5/2)*Sqrt[e])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1)
 - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)
], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n]
 && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 96

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*
c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*
x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n
, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m
, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 5802

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.) + (e_.)*(x_))^(m_.), x
_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(e*(m + 1)), x]
 - Dist[(b*c*n)/(e*(m + 1)), Int[((d + e*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n
 - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, e, m},
x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^3}\,dx = \int \left(-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}-ex\right)^3}-\frac{3e\left(a+b\cosh^{-1}(cx)\right)}{16d^2\left(\sqrt{-d}\sqrt{e}-ex\right)^2}-\frac{e^{3/2}\left(a+b\cosh^{-1}(cx)\right)}{8(-d)^{3/2}\left(\sqrt{-d}\sqrt{e}+ex\right)^3}\right.$$

$$=-\frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}-ex\right)^2}dx}{16d^2}-\frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{\left(\sqrt{-d}\sqrt{e}+ex\right)^2}dx}{16d^2}-\frac{(3e)\int \frac{a+b\cosh^{-1}(cx)}{-de-e^2x^2}dx}{8d^2}-\frac{e^{3/2}\int \frac{a+b}{(\sqrt{}}}{8(}$$

$$=-\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)^2}-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{16d^2\sqrt{e}\left(\sqrt{-d}-\sqrt{e}x\right)}+\frac{a+b\cosh^{-1}(cx)}{16(-d)^{3/2}\sqrt{e}\left(\sqrt{-d}+\sqrt{e}\right.}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

$$=-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}-\sqrt{e}x\right)}-\frac{bc\sqrt{-1+cx}\sqrt{1+cx}}{16(-d)^{3/2}\left(c^2d+e\right)\left(\sqrt{-d}+\sqrt{e}x\right)}-\frac{a+}{16(-d)^{3/2}}$$

Mathematica [C] time = 6.39, size = 1184, normalized size = 0.96

$$\frac{3ax}{8d^2\left(ex^2+d\right)}+\frac{ax}{4d\left(ex^2+d\right)^2}+\frac{3a\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{8d^{5/2}\sqrt{e}}+b\left(\frac{3\left(\frac{\cosh^{-1}(cx)}{\sqrt{e}x-i\sqrt{d}}+\frac{c\log\left(\frac{2e\left(\sqrt{d}xc^2+i\sqrt{e}-i\sqrt{-dc^2-e}\sqrt{cx-1}\sqrt{cx+1}\right)}{c\sqrt{-dc^2-e}\left(i\sqrt{e}x+\sqrt{d}\right)}\right)}{\sqrt{-dc^2-e}}\right)}{16d^2\sqrt{e}}-\frac{3\left(-\frac{\cos}{\sqrt{e}}\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^3,x]
```

```
[Out] (a*x)/(4*d*(d + e*x^2)^2) + (3*a*x)/(8*d^2*(d + e*x^2)) + (3*a*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(8*d^(5/2)*Sqrt[e]) + b*((3*(ArcCosh[c*x]/((-I)*Sqrt[d] +
```

```
Sqrt[e]*x) + (c*Log[(2*e*(I*Sqrt[e] + c^2*Sqrt[d]*x - I*Sqrt[-(c^2*d) - e]*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(Sqrt[d] + I*Sqrt[e]*x
))])/Sqrt[-(c^2*d) - e]))/(16*d^2*Sqrt[e]) - (3*(-(ArcCosh[c*x]/(I*Sqrt[d]
+ Sqrt[e]*x)) - (c*Log[(2*e*(-Sqrt[e] - I*c^2*Sqrt[d]*x + Sqrt[-(c^2*d) - e
]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c*Sqrt[-(c^2*d) - e]*(I*Sqrt[d] + Sqrt[e]
*x))])/Sqrt[-(c^2*d) - e]))/(16*d^2*Sqrt[e]) + ((I/16)*((c*Sqrt[-1 + c*x]*S
qrt[1 + c*x])/((c^2*d + e)*((-I)*Sqrt[d] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt
[e]*((-I)*Sqrt[d] + Sqrt[e]*x)^2) + (c^3*Sqrt[d]*(Log[4] + Log[(e*Sqrt[c^2*
d + e]*((-I)*Sqrt[e] - c^2*Sqrt[d]*x + Sqrt[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[
1 + c*x]))/(c^3*(d + I*Sqrt[d]*Sqrt[e]*x))]))/(Sqrt[e]*(c^2*d + e)^(3/2)))
/d^(3/2) - ((I/16)*((c*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/((c^2*d + e)*(I*Sqrt[d
] + Sqrt[e]*x)) - ArcCosh[c*x]/(Sqrt[e]*(I*Sqrt[d] + Sqrt[e]*x)^2) - (c^3*S
qrt[d]*(Log[4] + Log[(e*Sqrt[c^2*d + e]*((-I)*Sqrt[e] + c^2*Sqrt[d]*x + Sqr
t[c^2*d + e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(c^3*(d - I*Sqrt[d]*Sqrt[e]*x))
]))/(Sqrt[e]*(c^2*d + e)^(3/2))))/d^(3/2) + (((3*I)/32)*(ArcCosh[c*x]*(-Arc
Cosh[c*x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] - Sqrt[-(c^2*d
) - e])] + Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])])) + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])] + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(
c^2*d) - e]))]))/(d^(5/2)*Sqrt[e]) - (((3*I)/32)*(ArcCosh[c*x]*(-ArcCosh[c*
x] + 2*(Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d) -
e])] + Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2*d) - e])]
)) + 2*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/((-I)*c*Sqrt[d] + Sqrt[-(c^2*d
) - e]))] + 2*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(I*c*Sqrt[d] + Sqrt[-(c^2
*d) - e])]))/(d^(5/2)*Sqrt[e]))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b \operatorname{arcosh}(cx) + a}{e^3x^6 + 3\,de^2x^4 + 3\,d^2ex^2 + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] integral((b*arccosh(c*x) + a)/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^3, x)
```

maple [C] time = 3.13, size = 3128, normalized size = 2.53

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] -3/16*c*b/d^2/(c^2*d+e)*e*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_
R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+
1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+3/16*c*b/d^2/(c^2*
d+e)*e*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*
```

```
(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=R
ootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))+7/4*c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e)
)^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/(c^2*d+e)^2/e^2+7/4*c^5*b*((2*c^2*d+
2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/
2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/(c^2*d+e)^2/e^2-1/8*
c^5*b/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)+5/8*c^6*b/(
c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x-c^5*b*((2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d
+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)-c^5*b*(-(2*c^2*d-2*(c
^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))
*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)-1/8*c^5*
b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*e-3/8*c*b
*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*
(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d^2/(c^2*
d+e)^2/e*(c^2*d*(c^2*d+e))^(1/2)+3/4*c*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d
*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^2/d^2/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+3/
8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/d
^2/(c^2*d+e)^2/e*(c^2*d*(c^2*d+e))^(1/2)+3/8*c^6*b*e/d/(c^2*d+e)/(c^2*e*x^2
+c^2*d)^2*arccosh(c*x)*x^3+3/8*c^4*b/d^2/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arcc
osh(c*x)*x^3*e^2+5/8*c^4*b/d/(c^2*d+e)/(c^2*e*x^2+c^2*d)^2*arccosh(c*x)*x*e
+5/4*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c
*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/
2))/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)-c^3*b*(-(2*c^2*d-2*(c^2*d*(c^
2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*
c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2*d+
e))^(1/2)-5/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan(
(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*
e)^(1/2))/(c^2*d+e)^2/d/e^2*(c^2*d*(c^2*d+e))^(1/2)+c^3*b*((2*c^2*d+2*(c^2*
d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/(
(2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/d/(c^2*d+e)*(c^2*d*(c^2
*d+e))^(1/2)-3/4*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arcta
nh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)
-e)*e)^(1/2))/e^2/d^2/(c^2*d+e)*(c^2*d*(c^2*d+e))^(1/2)+3/8*a/d^2/(d*e)^(1/
2)*arctan(x*e/(d*e)^(1/2))+1/4*c^4*a*x/d/(c^2*e*x^2+c^2*d)^2+3/8*c^2*a/d^2*
x/(c^2*e*x^2+c^2*d)+3/16*c^3*b/d/(c^2*d+e)*sum(_R1/(_R1^2*e+2*c^2*d+e)*(arc
cosh(c*x)*ln((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x
-1)^(1/2)*(c*x+1)^(1/2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-3/1
6*c^3*b/d/(c^2*d+e)*sum(1/_R1/(_R1^2*e+2*c^2*d+e)*(arccosh(c*x)*ln((_R1-c*x
-(c*x-1)^(1/2)*(c*x+1)^(1/2))/_R1)+dilog((_R1-c*x-(c*x-1)^(1/2)*(c*x+1)^(1/
2))/_R1)),_R1=RootOf(e*_Z^4+(4*c^2*d+2*e)*_Z^2+e))-5/4*c^3*b*(-(2*c^2*d-2*(
c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2)
)*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/e^2/d/(c^2*d+e)-5/4*c
^3*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^2/d
/(c^2*d+e)+c^7*b*((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*
x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^
(1/2))/e^3/(c^2*d+e)^2*d+c^5*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(
1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d
+e))^(1/2)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*(c^2*d*(c^2*d+e))^(1/2)-c^5*b*((2*c
^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1
)^(1/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e^3/(c^2*d+e)^2
*(c^2*d*(c^2*d+e))^(1/2)-3/8*c*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)
^(1/2)*arctanh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)-e)*e)^(1/2))/e/d^2/(c^2*d+e)+3/4*c^3*b*((2*c^2*d+2*(c^2*d*(c^2
*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((2*c^2
*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/d/(c^2*d+e)^2/e-3/8*c*b*((2*c^2*d
```

```
+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctan((c*x+(c*x-1)^(1/2)*(c*x+1)^(1
/2))*e/((2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2))/e/d^2/(c^2*d+e)+3/4
*c^3*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arctanh((c*x+(c*x-1
)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2)-e)*e)^(1/2))/
d/(c^2*d+e)^2/e+c^7*b*(-(2*c^2*d-2*(c^2*d*(c^2*d+e))^(1/2)+e)*e)^(1/2)*arct
anh((c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e/((-2*c^2*d+2*(c^2*d*(c^2*d+e))^(1/2
)-e)*e)^(1/2))/e^3/(c^2*d+e)^2*d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{8}\,a\left(\frac{3\,ex^3+5\,dx}{d^2e^2x^4+2\,d^3ex^2+d^4}+\frac{3\,\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}\,d^2}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a*((3*e*x^3 + 5*d*x)/(d^2*e^2*x^4 + 2*d^3*e*x^2 + d^4) + 3*arctan(e*x/s
qrt(d*e))/(sqrt(d*e)*d^2)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.514 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)dx$

Optimal. Leaf size=23

$$\text{Int}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right),x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)dx=\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)dx$$

Mathematica [A] time = 5.78, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x]), x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{ex^2+d}\,(b\,\text{arcosh}\,(cx)+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\,\text{arcosh}\,(cx)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.56, size = 0, normalized size = 0.00

$$\int (a+b\,\text{arccosh}\,(cx))\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{ex^2+d}\,x+\frac{d\,\text{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a+b\int\sqrt{ex^2+d}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(e*x^2 + d)*x + d*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a + b*integrate(
sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int(a+b\,\text{acosh}(c\,x))\,\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))*(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\,\text{acosh}(cx))\,\sqrt{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))*sqrt(d + e*x**2), x)
```

3.515 $\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx = \int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 3.81, size = 0, normalized size = 0.00

$$\int \frac{a+b\cosh^{-1}(cx)}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCosh[c*x])/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{b\operatorname{arcosh}(cx)+a}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)/sqrt(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)/sqrt(e*x^2 + d), x)
```

maple [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{arccosh}(cx)}{\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$b \int \frac{\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{ex^2+d}}\, dx + \frac{a \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/sqrt(e*x^2 + d), x) + a*
arcsinh(e*x/sqrt(d*e))/sqrt(e)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{a + b \operatorname{acosh}(c\,x)}{\sqrt{e\,x^2 + d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}(cx)}{\sqrt{d + ex^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/sqrt(d + e*x**2), x)
```

3.516 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}$$

```
[Out] -b*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2))*(c^2*x^2-1)^(1/2)/d
/e^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {191, 5705, 12, 519, 444, 63, 217, 206}

$$\frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (x*(a + b*ArcCosh[c*x]))/(d*Sqrt[d + e*x^2]) - (b*Sqrt[-1 + c^2*x^2]*ArcTan
h[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(d*Sqrt[e]*Sqrt[-1 + c
*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{3/2}}\,dx &= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-(bc)\int \frac{x}{d\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}}\,dx\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}}\,dx}{d}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x}{\sqrt{-1+c^2x^2}\,\sqrt{d+ex^2}}\,dx}{d\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(bc\sqrt{-1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{-1+c^2x}\,\sqrt{d+ex}}\,dx,x,x^2\right)}{2d\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{d+\frac{e}{c^2}+\frac{ex^2}{c^2}}}\,dx,x,\sqrt{-1+c^2x^2}\right)}{cd\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{\left(b\sqrt{-1+c^2x^2}\right)\operatorname{Subst}\left(\int \frac{1}{1-\frac{ex^2}{c^2}}\,dx,x,\frac{\sqrt{-1+c^2x^2}}{\sqrt{d+ex^2}}\right)}{cd\sqrt{-1+cx}\,\sqrt{1+cx}}\\
&= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{d\sqrt{d+ex^2}}-\frac{b\sqrt{-1+c^2x^2}\,\tanh^{-1}\left(\frac{\sqrt{e}\,\sqrt{-1+c^2x^2}}{c\sqrt{d+ex^2}}\right)}{d\sqrt{e}\,\sqrt{-1+cx}\,\sqrt{1+cx}}
\end{aligned}
$$

Mathematica [C] time = 3.36, size = 556, normalized size = 5.50

$$ax+\frac{\frac{2b(cx-1)^{3/2}\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)\left(d+ex^2\right)}{de(cx-1)^2}}\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}\Pi\left(\frac{2c\sqrt{d}}{\sqrt{d}c+i\sqrt{e}};\sin^{-1}\left(\sqrt{-\frac{\frac{i\sqrt{e}x}{\sqrt{d}}+c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)-1}{2-2cx}}\right)|\frac{4ic\cdots}{\left(\sqrt{d}c\cdots\right.}\right.\right.}{c\sqrt{cx+1}\left(c^2d+e\right)\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}\cdots}}{d\sqrt{d+ex^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(3/2),x]
```

```
[Out] (a*x + b*x*ArcCosh[c*x] + (2*b*(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e
])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])*(-1 + c*x))]*((c*((-I)*c*Sqrt[d] + S
qrt[e])*(I*Sqrt[d] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*
Sqrt[e]*x)/Sqrt[d])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqr
t[e])/(c*Sqrt[d] + I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*
Sqrt[e])*Sqrt[((c^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I
*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(
2*c*Sqrt[d])/(c*Sqrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqr
t[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e
])/(c*Sqrt[d] + I*Sqrt[e])^2]))/(c*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (
I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(d*Sqrt[d
 + e*x^2])
```

fricas [A] time = 1.43, size = 332, normalized size = 3.29

$$\left[\frac{4\sqrt{ex^2+d}\,bex\log\left(cx+\sqrt{c^2x^2-1}\right)+4\sqrt{ex^2+d}\,aex+\left(bex^2+bd\right)\sqrt{e}\,\log\left(8\,c^4e^2x^4+c^4d^2-6\,c^2de+8\left(c^4d\cdots\right.\right.}{4\left(de^2x^2+d^2e\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*sqrt(e*x^2 + d)*b*e*x*log(c*x + sqrt(c^2*x^2 - 1)) + 4*sqrt(e*x^2 +
 d)*a*e*x + (b*e*x^2 + b*d)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e
 + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 -
 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2))/(d*e^2*x^2 + d^2*e), 1/2*(2*sqrt(e*x^2
+ d)*b*e*x*log(c*x + sqrt(c^2*x^2 - 1)) + 2*sqrt(e*x^2 + d)*a*e*x + (b*e*x^
2 + b*d)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sq
rt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)))/(d*e
^2*x^2 + d^2*e)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\operatorname{arcosh}(cx)+a}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(3/2), x)
```

maple [F] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccosh}\,(cx)}{\left(e\,x^2 + d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}\,(c\,x)}{\left(e\,x^2 + d\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{acosh}\,(cx)}{\left(d + ex^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**(3/2), x)
```

3.517 $\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}+\frac{2b\sqrt{1-c^2x^2}\,\tan^{-1}\left(\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}-\frac{bc\sqrt{cx-1}\sqrt{cx+1}}{3d\left(c^2d+e\right)\sqrt{d+ex^2}}$$

```
[Out] 1/3*x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(3/2)+2/3*b*arctan(e^(1/2)*(-c^2*x^2+1
)^(1/2)/c/(e*x^2+d)^(1/2))*(-c^2*x^2+1)^(1/2)/d^2/e^(1/2)/(c*x-1)^(1/2)/(c*
x+1)^(1/2)+2/3*x*(a+b*arccosh(c*x))/d^2/(e*x^2+d)^(1/2)-1/3*b*c*(c*x-1)^(1/
2)*(c*x+1)^(1/2)/d/(c^2*d+e)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 190, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {192, 191, 5705, 12, 519, 571, 78, 63, 217, 206}

$$\frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}}-\frac{2b\sqrt{c^2x^2-1}\,\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{3d^2\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}+\frac{bc\left(1-c^2x^2\right)}{3d\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (b*c*(1 - c^2*x^2))/(3*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d +
e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(3*d*(d + e*x^2)^(3/2)) + (2*x*(a + b*Ar
cCosh[c*x]))/(3*d^2*Sqrt[d + e*x^2]) - (2*b*Sqrt[-1 + c^2*x^2]*ArcTanh[(Sqr
t[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(3*d^2*Sqrt[e]*Sqrt[-1 + c*x
]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 571

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x
)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m,
 n, p, q, r}, x] && EqQ[m - n + 1, 0]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{5/2}}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - (bc)\int \frac{x\left(3d+2ex^2\right)}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+e\right.}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}}\,dx}{3d^2}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\int \frac{x\left(3d+2ex^2\right)}{\sqrt{-1+c^2x^2}\,\left(d+ex^2\right)^{3/2}}}{3d^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}(cx)\right)}{3d^2\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x^2}\right)\mathrm{Subst}\left(\int \frac{3d+2e}{\sqrt{-1+c^2x}\,(d}\right.}{6d^2\sqrt{-1+cx}\,\sqrt{1+cx}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}\right.}{3d^2\sqrt{d+ex^2}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}\right.}{3d^2\sqrt{d+ex^2}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}\right.}{3d^2\sqrt{d+ex^2}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{3d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+ex^2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{3d\left(d+ex^2\right)^{3/2}} + \frac{2x\left(a+b\cosh^{-1}\right.}{3d^2\sqrt{d+ex^2}}$$

Mathematica [C] time = 2.41, size = 633, normalized size = 3.48

$$\frac{ax\left(3d+2ex^2\right)}{d^2} + \frac{4b(cx-1)^{3/2}\left(d+ex^2\right)\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)\left(d+ex^2\right)}{de(cx-1)^2}}\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}\,\Pi\left(\frac{2c\sqrt{d}}{\sqrt{d}c+i\sqrt{e}};\sin^{-1}\left(\sqrt{-\frac{\frac{i\sqrt{e}x}{\sqrt{d}}+\cdots}{\cdots}}\right.\right.\right.}{cd^2\sqrt{cx+1}\left(c^2d+e\right)\sqrt{-\frac{c\left(x\cdots\right.}{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(5/2),x]
```

```
[Out] (-((b*c*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x^2))/(d*(c^2*d + e))) + (a*x*(
3*d + 2*e*x^2))/d^2 + (b*x*(3*d + 2*e*x^2)*ArcCosh[c*x])/d^2 + (4*b*(-1 + c
*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*Sqrt[e])
*(-1 + c*x))]*(d + e*x^2)*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d] + Sqrt[
e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d])/(1 - c
*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/S
```

```
qrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[
e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c^2*d + e)
*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((
I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*Sqrt[d] +
I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt[d])/Sqrt
[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I*Sqrt[e])
^2]))/(c*d^2*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] +
 c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(3*(d + e*x^2)^(3/2))
```

fricas [B] time = 0.87, size = 724, normalized size = 3.98

$$\left[\frac{\left(bc^2d^3 + \left(bc^2de^2 + be^3\right)x^4 + bd^2e + 2\left(bc^2d^2e + bde^2\right)x^2\right)\sqrt{e}\,\log\left(8\,c^4e^2x^4 + c^4d^2 - 6\,c^2de + 8\left(c^4de - c^2e^2\right)x^2 - 4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*((b*c^2*d^3 + (b*c^2*d*e^2 + b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b
*d*e^2)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4 + c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e -
 c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d - c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2
+ d)*sqrt(e) + e^2) + 2*(2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3*(b*c^2*d^2*e + b*d
*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) + 2*(2*(a*c^2*d*e^2 +
 a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d*e^2)*x - (b*c*d*e^2*x^2 + b*c*d^2*e)*sqr
t(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c^2*d^3*e^3 + d^2*
e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2), 1/3*((b*c^2*d^3 + (b*c^2*d*e^2 +
 b*e^3)*x^4 + b*d^2*e + 2*(b*c^2*d^2*e + b*d*e^2)*x^2)*sqrt(-e)*arctan(1/2*
(2*c^2*e*x^2 + c^2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e
^2*x^4 - c*d*e + (c^3*d*e - c*e^2)*x^2)) + (2*(b*c^2*d*e^2 + b*e^3)*x^3 + 3
*(b*c^2*d^2*e + b*d*e^2)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) +
(2*(a*c^2*d*e^2 + a*e^3)*x^3 + 3*(a*c^2*d^2*e + a*d*e^2)*x - (b*c*d*e^2*x^2
 + b*c*d^2*e)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^2*d^5*e + d^4*e^2 + (c
^2*d^3*e^3 + d^2*e^4)*x^4 + 2*(c^2*d^4*e^2 + d^3*e^3)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\,\mathrm{arcosh}\left(cx\right) + a}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(5/2), x)
```

maple [F] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{a + b\,\mathrm{arccosh}\left(cx\right)}{\left(e\,x^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a\left(\frac{2\,x}{\sqrt{ex^2 + d}\,d^2} + \frac{x}{\left(ex^2 + d\right)^{\frac{3}{2}}d}\right) + b\int \frac{\log\left(cx + \sqrt{cx + 1}\,\sqrt{cx - 1}\right)}{\left(ex^2 + d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*a*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + b*integrate(l
og(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b\,\mathrm{acosh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(5/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b \operatorname{acosh}{\left(cx \right)}}{\left(d + ex^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(5/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))/(d + e*x**2)**(5/2), x)
```

3.518 $\int \frac{a+b\cosh^{-1}(cx)}{(d+ex^2)^{7/2}}\,dx$

Optimal. Leaf size=284

$$\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}-\frac{8b\sqrt{c^2x^2-1}\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{c^2x^2-1}}{c\sqrt{d+ex^2}}\right)}{15d^3\sqrt{e}\sqrt{cx-1}\sqrt{cx+1}}+\frac{}{15d^2\sqrt{c}}$$

```
[Out] 1/5*x*(a+b*arccosh(c*x))/d/(e*x^2+d)^(5/2)+4/15*x*(a+b*arccosh(c*x))/d^2/(e
*x^2+d)^(3/2)+1/15*b*c*(-c^2*x^2+1)/d/(c^2*d+e)/(e*x^2+d)^(3/2)/(c*x-1)^(1/
2)/(c*x+1)^(1/2)-8/15*b*arctanh(e^(1/2)*(c^2*x^2-1)^(1/2)/c/(e*x^2+d)^(1/2)
)*(c^2*x^2-1)^(1/2)/d^3/e^(1/2)/(c*x-1)^(1/2)/(c*x+1)^(1/2)+8/15*x*(a+b*arc
cosh(c*x))/d^3/(e*x^2+d)^(1/2)+2/15*b*c*(3*c^2*d+2*e)*(-c^2*x^2+1)/d^2/(c^2
*d+e)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)/(e*x^2+d)^(1/2)
```

Rubi [A] time = 0.80, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 11, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.550, Rules used = {192, 191, 5705, 12, 519, 6715, 949, 78, 63, 217, 206}

$$\frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}}+\frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}}+\frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}}+\frac{2bc\left(1-c^2x^2\right)\left(3c^2d+2e\right)}{15d^2\sqrt{cx-1}\sqrt{cx+1}\left(c^2d+e\right)^2\sqrt{d+ex^2}}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] (b*c*(1 - c^2*x^2))/(15*d*(c^2*d + e)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(d + e*x
^2)^(3/2)) + (2*b*c*(3*c^2*d + 2*e)*(1 - c^2*x^2))/(15*d^2*(c^2*d + e)^2*Sq
rt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[d + e*x^2]) + (x*(a + b*ArcCosh[c*x]))/(5*d
*(d + e*x^2)^(5/2)) + (4*x*(a + b*ArcCosh[c*x]))/(15*d^2*(d + e*x^2)^(3/2))
 + (8*x*(a + b*ArcCosh[c*x]))/(15*d^3*Sqrt[d + e*x^2]) - (8*b*Sqrt[-1 + c^2
*x^2]*ArcTanh[(Sqrt[e]*Sqrt[-1 + c^2*x^2])/(c*Sqrt[d + e*x^2])])/(15*d^3*Sq
rt[e]*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p
_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol] :> Dist[((a1 + b1*x^(n/2)
)^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p
], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2,
b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] &&  !(EqQ
[n, 2] && IGtQ[q, 0])
```

Rule 949

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x
 + c*x^2)^p, d + e*x, x], R = PolynomialRemainder[(a + b*x + c*x^2)^p, d +
e*x, x]}, Simp[(R*(d + e*x)^(m + 1)*(f + g*x)^(n + 1))/((m + 1)*(e*f - d*g)
), x] + Dist[1/((m + 1)*(e*f - d*g)), Int[(d + e*x)^(m + 1)*(f + g*x)^n*Exp
andToSum[(m + 1)*(e*f - d*g)*Qx - g*R*(m + n + 2), x], x], x]] /; FreeQ[{a,
 b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c
*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[m, -1]
```

Rule 5705

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((d_) + (e_.)*(x_)^2)^(p_.), x_Symb
ol] :> With[{u = IntHide[(d + e*x^2)^p, x]}, Dist[a + b*ArcCosh[c*x], u, x]
 - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x]
, x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[c^2*d + e, 0] && (IGtQ[p, 0] || I
LtQ[p + 1/2, 0])
```

Rule 6715

```
Int[(u_)*(x_)^(m_.), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m +
 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionO
fQ[x^(m + 1), u, x]
```

Rubi steps

$$\int \frac{a+b\cosh^{-1}(cx)}{\left(d+ex^2\right)^{7/2}}\,dx = \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - (bc)\int \frac{x\,(}{15d^3\sqrt{}}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{(bc)\int \frac{x(15d^2+}{\sqrt{-1+cx}}}{1}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x}\right.}{15d^3}$$

$$= \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}(cx)\right)}{15d^2\left(d+ex^2\right)^{3/2}} + \frac{8x\left(a+b\cosh^{-1}(cx)\right)}{15d^3\sqrt{d+ex^2}} - \frac{\left(bc\sqrt{-1+c^2x}\right.}{}$$

$$= \frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}} + \frac{x\left(a+b\cosh^{-1}(cx)\right)}{5d\left(d+ex^2\right)^{5/2}} + \frac{4x\left(a+b\cosh^{-1}\right.}{15d^2\left(d+ex^2\right.}$$

$$= \frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}} + \frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}} + \frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}} + \frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+}}$$

$$= \frac{bc\left(1-c^2x^2\right)}{15d\left(c^2d+e\right)\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(d+ex^2\right)^{3/2}} + \frac{2bc\left(3c^2d+2e\right)\left(1-c^2x^2\right)}{15d^2\left(c^2d+e\right)^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{d+}}$$

Mathematica [C] time = 4.24, size = 685, normalized size = 2.41

$$\frac{\frac{ax\left(15d^2+20dex^2+8e^2x^4\right)}{d^3} + \frac{16b(cx-1)^{3/2}\left(d+ex^2\right)^2\sqrt{\frac{(cx+1)\left(c\sqrt{d}-i\sqrt{e}\right)}{(cx-1)\left(c\sqrt{d}+i\sqrt{e}\right)}}\left(c\sqrt{d}\left(-c\sqrt{d}+i\sqrt{e}\right)\sqrt{\frac{\left(c^2d+e\right)\left(d+ex^2\right)}{de(cx-1)^2}}\sqrt{-\frac{c\left(x+\frac{i\sqrt{d}}{\sqrt{e}}\right)+\frac{i\sqrt{e}x}{\sqrt{d}}-1}{1-cx}}\,\Pi\left(\frac{2c\sqrt{d}}{\sqrt{d}c+i\sqrt{e}};\sin^{-1}\left(\right.\right.\right.}{cd^3\sqrt{cx+1}\left(c^2d+\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])/(d + e*x^2)^(7/2),x]
```

```
[Out] ((a*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4))/d^3 - (b*c*Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]*(d + e*x^2)*(e*(5*d + 4*e*x^2) + c^2*d*(7*d + 6*e*x^2)))/(d^2*(c^2*d
```

```
+ e)^2) + (b*x*(15*d^2 + 20*d*e*x^2 + 8*e^2*x^4)*ArcCosh[c*x])/d^3 + (16*b*
(-1 + c*x)^(3/2)*Sqrt[((c*Sqrt[d] - I*Sqrt[e])*(1 + c*x))/((c*Sqrt[d] + I*S
qrt[e])*(-1 + c*x))]*(d + e*x^2)^2*((c*((-I)*c*Sqrt[d] + Sqrt[e])*(I*Sqrt[d
] + Sqrt[e]*x)*Sqrt[(1 + (I*c*Sqrt[d])/Sqrt[e] - c*x + (I*Sqrt[e]*x)/Sqrt[d
])/(1 - c*x)]*EllipticF[ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*S
qrt[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d]
+ I*Sqrt[e])^2])/(-1 + c*x) + c*Sqrt[d]*(-(c*Sqrt[d]) + I*Sqrt[e])*Sqrt[((c
^2*d + e)*(d + e*x^2))/(d*e*(-1 + c*x)^2)]*Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[
d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]*EllipticPi[(2*c*Sqrt[d])/(c*S
qrt[d] + I*Sqrt[e]), ArcSin[Sqrt[-((-1 + (I*Sqrt[e]*x)/Sqrt[d] + c*((I*Sqrt
[d])/Sqrt[e] + x))/(2 - 2*c*x))]], ((4*I)*c*Sqrt[d]*Sqrt[e])/(c*Sqrt[d] + I
*Sqrt[e])^2]))/(c*d^3*(c^2*d + e)*Sqrt[1 + c*x]*Sqrt[-((-1 + (I*Sqrt[e]*x)/
Sqrt[d] + c*((I*Sqrt[d])/Sqrt[e] + x))/(1 - c*x))]))/(15*(d + e*x^2)^(5/2))
```

fricas [B] time = 0.83, size = 1360, normalized size = 4.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/15*(2*(b*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^
5)*x^6 + b*d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*
(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(e)*log(8*c^4*e^2*x^4
+ c^4*d^2 - 6*c^2*d*e + 8*(c^4*d*e - c^2*e^2)*x^2 - 4*(2*c^3*e*x^2 + c^3*d
- c*e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(e) + e^2) + (8*(b*c^4*d^2*e^3
 + 2*b*c^2*d*e^4 + b*e^5)*x^5 + 20*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e
^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)
*log(c*x + sqrt(c^2*x^2 - 1)) + (8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*
x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e
+ 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*
c^3*d^2*e^3 + 2*b*c*d*e^4)*x^4 + (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sq
rt(c^2*x^2 - 1))*sqrt(e*x^2 + d))/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c
^4*d^5*e^4 + 2*c^2*d^4*e^5 + d^3*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4
+ d^4*e^5)*x^4 + 3*(c^4*d^7*e^2 + 2*c^2*d^6*e^3 + d^5*e^4)*x^2), 1/15*(4*(b
*c^4*d^5 + 2*b*c^2*d^4*e + (b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^6 + b*
d^3*e^2 + 3*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^4 + 3*(b*c^4*d^4*
e + 2*b*c^2*d^3*e^2 + b*d^2*e^3)*x^2)*sqrt(-e)*arctan(1/2*(2*c^2*e*x^2 + c^
2*d - e)*sqrt(c^2*x^2 - 1)*sqrt(e*x^2 + d)*sqrt(-e)/(c^3*e^2*x^4 - c*d*e +
(c^3*d*e - c*e^2)*x^2)) + (8*(b*c^4*d^2*e^3 + 2*b*c^2*d*e^4 + b*e^5)*x^5 +
20*(b*c^4*d^3*e^2 + 2*b*c^2*d^2*e^3 + b*d*e^4)*x^3 + 15*(b*c^4*d^4*e + 2*b*
c^2*d^3*e^2 + b*d^2*e^3)*x)*sqrt(e*x^2 + d)*log(c*x + sqrt(c^2*x^2 - 1)) +
(8*(a*c^4*d^2*e^3 + 2*a*c^2*d*e^4 + a*e^5)*x^5 + 20*(a*c^4*d^3*e^2 + 2*a*c^
2*d^2*e^3 + a*d*e^4)*x^3 + 15*(a*c^4*d^4*e + 2*a*c^2*d^3*e^2 + a*d^2*e^3)*x
 - (7*b*c^3*d^4*e + 5*b*c*d^3*e^2 + 2*(3*b*c^3*d^2*e^3 + 2*b*c*d*e^4)*x^4 +
 (13*b*c^3*d^3*e^2 + 9*b*c*d^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))*sqrt(e*x^2 + d)
)/(c^4*d^8*e + 2*c^2*d^7*e^2 + d^6*e^3 + (c^4*d^5*e^4 + 2*c^2*d^4*e^5 + d^3
*e^6)*x^6 + 3*(c^4*d^6*e^3 + 2*c^2*d^5*e^4 + d^4*e^5)*x^4 + 3*(c^4*d^7*e^2
+ 2*c^2*d^6*e^3 + d^5*e^4)*x^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b \operatorname{arcosh}(cx) + a}{\left(ex^2 + d\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)/(e*x^2 + d)^(7/2), x)
```

maple [F] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{a+b\,\mathrm{arccosh}\left(cx\right)}{\left(e\,x^{2}+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

```
[Out] int((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{15}\,a\left(\frac{8\,x}{\sqrt{ex^{2}+d}\,d^{3}}+\frac{4\,x}{\left(ex^{2}+d\right)^{\frac{3}{2}}d^{2}}+\frac{3\,x}{\left(ex^{2}+d\right)^{\frac{5}{2}}d}\right)+b\int\frac{\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{\left(ex^{2}+d\right)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))/(e*x^2+d)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/15*a*(8*x/(sqrt(e*x^2 + d)*d^3) + 4*x/((e*x^2 + d)^(3/2)*d^2) + 3*x/((e*x
^2 + d)^(5/2)*d)) + b*integrate(log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x
^2 + d)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{a+b\,\mathrm{acosh}\left(c\,x\right)}{{\left(e\,x^2+d\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))/(d + e*x^2)^(7/2),x)
```

```
[Out] int((a + b*acosh(c*x))/(d + e*x^2)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))/(e*x**2+d)**(7/2),x)
```

```
[Out] Timed out
```

3.519 $\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=558

$$\frac{d^3(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{3d^2e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{3de^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(\cdots\right.}{f\cdots}$$

```
[Out] d^3*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+3*d^2*e*(f*x)^(3+m)*(a+b*arccosh
(c*x))/f^3/(3+m)+3*d*e^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)+e^3*(f*x)
^(7+m)*(a+b*arccosh(c*x))/f^7/(7+m)+b*e*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c
^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*m^3+119*m^2+342*m+360))*(f*x)^(2+m)*(-c^
2*x^2+1)/c^5/f^2/(3+m)^2/(5+m)^2/(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*e^2*
(3*c^2*d*(7+m)^2+e*(m^2+11*m+30))*(f*x)^(4+m)*(-c^2*x^2+1)/c^3/f^4/(5+m)^2/
(7+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)+b*e^3*(f*x)^(6+m)*(-c^2*x^2+1)/c/f^6/(7
+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/2)-b*(c^6*d^3*(3+m)*(5+m)*(7+m)/(1+m)+e*(2+m
)*(3*c^2*d*e*(7+m)^2*(m^2+7*m+12)+3*c^4*d^2*(m^2+12*m+35)^2+e^2*(m^4+18*m^3
+119*m^2+342*m+360))/(m^3+15*m^2+71*m+105))*(f*x)^(2+m)*hypergeom([1/2, 1+1
/2*m],[2+1/2*m],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^5/f^2/(2+m)/(3+m)/(5+m)/(7+m)
/(c*x-1)^(1/2)/(c*x+1)^(1/2)
```

Rubi [A] time = 2.81, antiderivative size = 529, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {270, 5790, 12, 1610, 1809, 1267, 459, 365, 364}

$$\frac{3d^2e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{d^3(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{3de^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{e^3(fx)^{m+7}\left(\cdots\right.}{f\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) + 3*c^4*d^2*(35 + 12*m + m^2)^2
+ e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^5*f^2*(3 + m)^2*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e
^2*(3*c^2*d*(7 + m)^2 + e*(30 + 11*m + m^2))*(f*x)^(4 + m)*(1 - c^2*x^2))/(
c^3*f^4*(5 + m)^2*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^3*(f*x)^(6
 + m)*(1 - c^2*x^2))/(c*f^6*(7 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^3*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (3*d^2*e*(f*x)^(3 + m)*(a
 + b*ArcCosh[c*x]))/(f^3*(3 + m)) + (3*d*e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c
*x]))/(f^5*(5 + m)) + (e^3*(f*x)^(7 + m)*(a + b*ArcCosh[c*x]))/(f^7*(7 + m)
) - (b*c*(d^3/(2 + 3*m + m^2) + (e*(3*c^2*d*e*(7 + m)^2*(12 + 7*m + m^2) +
3*c^4*d^2*(35 + 12*m + m^2)^2 + e^2*(360 + 342*m + 119*m^2 + 18*m^3 + m^4))
)/(c^6*(3 + m)^2*(5 + m)^2*(7 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hype
rgeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*Sqrt
[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)\,dx = \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{3d^2e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)}$$

$$= \frac{be^3(fx)^{6+m}\left(1-c^2x^2\right)}{cf^6(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{d^3(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} +$$

$$= \frac{be^2\left(3c^2d(7+m)^2+e\left(30+11m+m^2\right)\right)(fx)^{4+m}\left(1-c^2x^2\right)}{c^3f^4(5+m)^2(7+m)^2\sqrt{-1+cx}\,\sqrt{1+cx}} + \frac{}{cf^6}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\right.}{c^5f^2(3+m)^2(5+m)^2(7+m}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\right.}{c^5f^2(3+m)^2(5+m)^2(7+m}$$

$$= \frac{be\left(3c^2de(7+m)^2\left(12+7m+m^2\right)+3c^4d^2\left(35+12m+m^2\right)^2+e^2\right.}{c^5f^2(3+m)^2(5+m)^2(7+m}$$

Mathematica [A] time = 1.42, size = 397, normalized size = 0.71

$$x(fx)^m\left(\frac{d^3\left(a+b\cosh^{-1}(cx)\right)}{m+1} + \frac{3d^2ex^2\left(a+b\cosh^{-1}(cx)\right)}{m+3} + \frac{3de^2x^4\left(a+b\cosh^{-1}(cx)\right)}{m+5} + \frac{e^3x^6\left(a+b\cosh^{-1}\right.}{m+7}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^3*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*((d^3*(a + b*ArcCosh[c*x]))/(1 + m) + (3*d^2*e*x^2*(a + b*ArcCosh
[c*x]))/(3 + m) + (3*d*e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) + (e^3*x^6*(a
+ b*ArcCosh[c*x]))/(7 + m) - (b*c*e^3*x^7*Sqrt[1 - c^2*x^2]*Hypergeometric2
F1[1/2, 4 + m/2, 5 + m/2, c^2*x^2])/((7 + m)*(8 + m)*Sqrt[-1 + c*x]*Sqrt[1
+ c*x]) - (b*c*d^3*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4
 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*
d^2*e*x^3*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^
2*x^2])/((12 + 7*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (3*b*c*d*e^2*x^5*
Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((
5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^3x^6 + 3\,ade^2x^4 + 3\,ad^2ex^2 + ad^3 + \left(be^3x^6 + 3\,bde^2x^4 + 3\,bd^2ex^2 + bd^3\right)\operatorname{arcosh}(cx)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3 + (b*e^3*x^6 +
3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2 + d\right)^3 (b\operatorname{arcosh}(cx) + a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^3*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\left(e\,x^2 + d\right)^3 (a + b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^3f^mx^7x^m}{m+7} + \frac{3\,ade^2f^mx^5x^m}{m+5} + \frac{3\,ad^2ef^mx^3x^m}{m+3} + \frac{\left(fx\right)^{m+1}ad^3}{f(m+1)} + \frac{\left(\left(m^3 + 9\,m^2 + 23\,m + 15\right)be^3f^mx^7 + 3\left(m^3 + 11\,m^2 -\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^3*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^3*f^m*x^7*x^m/(m + 7) + 3*a*d*e^2*f^m*x^5*x^m/(m + 5) + 3*a*d^2*e*f^m*x
^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d^3/(f*(m + 1)) + ((m^3 + 9*m^2 + 23*m + 1
5)*b*e^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*d*e^2*f^m*x^5 + 3*(m^3 +
13*m^2 + 47*m + 35)*b*d^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*d^3*f^m
*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^4 + 16*m^3 + 86*m^2 + 176
*m + 105) + integrate(((m^3 + 9*m^2 + 23*m + 15)*b*c*e^3*f^m*x^7 + 3*(m^3 +
 11*m^2 + 31*m + 21)*b*c*d*e^2*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c*d
^2*e*f^m*x^3 + (m^3 + 15*m^2 + 71*m + 105)*b*c*d^3*f^m*x)*x^m/((m^4 + 16*m^
3 + 86*m^2 + 176*m + 105)*c^3*x^3 - (m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c
*x + ((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*c^2*x^2 - m^4 - 16*m^3 - 86*m^2
 - 176*m - 105)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate(((m^3 + 9*m^2
+ 23*m + 15)*b*c^2*e^3*f^m*x^8 + 3*(m^3 + 11*m^2 + 31*m + 21)*b*c^2*d*e^2*f
^m*x^6 + 3*(m^3 + 13*m^2 + 47*m + 35)*b*c^2*d^2*e*f^m*x^4 + (m^3 + 15*m^2 +
 71*m + 105)*b*c^2*d^3*f^m*x^2)*x^m/((m^4 + 16*m^3 + 86*m^2 + 176*m + 105)*
c^2*x^2 - m^4 - 16*m^3 - 86*m^2 - 176*m - 105), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + b\operatorname{acosh}(c\,x))\left(f\,x\right)^m\left(e\,x^2 + d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3,x)
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m (a + b \operatorname{acosh}(cx)) \left(d + ex^2\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**3*(a+b*acosh(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2)**3, x)
```

3.520 $\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx$

Optimal. Leaf size=353

$$\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}+\frac{be^2\left(1-c^2x^2\right)}{cf^4(m+5)^2\sqrt{cx -}}$$

```
[Out] d^2*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+2*d*e*(f*x)^(3+m)*(a+b*arccosh(c
*x))/f^3/(3+m)+e^2*(f*x)^(5+m)*(a+b*arccosh(c*x))/f^5/(5+m)+b*e*(2*c^2*d*(5
+m)^2+e*(m^2+7*m+12))*(f*x)^(2+m)*(-c^2*x^2+1)/c^3/f^2/(3+m)^2/(5+m)^2/(c*x
-1)^(1/2)/(c*x+1)^(1/2)+b*e^2*(f*x)^(4+m)*(-c^2*x^2+1)/c/f^4/(5+m)^2/(c*x-1
)^(1/2)/(c*x+1)^(1/2)-b*(c^4*d^2*(3+m)*(5+m)/(1+m)+e*(2+m)*(2*c^2*d*(5+m)^2
+e*(m^2+7*m+12))/(3+m)/(5+m))*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m
],c^2*x^2)*(-c^2*x^2+1)^(1/2)/c^3/f^2/(2+m)/(3+m)/(5+m)/(c*x-1)^(1/2)/(c*x+
1)^(1/2)
```

Rubi [A] time = 0.56, antiderivative size = 332, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {270, 5790, 12, 520, 1267, 459, 365, 364}

$$\frac{d^2(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{2de(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}+\frac{e^2(fx)^{m+5}\left(a+b\cosh^{-1}(cx)\right)}{f^5(m+5)}-\frac{bc\sqrt{1-c^2x^2}\,(fx)^{n}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] (b*e*(2*c^2*d*(5 + m)^2 + e*(12 + 7*m + m^2))*(f*x)^(2 + m)*(1 - c^2*x^2))/
(c^3*f^2*(3 + m)^2*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (b*e^2*(f*x)^(
4 + m)*(1 - c^2*x^2))/(c*f^4*(5 + m)^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) + (d^2
*(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (2*d*e*(f*x)^(3 + m)*(a
+ b*ArcCosh[c*x]))/(f^3*(3 + m)) + (e^2*(f*x)^(5 + m)*(a + b*ArcCosh[c*x]))
/(f^5*(5 + m)) - (b*c*(d^2/(2 + 3*m + m^2) + (e*(2*c^2*d*(5 + m)^2 + e*(12
+ 7*m + m^2)))/(c^4*(3 + m)^2*(5 + m)^2))*(f*x)^(2 + m)*Sqrt[1 - c^2*x^2]*H
ypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/(f^2*Sqrt[-1 + c*x]*S
qrt[1 + c*x])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(b*e*(m + n*(p
 + 1) + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 520

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.) + (e_.)*(x_)^(n2_.))^(q_.)*((a1_) + (b1_
.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.), x_Symbol] :>
Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 +
 b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n + e*x^(2*n)
)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, n, p, q}, x] && EqQ[non2, n/
2] && EqQ[n2, 2*n] && EqQ[a2*b1 + a1*b2, 0]
```

Rule 1267

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(c^p*(f*x)^(m + 4*p - 1)*(d + e*x^2)^
(q + 1))/(e*f^(4*p - 1)*(m + 4*p + 2*q + 1)), x] + Dist[1/(e*(m + 4*p + 2*q
 + 1)), Int[(f*x)^m*(d + e*x^2)^q*ExpandToSum[e*(m + 4*p + 2*q + 1)*((a + b
*x^2 + c*x^4)^p - c^p*x^(4*p)) - d*c^p*(m + 4*p - 1)*x^(4*p - 2), x], x], x
] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0
] &&  !IntegerQ[q] && NeQ[m + 4*p + 2*q + 1, 0]
```

Rule 5790

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2)^(p_.), x_Symbol] :> With[{u = IntHide[(f*x)^m*(d + e*x^2)^p, x]}, Dis
t[a + b*ArcCosh[c*x], u, x] - Dist[b*c, Int[SimplifyIntegrand[u/(Sqrt[1 + c
*x]*Sqrt[-1 + c*x]), x], x], x]] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[
c^2*d + e, 0] && IntegerQ[p] && (GtQ[p, 0] || (IGtQ[(m - 1)/2, 0] && LeQ[m
+ p, 0]))
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right) dx = \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2}{}$$

$$= \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2de(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} + \frac{e^2}{}$$

$$= \frac{be^2(fx)^{4+m}\left(1-c^2x^2\right)}{cf^4(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{d^2(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{2c}{}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(}{cf^4(5+}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(}{cf^4(5+}$$

$$= \frac{be\left(2c^2d(5+m)^2+e\left(12+7m+m^2\right)\right)(fx)^{2+m}\left(1-c^2x^2\right)}{c^3f^2(3+m)^2(5+m)^2\sqrt{-1+cx}\sqrt{1+cx}} + \frac{be^2(}{cf^4(5+}$$

Mathematica [A] time = 0.49, size = 293, normalized size = 0.83

$$x(fx)^m\left(\frac{d^2\left(a+b\cosh^{-1}(cx)\right)}{m+1} + \frac{2dex^2\left(a+b\cosh^{-1}(cx)\right)}{m+3} + \frac{e^2x^4\left(a+b\cosh^{-1}(cx)\right)}{m+5} - \frac{bcd^2x\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m}{}\right.}{\left(m^2+3m+2\right)\sqrt{cx-}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)^2*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*((d^2*(a + b*ArcCosh[c*x]))/(1 + m) + (2*d*e*x^2*(a + b*ArcCosh[c
*x]))/(3 + m) + (e^2*x^4*(a + b*ArcCosh[c*x]))/(5 + m) - (b*c*d^2*x*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, c^2*x^2])/((2 + 3*m
 + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x]) - (2*b*c*d*e*x^3*Sqrt[1 - c^2*x^2]*Hy
pergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((12 + 7*m + m^2)*Sqrt
[-1 + c*x]*Sqrt[1 + c*x]) - (b*c*e^2*x^5*Sqrt[1 - c^2*x^2]*Hypergeometric2F
1[1/2, (6 + m)/2, (8 + m)/2, c^2*x^2])/((5 + m)*(6 + m)*Sqrt[-1 + c*x]*Sqrt
[1 + c*x]))
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(ae^2x^4 + 2\,adex^2 + ad^2 + \left(be^2x^4 + 2\,bdex^2 + bd^2\right)\mathrm{arcosh}\left(cx\right)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*d^
2)*arccosh(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)^2 (b\operatorname{arcosh}(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m \left(e\,x^2+d\right)^2 (a+b\operatorname{arccosh}(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ae^2f^mx^5x^m}{m+5}+\frac{2\,adef^mx^3x^m}{m+3}+\frac{\left(fx\right)^{m+1}ad^2}{f(m+1)}+\frac{\left(\left(m^2+4\,m+3\right)be^2f^mx^5+2\left(m^2+6\,m+5\right)bdef^mx^3+\left(m^2+8\,m\right.}{m^3+9\,m^2+23\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^2*(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] a*e^2*f^m*x^5*x^m/(m + 5) + 2*a*d*e*f^m*x^3*x^m/(m + 3) + (f*x)^(m + 1)*a*d
^2/(f*(m + 1)) + ((m^2 + 4*m + 3)*b*e^2*f^m*x^5 + 2*(m^2 + 6*m + 5)*b*d*e*f
^m*x^3 + (m^2 + 8*m + 15)*b*d^2*f^m*x)*x^m*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1))/(m^3 + 9*m^2 + 23*m + 15) + integrate(((m^2 + 4*m + 3)*b*c*e^2*f^m*x
^5 + 2*(m^2 + 6*m + 5)*b*c*d*e*f^m*x^3 + (m^2 + 8*m + 15)*b*c*d^2*f^m*x)*x^
m/((m^3 + 9*m^2 + 23*m + 15)*c^3*x^3 - (m^3 + 9*m^2 + 23*m + 15)*c*x + ((m^
3 + 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15)*sqrt(c*x + 1)*sqr
t(c*x - 1)), x) - integrate(((m^2 + 4*m + 3)*b*c^2*e^2*f^m*x^6 + 2*(m^2 + 6
*m + 5)*b*c^2*d*e*f^m*x^4 + (m^2 + 8*m + 15)*b*c^2*d^2*f^m*x^2)*x^m/((m^3 +
 9*m^2 + 23*m + 15)*c^2*x^2 - m^3 - 9*m^2 - 23*m - 15), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))\left(f\,x\right)^m\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m (a+b\operatorname{acosh}(cx))\left(d+ex^2\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**2*(a+b*acosh(c*x)),x)
```

```
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

3.521 $\int (fx)^m \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)\,dx$

Optimal. Leaf size=198

$$\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}-\frac{b\sqrt{1-c^2x^2}\,(fx)^{m+2}\left(c^2d(m+3)^2+e(m+1)(m+2)\right)}{cf^2(m+1)(m+2)(m+3)^2\sqrt{cx-1}}$$

```
[Out] d*(f*x)^(1+m)*(a+b*arccosh(c*x))/f/(1+m)+e*(f*x)^(3+m)*(a+b*arccosh(c*x))/f
^3/(3+m)-b*e*(f*x)^(2+m)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c/f^2/(3+m)^2-b*(e*(1+
m)*(2+m)+c^2*d*(3+m)^2)*(f*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1/2*m],c^2*
x^2)*(-c^2*x^2+1)^(1/2)/c/f^2/(1+m)/(2+m)/(3+m)^2/(c*x-1)^(1/2)/(c*x+1)^(1/
2)
```

Rubi [A] time = 0.20, antiderivative size = 187, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {5786, 460, 126, 365, 364}

$$\frac{d(fx)^{m+1}\left(a+b\cosh^{-1}(cx)\right)}{f(m+1)}+\frac{e(fx)^{m+3}\left(a+b\cosh^{-1}(cx)\right)}{f^3(m+3)}-\frac{b\sqrt{1-c^2x^2}\,(fx)^{m+2}\left(\frac{c^2d}{m^2+3m+2}+\frac{e}{(m+3)^2}\right){}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\right.}{cf^2\sqrt{cx-1}\,\sqrt{cx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] -((b*e*(f*x)^(2 + m)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(c*f^2*(3 + m)^2)) + (d*
(f*x)^(1 + m)*(a + b*ArcCosh[c*x]))/(f*(1 + m)) + (e*(f*x)^(3 + m)*(a + b*A
rcCosh[c*x]))/(f^3*(3 + m)) - (b*(e/(3 + m)^2 + (c^2*d)/(2 + 3*m + m^2))*(f
*x)^(2 + m)*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2,
c^2*x^2])/(c*f^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])
```

Rule 126

```
Int[((f_.)*(x_))^(p_.)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.
), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[m])/(a*c + b
*d*x^2)^FracPart[m], Int[(a*c + b*d*x^2)^m*(f*x)^p, x], x] /; FreeQ[{a, b,
c, d, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[m - n, 0]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*(e*x)^
(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/
2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
```

```
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 5786

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x
_)^2), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*ArcCosh[c*x]))/(f*(m + 1))
, x] + (-Dist[(b*c)/(f*(m + 1)*(m + 3)), Int[((f*x)^(m + 1)*(d*(m + 3) + e*
(m + 1)*x^2))/(Sqrt[1 + c*x]*Sqrt[-1 + c*x]), x], x] + Simp[(e*(f*x)^(m + 3
)*(a + b*ArcCosh[c*x]))/(f^3*(m + 3)), x]) /; FreeQ[{a, b, c, d, e, f, m},
x] && NeQ[c^2*d + e, 0] && NeQ[m, -1] && NeQ[m, -3]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)dx = \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(fx)^{3+m}\left(a+b\cosh^{-1}(cx)\right)}{f^3(3+m)} - \frac{(bc)}{}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

$$= -\frac{be(fx)^{2+m}\sqrt{-1+cx}\sqrt{1+cx}}{cf^2(3+m)^2} + \frac{d(fx)^{1+m}\left(a+b\cosh^{-1}(cx)\right)}{f(1+m)} + \frac{e(}{}$$

Mathematica [A] time = 0.61, size = 186, normalized size = 0.94

$$x(fx)^m\left(\frac{\frac{\left(d(m+3)+e(m+1)x^2\right)\left(a+b\cosh^{-1}(cx)\right)}{m+1} - \frac{bcex^3\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+4}{2};\frac{m+6}{2};c^2x^2\right)}{(m+4)\sqrt{cx-1}\sqrt{cx+1}}}{m+3} - \frac{bcdx\sqrt{1-c^2x^2}\,{}_2F_1\left(\frac{1}{2},\frac{m+2}{2};\frac{m+4}{2};c^2x^2\right)}{\left(m^2+3m+2\right)\sqrt{cx-1}\sqrt{cx+1}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*ArcCosh[c*x]),x]
```

```
[Out] x*(f*x)^m*(-((b*c*d*x*Sqrt[1 - c^2*x^2]*Hypergeometric2F1[1/2, (2 + m)/2, (
4 + m)/2, c^2*x^2])/((2 + 3*m + m^2)*Sqrt[-1 + c*x]*Sqrt[1 + c*x])) + (((d*
(3 + m) + e*(1 + m)*x^2)*(a + b*ArcCosh[c*x]))/(1 + m) - (b*c*e*x^3*Sqrt[1
- c^2*x^2]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, c^2*x^2])/((4 + m)*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]))/(3 + m))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(aex^2 + ad + \left(bex^2 + bd\right)\mathrm{arcosh}\left(cx\right)\right)\left(fx\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x))*(f*x)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)(b\,\mathrm{arcosh}\,(cx)+a)\left(fx\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)*(f*x)^m, x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m\left(e\,x^2+d\right)(a+b\,\mathrm{arccosh}\,(cx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
[Out] int((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{aef^m x^3 x^m}{m+3}+\frac{\left(bef^m(m+1)x^3+bdf^m(m+3)x\right)x^m\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)}{m^2+4\,m+3}+\frac{\left(fx\right)^{m+1}ad}{f(m+1)}+\int\frac{}{\left(m^2+4\,m+3\right)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] a*e*f^m*x^3*x^m/(m + 3) + (b*e*f^m*(m + 1)*x^3 + b*d*f^m*(m + 3)*x)*x^m*log
(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(m^2 + 4*m + 3) + (f*x)^(m + 1)*a*d/(f*
(m + 1)) + integrate((b*c*e*f^m*(m + 1)*x^3 + b*c*d*f^m*(m + 3)*x)*x^m/((m^
2 + 4*m + 3)*c^3*x^3 - (m^2 + 4*m + 3)*c*x + ((m^2 + 4*m + 3)*c^2*x^2 - m^2
 - 4*m - 3)*sqrt(c*x + 1)*sqrt(c*x - 1)), x) - integrate((b*c^2*e*f^m*(m +
1)*x^4 + b*c^2*d*f^m*(m + 3)*x^2)*x^m/((m^2 + 4*m + 3)*c^2*x^2 - m^2 - 4*m
- 3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\,\mathrm{acosh}\,(c\,x))\left(f\,x\right)^m\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))*(f*x)^m*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^m(a+b\,\mathrm{acosh}\,(cx))\left(d+ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(a+b*acosh(c*x)),x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))*(d + e*x**2), x)
```

3.522 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx = \int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Mathematica [A] time = 9.99, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2),x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2), x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arcosh}(cx)+a)\left(fx\right)^m}{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}(cx)+a)\left(fx\right)^m}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

maple [A] time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{arccosh}(cx))}{e\,x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)(fx)^m}{ex^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b \operatorname{acosh}(c\,x))\,(f\,x)^m}{e\,x^2 + d}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2),x)
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (a + b \operatorname{acosh}(cx))}{d + ex^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d),x)
[Out] Integral((f*x)**m*(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.523 $\int \frac{(fx)^m(a+b\cosh^{-1}(cx))}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx=\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 9.38, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^2, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e^2*x^4 + 2*d*e*x^2 + d^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

maple [A] time = 1.23, size = 0, normalized size = 0.00

$$\int \frac{\left(f x\right)^{m} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)}{\left(e \,x^{2}+d \right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{{\left(b \operatorname{arcosh}\left(c x\right) + a\right)} \left(f x\right)^{m}}{{\left(e x^{2} + d\right)}^{2}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(f\,x\right)}^m}{{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2,x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.524 $\int \frac{(fx)^m\left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3},x\right)$$

```
[Out] Unintegrable((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
[Out] Defer[Int][((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

Rubi steps

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx = \int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Mathematica [A] time = 20.40, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m \left(a+b\cosh^{-1}(cx)\right)}{\left(d+ex^2\right)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3,x]
[Out] Integrate[((f*x)^m*(a + b*ArcCosh[c*x]))/(d + e*x^2)^3, x]
```

fricas [A] time = 1.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="fricas")
[Out] integral((b*arccosh(c*x) + a)*(f*x)^m/(e^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2
+ d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\text{arcosh}\,(cx)+a)\left(fx\right)^m}{\left(ex^2+d\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{\left(fx\right)^{m} \left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}{\left(e\,x^{2}+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

```
[Out] int((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)\left(fx\right)^{m}}{\left(ex^{2}+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(a+b*arccosh(c*x))/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)*(f*x)^m/(e*x^2 + d)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\left(f\,x\right)^{m}}{\left(e\,x^{2}+d\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3,x)
```

```
[Out] int(((a + b*acosh(c*x))*(f*x)^m)/(d + e*x^2)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(a+b*acosh(c*x))/(e*x**2+d)**3,x)
```

```
[Out] Timed out
```

3.525 $\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)^2 dx$

Optimal. Leaf size=609

$$-\frac{32be^3\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{245c^7}-\frac{16bde^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^5}-\frac{16be^3x^2\sqrt{cx-1}\sqrt{cx}}{2}$$

```
[Out] 2*b^2*d^3*x+4/3*b^2*d^2*e*x/c^2+16/25*b^2*d*e^2*x/c^4+32/245*b^2*e^3*x/c^6+
2/9*b^2*d^2*e*x^3+8/75*b^2*d*e^2*x^3/c^2+16/735*b^2*e^3*x^3/c^4+6/125*b^2*d
*e^2*x^5+12/1225*b^2*e^3*x^5/c^2+2/343*b^2*e^3*x^7+d^3*x*(a+b*arccosh(c*x))
^2+d^2*e*x^3*(a+b*arccosh(c*x))^2+3/5*d*e^2*x^5*(a+b*arccosh(c*x))^2+1/7*e^
3*x^7*(a+b*arccosh(c*x))^2-2*b*d^3*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c-4/3*b*d^2*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-16/
25*b*d*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^5-32/245*b*e^3*
(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^7-2/3*b*d^2*e*x^2*(a+b*arc
cosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/25*b*d*e^2*x^2*(a+b*arccosh(c*x)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-16/245*b*e^3*x^2*(a+b*arccosh(c*x))*(c*x-
1)^(1/2)*(c*x+1)^(1/2)/c^5-6/25*b*d*e^2*x^4*(a+b*arccosh(c*x))*(c*x-1)^(1/2
)*(c*x+1)^(1/2)/c-12/245*b*e^3*x^4*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c^3-2/49*b*e^3*x^6*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 2.10, antiderivative size = 609, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{4bd^2e\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{3c^3}-\frac{8bde^2x^2\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{25c^3}-\frac{16bde^2\sqrt{cx-1}\sqrt{cx}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^3*x + (4*b^2*d^2*e*x)/(3*c^2) + (16*b^2*d*e^2*x)/(25*c^4) + (32*b^2
*e^3*x)/(245*c^6) + (2*b^2*d^2*e*x^3)/9 + (8*b^2*d*e^2*x^3)/(75*c^2) + (16*
b^2*e^3*x^3)/(735*c^4) + (6*b^2*d*e^2*x^5)/125 + (12*b^2*e^3*x^5)/(1225*c^2
) + (2*b^2*e^3*x^7)/343 - (2*b*d^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcC
osh[c*x]))/c - (4*b*d^2*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(3*c^3) - (16*b*d*e^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(25*c^5) - (32*b*e^3*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(24
5*c^7) - (2*b*d^2*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(3*c) - (8*b*d*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/(
25*c^3) - (16*b*e^3*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/
(245*c^5) - (6*b*d*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(25*c) - (12*b*e^3*x^4*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])
)/(245*c^3) - (2*b*e^3*x^6*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]
))/(49*c) + d^3*x*(a + b*ArcCosh[c*x])^2 + d^2*e*x^3*(a + b*ArcCosh[c*x])^2
 + (3*d*e^2*x^5*(a + b*ArcCosh[c*x])^2)/5 + (e^3*x^7*(a + b*ArcCosh[c*x])^2
)/7
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)^3 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx = \int \left(d^3 \left(a+b\cosh^{-1}(cx)\right)^2 + 3d^2ex^2 \left(a+b\cosh^{-1}(cx)\right)^2 + 3de^2x^4 \left(a+\right.\right.$$

$$= d^3 \int \left(a+b\cosh^{-1}(cx)\right)^2 \, dx + \left(3d^2e\right) \int x^2 \left(a+b\cosh^{-1}(cx)\right)^2 \, dx + \left(\right.$$

$$= d^3x \left(a+b\cosh^{-1}(cx)\right)^2 + d^2ex^3 \left(a+b\cosh^{-1}(cx)\right)^2 + \frac{3}{5}de^2x^5 \left(a+b\,\mathrm{co}\right.$$

$$= -\frac{2bd^3\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{2bd^2ex^2\sqrt{-1+cx}\,\sqrt{1+}}{3c}$$

$$= 2b^2d^3x + \frac{2}{9}b^2d^2ex^3 + \frac{6}{125}b^2de^2x^5 + \frac{2}{343}b^2e^3x^7 - \frac{2bd^3\sqrt{-1+cx}\,\sqrt{1+}}{}$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{6}{125}b^2de^2x^5 + \frac{12b^2e^3x^5}{1225c^2} -$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{16b^2e^3x^3}{735c^4} + \frac{}{1}$$

$$= 2b^2d^3x + \frac{4b^2d^2ex}{3c^2} + \frac{16b^2de^2x}{25c^4} + \frac{32b^2e^3x}{245c^6} + \frac{2}{9}b^2d^2ex^3 + \frac{8b^2de^2x^3}{75c^2} + \frac{1}{}$$

Mathematica [A] time = 0.85, size = 453, normalized size = 0.74

$$\frac{11025a^2c^7x\left(35d^3+35d^2ex^2+21de^2x^4+5e^3x^6\right)-210ab\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^6\left(3675d^3+1225d^2ex^2+441de^2x\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^3*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (11025*a^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) - 210*a
*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*
c^4*e*(1225*d^2 + 294*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^
2 + 441*d*e^2*x^4 + 75*e^3*x^6)) + 2*b^2*c*x*(25200*e^3 + 840*c^2*e^2*(147*
d + 5*e*x^2) + 210*c^4*e*(1225*d^2 + 98*d*e*x^2 + 9*e^2*x^4) + c^6*(385875*
d^3 + 42875*d^2*e*x^2 + 9261*d*e^2*x^4 + 1125*e^3*x^6)) - 210*b*(-105*a*c^7
*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*e^2*x^4 + 5*e^3*x^6) + b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(240*e^3 + 24*c^2*e^2*(49*d + 5*e*x^2) + 2*c^4*e*(1225*d^2 + 29
4*d*e*x^2 + 45*e^2*x^4) + c^6*(3675*d^3 + 1225*d^2*e*x^2 + 441*d*e^2*x^4 +
75*e^3*x^6)))*ArcCosh[c*x] + 11025*b^2*c^7*x*(35*d^3 + 35*d^2*e*x^2 + 21*d*
e^2*x^4 + 5*e^3*x^6)*ArcCosh[c*x]^2)/(385875*c^7)
```

fricas [A] time = 0.60, size = 586, normalized size = 0.96

$$\frac{1125\left(49\,a^2+2\,b^2\right)c^7e^3x^7+189\left(49\left(25\,a^2+2\,b^2\right)c^7de^2+20\,b^2c^5e^3\right)x^5+35\left(1225\left(9\,a^2+2\,b^2\right)c^7d^2e+1176\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/385875*(1125*(49*a^2 + 2*b^2)*c^7*e^3*x^7 + 189*(49*(25*a^2 + 2*b^2)*c^7*
d*e^2 + 20*b^2*c^5*e^3)*x^5 + 35*(1225*(9*a^2 + 2*b^2)*c^7*d^2*e + 1176*b^2
*c^5*d*e^2 + 240*b^2*c^3*e^3)*x^3 + 11025*(5*b^2*c^7*e^3*x^7 + 21*b^2*c^7*d
*e^2*x^5 + 35*b^2*c^7*d^2*e*x^3 + 35*b^2*c^7*d^3*x)*log(c*x + sqrt(c^2*x^2
- 1))^2 + 105*(3675*(a^2 + 2*b^2)*c^7*d^3 + 4900*b^2*c^5*d^2*e + 2352*b^2*c
```

```
^3*d*e^2 + 480*b^2*c*e^3)*x + 210*(525*a*b*c^7*e^3*x^7 + 2205*a*b*c^7*d*e^2
*x^5 + 3675*a*b*c^7*d^2*e*x^3 + 3675*a*b*c^7*d^3*x - (75*b^2*c^6*e^3*x^6 +
3675*b^2*c^6*d^3 + 2450*b^2*c^4*d^2*e + 1176*b^2*c^2*d*e^2 + 240*b^2*e^3 +
9*(49*b^2*c^6*d*e^2 + 10*b^2*c^4*e^3)*x^4 + (1225*b^2*c^6*d^2*e + 588*b^2*c
^4*d*e^2 + 120*b^2*c^2*e^3)*x^2)*sqrt(c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2
- 1)) - 210*(75*a*b*c^6*e^3*x^6 + 3675*a*b*c^6*d^3 + 2450*a*b*c^4*d^2*e + 1
176*a*b*c^2*d*e^2 + 240*a*b*e^3 + 9*(49*a*b*c^6*d*e^2 + 10*a*b*c^4*e^3)*x^4
 + (1225*a*b*c^6*d^2*e + 588*a*b*c^4*d*e^2 + 120*a*b*c^2*e^3)*x^2)*sqrt(c^2
*x^2 - 1))/c^7
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [**A**] time = 0.14, size = 632, normalized size = 1.04

$$\frac{\frac{a^2\left(\frac{1}{7}e^3c^7x^7+\frac{3}{5}c^7d\,e^2x^5+c^7d^2e\,x^3+x\,c^7d^3\right)}{c^6}+b^2\left(\frac{e^3\left(3675\operatorname{arccosh}(cx)^2c^7x^7-1050\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}\,c^6x^6-1260\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}\,c^4x^4+150c^7x^7-1680\ldots}{25725}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^6*(1/7*e^3*c^7*x^7+3/5*c^7*d*e^2*x^5+c^7*d^2*e*x^3+x*c^7*d^3)+b^
2/c^6*(1/25725*e^3*(3675*arccosh(c*x)^2*c^7*x^7-1050*arccosh(c*x)*(c*x-1)^(
1/2)*(c*x+1)^(1/2)*c^6*x^6-1260*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^
4*x^4+150*c^7*x^7-1680*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+252
*c^5*x^5-3360*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+560*c^3*x^3+3360*c*x
)+1/375*d*e^2*c^2*(225*arccosh(c*x)^2*c^5*x^5-90*arccosh(c*x)*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*c^4*x^4-120*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2
+18*c^5*x^5-240*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+40*c^3*x^3+240*c*x
)+1/9*c^4*d^2*e*(9*arccosh(c*x)^2*c^3*x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x
+1)^(1/2)*c^2*x^2-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*
c*x)+d^3*c^6*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)
+2*c*x))+2*a*b/c^6*(1/7*arccosh(c*x)*e^3*c^7*x^7+3/5*arccosh(c*x)*d*e^2*c^7
*x^5+arccosh(c*x)*c^7*d^2*e*x^3+arccosh(c*x)*c^7*x*d^3-1/3675*(c*x-1)^(1/2)
*(c*x+1)^(1/2)*(75*c^6*e^3*x^6+441*c^6*d*e^2*x^4+1225*c^6*d^2*e*x^2+90*c^4*
e^3*x^4+3675*c^6*d^3+588*c^4*d*e^2*x^2+2450*c^4*d^2*e+120*c^2*e^3*x^2+1176*
c^2*d*e^2+240*e^3)))
```

maxima [**A**] time = 0.49, size = 684, normalized size = 1.12

$$\frac{1}{7}\,b^2e^3x^7\,\operatorname{arcosh}(cx)^2+\frac{1}{7}\,a^2e^3x^7+\frac{3}{5}\,b^2de^2x^5\,\operatorname{arcosh}(cx)^2+\frac{3}{5}\,a^2de^2x^5+b^2d^2ex^3\,\operatorname{arcosh}(cx)^2+a^2d^2ex^3+b^2d^3x\,\operatorname{arcos}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^3*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/7*b^2*e^3*x^7*arccosh(c*x)^2 + 1/7*a^2*e^3*x^7 + 3/5*b^2*d*e^2*x^5*arccos
h(c*x)^2 + 3/5*a^2*d*e^2*x^5 + b^2*d^2*e*x^3*arccosh(c*x)^2 + a^2*d^2*e*x^3
```

```
 + b^2*d^3*x*arccosh(c*x)^2 + 2/3*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1
)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d^2*e - 2/9*(3*c*(sqrt(c^2*x^2 -
1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) - (c^2*x^3 + 6*x)/c^2)*b
^2*d^2*e + 2/25*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqr
t(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*a*b*d*e^2 - 2/375*(15*
(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2
 - 1)/c^6)*c*arccosh(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 120*x)/c^4)*b^2*d*e^2
 + 2/245*(35*x^7*arccosh(c*x) - (5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x
^2 - 1)*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c
)*a*b*e^3 - 2/25725*(105*(5*sqrt(c^2*x^2 - 1)*x^6/c^2 + 6*sqrt(c^2*x^2 - 1)
*x^4/c^4 + 8*sqrt(c^2*x^2 - 1)*x^2/c^6 + 16*sqrt(c^2*x^2 - 1)/c^8)*c*arccos
h(c*x) - (75*c^6*x^7 + 126*c^4*x^5 + 280*c^2*x^3 + 1680*x)/c^6)*b^2*e^3 + 2
*b^2*d^3*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*d^3*x + 2*(c*x*arccos
h(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^3/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))^2\,\left(e\,x^2+d\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^3,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^3, x)
```

sympy [A] time = 13.78, size = 996, normalized size = 1.64

$$\begin{cases} a^2d^3x + a^2d^2ex^3 + \frac{3a^2de^2x^5}{5} + \frac{a^2e^3x^7}{7} + 2abd^3x\operatorname{acosh}(cx) + 2abd^2ex^3\operatorname{acosh}(cx) + \frac{6abde^2x^5\operatorname{acosh}(cx)}{5} + \frac{2abe^3x^7\operatorname{aco}}{7} \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^3x + d^2ex^3 + \frac{3de^2x^5}{5} + \frac{e^3x^7}{7}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**3*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**3*x + a**2*d**2*e*x**3 + 3*a**2*d*e**2*x**5/5 + a**2*e**
3*x**7/7 + 2*a*b*d**3*x*acosh(c*x) + 2*a*b*d**2*e*x**3*acosh(c*x) + 6*a*b*d
*e**2*x**5*acosh(c*x)/5 + 2*a*b*e**3*x**7*acosh(c*x)/7 - 2*a*b*d**3*sqrt(c*
*2*x**2 - 1)/c - 2*a*b*d**2*e*x**2*sqrt(c**2*x**2 - 1)/(3*c) - 6*a*b*d*e**2
*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 2*a*b*e**3*x**6*sqrt(c**2*x**2 - 1)/(49*
c) - 4*a*b*d**2*e*sqrt(c**2*x**2 - 1)/(3*c**3) - 8*a*b*d*e**2*x**2*sqrt(c**
2*x**2 - 1)/(25*c**3) - 12*a*b*e**3*x**4*sqrt(c**2*x**2 - 1)/(245*c**3) - 1
6*a*b*d*e**2*sqrt(c**2*x**2 - 1)/(25*c**5) - 16*a*b*e**3*x**2*sqrt(c**2*x**
2 - 1)/(245*c**5) - 32*a*b*e**3*sqrt(c**2*x**2 - 1)/(245*c**7) + b**2*d**3*
x*acosh(c*x)**2 + 2*b**2*d**3*x + b**2*d**2*e*x**3*acosh(c*x)**2 + 2*b**2*d
**2*e*x**3/9 + 3*b**2*d*e**2*x**5*acosh(c*x)**2/5 + 6*b**2*d*e**2*x**5/125
+ b**2*e**3*x**7*acosh(c*x)**2/7 + 2*b**2*e**3*x**7/343 - 2*b**2*d**3*sqrt(
c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*d**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(
c*x)/(3*c) - 6*b**2*d*e**2*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) - 2*b
**2*e**3*x**6*sqrt(c**2*x**2 - 1)*acosh(c*x)/(49*c) + 4*b**2*d**2*e*x/(3*c*
*2) + 8*b**2*d*e**2*x**3/(75*c**2) + 12*b**2*e**3*x**5/(1225*c**2) - 4*b**2
*d**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(3*c**3) - 8*b**2*d*e**2*x**2*sqrt(c
**2*x**2 - 1)*acosh(c*x)/(25*c**3) - 12*b**2*e**3*x**4*sqrt(c**2*x**2 - 1)*
acosh(c*x)/(245*c**3) + 16*b**2*d*e**2*x/(25*c**4) + 16*b**2*e**3*x**3/(735
*c**4) - 16*b**2*d*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c**5) - 16*b**2*
e**3*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**5) + 32*b**2*e**3*x/(245*c
**6) - 32*b**2*e**3*sqrt(c**2*x**2 - 1)*acosh(c*x)/(245*c**7), Ne(c, 0)), (
(a + I*pi*b/2)**2*(d**3*x + d**2*e*x**3 + 3*d*e**2*x**5/5 + e**3*x**7/7), T
rue))
```

3.526 $\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=359

$$-\frac{16be^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{75c^5}-\frac{8bde\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{8be^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,(a}{75c^3}$$

```
[Out] 2*b^2*d^2*x+8/9*b^2*d*e*x/c^2+16/75*b^2*e^2*x/c^4+4/27*b^2*d*e*x^3+8/225*b^
2*e^2*x^3/c^2+2/125*b^2*e^2*x^5+d^2*x*(a+b*arccosh(c*x))^2+2/3*d*e*x^3*(a+b
*arccosh(c*x))^2+1/5*e^2*x^5*(a+b*arccosh(c*x))^2-2*b*d^2*(a+b*arccosh(c*x)
)*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/9*b*d*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*
(c*x+1)^(1/2)/c^3-16/75*b*e^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2
)/c^5-4/9*b*d*e*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c-8/75*b
*e^2*x^2*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/25*b*e^2*x^4*
(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 1.20, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{8bde\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{9c^3}-\frac{8be^2x^2\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{75c^3}-\frac{16be^2\sqrt{cx-1}\,\sqrt{cx+1}\,(a}{75c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d^2*x + (8*b^2*d*e*x)/(9*c^2) + (16*b^2*e^2*x)/(75*c^4) + (4*b^2*d*e*
x^3)/27 + (8*b^2*e^2*x^3)/(225*c^2) + (2*b^2*e^2*x^5)/125 - (2*b*d^2*Sqrt[-
1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (8*b*d*e*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(a + b*ArcCosh[c*x]))/(9*c^3) - (16*b*e^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(75*c^5) - (4*b*d*e*x^2*Sqrt[-1 + c*x]*Sqrt[1
 + c*x]*(a + b*ArcCosh[c*x]))/(9*c) - (8*b*e^2*x^2*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(75*c^3) - (2*b*e^2*x^4*Sqrt[-1 + c*x]*Sqrt[1 +
c*x]*(a + b*ArcCosh[c*x]))/(25*c) + d^2*x*(a + b*ArcCosh[c*x])^2 + (2*d*e*x
^3*(a + b*ArcCosh[c*x])^2)/3 + (e^2*x^5*(a + b*ArcCosh[c*x])^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \left(a+b\cosh^{-1}(cx)\right)^2 dx = \int \left(d^2\left(a+b\cosh^{-1}(cx)\right)^2 + 2dex^2\left(a+b\cosh^{-1}(cx)\right)^2 + e^2x^4\left(a+b\,\mathrm{co}\right.\right.$$

$$= d^2\int \left(a+b\cosh^{-1}(cx)\right)^2 dx + (2de)\int x^2\left(a+b\cosh^{-1}(cx)\right)^2 dx + e^2$$

$$= d^2x\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{2}{3}dex^3\left(a+b\cosh^{-1}(cx)\right)^2 + \frac{1}{5}e^2x^5\left(a+b\,\mathrm{co}\right.$$

$$= -\frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c} - \frac{4bdex^2\sqrt{-1+cx}\,\sqrt{1+c}}{9c}$$

$$= 2b^2d^2x + \frac{4}{27}b^2dex^3 + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh\right.}{c}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2bd^2\sqrt{-1+cx}}{}$$

$$= 2b^2d^2x + \frac{8b^2dex}{9c^2} + \frac{16b^2e^2x}{75c^4} + \frac{4}{27}b^2dex^3 + \frac{8b^2e^2x^3}{225c^2} + \frac{2}{125}b^2e^2x^5 - \frac{2b}{}$$

Mathematica [A] time = 0.54, size = 299, normalized size = 0.83

$$\frac{225a^2c^5x\left(15d^2+10dex^2+3e^2x^4\right) - 30ab\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^4\left(225d^2+50dex^2+9e^2x^4\right)+4c^2e\left(25d+3ex^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (225*a^2*c^5*x*(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4) - 30*a*b*Sqrt[-1 + c*x]*Sq
rt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^2) + c^4*(225*d^2 + 50*d*e*x^2
+ 9*e^2*x^4)) + 2*b^2*c*x*(360*e^2 + 60*c^2*e*(25*d + e*x^2) + c^4*(3375*d^
2 + 250*d*e*x^2 + 27*e^2*x^4)) - 30*b*(-15*a*c^5*x*(15*d^2 + 10*d*e*x^2 + 3
*e^2*x^4) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(24*e^2 + 4*c^2*e*(25*d + 3*e*x^
2) + c^4*(225*d^2 + 50*d*e*x^2 + 9*e^2*x^4)))*ArcCosh[c*x] + 225*b^2*c^5*x*
(15*d^2 + 10*d*e*x^2 + 3*e^2*x^4)*ArcCosh[c*x]^2)/(3375*c^5)
```

fricas [A] time = 0.72, size = 380, normalized size = 1.06

$$\frac{27\left(25\,a^2+2\,b^2\right)c^5e^2x^5+10\left(25\left(9\,a^2+2\,b^2\right)c^5de+12\,b^2c^3e^2\right)x^3+225\left(3\,b^2c^5e^2x^5+10\,b^2c^5dex^3+15\,b^2c^5d^2x\right)\mathrm{lo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*a^2 + 2*b^2)*c^5*e^2*x^5 + 10*(25*(9*a^2 + 2*b^2)*c^5*d*e +
12*b^2*c^3*e^2)*x^3 + 225*(3*b^2*c^5*e^2*x^5 + 10*b^2*c^5*d*e*x^3 + 15*b^2*
c^5*d^2*x)*log(c*x + sqrt(c^2*x^2 - 1))^2 + 15*(225*(a^2 + 2*b^2)*c^5*d^2 +
 200*b^2*c^3*d*e + 48*b^2*c*e^2)*x + 30*(45*a*b*c^5*e^2*x^5 + 150*a*b*c^5*d
*e*x^3 + 225*a*b*c^5*d^2*x - (9*b^2*c^4*e^2*x^4 + 225*b^2*c^4*d^2 + 100*b^2
*c^2*d*e + 24*b^2*e^2 + 2*(25*b^2*c^4*d*e + 6*b^2*c^2*e^2)*x^2)*sqrt(c^2*x^
2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 30*(9*a*b*c^4*e^2*x^4 + 225*a*b*c^4*
d^2 + 100*a*b*c^2*d*e + 24*a*b*e^2 + 2*(25*a*b*c^4*d*e + 6*a*b*c^2*e^2)*x^2
)*sqrt(c^2*x^2 - 1))/c^5
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.12, size = 402, normalized size = 1.12

$$\frac{\frac{a^2\left(\frac{1}{5}e^2c^5x^5+\frac{2}{3}c^5de\,x^3+x\,c^5d^2\right)}{c^4}+\frac{b^2\left(\frac{e^2\left(225\operatorname{arccosh}(cx)^2c^5x^5-90\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}\,c^4x^4-120\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}\,c^2x^2+18c^5x^5-240\operatorname{arccosh}(cx)\sqrt{cx-1}\right.}{1125}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^4*(1/5*e^2*c^5*x^5+2/3*c^5*d*e*x^3+x*c^5*d^2)+b^2/c^4*(1/1125*e^
2*(225*arccosh(c*x)^2*c^5*x^5-90*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c
^4*x^4-120*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2+18*c^5*x^5-240*
arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+40*c^3*x^3+240*c*x)+2/27*c^2*d*e*(
9*arccosh(c*x)^2*c^3*x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2
-12*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*c*x)+d^2*c^4*(arc
cosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))+2*a*b/c^
4*(1/5*arccosh(c*x)*e^2*c^5*x^5+2/3*arccosh(c*x)*c^5*d*e*x^3+arccosh(c*x)*c
^5*x*d^2-1/225*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(9*c^4*e^2*x^4+50*c^4*d*e*x^2+22
5*c^4*d^2+12*c^2*e^2*x^2+100*c^2*d*e+24*e^2)))
```

maxima [A] time = 0.43, size = 429, normalized size = 1.19

$$\frac{1}{5}b^2e^2x^5\operatorname{arcosh}(cx)^2+\frac{1}{5}a^2e^2x^5+\frac{2}{3}b^2dex^3\operatorname{arcosh}(cx)^2+\frac{2}{3}a^2dex^3+b^2d^2x\operatorname{arcosh}(cx)^2+\frac{4}{9}\left(3x^3\operatorname{arcosh}(cx)-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*e^2*x^5*arccosh(c*x)^2 + 1/5*a^2*e^2*x^5 + 2/3*b^2*d*e*x^3*arccosh(
c*x)^2 + 2/3*a^2*d*e*x^3 + b^2*d^2*x*arccosh(c*x)^2 + 4/9*(3*x^3*arccosh(c*
x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4))*a*b*d*e - 4/2
7*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4)*arccosh(c*x) -
 (c^2*x^3 + 6*x)/c^2)*b^2*d*e + 2/75*(15*x^5*arccosh(c*x) - (3*sqrt(c^2*x^2
 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c)*a
*b*e^2 - 2/1125*(15*(3*sqrt(c^2*x^2 - 1)*x^4/c^2 + 4*sqrt(c^2*x^2 - 1)*x^2/
c^4 + 8*sqrt(c^2*x^2 - 1)/c^6)*c*arccosh(c*x) - (9*c^4*x^5 + 20*c^2*x^3 + 1
20*x)/c^4)*b^2*e^2 + 2*b^2*d^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2
*d^2*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*d^2/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\operatorname{acosh}(c\,x))^2\,\left(e\,x^2+d\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^2, x)
```

sympy [A] time = 4.77, size = 602, normalized size = 1.68

$$\begin{cases} a^2d^2x+\frac{2a^2dex^3}{3}+\frac{a^2e^2x^5}{5}+2abd^2x\operatorname{acosh}(cx)+\frac{4abdex^3\operatorname{acosh}(cx)}{3}+\frac{2abe^2x^5\operatorname{acosh}(cx)}{5}-\frac{2abd^2\sqrt{c^2x^2-1}}{c}-\frac{4abdex^2\sqrt{c^2x^2-1}}{9c} \\ \left(a+\frac{i\pi b}{2}\right)^2\left(d^2x+\frac{2dex^3}{3}+\frac{e^2x^5}{5}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*x**3/3 + a**2*e**2*x**5/5 + 2*a*b*d**2*
x*acosh(c*x) + 4*a*b*d*e*x**3*acosh(c*x)/3 + 2*a*b*e**2*x**5*acosh(c*x)/5 -
 2*a*b*d**2*sqrt(c**2*x**2 - 1)/c - 4*a*b*d*e*x**2*sqrt(c**2*x**2 - 1)/(9*c
) - 2*a*b*e**2*x**4*sqrt(c**2*x**2 - 1)/(25*c) - 8*a*b*d*e*sqrt(c**2*x**2 -
 1)/(9*c**3) - 8*a*b*e**2*x**2*sqrt(c**2*x**2 - 1)/(75*c**3) - 16*a*b*e**2*
sqrt(c**2*x**2 - 1)/(75*c**5) + b**2*d**2*x*acosh(c*x)**2 + 2*b**2*d**2*x +
 2*b**2*d*e*x**3*acosh(c*x)**2/3 + 4*b**2*d*e*x**3/27 + b**2*e**2*x**5*acos
h(c*x)**2/5 + 2*b**2*e**2*x**5/125 - 2*b**2*d**2*sqrt(c**2*x**2 - 1)*acosh(
c*x)/c - 4*b**2*d*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c) - 2*b**2*e**2
*x**4*sqrt(c**2*x**2 - 1)*acosh(c*x)/(25*c) + 8*b**2*d*e*x/(9*c**2) + 8*b**
2*e**2*x**3/(225*c**2) - 8*b**2*d*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*c**3)
 - 8*b**2*e**2*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**3) + 16*b**2*e**2
*x/(75*c**4) - 16*b**2*e**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/(75*c**5), Ne(c,
 0)), ((a + I*pi*b/2)**2*(d**2*x + 2*d*e*x**3/3 + e**2*x**5/5), True))
```

3.527 $\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=168

$$-\frac{4be\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{1}{3}ex^3$$

```
[Out] 2*b^2*d*x+4/9*b^2*e*x/c^2+2/27*b^2*e*x^3+d*x*(a+b*arccosh(c*x))^2+1/3*e*x^3
*(a+b*arccosh(c*x))^2-2*b*d*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/
c-4/9*b*e*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c^3-2/9*b*e*x^2*(a
+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)^(1/2)/c
```

Rubi [A] time = 0.57, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5654, 5718, 8, 5662, 5759, 30}

$$-\frac{4be\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{9c^3}+dx\left(a+b\cosh^{-1}(cx)\right)^2-\frac{2bd\sqrt{cx-1}\sqrt{cx+1}\left(a+b\cosh^{-1}(cx)\right)}{c}+\frac{1}{3}ex^3$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*d*x + (4*b^2*e*x)/(9*c^2) + (2*b^2*e*x^3)/27 - (2*b*d*Sqrt[-1 + c*x]*
Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c - (4*b*e*Sqrt[-1 + c*x]*Sqrt[1 + c*x]
*(a + b*ArcCosh[c*x]))/(9*c^3) - (2*b*e*x^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a
 + b*ArcCosh[c*x]))/(9*c) + d*x*(a + b*ArcCosh[c*x])^2 + (e*x^3*(a + b*ArcC
osh[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5662

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
 :> Simp[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^n)/(d*(m + 1)), x] - Dist[(b*c
*n)/(d*(m + 1)), Int[((d*x)^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1
+ c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &&
 NeQ[m, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2\,dx = \int \left(d\left(a+b\cosh^{-1}(cx)\right)^2+ex^2\left(a+b\cosh^{-1}(cx)\right)^2\right)dx$$
$$= d\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx + e\int x^2\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$
$$= dx\left(a+b\cosh^{-1}(cx)\right)^2+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^2-(2bcd)\int \frac{x\left(a+b\right.}{\sqrt{-1+}}$$
$$= -\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{2bex^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(\right.}{9c}$$
$$= 2b^2dx+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}-\frac{4be\sqrt{-}}{}$$
$$= 2b^2dx+\frac{4b^2ex}{9c^2}+\frac{2}{27}b^2ex^3-\frac{2bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}$$

Mathematica [A] time = 0.28, size = 174, normalized size = 1.04

$$\frac{9a^2c^3x\left(3d+ex^2\right)-6ab\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2\left(9d+ex^2\right)+2e\right)-6b\cosh^{-1}(cx)\left(b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(c^2\left(9d+ex^2\right.\right.\right.}{27c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x*(3*d + e*x^2) - 6*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*
(9*d + e*x^2)) + 2*b^2*c*x*(6*e + c^2*(27*d + e*x^2)) - 6*b*(-3*a*c^3*x*(3*
d + e*x^2) + b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(2*e + c^2*(9*d + e*x^2)))*ArcC
osh[c*x] + 9*b^2*c^3*x*(3*d + e*x^2)*ArcCosh[c*x]^2)/(27*c^3)
```

fricas [A] time = 0.67, size = 209, normalized size = 1.24

$$\frac{\left(9\,a^2+2\,b^2\right)c^3ex^3+9\left(b^2c^3ex^3+3\,b^2c^3dx\right)\log\left(cx+\sqrt{c^2x^2-1}\right)^2+3\left(9\left(a^2+2\,b^2\right)c^3d+4\,b^2ce\right)x+6\left(3\,abc^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*((9*a^2 + 2*b^2)*c^3*e*x^3 + 9*(b^2*c^3*e*x^3 + 3*b^2*c^3*d*x)*log(c*x
 + sqrt(c^2*x^2 - 1))^2 + 3*(9*(a^2 + 2*b^2)*c^3*d + 4*b^2*c*e)*x + 6*(3*a*
b*c^3*e*x^3 + 9*a*b*c^3*d*x - (b^2*c^2*e*x^2 + 9*b^2*c^2*d + 2*b^2*e)*sqrt(
c^2*x^2 - 1))*log(c*x + sqrt(c^2*x^2 - 1)) - 6*(a*b*c^2*e*x^2 + 9*a*b*c^2*d
 + 2*a*b*e)*sqrt(c^2*x^2 - 1))/c^3
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

maple [A] time = 0.10, size = 217, normalized size = 1.29

$$\frac{\frac{a^2\left(\frac{1}{3}c^3x^3e+c^3dx\right)}{c^2}+\frac{b^2\left(\frac{e\left(9\operatorname{arccosh}(cx)^2c^3x^3-6\operatorname{arccosh}(cx)\sqrt{cx-1}\ \sqrt{cx+1}\ c^2x^2-12\operatorname{arccosh}(cx)\sqrt{cx-1}\ \sqrt{cx+1}+2c^3x^3+12cx\right)}{27}+c^2d\left(\operatorname{arccosh}(cx)^2cx-2\operatorname{arccosh}(cx)\sqrt{c}\right.}{c^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2/c^2*(1/3*c^3*x^3*e+c^3*d*x)+b^2/c^2*(1/27*e*(9*arccosh(c*x)^2*c^3*
x^3-6*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)*c^2*x^2-12*arccosh(c*x)*(c*x
-1)^(1/2)*(c*x+1)^(1/2)+2*c^3*x^3+12*c*x)+c^2*d*(arccosh(c*x)^2*c*x-2*arcco
sh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(1/2)+2*c*x))+2*a*b/c^2*(1/3*arccosh(c*x)*c^3
*x^3*e+arccosh(c*x)*c^3*d*x-1/9*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(c^2*e*x^2+9*c^
2*d+2*e)))
```

maxima [A] time = 0.51, size = 218, normalized size = 1.30

$$\frac{1}{3}b^2ex^3\operatorname{arcosh}(cx)^2+\frac{1}{3}a^2ex^3+b^2dx\operatorname{arcosh}(cx)^2+\frac{2}{9}\left(3x^3\operatorname{arcosh}(cx)-c\left(\frac{\sqrt{c^2x^2-1}\,x^2}{c^2}+\frac{2\sqrt{c^2x^2-1}}{c^4}\right)\right)abe-\frac{2}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*e*x^3*arccosh(c*x)^2 + 1/3*a^2*e*x^3 + b^2*d*x*arccosh(c*x)^2 + 2/9
*(3*x^3*arccosh(c*x) - c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c
^4))*a*b*e - 2/27*(3*c*(sqrt(c^2*x^2 - 1)*x^2/c^2 + 2*sqrt(c^2*x^2 - 1)/c^4
)*arccosh(c*x) - (c^2*x^3 + 6*x)/c^2)*b^2*e + 2*b^2*d*(x - sqrt(c^2*x^2 - 1
)*arccosh(c*x)/c) + a^2*d*x + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b*
d/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\operatorname{acosh}(c\,x))^2\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2), x)
```

sympy [A] time = 1.36, size = 286, normalized size = 1.70

$$\begin{cases} a^2dx + \frac{a^2ex^3}{3} + 2abdx\operatorname{acosh}(cx) + \frac{2abex^3\operatorname{acosh}(cx)}{3} - \frac{2abd\sqrt{c^2x^2-1}}{c} - \frac{2abex^2\sqrt{c^2x^2-1}}{9c} - \frac{4abe\sqrt{c^2x^2-1}}{9c^3} + b^2dx\operatorname{acosh}^2(c \\ \left(a + \frac{i\pi b}{2}\right)^2\left(dx + \frac{ex^3}{3}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*d*x + a**2*e*x**3/3 + 2*a*b*d*x*acosh(c*x) + 2*a*b*e*x**3*a
cosh(c*x)/3 - 2*a*b*d*sqrt(c**2*x**2 - 1)/c - 2*a*b*e*x**2*sqrt(c**2*x**2 -
 1)/(9*c) - 4*a*b*e*sqrt(c**2*x**2 - 1)/(9*c**3) + b**2*d*x*acosh(c*x)**2 +
 2*b**2*d*x + b**2*e*x**3*acosh(c*x)**2/3 + 2*b**2*e*x**3/27 - 2*b**2*d*sqr
t(c**2*x**2 - 1)*acosh(c*x)/c - 2*b**2*e*x**2*sqrt(c**2*x**2 - 1)*acosh(c*x
)/(9*c) + 4*b**2*e*x/(9*c**2) - 4*b**2*e*sqrt(c**2*x**2 - 1)*acosh(c*x)/(9*
c**3), Ne(c, 0)), ((a + I*pi*b/2)**2*(d*x + e*x**3/3), True))
```

3.528 $\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=51

$$-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+2b^2x$$

```
[Out] 2*b^2*x+x*(a+b*arccosh(c*x))^2-2*b*(a+b*arccosh(c*x))*(c*x-1)^(1/2)*(c*x+1)
^(1/2)/c
```

Rubi [A] time = 0.16, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {5654, 5718, 8}

$$-\frac{2b\sqrt{cx-1}\,\sqrt{cx+1}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+2b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] 2*b^2*x - (2*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x]))/c + x*(a
+ b*ArcCosh[c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\cosh^{-1}(cx)\right)^2\,dx &= x\left(a+b\cosh^{-1}(cx)\right)^2-(2bc)\int \frac{x\left(a+b\cosh^{-1}(cx)\right)}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx\\ &= -\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2+\left(2b^2\right)\int 1\,dx\\ &= 2b^2x-\frac{2b\sqrt{-1+cx}\,\sqrt{1+cx}\,\left(a+b\cosh^{-1}(cx)\right)}{c}+x\left(a+b\cosh^{-1}(cx)\right)^2\end{aligned}$$

Mathematica [A] time = 0.08, size = 84, normalized size = 1.65

$$x\left(a^2+2b^2\right)-\frac{2ab\sqrt{cx-1}\,\sqrt{cx+1}}{c}+\frac{2b\cosh^{-1}(cx)\left(acx-b\sqrt{cx-1}\,\sqrt{cx+1}\right)}{c}+b^2x\cosh^{-1}(cx)^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] (a^2 + 2*b^2)*x - (2*a*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/c + (2*b*(a*c*x - b*
Sqrt[-1 + c*x]*Sqrt[1 + c*x])*ArcCosh[c*x])/c + b^2*x*ArcCosh[c*x]^2
```

fricas [B] time = 0.70, size = 96, normalized size = 1.88

$$\frac{b^2cx\log\left(cx+\sqrt{c^2x^2-1}\right)^2+\left(a^2+2\,b^2\right)cx-2\,\sqrt{c^2x^2-1}\,ab+2\left(abcx-\sqrt{c^2x^2-1}\,b^2\right)\log\left(cx+\sqrt{c^2x^2-1}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*log(c*x + sqrt(c^2*x^2 - 1))^2 + (a^2 + 2*b^2)*c*x - 2*sqrt(c^2*x^
2 - 1)*a*b + 2*(a*b*c*x - sqrt(c^2*x^2 - 1)*b^2)*log(c*x + sqrt(c^2*x^2 - 1
)))/c
```

giac [B] time = 0.48, size = 111, normalized size = 2.18

$$2\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)-\frac{\sqrt{c^2x^2-1}}{c}\right)ab+\left(x\log\left(cx+\sqrt{c^2x^2-1}\right)^2+2\,c\left(\frac{x}{c}-\frac{\sqrt{c^2x^2-1}\,\log\left(cx+\sqrt{c^2x^2-\right.}}{c^2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] 2*(x*log(c*x + sqrt(c^2*x^2 - 1)) - sqrt(c^2*x^2 - 1)/c)*a*b + (x*log(c*x +
 sqrt(c^2*x^2 - 1))^2 + 2*c*(x/c - sqrt(c^2*x^2 - 1)*log(c*x + sqrt(c^2*x^2
 - 1))/c^2))*b^2 + a^2*x
```

maple [A] time = 0.07, size = 78, normalized size = 1.53

$$\frac{a^2cx+b^2\left(\operatorname{arccosh}(cx)^2cx-2\operatorname{arccosh}(cx)\sqrt{cx-1}\,\sqrt{cx+1}+2cx\right)+2ab\left(cx\operatorname{arccosh}(cx)-\sqrt{cx-1}\,\sqrt{cx+\right.}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(a^2*c*x+b^2*(arccosh(c*x)^2*c*x-2*arccosh(c*x)*(c*x-1)^(1/2)*(c*x+1)^(
1/2)+2*c*x)+2*a*b*(c*x*arccosh(c*x)-(c*x-1)^(1/2)*(c*x+1)^(1/2)))
```

maxima [A] time = 0.31, size = 72, normalized size = 1.41

$$b^2x\operatorname{arcosh}(cx)^2+2\,b^2\left(x-\frac{\sqrt{c^2x^2-1}\,\operatorname{arcosh}(cx)}{c}\right)+a^2x+\frac{2\left(cx\operatorname{arcosh}(cx)-\sqrt{c^2x^2-1}\right)ab}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] b^2*x*arccosh(c*x)^2 + 2*b^2*(x - sqrt(c^2*x^2 - 1)*arccosh(c*x)/c) + a^2*x
 + 2*(c*x*arccosh(c*x) - sqrt(c^2*x^2 - 1))*a*b/c
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int(a+b\operatorname{acosh}(c\,x))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2,x)
```

```
[Out] int((a + b*acosh(c*x))^2, x)
```

sympy [A] time = 0.27, size = 88, normalized size = 1.73

$$\begin{cases} a^2x + 2abx\operatorname{acosh}(cx) - \frac{2ab\sqrt{c^2x^2-1}}{c} + b^2x\operatorname{acosh}^2(cx) + 2b^2x - \frac{2b^2\sqrt{c^2x^2-1}\operatorname{acosh}(cx)}{c} & \text{for } c \neq 0 \\ x\left(a + \frac{i\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*acosh(c*x) - 2*a*b*sqrt(c**2*x**2 - 1)/c + b**2
*x*acosh(c*x)**2 + 2*b**2*x - 2*b**2*sqrt(c**2*x**2 - 1)*acosh(c*x)/c, Ne(c
, 0)), (x*(a + I*pi*b/2)**2, True))
```

3.529 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx$

Optimal. Leaf size=763

$$-\frac{b\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}_2\left(-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\left(a+b\cosh^{-1}(cx)\right)\mathrm{Li}}{\sqrt{-d}\,\sqrt{}}$$

```
[Out] 1/2*(a+b*arccosh(c*x))^2*ln(1-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2*(a+b*arccosh(c*x))^2*l
n(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2
)))/(-d)^(1/2)/e^(1/2)+1/2*(a+b*arccosh(c*x))^2*ln(1-(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-1/2
*(a+b*arccosh(c*x))^2*ln(1+(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d
)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b*(a+b*arccosh(c*x))*polylog(
2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)
))/(-d)^(1/2)/e^(1/2)+b*(a+b*arccosh(c*x))*polylog(2,(c*x+(c*x-1)^(1/2)*(c*
x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b*(
a+b*arccosh(c*x))*polylog(2,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(
-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)+b*(a+b*arccosh(c*x))*polylo
g(2,(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2
)))/(-d)^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(
1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x
+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)-(-c^2*d-e)^(1/2)))/(-d)
^(1/2)/e^(1/2)+b^2*polylog(3,-(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*
(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e^(1/2)-b^2*polylog(3,(c*x+(c*x-1)
^(1/2)*(c*x+1)^(1/2))*e^(1/2)/(c*(-d)^(1/2)+(-c^2*d-e)^(1/2)))/(-d)^(1/2)/e
^(1/2)
```

Rubi [A] time = 1.31, antiderivative size = 763, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5707, 5800, 5562, 2190, 2531, 2282, 6589}

$$-\frac{b\left(a+b\cosh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}+\frac{b\left(a+b\cosh^{-1}(cx)\right)\mathrm{PolyLog}\left(2,\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{c^2(-d)-e}}\right)}{\sqrt{-d}\sqrt{e}}-\frac{b\,(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]
```

```
[Out] ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt
[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log[1 + (S
qrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt
[e]) + ((a + b*ArcCosh[c*x])^2*Log[1 - (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d]
 + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e]) - ((a + b*ArcCosh[c*x])^2*Log
[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-
d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])
/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(a + b*ArcCos
h[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e
])])/(Sqrt[-d]*Sqrt[e]) - (b*(a + b*ArcCosh[c*x])*PolyLog[2, -((Sqrt[e]*E^A
rcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) + (b*(
a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[
-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[
c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLo
g[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sqrt[-(c^2*d) - e])])/(Sqrt[-d]
*Sqrt[e]) + (b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-
```

```
(c^2*d) - e]))])/(Sqrt[-d]*Sqrt[e]) - (b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*
x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(Sqrt[-d]*Sqrt[e])
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 5562

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[((e + f*x)^m*E^(c + d*x))/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x] + Int[((e + f*x)^m*E^(c + d*x))/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5800

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)/((d_.) + (e_.)*(x_)), x_Symbo
l] :> Subst[Int[((a + b*x)^n*Sinh[x])/(c*d + e*Cosh[x]), x], x, ArcCosh[c*x
]] /; FreeQ[{a, b, c, d, e}, x] && IGtQ[n, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{d+ex^2}\,dx = \int \left(\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}-\sqrt{e}\,x\right)}+\frac{\sqrt{-d}\,\left(a+b\cosh^{-1}(cx)\right)^2}{2d\left(\sqrt{-d}+\sqrt{e}\,x\right)}\right)dx$$

$$= -\frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}-\sqrt{e}\,x}\,dx}{2\sqrt{-d}}-\frac{\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{-d}+\sqrt{e}\,x}\,dx}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}-\sqrt{e}\cosh(x)}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{(a+bx)^2\sinh(x)}{c\sqrt{-d}+\sqrt{e}\cosh(x)}\,dx, x, \text{co}\cdots\right)}{2\sqrt{-d}}$$

$$= -\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}-\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\,dx, x, \cosh^{-1}(cx)\right)}{2\sqrt{-d}}-\frac{\text{Subst}\left(\int \frac{e^x(a+bx)^2}{c\sqrt{-d}+\sqrt{-c^2d-e}-\sqrt{e}\,e^x}\cdots\right)}{2\sqrt{-d}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\cdots}}{c\sqrt{-d}-\cdots}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\cdots}}{c\sqrt{-d}-\cdots}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\cdots}}{c\sqrt{-d}-\cdots}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

$$= \frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1-\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-c^2d-e}}\right)}{2\sqrt{-d}\,\sqrt{e}}-\frac{\left(a+b\cosh^{-1}(cx)\right)^2\log\left(1+\frac{\sqrt{e}\,e^{\cdots}}{c\sqrt{-d}-\cdots}\right)}{2\sqrt{-d}\,\sqrt{e}}$$

Mathematica [A] time = 0.57, size = 623, normalized size = 0.82

$$\frac{2b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{c\sqrt{-d}-\sqrt{-dc^2-e}}\right)-2b\left(a+b\cosh^{-1}(cx)\right)\text{Li}_2\left(\frac{\sqrt{e}\,e^{\cosh^{-1}(cx)}}{\sqrt{-dc^2-e}-c\sqrt{-d}}\right)-2b\left(a+b\cosh^{-1}(cx)\right)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2),x]
```

```
[Out] (-((a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] - Sq
rt[-(c^2*d) - e])]) + (a + b*ArcCosh[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x
])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + (a + b*ArcCosh[c*x])^2*Log[1 - (
Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])] - (a + b*ArcCosh
[c*x])^2*Log[1 + (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] - Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^A
rcCosh[c*x])/(-(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] - 2*b*(a + b*ArcCosh[c*x
])*PolyLog[2, -((Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))
] + 2*b*(a + b*ArcCosh[c*x])*PolyLog[2, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqrt[-d
] + Sqrt[-(c^2*d) - e])] - 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(c*Sqr
t[-d] - Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, (Sqrt[e]*E^ArcCosh[c*x])/(-
(c*Sqrt[-d]) + Sqrt[-(c^2*d) - e])] + 2*b^2*PolyLog[3, -((Sqrt[e]*E^ArcCosh
[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e]))] - 2*b^2*PolyLog[3, (Sqrt[e]*E^Ar
cCosh[c*x])/(c*Sqrt[-d] + Sqrt[-(c^2*d) - e])])/(2*Sqrt[-d]*Sqrt[e])
```

fricas [F] time = 0.66, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}{ex^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/(e*x^2 + d), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^2}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d), x)
```

maple [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(cx))^2}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{de}} + \int \frac{b^2 \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)^2}{ex^2 + d} + \frac{2\,ab \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{ex^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d),x, algorithm="maxima")
[Out] a^2*arctan(e*x/sqrt(d*e))/sqrt(d*e) + integrate(b^2*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))^2/(e*x^2 + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
)/(e*x^2 + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + b \operatorname{acosh}(c\,x))^2}{e\,x^2 + d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^2}{d + ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d),x)
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2), x)
```

3.530 $\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2,x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
[Out] Defer[Int][Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx=\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Mathematica [A] time = 15.86, size = 0, normalized size = 0.00

$$\int \sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2,x]
[Out] Integrate[Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)\sqrt{ex^2+d},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex^2+d}\,(b\operatorname{arcosh}(cx)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int (a+b\operatorname{arccosh}(cx))^2\sqrt{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2}\left(\sqrt{ex^2+d}\,x+\frac{d\,\mathrm{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}}\right)a^2+\int\sqrt{ex^2+d}\,b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)^2+2\,\sqrt{ex^2+d}\,ab\log\left(cx+\sqrt{cx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2*(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(e*x^2 + d)*x + d*arcsinh(e*x/sqrt(d*e))/sqrt(e))*a^2 + integrate(
sqrt(e*x^2 + d)*b^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2 + 2*sqrt(e*x^2
 + d)*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\,\mathrm{acosh}\left(c\,x\right))^2\,\sqrt{e\,x^2+d}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\,\mathrm{acosh}\,(cx))^2\,\sqrt{d+ex^2}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2*(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2*sqrt(d + e*x**2), x)
```

3.531 $\int \frac{(a+b\cosh^{-1}(cx))^2}{\sqrt{d+ex^2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Mathematica [A] time = 11.42, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\sqrt{d+ex^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2],x]
[Out] Integrate[(a + b*ArcCosh[c*x])^2/Sqrt[d + e*x^2], x]
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2\,\text{arcosh}\left(cx\right)^2+2\,ab\,\text{arcosh}\left(cx\right)+a^2}{\sqrt{ex^2+d}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)/sqrt(e*x^2 + d), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}{\sqrt{ex^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/sqrt(e*x^2 + d), x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b \,\mathrm{arccosh}\left(c x\right)\right)^{2}}{\sqrt{e \,x^{2}+d}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{a^{2} \operatorname{arsinh}\left(\frac{ex}{\sqrt{de}}\right)}{\sqrt{e}} + \int \frac{b^{2} \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)^{2}}{\sqrt{ex^{2}+d}} + \frac{2\,ab \log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)}{\sqrt{ex^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] a^2*arcsinh(e*x/sqrt(d*e))/sqrt(e) + integrate(b^2*log(c*x + sqrt(c*x + 1)*
sqrt(c*x - 1))^2/sqrt(e*x^2 + d) + 2*a*b*log(c*x + sqrt(c*x + 1)*sqrt(c*x -
 1))/sqrt(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}{\sqrt{e\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2}}{\sqrt{d + e x^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/sqrt(d + e*x**2), x)
```

3.532 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}},x\right)$$

[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]

[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Mathematica [A] time = 3.77, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2),x]

[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(3/2), x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\operatorname{arcosh}(cx)^2+2\,ab\operatorname{arcosh}(cx)+a^2\right)\sqrt{ex^2+d}}{e^2x^4+2\,dex^2+d^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d)/(e^2*x^4 + 2*d*e*x^2 + d^2), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\operatorname{arcosh}(cx)+a\right)^2}{\left(ex^2+d\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(3/2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{2}}{(e\,x^{2}+d)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-c^2*d>0)', see `assume?` for more
 details)Is e-c^2*d zero or nonzero?
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{2}}{(e\,x^{2}+d)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{acosh}\,(cx))^{2}}{(d+ex^{2})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**2/(d + e*x**2)**(3/2), x)
```

3.533 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Mathematica [A] time = 7.53, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^2}{\left(d+ex^2\right)^{5/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^2/(d + e*x^2)^(5/2), x]
```

fricas [A] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2\right)\sqrt{ex^2+d}}{e^3x^6+3\,de^2x^4+3\,d^2ex^2+d^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="fricas")
[Out] integral((b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2)*sqrt(e*x^2 + d)/(e
^3*x^6 + 3*d*e^2*x^4 + 3*d^2*e*x^2 + d^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\,(cx)+a\right)^2}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccosh(c*x) + a)^2/(e*x^2 + d)^(5/2), x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^2}{\left(e\,x^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
[Out] int((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{3}\,a^2\left(\frac{2\,x}{\sqrt{ex^2+d}\,d^2}+\frac{x}{\left(ex^2+d\right)^{\frac{3}{2}}d}\right)+\int \frac{b^2\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\,\right)^2}{\left(ex^2+d\right)^{\frac{5}{2}}}+\frac{2\,ab\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\,\right)}{\left(ex^2+d\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^2/(e*x^2+d)^(5/2),x, algorithm="maxima")
[Out] 1/3*a^2*(2*x/(sqrt(e*x^2 + d)*d^2) + x/((e*x^2 + d)^(3/2)*d)) + integrate(b
^2*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))^2/(e*x^2 + d)^(5/2) + 2*a*b*log(c
*x + sqrt(c*x + 1)*sqrt(c*x - 1))/(e*x^2 + d)^(5/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^2}{\left(e\,x^2+d\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^2/(d + e*x^2)^(5/2),x)
[Out] int((a + b*acosh(c*x))^2/(d + e*x^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**2/(e*x**2+d)**(5/2),x)
[Out] Timed out
```

3.534 $\int \frac{(d+ex^2)^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=388

$$-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8bc^5}-\frac{3e^2\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}-\frac{e^2\sinh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5(a+b\cosh^{-1}(cx))}{b}\right)}{16bc^5}+\frac{e^2\,\text{co}}{}$$

```
[Out] d^2*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c+1/2*d*e*cosh(a/b)*Shi((a+b*arcc
osh(c*x))/b)/b/c^3+1/8*e^2*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^5+1/2*d*
e*cosh(3*a/b)*Shi(3*(a+b*arccosh(c*x))/b)/b/c^3+3/16*e^2*cosh(3*a/b)*Shi(3*
(a+b*arccosh(c*x))/b)/b/c^5+1/16*e^2*cosh(5*a/b)*Shi(5*(a+b*arccosh(c*x))/b
)/b/c^5-d^2*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b/c-1/2*d*e*Chi((a+b*arccos
h(c*x))/b)*sinh(a/b)/b/c^3-1/8*e^2*Chi((a+b*arccosh(c*x))/b)*sinh(a/b)/b/c^
5-1/2*d*e*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b/c^3-3/16*e^2*Chi(3*(a+b
*arccosh(c*x))/b)*sinh(3*a/b)/b/c^5-1/16*e^2*Chi(5*(a+b*arccosh(c*x))/b)*si
nh(5*a/b)/b/c^5
```

Rubi [A] time = 0.79, antiderivative size = 380, normalized size of antiderivative = 0.98, number of steps used = 27, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5707, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{de\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{2bc^3}-\frac{de\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{2bc^3}-\frac{e^2\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8bc^5}-$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(d*e*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(2*b*c^3) - (e^2*CoshInte
gral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(8*b*c^5) - (d^2*CoshIntegral[(a + b*Ar
cCosh[c*x])/b]*Sinh[a/b])/(b*c) - (d*e*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x
]]*Sinh[(3*a)/b])/(2*b*c^3) - (3*e^2*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]]
*Sinh[(3*a)/b])/(16*b*c^5) - (e^2*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]*Si
nh[(5*a)/b])/(16*b*c^5) + (d*e*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/
(2*b*c^3) + (e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b*c^5) + (d
*e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(2*b*c^3) + (3*e^2
*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b*c^5) + (e^2*Co
sh[(5*a)/b]*SinhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b*c^5) + (d^2*Cosh[
a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{\left(d+ex^2\right)^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d^2}{a+b\cosh^{-1}(cx)}+\frac{2dex^2}{a+b\cosh^{-1}(cx)}+\frac{e^2x^4}{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d^2\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx+(2de)\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx+e^2\int \frac{x^4}{a+b\cosh^{-1}(cx)}\,dx$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\cosh^{-1}(cx)\right)}{bc}+\frac{(2de)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx,x,\text{co}\right.}{c^3}$$

$$= \frac{(2de)\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx,x,\cosh^{-1}(cx)\right)}{c^3}+\frac{e^2\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{8(a+bx)}+\frac{3\sinh(3x)}{16(a+bx)}\right.\right.}{c^5}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{(de)\,\text{Subst}\left(\int \frac{\sinh(x}{a+bx}\right.}{2c}$$

$$= -\frac{d^2\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc}+\frac{d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}+\frac{\left(de\cosh\left(\frac{a}{b}\right)\right)\text{Subs}}{}$$

$$= -\frac{de\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{2bc^3}-\frac{e^2\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{8bc^5}-\frac{d^2\text{Chi}\left(\frac{a+b\cos}{b}\right.}{}$$

Mathematica [A] time = 0.53, size = 254, normalized size = 0.65

$$16c^4d^2\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\sinh\left(\frac{3a}{b}\right)\left(8c^2d+3e\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+8c^2de\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-2*(8*c^4*d^2 + 4*c^2*d*e + e^2)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b
] - e*(8*c^2*d + 3*e)*CoshIntegral[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] -
e^2*CoshIntegral[5*(a/b + ArcCosh[c*x])]*Sinh[(5*a)/b] + 16*c^4*d^2*Cosh[a/
b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*d*e*Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 2*e^2*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*c^2*
d*e*Cosh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 3*e^2*Cosh[(3*a)/b
]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + e^2*Cosh[(5*a)/b]*SinhIntegral[5*(
a/b + ArcCosh[c*x])])/(16*b*c^5)
```

fricas [F] time = 1.15, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4 + 2\,dex^2 + d^2}{b\,\text{arcosh}\,(cx) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.44, size = 380, normalized size = 0.98

$$-\frac{e^2\mathrm{e}^{-\frac{5a}{b}}\,\mathrm{Ei}\left(1,-5\,\text{arccosh}(cx)-\frac{5a}{b}\right)}{32c^4b} + \frac{e^2\mathrm{e}^{\frac{5a}{b}}\,\mathrm{Ei}\left(1,5\,\text{arccosh}(cx)+\frac{5a}{b}\right)}{32c^4b} + \frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)d^2}{2b} + \frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)de}{4c^2b} + \frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\text{ar}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(-1/32/c^4*e^2/b*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)-5*a/b)+1/32/c^4*e^2/b
*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b
)*d^2+1/4/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d*e+1/16/c^4/b*exp(a/b)*Ei(
1,arccosh(c*x)+a/b)*e^2-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d^2-1/4/c^2
/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d*e-1/16/c^4/b*exp(-a/b)*Ei(1,-arccosh
(c*x)-a/b)*e^2+1/4/c^2*e/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)*d+3/32/c^4
*e^2/b*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e/b*exp(-3*a/b)*Ei(1,-
3*arccosh(c*x)-3*a/b)*d-3/32/c^4*e^2/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a
/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2 + d\right)^2}{b\,\text{arcosh}\,(cx) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{a+b\,\mathrm{acosh}\left(c\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x)),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{a+b\,\mathrm{acosh}\left(cx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x)), x)
```

3.535 $\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=139

$$-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^3}+\frac{e\cosh\left(\frac{3a}{b}\right)\text{Shi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4bc^3}-\frac{\sinh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4bc^3}+$$

```
[Out] 1/4*(4*c^2*d+e)*cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c^3+1/4*e*cosh(3*a/b)
*Shi(3*(a+b*arccosh(c*x))/b)/b/c^3-1/4*(4*c^2*d+e)*Chi((a+b*arccosh(c*x))/b
)*sinh(a/b)/b/c^3-1/4*e*Chi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b/c^3
```

Rubi [A] time = 0.38, antiderivative size = 176, normalized size of antiderivative = 1.27, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5658, 3303, 3298, 3301, 5670, 5448}

$$-\frac{e\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}-\frac{e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4bc^3}+\frac{e\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4bc^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] -(e*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b])/(4*b*c^3) - (d*CoshIntegral
[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c) - (e*CoshIntegral[(3*a)/b + 3*Arc
Cosh[c*x]]*Sinh[(3*a)/b])/(4*b*c^3) + (e*Cosh[a/b]*SinhIntegral[a/b + ArcCo
sh[c*x]])/(4*b*c^3) + (e*Cosh[(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x
]])/(4*b*c^3) + (d*Cosh[a/b]*SinhIntegral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
```

```
, b, c, n}, x]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{a+b\cosh^{-1}(cx)}\,dx = \int \left(\frac{d}{a+b\cosh^{-1}(cx)}+\frac{ex^2}{a+b\cosh^{-1}(cx)}\right)dx$$

$$= d\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx + e\int \frac{x^2}{a+b\cosh^{-1}(cx)}\,dx$$

$$= -\frac{d\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{a+bx}\,dx, x, \cosh^{-1}(\right.}{c^3}$$

$$= \frac{e\,\text{Subst}\left(\int \left(\frac{\sinh(x)}{4(a+bx)}+\frac{\sinh(3x)}{4(a+bx)}\right)dx, x, \cosh^{-1}(cx)\right)}{c^3} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x,\right.}{bc}$$

$$= -\frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\sinh(x)}{a+bx}\,dx,\right.}{4c^3}$$

$$= -\frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc} + \frac{\left(e\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{e\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)\sinh\left(\frac{a}{b}\right)}{4bc^3} - \frac{d\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} - \frac{e\text{Chi}\left(\frac{3a}{b}+3\cosh^{-}\right.}{4bc^{}}$$

Mathematica [A] time = 0.23, size = 125, normalized size = 0.90

$$\frac{-\sinh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4c^2d\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-e\sinh\left(\frac{3a}{b}\right)\text{Chi}\left(3\left(\frac{a}{b}+\cosh\right.\right.}{4bc^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x]),x]
```

```
[Out] (-((4*c^2*d + e)*CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b]) - e*CoshIntegr
al[3*(a/b + ArcCosh[c*x])]*Sinh[(3*a)/b] + 4*c^2*d*Cosh[a/b]*SinhIntegral[a
/b + ArcCosh[c*x]] + e*Cosh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + e*Cosh[
(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])])/(4*b*c^3)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2+d}{b\,\text{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.42, size = 178, normalized size = 1.28

$$\frac{-\frac{e\,\mathrm{e}^{-\frac{3a}{b}}\,\mathrm{Ei}\left(1,-3\operatorname{arccosh}(cx)-\frac{3a}{b}\right)}{8c^2b}+\frac{e\,\mathrm{e}^{\frac{3a}{b}}\,\mathrm{Ei}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)d}{2b}+\frac{\mathrm{e}^{\frac{a}{b}}\,\mathrm{Ei}\left(1,\operatorname{arccosh}(cx)+\frac{a}{b}\right)e}{8c^2b}-\frac{\mathrm{e}^{-\frac{a}{b}}\,\mathrm{Ei}\left(1,-\operatorname{arc}\right.}{2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x)),x)
[Out] 1/c*(-1/8/c^2*e/b*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/8/c^2*e/b*exp(3
*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)+1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d+1/
8/c^2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*x
)-a/b)*d-1/8/c^2/b*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{b \operatorname{arcosh}(cx) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2 + d}{a + b\,\mathrm{acosh}\left(c\,x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x)),x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{a + b \operatorname{acosh}(cx)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x)),x)
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x)), x)
```

3.536 $\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=54

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

```
[Out] cosh(a/b)*Shi((a+b*arccosh(c*x))/b)/b/c-Chi((a+b*arccosh(c*x))/b)*sinh(a/b)
/b/c
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5658, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-1),x]
```

```
[Out] -((CoshIntegral[(a + b*ArcCosh[c*x])/b]*Sinh[a/b])/(b*c)) + (Cosh[a/b]*Sinh
Integral[(a + b*ArcCosh[c*x])/b])/(b*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{1}{a+b\cosh^{-1}(cx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}$$
$$= \frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sinh\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} - \frac{\sinh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{x}{b}\right)}{x}\,dx, x,\right.}{bc}$$
$$= -\frac{\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)\sinh\left(\frac{a}{b}\right)}{bc} + \frac{\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{bc}$$

Mathematica [A] time = 0.06, size = 46, normalized size = 0.85

$$-\frac{\sinh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\cosh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-1),x]
```

```
[Out] -((CoshIntegral[a/b + ArcCosh[c*x]]*Sinh[a/b] - Cosh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]])/(b*c))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b\,\text{arcosh}\,(cx)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(b*arccosh(c*x) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.04, size = 56, normalized size = 1.04

$$\frac{\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x)),x)
```

```
[Out] 1/c*(1/2/b*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*exp(-a/b)*Ei(1,-arccosh(c*
x)-a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{a + b \operatorname{acosh}(c x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x)),x)
```

```
[Out] int(1/(a + b*acosh(c*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \operatorname{acosh}(cx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x)),x)
```

```
[Out] Integral(1/(a + b*acosh(c*x)), x)
```

3.537 $\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{aex^2+ad+(bex^2+bd)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(1/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)), x)
```

3.538 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arcosh}\,(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*b*d*e*x^2 + b*
d^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^2\left(b\,\text{arcosh}\,(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a +b \,\mathrm{arccosh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(ex^{2} + d\right)}^{2}{\left(b \operatorname{arcosh}\left(cx\right) + a\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x)),x)
```

```
[Out] Timed out
```

3.539 $\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx = \int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Mathematica [A] time = 1.18, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{a+b\cosh^{-1}(cx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]),x]
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x]), x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b\,\text{arcosh}\,(cx)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^{2}+d}}{a+b\,\mathrm{arccosh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^{2}+d}}{b\,\mathrm{arcosh}\left(cx\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^{2}+d}}{a+b\,\mathrm{acosh}\left(c\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x)),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^{2}}}{a+b\,\mathrm{acosh}\left(cx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x)),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x)), x)
```

3.540 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.11, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])),x]
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{aex^2+ad+\left(bex^2+bd\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a*e*x^2 + a*d + (b*e*x^2 + b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^2+d}\,(b\operatorname{arcosh}(cx)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
[Out] int(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^{2}+d}\,(b\,\mathrm{arcosh}\left(cx\right)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))/(e*x^2+d)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,\sqrt{e\,x^2+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(1/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)\sqrt{d+ex^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))/(e*x**2+d)**(1/2),x)
[Out] Integral(1/((a + b*acosh(c*x))*sqrt(d + e*x**2)), x)
```

3.541 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 1.56, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{ae^2x^4+2\,adex^2+ad^2+\left(be^2x^4+2\,bdex^2+bd^2\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^2*x^4 + 2*a*d*e*x^2 + a*d^2 + (b*e^2*x^4 + 2*
b*d*e*x^2 + b*d^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)^{\frac{3}{2}} \left(b\,\mathrm{arcosh}\left(cx\right)+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right) \left(d+ex^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(3/2)), x)
```

3.542 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Mathematica [A] time = 3.92, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])),x]
```

```
[Out] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{ex^2+d}}{ae^3x^6+3\,ade^2x^4+3\,ad^2ex^2+ad^3+\left(be^3x^6+3\,bde^2x^4+3\,bd^2ex^2+bd^3\right)\operatorname{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a*e^3*x^6 + 3*a*d*e^2*x^4 + 3*a*d^2*e*x^2 + a*d^3
 + (b*e^3*x^6 + 3*b*d*e^2*x^4 + 3*b*d^2*e*x^2 + b*d^3)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\operatorname{arcosh}(cx)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(e x^{2} + d\right)}^{\frac{5}{2}} {\left(b \operatorname{arcosh}\left(c x\right) + a\right)}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x)),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(5/2)),x)
[Out] int(1/((a + b*acosh(c*x))*(d + e*x^2)^(5/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right) \left(d + e x^{2}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x)),x)
[Out] Integral(1/((a + b*acosh(c*x))*(d + e*x**2)**(5/2)), x)
```

3.543 $\int \frac{(d+ex^2)^2}{(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=510

$$\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{8b^2c^5}+\frac{9e^2\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^5}+\frac{5e^2\cosh\left(\frac{5a}{b}\right)\text{Chi}\left(\frac{5\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{16b^2c^5}-\frac{e^2\,\mathrm{s}}{}$$

```
[Out] d^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/2*d*e*Chi((a+b*arccosh(c*x)
)/b)*cosh(a/b)/b^2/c^3+1/8*e^2*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c^5+
3/2*d*e*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3+9/16*e^2*Chi(3*(a+b
*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^5+5/16*e^2*Chi(5*(a+b*arccosh(c*x))/b)*
cosh(5*a/b)/b^2/c^5-d^2*Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-1/2*d*e*S
hi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c^3-1/8*e^2*Shi((a+b*arccosh(c*x))/b
)*sinh(a/b)/b^2/c^5-3/2*d*e*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3
-9/16*e^2*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^5-5/16*e^2*Shi(5*(a
+b*arccosh(c*x))/b)*sinh(5*a/b)/b^2/c^5-d^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c
/(a+b*arccosh(c*x))-2*d*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(
c*x))-e^2*x^4*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.95, antiderivative size = 498, normalized size of antiderivative = 0.98, number of steps used = 26, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {5707, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{de\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{2b^2c^3}+\frac{3de\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{2b^2c^3}+\frac{e^2\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{8b^2c^5}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (2*d*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) - (e^2*x^4*Sqrt
[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d^2*Cosh[a/b]*CoshI
ntegral[a/b + ArcCosh[c*x]])/(b^2*c) + (d*e*Cosh[a/b]*CoshIntegral[a/b + Ar
cCosh[c*x]])/(2*b^2*c^3) + (e^2*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]])
/(8*b^2*c^5) + (3*d*e*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])
/(2*b^2*c^3) + (9*e^2*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh[c*x]])
/(16*b^2*c^5) + (5*e^2*Cosh[(5*a)/b]*CoshIntegral[(5*a)/b + 5*ArcCosh[c*x]]
)/(16*b^2*c^5) - (d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^2*c) -
 (d*e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(2*b^2*c^3) - (e^2*Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(8*b^2*c^5) - (3*d*e*Sinh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(2*b^2*c^3) - (9*e^2*Sinh[(3*a)/b]*Si
nhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(16*b^2*c^5) - (5*e^2*Sinh[(5*a)/b]*S
inhIntegral[(5*a)/b + 5*ArcCosh[c*x]])/(16*b^2*c^5)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{(a+b\cosh^{-1}(cx))^2}\,dx = \int \left(\frac{d^2}{(a+b\cosh^{-1}(cx))^2}+\frac{2dex^2}{(a+b\cosh^{-1}(cx))^2}+\frac{e^2x^4}{(a+b\cosh^{-1}(cx))^2}\right)dx$$

$$= d^2\int \frac{1}{(a+b\cosh^{-1}(cx))^2}\,dx + (2de)\int \frac{x^2}{(a+b\cosh^{-1}(cx))^2}\,dx + e^2\int \frac{x}{(a+b\cos}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} + \frac{(c}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} + \frac{d^2}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} + \frac{(c}{}$$

$$= -\frac{d^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{2dex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} - \frac{e^2x^4\sqrt{-1+cx}\sqrt{1+cx}}{bc(a+b\cosh^{-1}(cx))} + \frac{d^2}{}$$

Mathematica [A] time = 2.06, size = 663, normalized size = 1.30

$$-\frac{16ac^4d^2\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+16bc^4d^2\sinh\left(\frac{a}{b}\right)\cosh^{-1}(cx)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-3e\cosh\left(\frac{3a}{b}\right)(8}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/16*(16*b*c^4*d^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c^5*d^2*x*Sqrt[(-1 +
c*x)/(1 + c*x)] + 32*b*c^4*d*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 32*b*c^5*d*
e*x^3*Sqrt[(-1 + c*x)/(1 + c*x)] + 16*b*c^4*e^2*x^4*Sqrt[(-1 + c*x)/(1 + c*
x)] + 16*b*c^5*e^2*x^5*Sqrt[(-1 + c*x)/(1 + c*x)] - 2*(8*c^4*d^2 + 4*c^2*d*
e + e^2)*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshIntegral[a/b + ArcCosh[c*x]] -
3*e*(8*c^2*d + 3*e)*(a + b*ArcCosh[c*x])*Cosh[(3*a)/b]*CoshIntegral[3*(a/b
+ ArcCosh[c*x])] - 5*a*e^2*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c*x]
)] - 5*b*e^2*ArcCosh[c*x]*Cosh[(5*a)/b]*CoshIntegral[5*(a/b + ArcCosh[c*x])
] + 16*a*c^4*d^2*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 8*a*c^2*d*e*S
inh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 2*a*e^2*Sinh[a/b]*SinhIntegral[
a/b + ArcCosh[c*x]] + 16*b*c^4*d^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b
+ ArcCosh[c*x]] + 8*b*c^2*d*e*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + Arc
Cosh[c*x]] + 2*b*e^2*ArcCosh[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]
] + 24*a*c^2*d*e*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 9*a*e
^2*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 24*b*c^2*d*e*ArcCos
h[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 9*b*e^2*ArcCosh
[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b + ArcCosh[c*x])] + 5*a*e^2*Sinh[(5*
a)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])] + 5*b*e^2*ArcCosh[c*x]*Sinh[(5*a
)/b]*SinhIntegral[5*(a/b + ArcCosh[c*x])])/(b^2*c^5*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{e^2x^4+2\,dex^2+d^2}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e^2*x^4 + 2*d*e*x^2 + d^2)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*
x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\left(b\operatorname{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/(b*arccosh(c*x) + a)^2, x)
```

maple [B] time = 0.57, size = 1102, normalized size = 2.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/32*(-16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^4*c^4+12*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2)+16*c^5*x^5-20*c^3*x^3+5*c*x)*e^2
/c^4/b/(a+b*arccosh(c*x))-5/32*e^2/c^4/b^2*exp(5*a/b)*Ei(1,5*arccosh(c*x)+5
*a/b)-1/32/b*e^2/c^4*(16*c^5*x^5-20*c^3*x^3+16*(c*x+1)^(1/2)*(c*x-1)^(1/2)*
x^4*c^4+5*c*x-12*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^
(1/2))/(a+b*arccosh(c*x))-5/32/b^2*e^2/c^4*exp(-5*a/b)*Ei(1,-5*arccosh(c*x)
-5*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*d^2/b/(a+b*arccosh(c*x))-1/2
*d^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)+1/4*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+
c*x)*d*e/c^2/b/(a+b*arccosh(c*x))-1/4/c^2*d*e/b^2*exp(a/b)*Ei(1,arccosh(c*x
)+a/b)+1/16*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e^2/c^4/b/(a+b*arccosh(c*x))
-1/16/c^4*e^2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)-1/2/b*d^2*(c*x+(c*x-1)^(1
/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*d^2*exp(-a/b)*Ei(1,-arccosh(c
*x)-a/b)-1/4/c^2/b*d*e*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
-1/4/c^2/b^2*d*e*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)-1/16/c^4/b*e^2*(c*x+(c*x
-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/16/c^4/b^2*e^2*exp(-a/b)*Ei(1
,-arccosh(c*x)-a/b)+1/4*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/
2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*x)*d*e/c^2/b/(a+b*arccosh(c*x))+3/32*(-4*(c*
x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2)+4*c^3*x^3-3*c*
x)*e^2/c^4/b/(a+b*arccosh(c*x))-3/4*e/c^2/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x
)+3*a/b)*d-9/32*e^2/c^4/b^2*exp(3*a/b)*Ei(1,3*arccosh(c*x)+3*a/b)-1/4/c^2*e
/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*
x+1)^(1/2))/(a+b*arccosh(c*x))*d-3/32/c^4*e^2/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^
(1/2)*(c*x-1)^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))
-3/4/c^2*e/b^2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)*d-9/32/c^4*e^2/b^2*e
xp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3e^2x^7+\left(2\,c^3de-ce^2\right)x^5-cd^2x+\left(c^3d^2-2\,cde\right)x^3+\left(c^2e^2x^6+\left(2\,c^2de-e^2\right)x^4+\left(c^2d^2-2\,de\right)x^2-d^2\right)\sqrt{cx+1}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e^2*x^7 + (2*c^3*d*e - c*e^2)*x^5 - c*d^2*x + (c^3*d^2 - 2*c*d*e)*x^3
 + (c^2*e^2*x^6 + (2*c^2*d*e - e^2)*x^4 + (c^2*d^2 - 2*d*e)*x^2 - d^2)*sqrt
```

```
(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2
*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((5*c^5*e^2*x^8 + 2*(3*c
^5*d*e - 5*c^3*e^2)*x^6 + (c^5*d^2 - 12*c^3*d*e + 5*c*e^2)*x^4 + (5*c^3*e^2
*x^6 + 3*(2*c^3*d*e - c*e^2)*x^4 + c*d^2 + (c^3*d^2 - 2*c*d*e)*x^2)*(c*x +
1)*(c*x - 1) + c*d^2 - 2*(c^3*d^2 - 3*c*d*e)*x^2 + (10*c^4*e^2*x^7 + (12*c^
4*d*e - 13*c^2*e^2)*x^5 + 2*(c^4*d^2 - 7*c^2*d*e + 2*e^2)*x^3 - (c^2*d^2 -
4*d*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^5*x^4 + (c*x + 1)*(c*x - 1)*a
*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)*sqrt(c*x +
 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x^2 - 2*b^2*
c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*
log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)**2/(a + b*acosh(c*x))**2, x)
```

3.544 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=257

$$\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right)}{4b^2c^3}-\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{4b^2c^3}-\frac{3e\sinh\left(\frac{3a}{b}\right)\text{Sh}}{4}$$

```
[Out] d*Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c+1/4*e*Chi((a+b*arccosh(c*x))/b)
*cosh(a/b)/b^2/c^3+3/4*e*Chi(3*(a+b*arccosh(c*x))/b)*cosh(3*a/b)/b^2/c^3-d*
Shi((a+b*arccosh(c*x))/b)*sinh(a/b)/b^2/c-1/4*e*Shi((a+b*arccosh(c*x))/b)*s
inh(a/b)/b^2/c^3-3/4*e*Shi(3*(a+b*arccosh(c*x))/b)*sinh(3*a/b)/b^2/c^3-d*(c
*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))-e*x^2*(c*x-1)^(1/2)*(c*x+1
)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.60, antiderivative size = 249, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {5707, 5656, 5781, 3303, 3298, 3301, 5666}

$$\frac{e\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}+\frac{3e\cosh\left(\frac{3a}{b}\right)\text{Chi}\left(\frac{3a}{b}+3\cosh^{-1}(cx)\right)}{4b^2c^3}-\frac{e\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{4b^2c^3}-\frac{3e\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -((d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) - (e*x^2*Sqr
t[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x])) + (d*Cosh[a/b]*CoshIn
tegral[a/b + ArcCosh[c*x]])/(b^2*c) + (e*Cosh[a/b]*CoshIntegral[a/b + ArcCo
sh[c*x]])/(4*b^2*c^3) + (3*e*Cosh[(3*a)/b]*CoshIntegral[(3*a)/b + 3*ArcCosh
[c*x]])/(4*b^2*c^3) - (d*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(b^2*c
) - (e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]])/(4*b^2*c^3) - (3*e*Sinh[
(3*a)/b]*SinhIntegral[(3*a)/b + 3*ArcCosh[c*x]])/(4*b^2*c^3)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
```

```
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^2}+\frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\right)dx$$

$$= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx + e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

$$= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{(cd)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(c\right.}}{b}$$

$$= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\,\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx, x, \cosh^{-}\right.}{bc}$$

$$= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{\left(d\cosh\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+\right.}{a+bx}\right.}{bc}$$

$$= -\frac{d\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} - \frac{ex^2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)} + \frac{d\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(c\right.}{b^2c}$$

Mathematica [A] time = 0.97, size = 338, normalized size = 1.32

$$-\frac{-\cosh\left(\frac{a}{b}\right)\left(4c^2d+e\right)\left(a+b\cosh^{-1}(cx)\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4ac^2d\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)+4bc^2}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] -1/4*(4*b*c^2*d*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*d*x*Sqrt[(-1 + c*x)/(1
 + c*x)] + 4*b*c^2*e*x^2*Sqrt[(-1 + c*x)/(1 + c*x)] + 4*b*c^3*e*x^3*Sqrt[(-
1 + c*x)/(1 + c*x)] - (4*c^2*d + e)*(a + b*ArcCosh[c*x])*Cosh[a/b]*CoshInte
gral[a/b + ArcCosh[c*x]] - 3*e*(a + b*ArcCosh[c*x])*Cosh[(3*a)/b]*CoshInteg
ral[3*(a/b + ArcCosh[c*x])] + 4*a*c^2*d*Sinh[a/b]*SinhIntegral[a/b + ArcCos
h[c*x]] + a*e*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 4*b*c^2*d*ArcCos
h[c*x]*Sinh[a/b]*SinhIntegral[a/b + ArcCosh[c*x]] + b*e*ArcCosh[c*x]*Sinh[a
/b]*SinhIntegral[a/b + ArcCosh[c*x]] + 3*a*e*Sinh[(3*a)/b]*SinhIntegral[3*(
a/b + ArcCosh[c*x])] + 3*b*e*ArcCosh[c*x]*Sinh[(3*a)/b]*SinhIntegral[3*(a/b
 + ArcCosh[c*x])])/(b^2*c^3*(a + b*ArcCosh[c*x]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{ex^2 + d}{b^2 \operatorname{arcosh}(cx)^2 + 2\,ab \operatorname{arcosh}(cx) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((e*x^2 + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{(b \operatorname{arcosh}(cx) + a)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 0.47, size = 465, normalized size = 1.81

$$\frac{\frac{\left(-4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2+\sqrt{cx-1}\,\sqrt{cx+1}+4c^3x^3-3cx\right)e}{8c^2b(a+b\operatorname{arccosh}(cx))}-\frac{3e\,e^{\frac{3a}{b}}\operatorname{Ei}\left(1,3\operatorname{arccosh}(cx)+\frac{3a}{b}\right)}{8c^2b^2}-\frac{e\left(4c^3x^3-3cx+4\sqrt{cx+1}\,\sqrt{cx-1}\,x^2c^2-\sqrt{cx-1}\,\sqrt{cx+1}\right)}{8c^2b(a+b\operatorname{arccosh}(cx))}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(1/8*(-4*(c*x+1)^(1/2)*(c*x-1)^(1/2)*x^2*c^2+(c*x-1)^(1/2)*(c*x+1)^(1/2
)+4*c^3*x^3-3*c*x)*e/c^2/b/(a+b*arccosh(c*x))-3/8/c^2*e/b^2*exp(3*a/b)*Ei(1
,3*arccosh(c*x)+3*a/b)-1/8/c^2*e/b*(4*c^3*x^3-3*c*x+4*(c*x+1)^(1/2)*(c*x-1)
^(1/2)*x^2*c^2-(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-3/8/c^2*e/b^
2*exp(-3*a/b)*Ei(1,-3*arccosh(c*x)-3*a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)
+c*x)*d/b/(a+b*arccosh(c*x))+1/8*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)*e/c^2/b
/(a+b*arccosh(c*x))-1/2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b)*d-1/8/c^2/b^2*e
xp(a/b)*Ei(1,arccosh(c*x)+a/b)*e-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a
+b*arccosh(c*x))*d-1/8/c^2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh
(c*x))*e-1/2/b^2*exp(-a/b)*Ei(1,-arccosh(c*x)-a/b)*d-1/8/c^2/b^2*exp(-a/b)*
Ei(1,-arccosh(c*x)-a/b)*e)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3ex^5 + \left(c^3d - ce\right)x^3 - cdx + \left(c^2ex^4 + \left(c^2d - e\right)x^2 - d\right)\sqrt{cx+1}\,\sqrt{cx-1}}{abc^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*e*x^5 + (c^3*d - c*e)*x^3 - c*d*x + (c^2*e*x^4 + (c^2*d - e)*x^2 - d)
*sqrt(c*x + 1)*sqrt(c*x - 1))/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*
b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^
2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))) + integrate((3*c^5*e*x^6 + (c^
5*d - 6*c^3*e)*x^4 + (3*c^3*e*x^4 + (c^3*d - c*e)*x^2 + c*d)*(c*x + 1)*(c*x
 - 1) - (2*c^3*d - 3*c*e)*x^2 + (6*c^4*e*x^5 + (2*c^4*d - 7*c^2*e)*x^3 - (c
^2*d - 2*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/(a*b*c^5*x^4 + (c*x + 1)*
(c*x - 1)*a*b*c^3*x^2 - 2*a*b*c^3*x^2 + a*b*c + 2*(a*b*c^4*x^3 - a*b*c^2*x)
*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^3*x
^2 - 2*b^2*c^3*x^2 + b^2*c + 2*(b^2*c^4*x^3 - b^2*c^2*x)*sqrt(c*x + 1)*sqrt
(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^2,x)
```

```
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^{2}}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

3.545 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a+b\cosh^{-1}(cx)}{b}\right)}{b^2c}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

```
[Out] Chi((a+b*arccosh(c*x))/b)*cosh(a/b)/b^2/c-Shi((a+b*arccosh(c*x))/b)*sinh(a/
b)/b^2/c-(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
```

Rubi [A] time = 0.32, antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5656, 5781, 3303, 3298, 3301}

$$\frac{\cosh\left(\frac{a}{b}\right)\mathrm{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\mathrm{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sqrt{cx-1}\,\sqrt{cx+1}}{bc\left(a+b\cosh^{-1}(cx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] -((Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*(a + b*ArcCosh[c*x]))) + (Cosh[a/b]*C
oshIntegral[a/b + ArcCosh[c*x]])/(b^2*c) - (Sinh[a/b]*SinhIntegral[a/b + Ar
cCosh[c*x]])/(b^2*c)
```

Rule 3298

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[(I*SinhIntegral[(c*f*fz)/d + f*fz*x])/d, x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3301

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[(c*f*fz)/d + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
```

```
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx = -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{c\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\left(a+b\cosh^{-1}(cx)\right)}\,dx}{b}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\text{Subst}\left(\int \frac{\cosh(x)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cosh\left(\frac{a}{b}+x\right)}{a+bx}\,dx,x,\cosh^{-1}(cx)\right)}{bc}-\frac{\sinh(}{}$$

$$= -\frac{\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\left(a+b\cosh^{-1}(cx)\right)}+\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)}{b^2c}-\frac{\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cos\right.}{b^2c}$$

Mathematica [A] time = 0.35, size = 80, normalized size = 0.89

$$\frac{\cosh\left(\frac{a}{b}\right)\text{Chi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\sinh\left(\frac{a}{b}\right)\text{Shi}\left(\frac{a}{b}+\cosh^{-1}(cx)\right)-\frac{b\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)}{a+b\cosh^{-1}(cx)}}{b^2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-2),x]
```

```
[Out] (-((b*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x))/(a + b*ArcCosh[c*x])) + Cosh[a/
b]*CoshIntegral[a/b + ArcCosh[c*x]] - Sinh[a/b]*SinhIntegral[a/b + ArcCosh[
c*x]])/(b^2*c)
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-2), x)
```

maple [A] time = 0.05, size = 125, normalized size = 1.39

$$\frac{-\frac{cx+\sqrt{cx-1}\,\sqrt{cx+1}}{2b(a+b\,\text{arccosh}(cx))}-\frac{e^{-\frac{a}{b}}\,\text{Ei}\left(1,-\text{arccosh}(cx)-\frac{a}{b}\right)}{2b^2}+\frac{-\sqrt{cx-1}\,\sqrt{cx+1}+cx}{2b(a+b\,\text{arccosh}(cx))}-\frac{e^{\frac{a}{b}}\,\text{Ei}\left(1,\text{arccosh}(cx)+\frac{a}{b}\right)}{2b^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2,x)
```

```
[Out] 1/c*(-1/2/b*(c*x+(c*x-1)^(1/2)*(c*x+1)^(1/2))/(a+b*arccosh(c*x))-1/2/b^2*ex
p(-a/b)*Ei(1,-arccosh(c*x)-a/b)+1/2*(-(c*x-1)^(1/2)*(c*x+1)^(1/2)+c*x)/b/(a
+b*arccosh(c*x))-1/2/b^2*exp(a/b)*Ei(1,arccosh(c*x)+a/b))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^3x^3 + \left(c^2x^2 - 1\right)\sqrt{cx+1}\,\sqrt{cx-1} - cx}{abc^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x - abc + \left(b^2c^3x^2 + \sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x - b^2c\right)\log\left(cx + \sqrt{cx+1}\,\sqrt{cx-1}\right)} + \int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*x^2 +
 sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x^2 + sqrt(c*x +
1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
 + integrate((c^4*x^4 - 2*c^2*x^2 + (c^2*x^2 + 1)*(c*x + 1)*(c*x - 1) + (2*
c^3*x^3 - c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + 1)/(a*b*c^4*x^4 + (c*x + 1)*(c
*x - 1)*a*b*c^2*x^2 - 2*a*b*c^2*x^2 + 2*(a*b*c^3*x^3 - a*b*c*x)*sqrt(c*x +
1)*sqrt(c*x - 1) + a*b + (b^2*c^4*x^4 + (c*x + 1)*(c*x - 1)*b^2*c^2*x^2 - 2
*b^2*c^2*x^2 + 2*(b^2*c^3*x^3 - b^2*c*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + b^2)
*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^2,x)
```

```
[Out] int(1/(a + b*acosh(c*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-2), x)
```

3.546 $\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 178.62, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\text{arcosh}(cx)^2+2\left(abex^2+abd\right)\text{arcosh}(cx)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccosh(c*x)^2 + 2*(a*b
*e*x^2 + a*b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)(b\,\text{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{3}+\left(c^{2}x^{2}-1\right)\sqrt{cx+1}\,\sqrt{cx-1}-cx}{abc^{3}ex^{4}+\left(c^{3}d-ce\right)abx^{2}-abcd+\left(abc^{2}ex^{3}+abc^{2}dx\right)\sqrt{cx+1}\,\sqrt{cx-1}+\left(b^{2}c^{3}ex^{4}+\left(c^{3}d-ce\right)b^{2}x^{2}-b^{2}cd+\left(b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e*x^4
 + (c^3*d - c*e)*a*b*x^2 - a*b*c*d + (a*b*c^2*e*x^3 + a*b*c^2*d*x)*sqrt(c*x
 + 1)*sqrt(c*x - 1) + (b^2*c^3*e*x^4 + (c^3*d - c*e)*b^2*x^2 - b^2*c*d + (b
^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x
 + 1)*sqrt(c*x - 1))) - integrate((c^5*e*x^6 - (c^5*d + 2*c^3*e)*x^4 + (c^3
*e*x^4 - (c^3*d + 3*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d + c*e)*x
^2 + (2*c^4*e*x^5 - (2*c^4*d + 5*c^2*e)*x^3 + (c^2*d + 2*e)*x)*sqrt(c*x + 1
)*sqrt(c*x - 1) - c*d)/(a*b*c^5*e^2*x^8 + 2*(c^5*d*e - c^3*e^2)*a*b*x^6 + (
c^5*d^2 - 4*c^3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 - 2*(c^3*d^2 - c*d*e)*a*b*
x^2 + (a*b*c^3*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*
x - 1) + 2*(a*b*c^4*e^2*x^7 + (2*c^4*d*e - c^2*e^2)*a*b*x^5 - a*b*c^2*d^2*x
 + (c^4*d^2 - 2*c^2*d*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^
2*x^8 + 2*(c^5*d*e - c^3*e^2)*b^2*x^6 + (c^5*d^2 - 4*c^3*d*e + c*e^2)*b^2*x
^4 + b^2*c*d^2 - 2*(c^3*d^2 - c*d*e)*b^2*x^2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3
*d*e*x^4 + b^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^7 + (2*c
^4*d*e - c^2*e^2)*b^2*x^5 - b^2*c^2*d^2*x + (c^4*d^2 - 2*c^2*d*e)*b^2*x^3)*
sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2} \left(d + ex^{2}\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)), x)
```

3.547 $\int \frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2}\,dx = \int \frac{1}{(d+ex^2)^2(a+b\cosh^{-1}(cx))^2}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.78, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{a^2e^2x^4+2a^2dex^2+a^2d^2+(b^2e^2x^4+2b^2dex^2+b^2d^2)\,\text{arcosh}(cx)^2+2(abe^2x^4+2abdex^2+abd^2)\,\ldots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*x^4 + 2*b^2*d*
e*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*x^2 + a*b*d^2)
*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)^2(b\,\text{arcosh}(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.71, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}}{abc^{3}e^{2}x^{6}+\left(2\,c^{3}de-ce^{2}\right)abx^{4}-abcd^{2}+\left(c^{3}d^{2}-2\,cde\right)abx^{2}+\left(abc^{2}e^{2}x^{5}+2\,abc^{2}dex^{3}+abc^{2}d^{2}x\right)\sqrt{cx+1}\,\sqrt{cx-}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/(a*b*c^3*e^2*x
^6 + (2*c^3*d*e - c*e^2)*a*b*x^4 - a*b*c*d^2 + (c^3*d^2 - 2*c*d*e)*a*b*x^2
+ (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(
c*x - 1) + (b^2*c^3*e^2*x^6 + (2*c^3*d*e - c*e^2)*b^2*x^4 - b^2*c*d^2 + (c^
3*d^2 - 2*c*d*e)*b^2*x^2 + (b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d
^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)))
- integrate((3*c^5*e*x^6 - (c^5*d + 6*c^3*e)*x^4 + (3*c^3*e*x^4 - (c^3*d +
5*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + (2*c^3*d + 3*c*e)*x^2 + (6*c^4*e*x^
5 - (2*c^4*d + 11*c^2*e)*x^3 + (c^2*d + 4*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1)
 - c*d)/(a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*
e - 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 - 6*c^3*d^2*e + 3*c*d*e^2)*a*b*
x^4 + a*b*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8 + 3*a*
b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1
) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*a*b*x^7 - a*b*c^2*d^3*x +
3*(c^4*d^2*e - c^2*d*e^2)*a*b*x^5 + (c^4*d^3 - 3*c^2*d^2*e)*a*b*x^3)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*b^2*x
^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 - 6*c^3*d^2*e +
 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*b^2*x^2 + (b^2*c^
3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c
*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*b^2*x^7 -
b^2*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*b^2*x^5 + (c^4*d^3 - 3*c^2*d^2*e)
*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1
))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**2,x)
```

```
[Out] Timed out
```

3.548 $\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 28.23, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2,x]
```

```
[Out] Integrate[Sqrt[d + e*x^2]/(a + b*ArcCosh[c*x])^2, x]
```

fricas [A] time = 0.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{b^2\,\text{arcosh}\,(cx)^2+2\,ab\,\text{arcosh}\,(cx)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(b^2*arccosh(c*x)^2 + 2*a*b*arccosh(c*x) + a^2), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ex^2+d}}{(b\,\text{arcosh}\,(cx)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*x^2 + d)/(b*arccosh(c*x) + a)^2, x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\,x^2+d}}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
[Out] int((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(c^3x^3+\left(c^2x^2-1\right)\sqrt{cx+1}\,\sqrt{cx-1}\,-cx\right)\sqrt{ex^2+d}}{abc^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,abc^2x-abc+\left(b^2c^3x^2+\sqrt{cx+1}\,\sqrt{cx-1}\,b^2c^2x-b^2c\right)\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\,\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)*sqrt(e*x^2 + d
)/(a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a*b*c^2*x - a*b*c + (b^2*c^3*x
^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*log(c*x + sqrt(c*x + 1)
*sqrt(c*x - 1))) + integrate((2*c^5*e*x^6 + (c^5*d - 4*c^3*e)*x^4 + (2*c^3*
e*x^4 + c^3*d*x^2 + c*d)*(c*x + 1)*(c*x - 1) - 2*(c^3*d - c*e)*x^2 + (4*c^4
*e*x^5 + 2*(c^4*d - 2*c^2*e)*x^3 - (c^2*d - e)*x)*sqrt(c*x + 1)*sqrt(c*x -
1) + c*d)*sqrt(e*x^2 + d)/(a*b*c^5*e*x^6 + (c^5*d - 2*c^3*e)*a*b*x^4 - (2*c
^3*d - c*e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c*x + 1)*(
c*x - 1) + 2*(a*b*c^4*e*x^5 - a*b*c^2*d*x + (c^4*d - c^2*e)*a*b*x^3)*sqrt(c
*x + 1)*sqrt(c*x - 1) + (b^2*c^5*e*x^6 + (c^5*d - 2*c^3*e)*b^2*x^4 - (2*c^3
*d - c*e)*b^2*x^2 + b^2*c*d + (b^2*c^3*e*x^4 + b^2*c^3*d*x^2)*(c*x + 1)*(c*
x - 1) + 2*(b^2*c^4*e*x^5 - b^2*c^2*d*x + (c^4*d - c^2*e)*b^2*x^3)*sqrt(c*x
 + 1)*sqrt(c*x - 1))*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1))), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{e\,x^2+d}}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x))^2,x)
[Out] int((d + e*x^2)^(1/2)/(a + b*acosh(c*x))^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*acosh(c*x))**2, x)
```

3.549 $\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [A] time = 24.84, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Integrate[1/(Sqrt[d + e*x^2]*(a + b*ArcCosh[c*x])^2), x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2ex^2+a^2d+\left(b^2ex^2+b^2d\right)\text{arcosh}\left(cx\right)^2+2\left(abex^2+abd\right)\text{arcosh}\left(cx\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e*x^2 + a^2*d + (b^2*e*x^2 + b^2*d)*arccosh(c
*x)^2 + 2*(a*b*e*x^2 + a*b*d)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex^2+d}\,(b\,\text{arcosh}\left(cx\right)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*x^2 + d)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{arccosh}\left(cx\right)\right)^{2}\sqrt{e\,x^{2}+d}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3}x^{3}+\left(c^{2}x^{2}-1\right)\sqrt{cx+1}\,\sqrt{cx-1}-cx}{\left(b^{2}c^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,b^{2}c^{2}x-b^{2}c\right)\sqrt{ex^{2}+d}\,\log\left(cx+\sqrt{cx+1}\,\sqrt{cx-1}\right)+\left(abc^{3}x^{2}+\sqrt{cx+1}\,\sqrt{cx-1}\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^2/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*x^2
+ sqrt(c*x + 1)*sqrt(c*x - 1)*b^2*c^2*x - b^2*c)*sqrt(e*x^2 + d)*log(c*x +
sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^3*x^2 + sqrt(c*x + 1)*sqrt(c*x - 1)*a
*b*c^2*x - a*b*c)*sqrt(e*x^2 + d)) + integrate((c^5*d*x^4 - 2*c^3*d*x^2 + (
(c^3*d + 2*c*e)*x^2 + c*d)*(c*x + 1)*(c*x - 1) + (2*(c^4*d + c^2*e)*x^3 - (
c^2*d + e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) + c*d)/((b^2*c^5*e*x^6 + (c^5*d -
 2*c^3*e)*b^2*x^4 - (2*c^3*d - c*e)*b^2*x^2 + b^2*c*d + (b^2*c^3*e*x^4 + b^
2*c^3*d*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e*x^5 - b^2*c^2*d*x + (c^4*d
- c^2*e)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sq
rt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*e*x^6 + (c^5*d - 2*c^3*e)*a*b*x^4 - (
2*c^3*d - c*e)*a*b*x^2 + a*b*c*d + (a*b*c^3*e*x^4 + a*b*c^3*d*x^2)*(c*x + 1
)*(c*x - 1) + 2*(a*b*c^4*e*x^5 - a*b*c^2*d*x + (c^4*d - c^2*e)*a*b*x^3)*sqr
t(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,\sqrt{e\,x^2+d}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(1/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(cx \right)}\right)^{2} \sqrt{d + ex^{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**2/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*sqrt(d + e*x**2)), x)
```

3.550 $\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

[Out] Unintegrable(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] Defer[Int][1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{3/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*ArcCosh[c*x])^2),x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.72, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^2x^4+2a^2dex^2+a^2d^2+\left(b^2e^2x^4+2b^2dex^2+b^2d^2\right)\operatorname{arcosh}(cx)^2+2\left(abe^2x^4+2abdex^2+abd^2\right)\text{arc}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^2*x^4 + 2*a^2*d*e*x^2 + a^2*d^2 + (b^2*e^2*
x^4 + 2*b^2*d*e*x^2 + b^2*d^2)*arccosh(c*x)^2 + 2*(a*b*e^2*x^4 + 2*a*b*d*e*
x^2 + a*b*d^2)*arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{3}{2}}\left(b\operatorname{arcosh}(cx)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^(3/2)*(b*arccosh(c*x) + a)^2), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{3}{2}} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
[Out] int(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c^{3} x^{3} + \left(c^{2} x^{2} - 1\right) \sqrt{c x + 1} \sqrt{c x}}{\left(b^{2} c^{3} e x^{4} + \left(c^{3} d - c e\right) b^{2} x^{2} - b^{2} c d + \left(b^{2} c^{2} e x^{3} + b^{2} c^{2} d x\right) \sqrt{c x + 1} \sqrt{c x - 1}\right) \sqrt{e x^{2} + d} \log\left(c x + \sqrt{c x + 1} \sqrt{c x - 1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")
[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*e*x^
4 + (c^3*d - c*e)*b^2*x^2 - b^2*c*d + (b^2*c^2*e*x^3 + b^2*c^2*d*x)*sqrt(c*
x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)
) + (a*b*c^3*e*x^4 + (c^3*d - c*e)*a*b*x^2 - a*b*c*d + (a*b*c^2*e*x^3 + a*b
*c^2*d*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)) - integrate((2*c^5*
e*x^6 - (c^5*d + 4*c^3*e)*x^4 + (2*c^3*e*x^4 - (c^3*d + 4*c*e)*x^2 - c*d)*(
c*x + 1)*(c*x - 1) + 2*(c^3*d + c*e)*x^2 + (4*c^4*e*x^5 - 2*(c^4*d + 4*c^2*
e)*x^3 + (c^2*d + 3*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*c^5*e^2*
x^8 + 2*(c^5*d*e - c^3*e^2)*b^2*x^6 + (c^5*d^2 - 4*c^3*d*e + c*e^2)*b^2*x^4
 + b^2*c*d^2 - 2*(c^3*d^2 - c*d*e)*b^2*x^2 + (b^2*c^3*e^2*x^6 + 2*b^2*c^3*d
*e*x^4 + b^2*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^2*x^7 + (2*c^4
*d*e - c^2*e^2)*b^2*x^5 - b^2*c^2*d^2*x + (c^4*d^2 - 2*c^2*d*e)*b^2*x^3)*sq
rt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x
 - 1)) + (a*b*c^5*e^2*x^8 + 2*(c^5*d*e - c^3*e^2)*a*b*x^6 + (c^5*d^2 - 4*c^
3*d*e + c*e^2)*a*b*x^4 + a*b*c*d^2 - 2*(c^3*d^2 - c*d*e)*a*b*x^2 + (a*b*c^3
*e^2*x^6 + 2*a*b*c^3*d*e*x^4 + a*b*c^3*d^2*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*
b*c^4*e^2*x^7 + (2*c^4*d*e - c^2*e^2)*a*b*x^5 - a*b*c^2*d^2*x + (c^4*d^2 -
2*c^2*d*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(3/2)),x)
[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(3/2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{2} \left(d + e x^{2}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(a+b*acosh(c*x))**2,x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)**(3/2)), x)
```

3.551 $\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] Defer[Int][1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx=\int \frac{1}{\left(d+ex^2\right)^{5/2}\left(a+b\cosh^{-1}(cx)\right)^2}\,dx$$

Mathematica [F] time = 180.00, size = 0, normalized size = 0.00

$Aborted

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^(5/2)*(a + b*ArcCosh[c*x])^2),x]
[Out] $Aborted
```

fricas [A] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{ex^2+d}}{a^2e^3x^6+3\,a^2de^2x^4+3\,a^2d^2ex^2+a^2d^3+\left(b^2e^3x^6+3\,b^2de^2x^4+3\,b^2d^2ex^2+b^2d^3\right)\text{arcosh}\left(cx\right)^2+2\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(e*x^2 + d)/(a^2*e^3*x^6 + 3*a^2*d*e^2*x^4 + 3*a^2*d^2*e*x^2 +
 a^2*d^3 + (b^2*e^3*x^6 + 3*b^2*d*e^2*x^4 + 3*b^2*d^2*e*x^2 + b^2*d^3)*arcc
osh(c*x)^2 + 2*(a*b*e^3*x^6 + 3*a*b*d*e^2*x^4 + 3*a*b*d^2*e*x^2 + a*b*d^3)*
arccosh(c*x)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^{\frac{5}{2}}\left(b\,\text{arcosh}\left(cx\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="giac")

[Out] integrate(1/((e*x^2 + d)^(5/2)*(b*arccosh(c*x) + a)^2), x)

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{\frac{5}{2}} \left(a +b \,\mathrm{arccosh}\left(c x \right)\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)

[Out] int(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{}{\left(b^{2} c^{3} e^{2} x^{6} + \left(2 \, c^{3} d e - c e^{2}\right) b^{2} x^{4} - b^{2} c d^{2} + \left(c^{3} d^{2} - 2 \, c d e\right) b^{2} x^{2} + \left(b^{2} c^{2} e^{2} x^{5} + 2 \, b^{2} c^{2} d e x^{3} + b^{2} c^{2} d^{2} x\right) \sqrt{c x + 1} \, \sqrt{c x -}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)^(5/2)/(a+b*arccosh(c*x))^2,x, algorithm="maxima")

[Out] -(c^3*x^3 + (c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*x)/((b^2*c^3*e^2*x^6 + (2*c^3*d*e - c*e^2)*b^2*x^4 - b^2*c*d^2 + (c^3*d^2 - 2*c*d*e)*b^2*x^2 + (b^2*c^2*e^2*x^5 + 2*b^2*c^2*d*e*x^3 + b^2*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^3*e^2*x^6 + (2*c^3*d*e - c*e^2)*a*b*x^4 - a*b*c*d^2 + (c^3*d^2 - 2*c*d*e)*a*b*x^2 + (a*b*c^2*e^2*x^5 + 2*a*b*c^2*d*e*x^3 + a*b*c^2*d^2*x)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)) - integrate((4*c^5*e*x^6 - (c^5*d + 8*c^3*e)*x^4 + (4*c^3*e*x^4 - (c^3*d + 6*c*e)*x^2 - c*d)*(c*x + 1)*(c*x - 1) + 2*(c^3*d + 2*c*e)*x^2 + (8*c^4*e*x^5 - 2*(c^4*d + 7*c^2*e)*x^3 + (c^2*d + 5*e)*x)*sqrt(c*x + 1)*sqrt(c*x - 1) - c*d)/((b^2*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*b^2*x^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*b^2*x^6 + (c^5*d^3 - 6*c^3*d^2*e + 3*c*d*e^2)*b^2*x^4 + b^2*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*b^2*x^2 + (b^2*c^3*e^3*x^8 + 3*b^2*c^3*d*e^2*x^6 + 3*b^2*c^3*d^2*e*x^4 + b^2*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(b^2*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*b^2*x^7 - b^2*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*b^2*x^5 + (c^4*d^3 - 3*c^2*d^2*e)*b^2*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)*log(c*x + sqrt(c*x + 1)*sqrt(c*x - 1)) + (a*b*c^5*e^3*x^10 + (3*c^5*d*e^2 - 2*c^3*e^3)*a*b*x^8 + (3*c^5*d^2*e - 6*c^3*d*e^2 + c*e^3)*a*b*x^6 + (c^5*d^3 - 6*c^3*d^2*e + 3*c*d*e^2)*a*b*x^4 + a*b*c*d^3 - (2*c^3*d^3 - 3*c*d^2*e)*a*b*x^2 + (a*b*c^3*e^3*x^8 + 3*a*b*c^3*d*e^2*x^6 + 3*a*b*c^3*d^2*e*x^4 + a*b*c^3*d^3*x^2)*(c*x + 1)*(c*x - 1) + 2*(a*b*c^4*e^3*x^9 + (3*c^4*d*e^2 - c^2*e^3)*a*b*x^7 - a*b*c^2*d^3*x + 3*(c^4*d^2*e - c^2*d*e^2)*a*b*x^5 + (c^4*d^3 - 3*c^2*d^2*e)*a*b*x^3)*sqrt(c*x + 1)*sqrt(c*x - 1))*sqrt(e*x^2 + d)), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^2\,{\left(e\,x^2+d\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(5/2)),x)

[Out] int(1/((a + b*acosh(c*x))^2*(d + e*x^2)^(5/2)), x)

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+b\operatorname{acosh}(cx))^2\left(d+ex^2\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(5/2)/(a+b*acosh(c*x))**2,x)
[Out] Integral(1/((a + b*acosh(c*x))**2*(d + e*x**2)**(5/2)), x)
```

3.552 $\int (d+ex^2)^2 \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=672

$$-\frac{\sqrt{\pi}\sqrt{b}e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^5}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{64c^5}-\frac{\sqrt{\frac{\pi}{5}}\sqrt{b}e^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{320c^5}-\ldots$$

```
[Out] -1/1600*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2
)*5^(1/2)*Pi^(1/2)/c^5-1/1600*e^2*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)-1/72*d*e*exp(3*a/b)*erf(3^(1/
2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^3-1/192*e^2
*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*P
i^(1/2)/c^5-1/72*d*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)
*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/192*e^2*erfi(3^(1/2)*(a+b*arccosh(c*x))^
(1/2)/b^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)-1/4*d^2*exp(a/b)*erf
((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/8*d*e*exp(a/b)*erf(
(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/32*e^2*exp(a/b)*er
f((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5-1/4*d^2*erfi((a+b*
arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)-1/8*d*e*erfi((a+b*
arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)-1/32*e^2*erfi((a
+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^5/exp(a/b)+d^2*x*(a+b*ar
ccosh(c*x))^(1/2)+2/3*d*e*x^3*(a+b*arccosh(c*x))^(1/2)+1/5*e^2*x^5*(a+b*arc
cosh(c*x))^(1/2)
```

Rubi [A] time = 2.40, antiderivative size = 672, normalized size of antiderivative = 1.00, number of steps used = 42, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {5707, 5654, 5781, 3307, 2180, 2204, 2205, 5664, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{24c^3}-\frac{\sqrt{\pi}\sqrt{b}dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c^3}-\ldots$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d^2*x*Sqrt[a + b*ArcCosh[c*x]] + (2*d*e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 + (
e^2*x^5*Sqrt[a + b*ArcCosh[c*x]])/5 - (Sqrt[b]*d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqr
t[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*d*e*E^(a/b)*Sqrt[Pi]*Erf[S
qrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3) - (Sqrt[b]*e^2*E^(a/b)*Sqrt[Pi]*E
rf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5) - (Sqrt[b]*d*e*E^((3*a)/b)*S
qrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3) - (Sqrt
[b]*e^2*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[
b]])/(64*c^5) - (Sqrt[b]*e^2*E^((5*a)/b)*Sqrt[Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b
*ArcCosh[c*x]])/Sqrt[b]])/(320*c^5) - (Sqrt[b]*d^2*Sqrt[Pi]*Erfi[Sqrt[a + b
*ArcCosh[c*x]]/Sqrt[b]])/(4*c*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi]*Erfi[Sqrt[a
+ b*ArcCosh[c*x]]/Sqrt[b]])/(8*c^3*E^(a/b)) - (Sqrt[b]*e^2*Sqrt[Pi]*Erfi[Sq
rt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^5*E^(a/b)) - (Sqrt[b]*d*e*Sqrt[Pi/3]
*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(24*c^3*E^((3*a)/b)) - (
Sqrt[b]*e^2*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(6
4*c^5*E^((3*a)/b)) - (Sqrt[b]*e^2*Sqrt[Pi/5]*Erfi[(Sqrt[5]*Sqrt[a + b*ArcCo
sh[c*x]])/Sqrt[b]])/(320*c^5*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}\,dx = \int \left(d^2\sqrt{a+b\cosh^{-1}(cx)} + 2dex^2\sqrt{a+b\cosh^{-1}(cx)} + e^2x^4\sqrt{a+b\cosh^{-1}\right.$$

$$= d^2\int \sqrt{a+b\cosh^{-1}(cx)}\,dx + (2de)\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx + e^2\int x^4$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

$$= d^2x\sqrt{a+b\cosh^{-1}(cx)} + \frac{2}{3}dex^3\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{5}e^2x^5\sqrt{a+b\cosh^{-1}}$$

Mathematica [A] time = 6.69, size = 536, normalized size = 0.80

$$be^{-\frac{5a}{b}}\left(450e^{\frac{6a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)\left(-be\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\sqrt{-\frac{\left(a+b\cosh^{-1}(cx)\right)^2}{b^2}}+8ac^4d^2\sqrt{\frac{a}{b}+\cosh^{-1}(cx\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (b*(450*E^((6*a)/b)*(8*a*c^4*d^2*Sqrt[a/b + ArcCosh[c*x]] + 8*b*c^4*d^2*Arc
Cosh[c*x]*Sqrt[a/b + ArcCosh[c*x]] - b*e*(4*c^2*d + e)*Sqrt[-((a + b*ArcCos
h[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, a/b + ArcCosh[c
*x]] - 9*Sqrt[5]*b*e^2*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])
^2/b^2)]*Gamma[3/2, (-5*(a + b*ArcCosh[c*x]))/b] - E^((2*a)/b)*(25*Sqrt[3]*
b*e*(8*c^2*d + 3*e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a + b*ArcCosh[c*x])^2/
b^2)]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 450*E^((2*a)/b)*(8*a*c^4*d^
2*Sqrt[-((a + b*ArcCosh[c*x])/b)] + 8*b*c^4*d^2*ArcCosh[c*x]*Sqrt[-((a + b*
ArcCosh[c*x])/b)] + b*e*(4*c^2*d + e)*Sqrt[a/b + ArcCosh[c*x]]*Sqrt[-((a +
b*ArcCosh[c*x])^2/b^2)])*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] + b*e*E^((6*
a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)]*(
25*Sqrt[3]*(8*c^2*d + 3*e)*Gamma[3/2, (3*(a + b*ArcCosh[c*x]))/b] + 9*Sqrt[
5]*e*E^((2*a)/b)*Gamma[3/2, (5*(a + b*ArcCosh[c*x]))/b]))))/(7200*c^5*E^((5
*a)/b)*(a + b*ArcCosh[c*x])^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\mathrm{arccosh}\left(c x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int {\left(ex^{2} + d\right)}^{2} \sqrt{b \operatorname{arcosh}\left(cx\right) + a}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,{\left(e\,x^2+d\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}{\left(cx \right)}} \left(d + ex^{2}\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2)**2, x)
```

3.553 $\int \left(d+ex^2\right)\sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=322

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}}{}$$

```
[Out] -1/144*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*3
^(1/2)*Pi^(1/2)/c^3-1/144*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*
b^(1/2)*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b)-1/4*d*exp(a/b)*erf((a+b*arccosh(c*x
))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/16*e*exp(a/b)*erf((a+b*arccosh(c*x))
^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c^3-1/4*d*erfi((a+b*arccosh(c*x))^(1/2)/b^
(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)-1/16*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(
1/2))*b^(1/2)*Pi^(1/2)/c^3/exp(a/b)+d*x*(a+b*arccosh(c*x))^(1/2)+1/3*e*x^3*
(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 1.29, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {5707, 5654, 5781, 3307, 2180, 2204, 2205, 5664, 3312}

$$-\frac{\sqrt{\pi}\sqrt{b}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{48c^3}-\frac{\sqrt{\pi}\sqrt{b}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16c^3}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{b}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] d*x*Sqrt[a + b*ArcCosh[c*x]] + (e*x^3*Sqrt[a + b*ArcCosh[c*x]])/3 - (Sqrt[b
]*d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b
]*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*c^3) - (Sqr
t[b]*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(48*c^3) - (Sqrt[b]*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/
(4*c*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(16*c^3*E^(a/b)) - (Sqrt[b]*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(48*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int (d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}\,dx = \int \left(d\sqrt{a+b\cosh^{-1}(cx)} + ex^2\sqrt{a+b\cosh^{-1}(cx)}\right) dx$$
$$= d\int \sqrt{a+b\cosh^{-1}(cx)}\,dx + e\int x^2\sqrt{a+b\cosh^{-1}(cx)}\,dx$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{2}(bcd)\int \frac{}{\sqrt{-1+cx}\sqrt{1}}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx,\right.}{2c}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{(bd)\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx,\right.}{4c}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{d\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x,\right.}{2c}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh}}{\sqrt{}}\right.}{4c}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh}}{\sqrt{}}\right.}{4c}$$
$$= dx\sqrt{a+b\cosh^{-1}(cx)} + \frac{1}{3}ex^3\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}\,de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh}}{\sqrt{}}\right.}{4c}$$

Mathematica [A] time = 2.82, size = 317, normalized size = 0.98

$$\frac{ee^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{3}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{3}{2},-\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right.\right.}{72c^3\sqrt{-\frac{\left(a+b\cosh^{-1}(cx)\right)}{b^2}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (e*Sqrt[a + b*ArcCosh[c*x]]*(9*E^((4
*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] + Sqr
t[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] + 9*E
^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b)] +
 Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a + b*A
rcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^2)])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)\sqrt{b\operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arccosh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int (e\,x^2 + d)\sqrt{a + b\operatorname{arccosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (ex^2 + d)\sqrt{b\operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)*sqrt(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + b\operatorname{acosh}(c\,x)}\,(e\,x^2 + d)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\operatorname{acosh}(cx)}\,(d + ex^2)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))*(d + e*x**2), x)
```

3.554 $\int \sqrt{a+b\cosh^{-1}(cx)}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

```
[Out] -1/4*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c-1/4*
erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*b^(1/2)*Pi^(1/2)/c/exp(a/b)+x*(a+b*a
rccosh(c*x))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5654, 5781, 3307, 2180, 2204, 2205}

$$-\frac{\sqrt{\pi}\sqrt{b}e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}-\frac{\sqrt{\pi}\sqrt{b}e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c}+x\sqrt{a+b\cosh^{-1}(cx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCosh[c*x]] - (Sqrt[b]*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCo
sh[c*x]]/Sqrt[b]])/(4*c) - (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(4*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \sqrt{a+b\cosh^{-1}(cx)}\,dx = x\sqrt{a+b\cosh^{-1}(cx)} - \frac{1}{2}(bc)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{2c}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{b\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{4c} - \frac{b\,\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\,dx\right.}{4c}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{2c} - \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+}\right.}{}$$

$$= x\sqrt{a+b\cosh^{-1}(cx)} - \frac{\sqrt{b}\,e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4c} - \frac{\sqrt{b}\,e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{b}}\right.}{4c}$$

Mathematica [A] time = 0.20, size = 100, normalized size = 0.98

$$\frac{e^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/Sqr
t[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a + b
*ArcCosh[c*x])/b)]))/(2*c*E^(a/b))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\,\text{arcosh}\,(cx)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{arccosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \operatorname{arcosh}(cx) + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + b \operatorname{acosh}(c\, x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2),x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \operatorname{acosh}(cx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x)), x)
```

3.555 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx=\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Mathematica [A] time = 4.89, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2),x]
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\text{arcosh}\left(cx\right)+a}}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

maple [A] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{arccosh}\,(cx)}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\mathrm{arcosh}\,(cx)+a}}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+b\,\mathrm{acosh}\,(c\,x)}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\,\mathrm{acosh}\,(cx)}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2), x)
```

3.556 $\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]
[Out] Defer[Int][Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx=\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Mathematica [A] time = 21.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\cosh^{-1}(cx)}}{(d+ex^2)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2,x]
[Out] Integrate[Sqrt[a + b*ArcCosh[c*x]]/(d + e*x^2)^2, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\mathrm{arcosh}\,(cx)+a}}{(ex^2+d)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

maple [A] time = 1.43, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\,\mathrm{arccosh}\left(c x\right)}}{\left(e\,x^{2} + d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b\,\mathrm{arcosh}\left(cx\right) + a}}{\left(ex^{2} + d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(1/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*arccosh(c*x) + a)/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a + b\,\mathrm{acosh}\left(c\,x\right)}}{\left(e\,x^{2} + d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^(1/2)/(d + e*x^2)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\,\mathrm{acosh}\left(cx\right)}}{\left(d + ex^{2}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(1/2)/(e*x**2+d)**2,x)
```

```
[Out] Integral(sqrt(a + b*acosh(c*x))/(d + e*x**2)**2, x)
```

3.557 $\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=442

$$-\frac{3\sqrt{\pi}\,b^{3/2}ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}\,b^{3/2}ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+$$

```
[Out] d*x*(a+b*arccosh(c*x))^(3/2)+1/3*e*x^3*(a+b*arccosh(c*x))^(3/2)-1/288*b^(3/
2)*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1
/2)/c^3+1/288*b^(3/2)*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1
/2)*Pi^(1/2)/c^3/exp(3*a/b)-3/8*b^(3/2)*d*exp(a/b)*erf((a+b*arccosh(c*x))^(
1/2)/b^(1/2))*Pi^(1/2)/c-3/32*b^(3/2)*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1/
2)/b^(1/2))*Pi^(1/2)/c^3+3/8*b^(3/2)*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2
))*Pi^(1/2)/c/exp(a/b)+3/32*b^(3/2)*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)-3/2*b*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*
x))^(1/2)/c-1/3*b*e*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2)/c^
3-1/6*b*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2)/c
```

Rubi [A] time = 1.73, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {5707, 5654, 5718, 5658, 3308, 2180, 2205, 2204, 5664, 5759, 5670, 5448}

$$-\frac{3\sqrt{\pi}\,b^{3/2}ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}-\frac{\sqrt{\frac{\pi}{3}}\,b^{3/2}ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{96c^3}+\frac{3\sqrt{\pi}\,b^{3/2}ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32c^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) - (b*e
*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(3*c^3) - (b*e*x^2*
Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(6*c) + d*x*(a + b*A
rcCosh[c*x])^(3/2) + (e*x^3*(a + b*ArcCosh[c*x])^(3/2))/3 - (3*b^(3/2)*d*E^
(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*c) - (3*b^(3/2)*e*
E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(32*c^3) - (b^(3/2)
*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(96*c^3) + (3*b^(3/2)*d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8
*c*E^(a/b)) + (3*b^(3/2)*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])
/(32*c^3*E^(a/b)) + (b^(3/2)*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[
c*x]])/Sqrt[b]])/(96*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5664

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^(m + 1)*(a + b*ArcCosh[c*x])^n)/(m + 1), x] - Dist[(b*c*n)/(m + 1), Int[
(x^(m + 1)*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt[-1 + c*x]*Sqrt[1 + c*x]), x]
, x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rule 5759

```
Int[(((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_))/(Sqrt[(d1
_) + (e1_.)*(x_)]*Sqrt[(d2_) + (e2_.)*(x_)]), x_Symbol] :> Simp[(f*(f*x)^(m
 - 1)*Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]*(a + b*ArcCosh[c*x])^n)/(e1*e2*m), x]
 + (Dist[(f^2*(m - 1))/(c^2*m), Int[((f*x)^(m - 2)*(a + b*ArcCosh[c*x])^n)/
(Sqrt[d1 + e1*x]*Sqrt[d2 + e2*x]), x], x] + Dist[(b*f*n*Sqrt[d1 + e1*x]*Sqr
t[d2 + e2*x])/(c*d1*d2*m*Sqrt[1 + c*x]*Sqrt[-1 + c*x]), Int[(f*x)^(m - 1)*(
a + b*ArcCosh[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c, d1, e1, d2, e2, f},
 x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && GtQ[m, 1] &&
IntegerQ[m]
```

Rubi steps

$$\int \left(d+ex^2\right)\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx = \int \left(d\left(a+b\cosh^{-1}(cx)\right)^{3/2}+ex^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\right)dx$$

$$= d\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx + e\int x^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$$

$$= dx\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{3}ex^3\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bcd)\int \frac{x\sqrt{}}{\sqrt{}}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{bex^2\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{}}{6c}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

$$= -\frac{3bd\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}-\frac{be\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a}}{3c^3}$$

Mathematica [A] time = 3.48, size = 812, normalized size = 1.84

$$\frac{ade^{-\frac{a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(\frac{e^{\frac{2a}{b}}\Gamma\left(\frac{3}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)}{\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}}+\frac{\Gamma\left(\frac{3}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)}{\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}}\right)}{2c}+\frac{aee^{-\frac{3a}{b}}\sqrt{a+b\cosh^{-1}(cx)}\left(9e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}}{b}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*d*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])
/Sqrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a
 + b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (a*e*Sqrt[a + b*ArcCosh[c*x]]*(9*E
^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, a/b + ArcCosh[c*x]] +
 Sqrt[3]*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, (-3*(a + b*ArcCosh[c*x]))/b] +
 9*E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[3/2, -((a + b*ArcCosh[c*x])/b
)] + Sqrt[3]*E^((6*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[3/2, (3*(a +
 b*ArcCosh[c*x]))/b]))/(72*c^3*E^((3*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])^2/b^
2)]) + (b*d*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*
x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c*x]] + ((2*a + 3*b)*Sqrt[Pi]*E
rfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] - Sinh[a/b]))/Sqrt[b] + ((
2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh
[a/b]))/Sqrt[b]))/(8*c) + (Sqrt[b]*e*(9*(-12*Sqrt[b]*Sqrt[(-1 + c*x)/(1 + c
*x)]*(1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*Sqrt[b]*c*x*ArcCosh[c*x]*Sqrt[a
 + b*ArcCosh[c*x]] + (2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqr
t[b]]*(Cosh[a/b] - Sinh[a/b]) + (2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b])) + (2*a + b)*Sqrt[3*Pi]*Erfi[(Sqrt[
3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*a)/b] - Sinh[(3*a)/b]) + (2*
a - b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]]*(Cosh[(3*
a)/b] + Sinh[(3*a)/b]) + 12*Sqrt[b]*Sqrt[a + b*ArcCosh[c*x]]*(2*ArcCosh[c*x
]*Cosh[3*ArcCosh[c*x]] - Sinh[3*ArcCosh[c*x]])))/(288*c^3)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Simplification assuming c near 0Simplification assuming c
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/2)>(-8*pi/(sign(c^2*t_nostep^2-1)-1)/2)Unable to check sign: (8*pi/(sign(c
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```

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sign(c^2*x^2-1)-1)/2)>(-8*pi/(sign(c^2*x^2-1)-1)/2)Unable to check sign: (8
*pi/(sign(c^2*x^2-1)-1)/2)>(-8*pi/(sign(c^2*x^2-1)-1)/2)Evaluation time: 20
.4sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error:
Bad Argument Value
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \left(e\,x^2+d\right)\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
[Out] int((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex^2+d\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)*(d + e*x^2),x)
[Out] int((a + b*acosh(c*x))^(3/2)*(d + e*x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{\frac{3}{2}}\left(d+ex^2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(a+b*acosh(c*x))**(3/2),x)
[Out] Integral((a + b*acosh(c*x))**(3/2)*(d + e*x**2), x)
```

3.558 $\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\big($$

```
[Out] x*(a+b*arccosh(c*x))^(3/2)-3/8*b^(3/2)*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2
)/b^(1/2))*Pi^(1/2)/c+3/8*b^(3/2)*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi
^(1/2)/c/exp(a/b)-3/2*b*(c*x-1)^(1/2)*(c*x+1)^(1/2)*(a+b*arccosh(c*x))^(1/2
)/c
```

Rubi [A] time = 0.40, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {5654, 5718, 5658, 3308, 2180, 2205, 2204}

$$-\frac{3\sqrt{\pi}\,b^{3/2}e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}+\frac{3\sqrt{\pi}\,b^{3/2}e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8c}-\frac{3b\sqrt{cx-1}\,\sqrt{cx+1}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\big($$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*Sqrt[a + b*ArcCosh[c*x]])/(2*c) + x*(a +
 b*ArcCosh[c*x])^(3/2) - (3*b^(3/2)*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh
[c*x]]/Sqrt[b]])/(8*c) + (3*b^(3/2)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/
Sqrt[b]])/(8*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5654

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*A
rcCosh[c*x])^n, x] - Dist[b*c*n, Int[(x*(a + b*ArcCosh[c*x])^(n - 1))/(Sqrt
[-1 + c*x]*Sqrt[1 + c*x]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5718

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d1_) + (e1_.)*(x_))^(p
_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Simp[((d1 + e1*x)^(p + 1)*(d2
 + e2*x)^(p + 1)*(a + b*ArcCosh[c*x])^n)/(2*e1*e2*(p + 1)), x] - Dist[(b*n*
(-(d1*d2))^IntPart[p]*(d1 + e1*x)^FracPart[p]*(d2 + e2*x)^FracPart[p])/(2*c
*(p + 1)*(1 + c*x)^FracPart[p]*(-1 + c*x)^FracPart[p]), Int[(-1 + c^2*x^2)^
(p + 1/2)*(a + b*ArcCosh[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d1, e1, d
2, e2, p}, x] && EqQ[e1 - c*d1, 0] && EqQ[e2 + c*d2, 0] && GtQ[n, 0] && NeQ
[p, -1] && IntegerQ[p + 1/2]
```

Rubi steps

$$\int \left(a+b\cosh^{-1}(cx)\right)^{3/2}\,dx = x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{-1+cx}\,\sqrt{1+cx}}\,dx$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}+\frac{1}{4}\left(3b^2\right)\int$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b)\,\text{Subs}}{}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b)\,\text{Subs}}{}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{(3b)\,\text{Subs}}{}$$

$$= -\frac{3b\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}{2c}+x\left(a+b\cosh^{-1}(cx)\right)^{3/2}-\frac{3b^{3/2}e^{a/b}}{}$$

Mathematica [A] time = 0.71, size = 269, normalized size = 1.92

$$\frac{b\left(\frac{\sqrt{\pi}(2a-3b)\left(\sinh\left(\frac{a}{b}\right)+\cosh\left(\frac{a}{b}\right)\right)\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}}+\frac{\sqrt{\pi}(2a+3b)\left(\cosh\left(\frac{a}{b}\right)-\sinh\left(\frac{a}{b}\right)\right)\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{\sqrt{b}}-12\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)\sqrt{a}\right.}{8c}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (a*Sqrt[a + b*ArcCosh[c*x]]*((E^((2*a)/b)*Gamma[3/2, a/b + ArcCosh[c*x]])/S
qrt[a/b + ArcCosh[c*x]] + Gamma[3/2, -((a + b*ArcCosh[c*x])/b)]/Sqrt[-((a +
 b*ArcCosh[c*x])/b)]))/(2*c*E^(a/b)) + (b*(-12*Sqrt[(-1 + c*x)/(1 + c*x)]*(
1 + c*x)*Sqrt[a + b*ArcCosh[c*x]] + 8*c*x*ArcCosh[c*x]*Sqrt[a + b*ArcCosh[c
```

```
*x]] + ((2*a + 3*b)*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]]*(Cosh[a
/b] - Sinh[a/b]))/Sqrt[b] + ((2*a - 3*b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*
x]]/Sqrt[b]]*(Cosh[a/b] + Sinh[a/b]))/Sqrt[b]))/(8*c)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{arccosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b \operatorname{acosh}(c\,x))^{3/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int((a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \operatorname{acosh}(cx))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2), x)
```

3.559 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Mathematica [A] time = 2.34, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{d+ex^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2),x]
[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\text{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}{ex^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{arccosh}(cx))^{\frac{3}{2}}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}}{ex^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a + b \operatorname{acosh}(c x))^{3/2}}{e x^2 + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2),x)
```

```
[Out] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}}{d + ex^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(3/2)/(d + e*x**2), x)
```

3.560 $\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2},x\right)$$

```
[Out] Unintegrable((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]
[Out] Defer[Int][(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

Rubi steps

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx=\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Mathematica [A] time = 14.26, size = 0, normalized size = 0.00

$$\int \frac{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}{\left(d+ex^2\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2,x]
[Out] Integrate[(a + b*ArcCosh[c*x])^(3/2)/(d + e*x^2)^2, x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}{\left(ex^2+d\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

maple [A] time = 1.35, size = 0, normalized size = 0.00

$$\int \frac{(a+b\,\mathrm{arccosh}\,(cx))^{\frac{3}{2}}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

```
[Out] int((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\,\mathrm{arcosh}\,(cx)+a)^{\frac{3}{2}}}{\left(ex^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*arccosh(c*x))^(3/2)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(3/2)/(e*x^2 + d)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(a+b\,\mathrm{acosh}\,(c\,x))^{3/2}}{\left(e\,x^{2}+d\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2)^2,x)
```

```
[Out] int((a + b*acosh(c*x))^(3/2)/(d + e*x^2)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*acosh(c*x))**(3/2)/(e*x**2+d)**2,x)
```

```
[Out] Timed out
```

3.561 $\int \frac{(d+ex^2)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=608

$$-\frac{\sqrt{\pi}\,e^2e^{a/b}\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{16\sqrt{b}\,c^5}-\frac{\sqrt{3\pi}\,e^2e^{\frac{3a}{b}}\text{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,c^5}-\frac{\sqrt{\frac{\pi}{5}}\,e^2e^{\frac{5a}{b}}\text{erf}\left(\frac{\sqrt{5}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{32\sqrt{b}\,c^5}+\frac{\sqrt{\pi}\,e^2e^{-\frac{a}{b}}}{}$$

```
[Out] -1/160*e^2*exp(5*a/b)*erf(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*5^(1/2)
*Pi^(1/2)/c^5/b^(1/2)+1/160*e^2*erfi(5^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/
2))*5^(1/2)*Pi^(1/2)/c^5/exp(5*a/b)/b^(1/2)-1/12*d*e*exp(3*a/b)*erf(3^(1/2)
*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*d*e*er
fi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^3/exp(3*a/b
)/b^(1/2)-1/2*d^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c
/b^(1/2)-1/4*d*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^
3/b^(1/2)-1/16*e^2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/
c^5/b^(1/2)+1/2*d^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a
/b)/b^(1/2)+1/4*d*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^3/exp
(a/b)/b^(1/2)+1/16*e^2*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c^5/
exp(a/b)/b^(1/2)-1/32*e^2*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b
^(1/2))*3^(1/2)*Pi^(1/2)/c^5/b^(1/2)+1/32*e^2*erfi(3^(1/2)*(a+b*arccosh(c*x
))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/c^5/exp(3*a/b)/b^(1/2)
```

Rubi [A] time = 1.15, antiderivative size = 608, normalized size of antiderivative = 1.00, number of steps used = 39, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {5707, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{\sqrt{\pi}\,dee^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}-\frac{\sqrt{\frac{\pi}{3}}\,dee^{\frac{3a}{b}}\text{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{\sqrt{\pi}\,dee^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,dee^{-\frac{3a}{b}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(d^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c)
 - (d*e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*
c^3) - (e^2*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqr
t[b]*c^5) - (d*e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(4*Sqrt[b]*c^3) - (e^2*E^((3*a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sq
rt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) - (e^2*E^((5*a)/b)*Sqrt[
Pi/5]*Erf[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5) + (
d^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
+ (d*e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*Sqrt[b]*c^3*E^(a
/b)) + (e^2*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(16*Sqrt[b]*c^
5*E^(a/b)) + (d*e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b
]])/(4*Sqrt[b]*c^3*E^((3*a)/b)) + (e^2*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*
ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((3*a)/b)) + (e^2*Sqrt[Pi/5]*Erf
i[(Sqrt[5]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(32*Sqrt[b]*c^5*E^((5*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{(d+ex^2)^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d^2}{\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2dex^2}{\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^2x^4}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d^2\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx + (2de)\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx + e^2\int \frac{x^4}{\sqrt{a+b\cos}}$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{(2de)\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx\right.}{c^3}$$

$$= -\frac{d^2\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{d^2\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,c\right.}{2bc}$$

$$= -\frac{d^2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{d^2\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b}\right.}{bc}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{(de)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{d^2e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{(de)\,\text{Subst}\left(\int e\right.}{}$$

$$= -\frac{d^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{dee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4\sqrt{b}\,c^3} - \frac{e^2e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{}}{}\right.}{16\sqrt{b}}$$

Mathematica [A] time = 1.14, size = 530, normalized size = 0.87

$$e^{-\frac{5a}{b}}\left(240c^4d^2e^{\frac{4a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{a+b\cosh^{-1}(cx)}{b}\right) + 40\sqrt{3}\,c^2dee^{\frac{2a}{b}}\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{3\left(a+b\cosh^{-1}(cx)\right)}{b}\right) + \right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)^2/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (30*(8*c^4*d^2 + 4*c^2*d*e + e^2)*E^((6*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamm
a[1/2, a/b + ArcCosh[c*x]] + 3*Sqrt[5]*e^2*Sqrt[-((a + b*ArcCosh[c*x])/b)]*
Gamma[1/2, (-5*(a + b*ArcCosh[c*x]))/b] + 40*Sqrt[3]*c^2*d*e*E^((2*a)/b)*Sq
rt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b*ArcCosh[c*x]))/b] + 15*
Sqrt[3]*e^2*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
 b*ArcCosh[c*x]))/b] + 240*c^4*d^2*E^((4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/
b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 120*c^2*d*e*E^((4*a)/b)*Sqrt[-(
(a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + 30*e^2*E^(
(4*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/
b)] + 40*Sqrt[3]*c^2*d*e*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, (3
*(a + b*ArcCosh[c*x]))/b] + 15*Sqrt[3]*e^2*E^((8*a)/b)*Sqrt[a/b + ArcCosh[c
*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 3*Sqrt[5]*e^2*E^((10*a)/b)*Sq
rt[a/b + ArcCosh[c*x]]*Gamma[1/2, (5*(a + b*ArcCosh[c*x]))/b])/(480*c^5*E^(
(5*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{b\operatorname{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arccosh(c*x) + a), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{arccosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

```
[Out] int((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex^2+d\right)^2}{\sqrt{b\operatorname{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x^2 + d)^2/sqrt(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)^2}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2/(a + b*acosh(c*x))^(1/2),x)
```

```
[Out] int((d + e*x^2)^2/(a + b*acosh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^2\right)^2}{\sqrt{a+b\operatorname{acosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
```

```
[Out] Integral((d + e*x**2)**2/sqrt(a + b*acosh(c*x)), x)
```

3.562 $\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=287

$$-\frac{\sqrt{\pi}\,ee^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\operatorname{erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\operatorname{erfi}\left(\right.}{8\sqrt{}}$$

```
[Out] -1/24*e*exp(3*a/b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi
^(1/2)/c^3/b^(1/2)+1/24*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^
(1/2)*Pi^(1/2)/c^3/exp(3*a/b)/b^(1/2)-1/2*d*exp(a/b)*erf((a+b*arccosh(c*x))
^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)-1/8*e*exp(a/b)*erf((a+b*arccosh(c*x))^(1
/2)/b^(1/2))*Pi^(1/2)/c^3/b^(1/2)+1/2*d*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/
2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)+1/8*e*erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2)
)*Pi^(1/2)/c^3/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5707, 5658, 3308, 2180, 2205, 2204, 5670, 5448}

$$-\frac{\sqrt{\pi}\,ee^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}-\frac{\sqrt{\frac{\pi}{3}}\,ee^{\frac{3a}{b}}\operatorname{Erf}\left(\frac{\sqrt{3}\,\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3}+\frac{\sqrt{\frac{\pi}{3}}\,ee^{-\frac{3a}{b}}\operatorname{Erfi}\left(\right.}{8}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(d*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) -
 (e*E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(8*Sqrt[b]*c^3)
 - (e*E^((3*a)/b)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]
])/(8*Sqrt[b]*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2
*Sqrt[b]*c*E^(a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(
8*Sqrt[b]*c^3*E^(a/b)) + (e*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x
]])/Sqrt[b]])/(8*Sqrt[b]*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5448

```
Int[Cosh[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sinh[(a_.) +
(b_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sinh[a +
b*x]^n*Cosh[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] &
& IGtQ[p, 0]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rule 5670

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[
1/c^(m + 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \left(\frac{d}{\sqrt{a+b\cosh^{-1}(cx)}} + \frac{ex^2}{\sqrt{a+b\cosh^{-1}(cx)}}\right)dx$$

$$= d\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx + e\int \frac{x^2}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

$$= -\frac{d\,\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc} + \frac{e\,\text{Subst}\left(\int \frac{\cosh^2(x)\sinh(x)}{\sqrt{a+bx}}\,dx, x, \text{cc}\right.}{c^3}$$

$$= -\frac{d\,\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{d\,\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\,\text{co}\right.}{2bc}$$

$$= -\frac{d\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{d\,\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\,\text{c}}\right.}{bc}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{e\,\text{Subst}\left(\int \frac{e^{-3x}}{\sqrt{a+bx}}\right.}{8}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{de^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{e\,\text{Subst}\left(\int e^{\frac{3a}{b}-\frac{3}{}}\right.}{}$$

$$= -\frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} - \frac{ee^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{8\sqrt{b}\,c^3} - \frac{ee^{\frac{3a}{b}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{}}{}\right.}{8\sqrt{b}\,c}$$

Mathematica [A] time = 0.64, size = 213, normalized size = 0.74

$$\frac{e^{-\frac{3a}{b}}\left(3e^{\frac{4a}{b}}\left(4c^2d+e\right)\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+3e^{\frac{2a}{b}}\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh}{b}\right.\right.}{24c^3\sqrt{a+b\,\text{cosl}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (3*(4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]] + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a + b
*ArcCosh[c*x]))/b] + 3*(4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x]
)/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] + Sqrt[3]*e*E^((6*a)/b)*Sqrt[a/
b + ArcCosh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b])/(24*c^3*E^((3*a)/
b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{b \operatorname{arcosh}(cx) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/sqrt(b*arccosh(c*x) + a), x)
```

maple **[F(-2)]** time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2 + d}{\sqrt{a + b \operatorname{arccosh}(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

maxima **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{\sqrt{b \operatorname{arcosh}(cx) + a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/sqrt(b*arccosh(c*x) + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{\sqrt{a + b \operatorname{acosh}(c\,x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^(1/2),x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\sqrt{a + b \operatorname{acosh}(cx)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
[Out] Integral((d + e*x**2)/sqrt(a + b*acosh(c*x)), x)
```

3.563 $\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=88

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

```
[Out] -1/2*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/b^(1/2)+1/2*
erfi((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/c/exp(a/b)/b^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {5658, 3308, 2180, 2205, 2204}

$$\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\text{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}-\frac{\sqrt{\pi}\,e^{a/b}\text{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] -(E^(a/b)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c) + (
Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(2*Sqrt[b]*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 5658

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Dist[(b*c)^(-1)
, Subst[Int[x^n*Sinh[a/b - x/b], x], x, a + b*ArcCosh[c*x]], x] /; FreeQ[{a
, b, c, n}, x]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\cosh^{-1}(cx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\sinh\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{bc}$$
$$= -\frac{\text{Subst}\left(\int \frac{e^{-i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(cx)\right)}{2bc} + \frac{\text{Subst}\left(\int \frac{e^{i\left(\frac{ia}{b}-\frac{ix}{b}\right)}}{\sqrt{x}}\,dx, x, a+b\cosh^{-1}(c\right.}{2bc}$$
$$= -\frac{\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{bc} + \frac{\text{Subst}\left(\int e^{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}}\right.}{bc}$$
$$= -\frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}\,c}$$

Mathematica [A] time = 0.12, size = 100, normalized size = 1.14

$$\frac{e^{-\frac{a}{b}}\left(e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right)}{2c\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + b*ArcCosh[c*x]],x]
```

```
[Out] (E^((2*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt
[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(2*c*E^(
a/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\text{arcosh}\,(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\text{arccosh}\,(cx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(1/2),x)
[Out] int(1/(a+b*arccosh(c*x))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\,\operatorname{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arccosh(c*x) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^(1/2),x)
[Out] int(1/(a + b*acosh(c*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\operatorname{acosh}\left(cx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*acosh(c*x)), x)
```

3.564 $\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx = \int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Mathematica [A] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\sqrt{a+b\cosh^{-1}(cx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)*Sqrt[a + b*ArcCosh[c*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)\sqrt{b\,\text{arcosh}(cx)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arccosh(c*x) + a)), x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\sqrt{a+b\,\mathrm{arccosh}\left(c x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\sqrt{b\,\mathrm{arcosh}\left(cx\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)*sqrt(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,\left(e\,x^{2}+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)),x)
[Out] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(cx\right)}\,\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*acosh(c*x))*(d + e*x**2)), x)
```

3.565 $\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Defer[Int][1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx = \int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Mathematica [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^2 \sqrt{a+b\cosh^{-1}(cx)}} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]),x]
[Out] Integrate[1/((d + e*x^2)^2*Sqrt[a + b*ArcCosh[c*x]]), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^2 \sqrt{b\,\operatorname{arcosh}\left(cx\right)+a}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a)), x)
```

maple [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \sqrt{a+b\,\mathrm{arccosh}\left(c x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)^{2} \sqrt{b\,\mathrm{arcosh}\left(cx\right)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/((e*x^2 + d)^2*sqrt(b*arccosh(c*x) + a)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(c\,x\right)}\,\left(e\,x^{2}+d\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2),x)
[Out] int(1/((a + b*acosh(c*x))^(1/2)*(d + e*x^2)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\,\mathrm{acosh}\left(cx\right)}\,\left(d+ex^{2}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(1/2),x)
[Out] Integral(1/(sqrt(a + b*acosh(c*x))*(d + e*x**2)**2), x)
```

3.566 $\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=358

$$\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{\frac{3a}{b}}\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{-\frac{3a}{b}}\mathrm{erfi}\left(\frac{\sqrt{3}}{}\right.}{4b^{3/2}}$$

```
[Out] d*exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+1/4*e*e
xp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3+d*erfi((
a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)+1/4*e*erfi((a+
b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c^3/exp(a/b)+1/4*e*exp(3*a/
b)*erf(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^(3/2)/c
^3+1/4*e*erfi(3^(1/2)*(a+b*arccosh(c*x))^(1/2)/b^(1/2))*3^(1/2)*Pi^(1/2)/b^
(3/2)/c^3/exp(3*a/b)-2*d*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))
^(1/2)-2*e*x^2*(c*x-1)^(1/2)*(c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 0.83, antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {5707, 5656, 5781, 3307, 2180, 2204, 2205, 5666}

$$\frac{\sqrt{\pi}\,ee^{a/b}\mathrm{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{\frac{3a}{b}}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{\pi}\,ee^{-\frac{a}{b}}\mathrm{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{4b^{3/2}c^3}+\frac{\sqrt{3\pi}\,ee^{-\frac{3a}{b}}\mathrm{Erfi}\left(\right.}{4b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-2*d*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) - (2*e*x
^2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (d*E^(a/b
)*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (e*E^(a/b)*
Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3) + (e*E^((3*
a)/b)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/(4*b^(3/2
)*c^3) + (d*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(
a/b)) + (e*Sqrt[Pi]*Erfi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(4*b^(3/2)*c^3*
E^(a/b)) + (e*Sqrt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[a + b*ArcCosh[c*x]])/Sqrt[b]])/
(4*b^(3/2)*c^3*E^((3*a)/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5666

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[
(x^m*Sqrt[-1 + c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1
)), x] + Dist[1/(b*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[(a + b*x)
^(n + 1)*Cosh[x]^(m - 1)*(m - (m + 1)*Cosh[x]^2), x], x], x, ArcCosh[c*x]],
 x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 5707

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*((d_) + (e_.)*(x_)^2)^(p_.),
x_Symbol] :> Int[ExpandIntegrand[(a + b*ArcCosh[c*x])^n, (d + e*x^2)^p, x],
 x] /; FreeQ[{a, b, c, d, e, n}, x] && NeQ[c^2*d + e, 0] && IntegerQ[p] &&
(p > 0 || IGtQ[n, 0])
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{d+ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = \int \left(\frac{d}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}} + \frac{ex^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\right) dx$$

$$= d\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx + e\int \frac{x^2}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2cd)\int \frac{x}{\sqrt{-1+cx}\sqrt{1+cx}\sqrt{a+b\cosh^{-1}}}}{b}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}dx, x, \cos\right.}{bc}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{d\,\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}dx, x, \cosh^{-}\right.}{bc}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2d)\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}dx, x, \sqrt{a}\right.}{b^2c}$$

$$= -\frac{2d\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} - \frac{2ex^2\sqrt{-1+cx}\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{de^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} +$$

Mathematica [A] time = 1.97, size = 268, normalized size = 0.75

$$\frac{e^{-\frac{3a}{b}}\left(-\left(e^{\frac{4a}{b}}\left(4c^2d+e\right)\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)\right)+e^{\frac{2a}{b}}\left(4c^2d+e\right)\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(}{b}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d + e*x^2)/(a + b*ArcCosh[c*x])^(3/2),x]
```

```
[Out] (-((4*c^2*d + e)*E^((4*a)/b)*Sqrt[a/b + ArcCosh[c*x]]*Gamma[1/2, a/b + ArcC
osh[c*x]]) + Sqrt[3]*e*Sqrt[-((a + b*ArcCosh[c*x])/b)]*Gamma[1/2, (-3*(a +
b*ArcCosh[c*x]))/b] + (4*c^2*d + e)*E^((2*a)/b)*Sqrt[-((a + b*ArcCosh[c*x])
/b)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)] - E^((3*a)/b)*(2*(4*c^2*d + e)*S
qrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) + Sqrt[3]*e*E^((3*a)/b)*Sqrt[a/b + ArcC
osh[c*x]]*Gamma[1/2, (3*(a + b*ArcCosh[c*x]))/b] + 2*e*Sinh[3*ArcCosh[c*x]]
))/(4*b*c^3*E^((3*a)/b)*Sqrt[a + b*ArcCosh[c*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^(3/2), x)
```

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2 + d}{(a + b \operatorname{arccosh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
[Out] int((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{ex^2 + d}{(b \operatorname{arcosh}(cx) + a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((e*x^2 + d)/(b*arccosh(c*x) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2 + d}{(a + b \operatorname{acosh}(c\,x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a + b*acosh(c*x))^(3/2),x)
[Out] int((d + e*x^2)/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{(a + b \operatorname{acosh}(cx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
[Out] Integral((d + e*x**2)/(a + b*acosh(c*x))**(3/2), x)
```

3.567 $\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=120

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

```
[Out] exp(a/b)*erf((a+b*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c+erfi((a+b
*arccosh(c*x))^(1/2)/b^(1/2))*Pi^(1/2)/b^(3/2)/c/exp(a/b)-2*(c*x-1)^(1/2)*(
c*x+1)^(1/2)/b/c/(a+b*arccosh(c*x))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {5656, 5781, 3307, 2180, 2204, 2205}

$$\frac{\sqrt{\pi}\,e^{a/b}\operatorname{Erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}+\frac{\sqrt{\pi}\,e^{-\frac{a}{b}}\operatorname{Erfi}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}-\frac{2\sqrt{cx-1}\,\sqrt{cx+1}}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*Sqrt[-1 + c*x]*Sqrt[1 + c*x])/(b*c*Sqrt[a + b*ArcCosh[c*x]]) + (E^(a/b)
*Sqrt[Pi]*Erf[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c) + (Sqrt[Pi]*Er
fi[Sqrt[a + b*ArcCosh[c*x]]/Sqrt[b]])/(b^(3/2)*c*E^(a/b))
```

Rule 2180

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - (c*f)/d) + (f*g*x^2)/d), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !$UseGamma === True
```

Rule 2204

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erfi[(c + d*x)*Rt[b*Log[F], 2]])/(2*d*Rt[b*Log[F], 2]), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2205

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[(F^a*Sqr
t[Pi]*Erf[(c + d*x)*Rt[-(b*Log[F]), 2]])/(2*d*Rt[-(b*Log[F]), 2]), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 5656

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Sqrt[-1 +
c*x]*Sqrt[1 + c*x]*(a + b*ArcCosh[c*x])^(n + 1))/(b*c*(n + 1)), x] - Dist[c
```

```
/(b*(n + 1)), Int[(x*(a + b*ArcCosh[c*x])^(n + 1))/(Sqrt[-1 + c*x]*Sqrt[1 +
 c*x]), x], x] /; FreeQ[{a, b, c}, x] && LtQ[n, -1]
```

Rule 5781

```
Int[((a_.) + ArcCosh[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d1_) + (e1_.)*(x
_))^(p_.)*((d2_) + (e2_.)*(x_))^(p_.), x_Symbol] :> Dist[(-(d1*d2))^p/c^(m
+ 1), Subst[Int[(a + b*x)^n*Cosh[x]^m*Sinh[x]^(2*p + 1), x], x, ArcCosh[c*x
]], x] /; FreeQ[{a, b, c, d1, e1, d2, e2, n}, x] && EqQ[e1 - c*d1, 0] && Eq
Q[e2 + c*d2, 0] && IntegerQ[p + 1/2] && GtQ[p, -1] && IGtQ[m, 0] && (GtQ[d1
, 0] && LtQ[d2, 0])
```

Rubi steps

$$\int \frac{1}{\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx = -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{(2c)\int \frac{x}{\sqrt{-1+cx}\,\sqrt{1+cx}\,\sqrt{a+b\cosh^{-1}(cx)}}\,dx}{b}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\text{Subst}\left(\int \frac{\cosh(x)}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{\text{Subst}\left(\int \frac{e^{-x}}{\sqrt{a+bx}}\,dx, x, \cosh^{-1}(cx)\right)}{bc} + \frac{\text{Subst}\left(\int \frac{e^{x}}{\sqrt{a+bx}}\right.}{}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{2\,\text{Subst}\left(\int e^{\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\cosh^{-1}(cx)}\right)}{b^2c} + \frac{2\,\text{Subs}}{}$$

$$= -\frac{2\sqrt{-1+cx}\,\sqrt{1+cx}}{bc\sqrt{a+b\cosh^{-1}(cx)}} + \frac{e^{a/b}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{a+b\cosh^{-1}(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{e^{-\frac{a}{b}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{a+b\cosh^{-1}}}{\sqrt{b}}\right.}{b^{3/2}c}$$

Mathematica [A] time = 0.14, size = 132, normalized size = 1.10

$$\frac{e^{-\frac{a}{b}}\left(-2e^{a/b}\sqrt{\frac{cx-1}{cx+1}}\,(cx+1)-e^{\frac{2a}{b}}\sqrt{\frac{a}{b}+\cosh^{-1}(cx)}\,\Gamma\left(\frac{1}{2},\frac{a}{b}+\cosh^{-1}(cx)\right)+\sqrt{-\frac{a+b\cosh^{-1}(cx)}{b}}\,\Gamma\left(\frac{1}{2},-\frac{a+b\cosh^{-1}(cx)}{b}\right)\right.}{bc\sqrt{a+b\cosh^{-1}(cx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*ArcCosh[c*x])^(-3/2),x]
```

```
[Out] (-2*E^(a/b)*Sqrt[(-1 + c*x)/(1 + c*x)]*(1 + c*x) - E^((2*a)/b)*Sqrt[a/b + A
rcCosh[c*x]]*Gamma[1/2, a/b + ArcCosh[c*x]] + Sqrt[-((a + b*ArcCosh[c*x])/b
)]*Gamma[1/2, -((a + b*ArcCosh[c*x])/b)])/(b*c*E^(a/b)*Sqrt[a + b*ArcCosh[c
*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\,\mathrm{arccosh}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\operatorname{arcosh}\left(cx\right) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccosh(c*x) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*acosh(c*x))^(3/2),x)
```

```
[Out] int(1/(a + b*acosh(c*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \operatorname{acosh}{\left(c x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acosh(c*x))**(-3/2), x)
```

3.568 $\int \frac{1}{(d+ex^2)(a+b\cosh^{-1}(cx))^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)*(a + b*ArcCosh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(ex^2+d)(b\,\text{arcosh}(cx)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2)), x)
```

maple [A] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)\left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^{2}+d\right)\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)*(b*arccosh(c*x) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\,\mathrm{acosh}\left(cx\right)\right)^{\frac{3}{2}}\left(d+ex^{2}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Integral(1/((a + b*acosh(c*x))**(3/2)*(d + e*x**2)), x)
```

3.569 $\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}},x\right)$$

```
[Out] Unintegrable(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Defer[Int][1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx=\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Mathematica [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d+ex^2\right)^2\left(a+b\cosh^{-1}(cx)\right)^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)),x]
[Out] Integrate[1/((d + e*x^2)^2*(a + b*ArcCosh[c*x])^(3/2)), x]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex^2+d\right)^2\left(b\,\mathrm{arcosh}\left(cx\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^(3/2)), x)
```

maple [A] time = 1.30, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\,x^{2}+d\right)^{2} \left(a+b\,\mathrm{arccosh}\left(c x\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

```
[Out] int(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{{\left(e x^{2}+d\right)}^{2} {\left(b \operatorname{arcosh}\left(c x\right)+a\right)}^{\frac{3}{2}}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^2/(a+b*arccosh(c*x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*x^2 + d)^2*(b*arccosh(c*x) + a)^(3/2)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\mathrm{acosh}\left(c\,x\right)\right)}^{3/2}\,{\left(e\,x^2+d\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)^2),x)
```

```
[Out] int(1/((a + b*acosh(c*x))^(3/2)*(d + e*x^2)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**2/(a+b*acosh(c*x))**(3/2),x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```